Transport Phenomena.
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Lecture-53.
Boundary Layer-Analogy.

I will quickly go through what I have covered in the last class as a prelude to establishing the different analogies between heat, mass and momentum transfer. So we started with the conservation equations, nondimensionalized those conservation equations using standard nondimensionalizing parameters and there we saw that the form of the equation are slowly coming to mimic one another. And more importantly, the emergence of dimensionless groups out of the nondimensionalizing process is automatic. So what you are going to see is, you would be able to identify the dimensionless similarity parameters which are going to be relevant in each of these cases.

So we saw that Reynolds number is going to be important in the case of momentum transfer, both Reynolds and Prandlt number in the case of heat transfer and Reynolds and Schmidt number in the case of mass transfer. So we wrote those equations and we also wrote what are the boundary conditions which can be used to solve this specific process. And we have divided the boundary conditions into 2 different types of categories, one is what is going to happen at the free stream and what is going to happen at the intersection, at the interface between the liquid and the solid surface.

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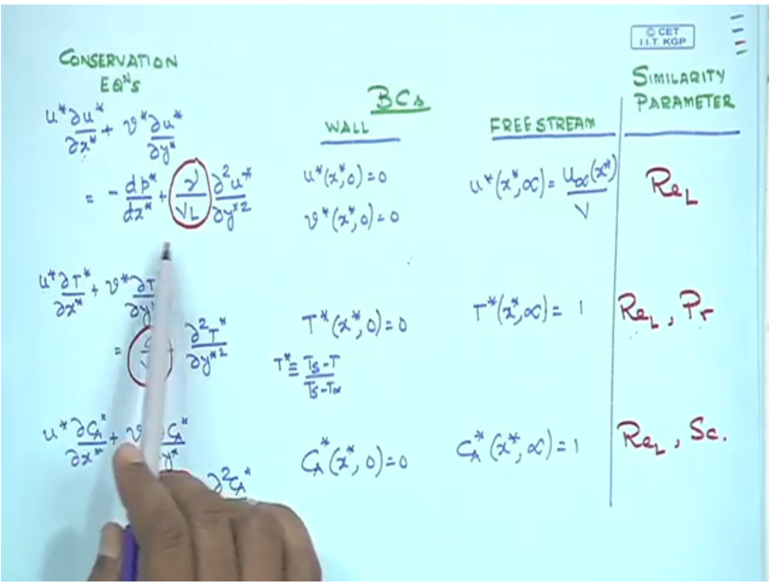

So one, one set of boundary condition for the solid surface, the other is in the free stream. And we would, we have written the similarity parameters for each of these processes. So this is what we have seen, starting with the conservation equation which is the momentum transfer, the Naviar Stokes equation, the X component of Naviar Stokes equation written for flow inside the boundary layer, so utilising all the other missions and approximations, the boundary layer approximations that I have discussed before. And the circled one is the dimensionless, similarity parameter which is Reynolds number.

The sets of boundary conditions at wall and free stream are no slip and the velocity is equal, velocity is going to be, velocity is going to be equal to U infinity by V where U infinity is the velocity, is the free stream velocity at the given X location and V is the approach velocity. For the case of the flow over a flat plate, in absence of any DP D X, that is pressure gradient, U infinity would be equal to V. But in order to maintain the gentle nature of the boundary conditions we have kept it in the form of U infinity by V. Now next let us look at the energy equation which again is same, that is the advection, these terms are on the left-hand side and the diffusive terms, diffusion term is on the right-hand side and again the circled one would resolve into the product of Reynolds, I mean the similarity parameters of the Reynolds number and Prandlt number.

The conditions are, as since we have defined T star in this form, so the value of the dimensionless temperature at any axial location on the solid plate, that is Y equal to 0 would be 0 and the temperature at a point far from the solid plate would be equal to 1. Similarly for mass transfer, species balance equation, this is nothing but Reynolds and Schmidt number, the conditions are identical as this. So these equations are more or less look like the same, except they have Reynolds number 1 by Reynolds number, 1 by Reynolds and Prandlt number and 1 by Reynolds and Schmidt number.

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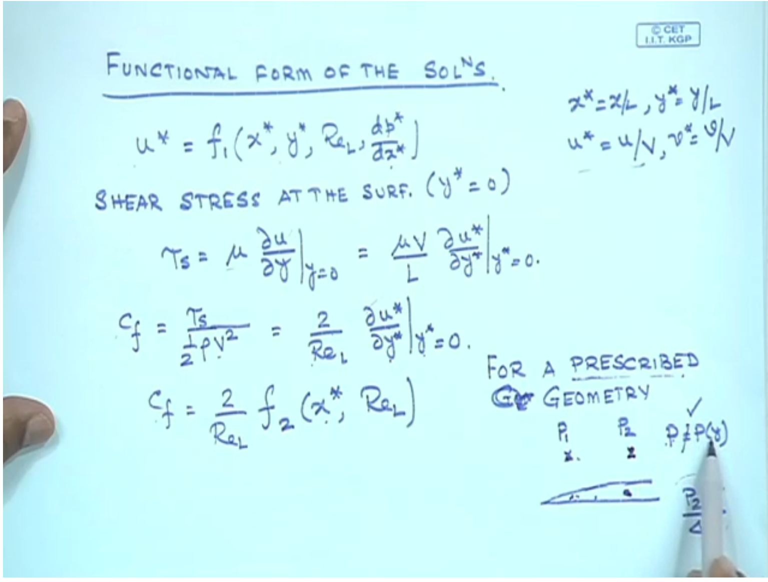

And if you look at the boundary conditions, they also look the same, except that in the case of flow over a, flow over a solid surface hydrodynamic boundary layer, this dimensionless velocity at infinite distance, at a large distance from the solid plate would simply be equal to U infinity by V. So with this we proceed with the functional form of the solutions, we saw that the velocity is going to be a function of F1 of X, Y, Reynolds number and DP DX. And the shear stress is velocity gradient multiplied by mu at Y equals 0 since I am specifying the value of Y. So and then nondimensionalizing it, so the function, the form of CF, the friction coefficient by definition is Tao S by half rho V square and when you put this form of Tao S in here, what you get is this is the form of the friction coefficient.

And the friction coefficient would obviously be a function of X star, it will not be a function of Y star since value of Y, specific value of Y is mentioned here. It is going to be a function of Reynolds number but if you find the geometry, if you prescribe geometry, then DP star, DX star, the pressure gradient can be obtained by through the use of let say Bernoulli’s equation in the inviscid flow region outside of the boundary layer. And P is a function, P is not a function, P is a function, P is not a function of Y, so therefore the pressure difference between these 2 points outside of the boundary layers that can be solved, that can be evaluated through the use of inviscid flow theory would be equal to the pressure difference between the 2 points inside the boundary layer, since P is not a function of Y.

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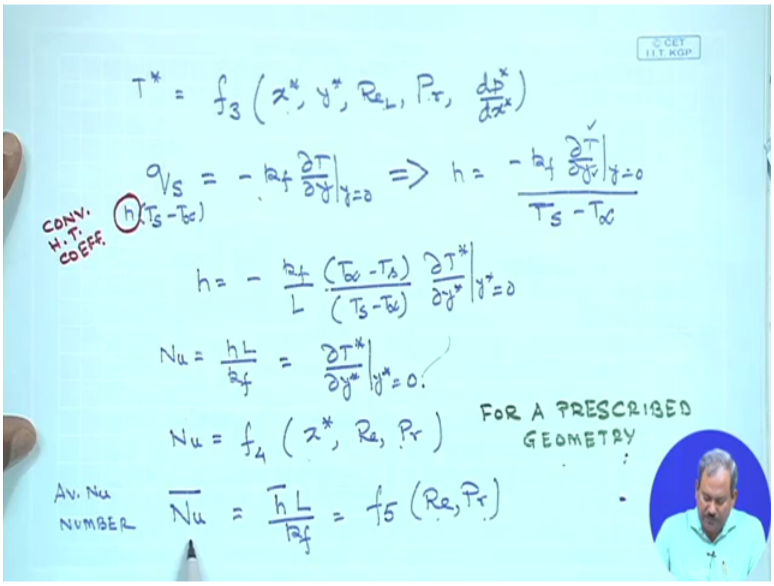

So CF for a prescribed geometry is going to be function of X star and Reynolds number. Similarly we started with the energy equation and we identified that T star is going to be all this, additional would be Prandlt number and then we start with the equality of the convective, equality of the convection and conduction at the solid liquid interface and therefore from that we obtained the value, expression for H and finally the expression for Nusselt number which is the dimensionless pressure gradient at the wall, that means at Y equals to 0. Since I am specify, I have specified the value of Y, so Nusselt number would be function, the functional transform of Nusselt number should contain which I call it as F4, X star, Y star is specified, Reynolds number, Prandlt number, and if I specify the geometry, then DP DX would not appear in the functional form of the Nusselt number.

So this is what Nusselt number is and if you find the average value, that means length average value of Nusselt number, so your X are would also not be there since you are integrating this from 0 to X star, from 0 to L, the entire length of the of the surface. So in that case Nusselt number, average value of Nusselt number is simply going to be function of Reynolds and Prandlt.

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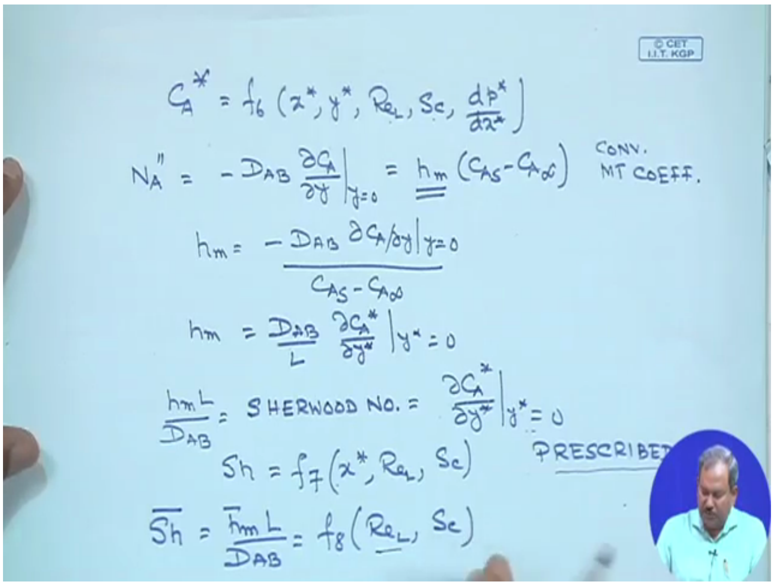

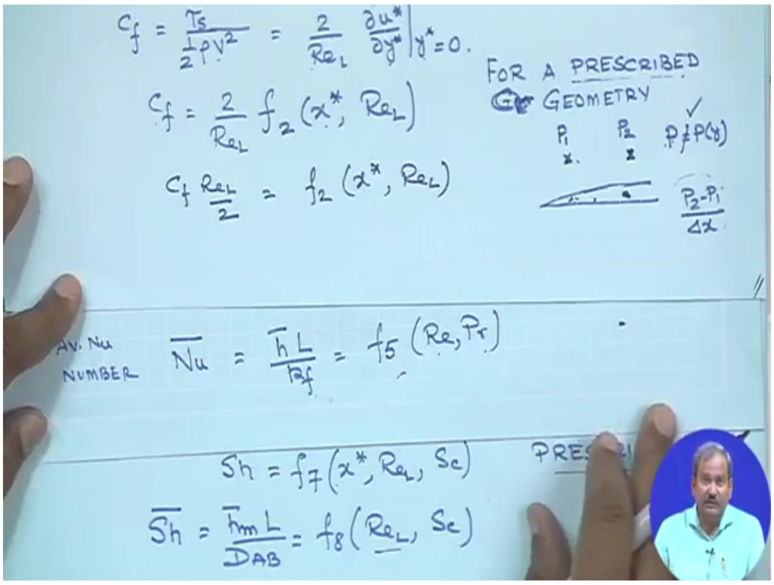

Exactly the same way I defined what is HML by DAB, that is Sherwood number starting with the functional form of C A star, the same way as in heat transfer where HM is the convective mass transfer coefficient. So and this is Sherwood number is by definition then is that dimensionless concentration gradient at the interface. Therefore for a prescribed geometry, Sherwood number would simply be, I call this function as F7 which we still do not know but the F7 should contain extra, Y star would not be there, it is Reynolds number and Schmidt number.

And same as in the case of average value of Nusselt number, if I find out what is the average value of Sherwood number, that means if I integrate over the entire length, I denote the convective mass transfer coefficient as H bar M, unlike this H M, so H bar M by DAB, which is the average Sherwood number, it would be a function of Reynolds and Schmidt.

So just to show these 2 together, here you can see that the form of Nusselt number, F5 and F8, in one case it is Reynolds and Prandlt, in the other case is this Reynolds and Schmidt. And if you compare that with CF, the friction coefficient rather CF REL by 2, this would simply be equals F2, X star, REL. Okay. So these 3 F2, F4 and F5, F2, F5 and F8 together would give you some idea about how we can obtain, how we can try to get a relationship between, relationship between all these things. If I concentrate on let us say F1 and F3, I think it is visible.

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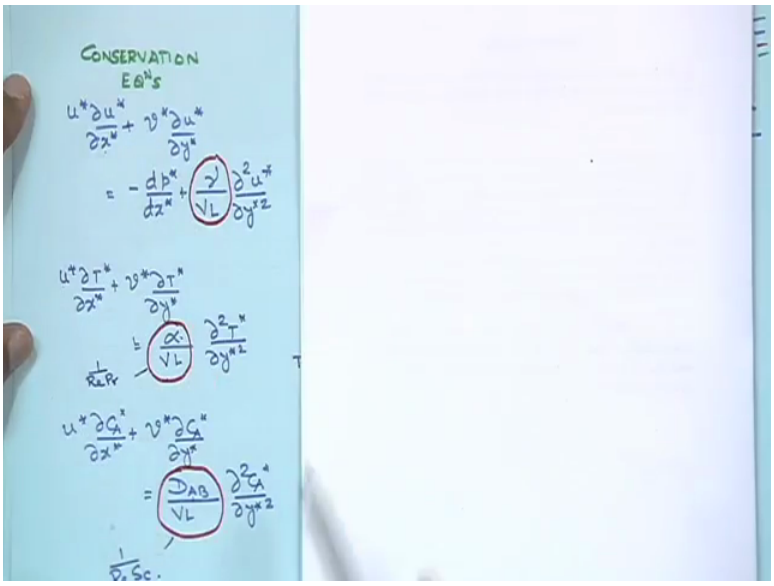

So F1, what I have in here is the functional form of velocity, F3 is the functional form of temperature and similarly I have F6 in here which is the functional form of the dimensionless concentration, dimensionless concentration. Now let us go back to this equation, this case again before we go into Reynolds analogy. I have these as conservation equations, in this I have 1 by Reynolds times Prandlt, in here I have 1 by Reynolds times Schmidt, so this is nothing but 1 by Reynolds times Prandlt number, this is 1 by Reynolds times Schmidt number. So forgetting about this part, if I only consider these 3 cases, then I can say that these 3 equations well be similar only when the Prandlt number is equal to 1 and Schmidt number is equal to1.

And for the case where DP DX is 0, that means we are dealing with a flat plate. So think about the implication of our assumption of our statement here. I would like to make these 3 equations same, identical, all are going to have an advection term on the left and the diffusion term on the right. And when I can, I can say that we 3 equations are equivalent? Only when DP DX is 0 because you do not have any DP DX in these 2 equations, so DP DX has to be 0. And DP DX to be 0, signifies that the flow is taking over a flat plate. And you now you see these other terms which are there. The 1st equation, momentum equation contains 1 by Reynolds number, the energy equation contains 1 by Reynolds number times Prandlt number and the 3rd equation, species balance equation contains 1 by Reynolds number into Schmidt number.

So in order for these 3 equations to become identical, the additional constraint that one has to put is that Schmidt number is equal to 1 and Prandlt number is equal to1. So only in that very restrictive condition where Prandlt number of the fluid which is which is flowing, having momentum, heat and mass transfer, its value should be equal to 1 and Schmidt number should be equal to 1 and the flow is going to take place over a flat plate such that the pressure gradient, DP star DX star is equal to 0. When these conditions are met, the momentum transfer equation, the energy transport equation and the species transport equation, all will look identical.

So in addition if we can show that the boundary conditions are also identical, then these 3 systems, one having heat transfer, the other having momentum transfer and the other one having mass transfer, these can be expressed in terms of the, of same expressions. That is what analogy is all about, such that the expression, if it can be obtained by experimental or other means for one type of transport process, I should be able to use it for the other transport process by simply making certain obvious substitutions. So once again these 3 equations are now identical, since I have assumed DP DX to be 0, the Prandlt number is equal to 1 and the Schmidt number is equal to 1.

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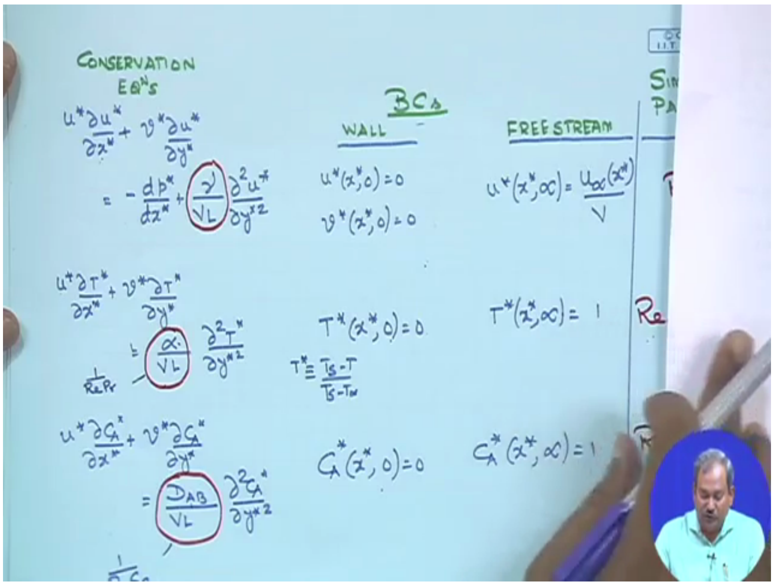

Now let us look at whatever, what happens to the boundary conditions. U star 0, T star 0, CA Star 0, so at the wall the 3 boundary conditions are identical. At the free stream, CA star is equal to 1, T star is equal to 1, however U star which was equal to infinity, U infinity by V now would be equal to 1 since I have assumed DP D X to be equal to 1. And DP DX to be 1, essentially show, essentially is the case of flow over a flat plate. And we understand for flow over a flat plate, the value of the approach velocity would be equal to the value of the free stream velocity. So by making the assumption DP star DX to be equal to1, what I am, what I am doing over here is that I am making all the boundary conditions identical, both at the wall as well at the free stream.

So this would be, this would give us the 1st type of analogy that we are going to get but once again when the equations are same and the boundary conditions are same, then all these 3 systems would be called dynamically similar. So if 3 equations, these 3 governing equations are dynamically similar, then it allows us to use the correlation for 1 as the correlation for 2nd and for the 3rd and provided we simply use the right kind of variables for which are specific to the heat transfer, mass transfer or momentum transfer process. So the analogy, the most restrictive form of analogy that one can choose is for the case where the flow is taking over a flat plate, it is laminar flow and the value of the dimensionless similarity parameters, namely Prandlt number and Schmidt number are equal to1.

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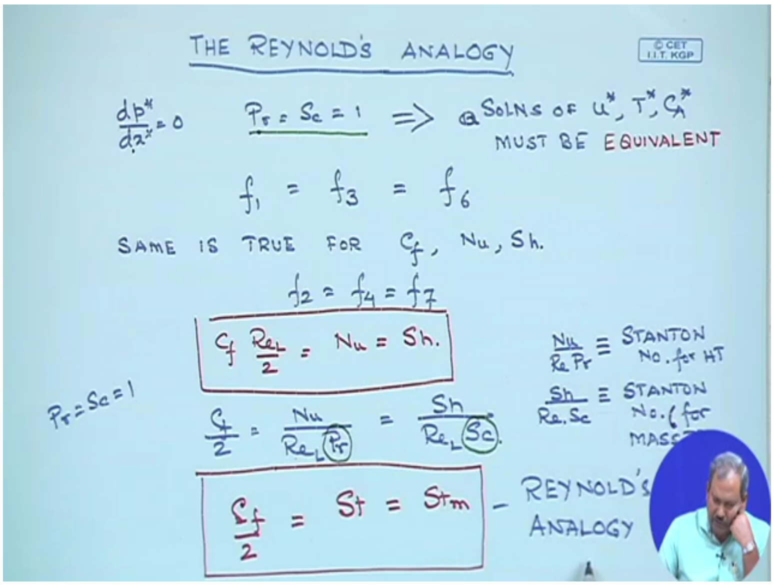

This analogy is called the Reynolds analogy and as I mentioned it is the most restrictive analogy form of the analogy between heat, mass and momentum transfer. So we would start 1st with the Reynolds analogy which simply tells you that it is going to be valid when DP star by DX star is equal to 0 and Prandlt number equal to Schmidt number is equal to 1 and then the conservation equations are all of the same form. So this would give you, conservation equation is same, boundary condition is same, so solutions, solutions of U star, T star and CA star, the dependent variables, must be equivalent.

So this is what analogy is and if you look at your class notes, the U star is essentially expressed in terms of function F1, T star was expressed as F3 and CA star was expressed in terms of F6. So F1 must be equal to F3, and is equal to going to be F6. So since these, solutions of these are equivalent, same is true for CF, the friction coefficient, Nusselt number and Sherwood number. So what are the coefficients for friction, so CF is CF times REL is F2, Sherwood number is equal to, Sherwood, sorry Sherwood number is equal to F7 and Nusselt number I think is going to be equal to F4.

So these are going to be equal, going to be the same, so therefore F2 would be same as F4 as same as F7. So CF times RE, so F2 is nothing but, if you again see the definition of F2, F2 is CF REL by 2, F7, F7 is Sherwood number and F4 is Nusselt number. So if F1, if F2, F4 and F7 are equal, which tells you that CF REL by 2 is equal to Nusselt number is equal to Sherwood number. So this is the relation that you would get in the case of Reynolds analogy. Sometimes this analogy is slightly modified, slightly modified, that it is expressed as CF by 2 equals Nusselt by Reynolds equals Sherwood by Reynolds, REL and since we understand that PR is equal to SC is equal to 1, therefore nothing the generality of the solution, generality object which would not be disturbed if I bring in these 2 terms here as well.

So initially from here what you get is CF by 2 equals Nu by REL and Sherwood by REL but you just bring in these, bring in these additional numbers knowing fully well that in order to have Reynolds analogy I have assumed Prandlt is equal to Schmidt number and its numerical value is equal to 1. So therefore I am expressing CF in terms of Nusselt, Reynolds, Prandlt or Sherwood, Reynolds, Schmidt would not change this, the generality of this. And this Nusselt by Reynolds times Prandlt, this, this is called the Stanton number and Sherwood, Reynolds, Schmidt is again also called Stanton number, Stanton number and this is either called Stanton number for mass transfer.

So this is Stanton number for heat transfer and this is Stanton number for mass transfer. These are just definitions, so a more generalised and modified form of Reynolds analogy is written as CF by 2 is equal to Stanton number based on heat is equal to Stanton number based on mass. So this is the form of Reynolds analogy that can be used when you have this condition, DP DX to be equal to 0, Prandlt number is equal to 1 and Schmidt number is equal to 1. So this analogy relates the key engineering parameters of velocity, thermal and concentration boundary layer.

So the significance of this simple relation is enormous. It tells you now that the engineering important parameter, for example the friction factor in the case of momentum transfer, the heat transfer coefficient, convective heat transfer coefficient and therefore Nusselt number in the case of heat transfer and convective mass transfer coefficient or its dimensionless form in Sherwood number, they all can be expressed, they all are related by simple equality sign. So therefore if and we know we have expressions for CF in laminar flow, expressions for CF in turbulent flow. We have some idea of what is going to be the expression of heat transfer in laminar flow but we have not studied heat transfer in turbulent flow inside the boundary layer or mass transfer in turbulent flow inside the boundary layer.

And we do not need to, because with this analogy available to me and with our knowledge of the exact expressions for CF in turbulent flow as well as in laminar flow and if I use this analogy, what you see here is CF by 2 is equal to Stanton number based on, Stanton number based on heat or heat transfer Stanton number. So an expression for CF in turbulent flow is available, so with this analogy, then I should be able to obtain an expression for Nusselt number or for that matter an expression for Sherwood number for the case of turbulent flow using a simple analogy which is known as a Reynolds analogy.

So the hydrodynamic boundary layer is well researched, it is comparatively easier to analyse, since as I as I said before, the heat transfer and the mass transfer are coupled to the hydrodynamic boundary layer but the hydrodynamic boundary layer solution of the hydrodynamic boundary layer, the equation of the hydrodynamic boundary layer is not coupled with the temperature with the concentration as long as the properties remain constant.

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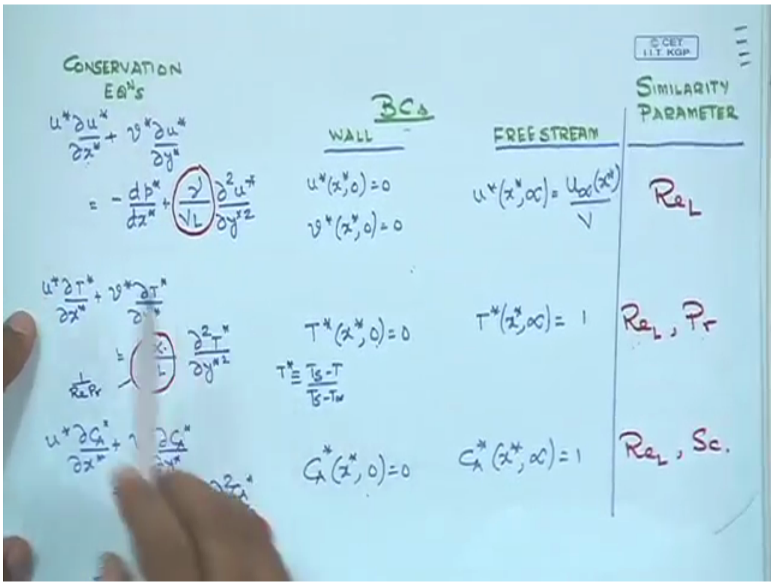

So which would become even more clear if you look at this expression once again. Here I have the velocity but nobody this expression, either the temperature or the concentration appear. On the other hand if you look at the thermal boundary layer equation or the species boundary layer equation, you have the presence of U in both cases. So in that way the, this momentum transfer can be solved independent to the solution of the thermal boundary layer or the species boundary layer.

So that is the reason why that this has been explored in greater details, both for the case of laminar flow as we have seen in the case of Blassius solution and the case of the turbulent flow in which we have used the one 7^{th} power law, the Blassius correlation and so many other things which gave us a compressive idea of the variation of the engineering parameters which is friction coefficient for the case of laminar as well as turbulent flow.

What we do not have that luxury in the case of heat transfer boundary layer or species boundary layer, they are more complicated because of the appearance of U and V in here. So the simultaneous solution of these 2 are needed. But we do not have to do that anymore, since we have the Reynolds analogy which directly relates Nusselt number with the expression of the friction coefficient either in laminar flow or in turbulent flow. So an expression for Nusselt number can thus be obtained from the expression of CF in the various types of flow.

So that is big big advantage but let us think about the disadvantage now. What is the disadvantage? The conditions which I have 2 specify not to obtain this Reynolds analogy is extremely strict, it is unrealistic. Because it is unlikely that you are going to get a liquid whose Prandlt number equal to1 or whose Schmidt number, and its Schmidt number is also equal to1. That is that is a very restrictive unrealistic boundary condition to have. So what is the solution? You may have DP DX to be equal to 0 if it is a flow over a flat plate and the case of turbulent flow, the pressure drop, it does not depend on the, it does not depend that much on the shape of the surface over which the flow is taking place.

So DP DX to be equal to 0 is also approximately valid for the case of turbulent flow. So we need not worry too much about DP DX to be 0 or nonzero, specially for the case of turbulent flow but we have to think about the value of Prandlt and Schmidt number which are going to be, which are not going to be equal to1 and it can have various values, for example the Prandlt number for air is about 0.7 and, that is large, there is a significantly large range over which the Prandlt number and Schmidt number of commonly encountered fluids can be.

So these numbers can lie over a significantly wide range for most of the common fluids that we encounter and therefore a correction to the Reynolds analogy must be provided which would allow the use, the exception of this analogy for real fluids. That is, that is done through the use of extensive experiments and with an empirical approach, the new analogy, modified analogy has been proposed which is now available over a large range of Prandlt and Schmidt number.

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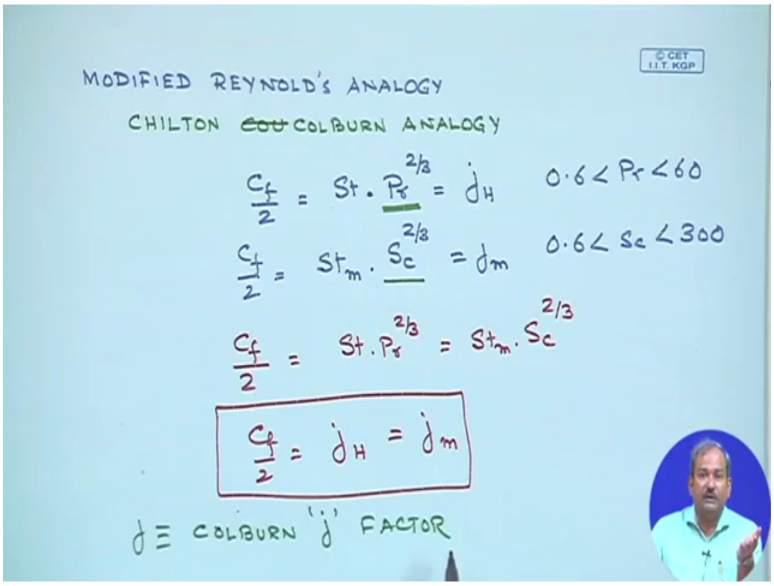

So that is what we are going to look at next which is called the modified, it is also called Chilton, both are same, they are modified Reynolds analogy or Chilton Colburn analogy. What it says is that CF by 2, it was Stanton number based on heat which is only valid for Nusselt number equal, for Prandlt number equal to 1 but the range of this can be extended if a Prandlt number is added to this relation which is called this is called JH and its extends the Prandlt number from 0.6 to about 60. So this is the Prandlt number correction factor added to Reynolds analogy to modify it for a wide range of Prandlt number also called as Chilton Colburn analogy.

And for the case of mass transfer, it is going to be CF by 2 is equal to Stanton number based on mass and Schmidt number correction factor exactly the same way as the case of Prandlt number is added and this product is, this is called JM and it extends the value of Schmidt number from 0.6 to Schmidt number from 0.6 to 300. Therefore the modified Reynolds analogy or Chilton Colburn analogy which is valid over a wide range of Prandlt number and Schmidt number is expressed as CF, the friction factor by 2 is equal to Stanton number times Prandlt number to the power 2 3rd equals Stanton number based on mass, Schmidt number to the power 2 3rd or CF by 2 is equal to JH equal to JM. So these, this is the Chilton Colburn analogy and J is called the Colburn J factor.

So this is the analogy which we are going to use for all our calculations in the subsequent exercises that we are going to see. So this is a unique relation which connects all 3 transport processes together. And it gives a tremendous advantage to experimentalists, practising engineers and so on where one set of experiment which is possible, let us say the situation is such that you can measure temperature but you cannot measure concentration. Okay. So on a prototype which is , where the actual is going to our experience both heat and mass transfer and you would like to know how, what would be the relations to be used, what are the design equations for that.

You build a prototype, you do an experiment but while doing the experiment for some reason let us say you see that it is possible to do measure the temperature and therefore do the heat transfer experiment but you cannot measure concentration, so therefore no mass transfer prototype experiment can be obtained with the system that you have right now. So what you do is you perform the heat transfer experiments on the prototype, obtain a relation between let us see Nusselt number, Reynolds number, Prandlt number, whatever from your experimental data. Now what expression are you going to use for mass transfer? Since you cannot perform any mass transfer experiments, then you are simply going to use Chilton Colburn analogy.

Make sure that the fluid you are dealing with is having the Prandlt number and Schmidt number within the range specified for Chilton Colburn analogy and simply write the expression, equivalent expression, equivalent mass transfer experiment, mass transfer expression of the heat transfer expression that you have obtained experimentally. So it reduces your work significantly, it gives you a tool to predict the performance, the mass transfer performance of the device without even doing a single experiment on mass transfer. So the utility of Chilton, these analogies are, they are extremely useful to all of us.

So whenever you think of an analogy, whenever you try to apply an analogy, always keep in mind what is the range over which, range of these parameters over which this this analogy can safely be used. And if you are satisfied, then go ahead and use this analogy and it would really give you the expression for heat transfer coefficient and mass transfer coefficient in turbulent flow without solving for the complicated boundary layer equations, governing equations that you have starting from the energy equation or starting from the species balance equation.

So what we will do in the next class is, just to give you a brief few more ideas, fundamentals about the different thickness, the thicknesses of the different layers, how they are related, for example what is the, what is the thickness of the hydrodynamic boundary layer, the ratio of the hydrodynamic boundary layer and heat transfer boundary layer or mass transfer boundary layer and so on. But after that we are going to solve different problems, various problems which directly use the concept of analogy and that would clarify your concepts even more.