Transport Phenomena.
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Lecture-51.
Convection Transfer Equations.

So we will start the last part of this course, last major part of this course which is to see the analogy between heat, mass and momentum transfer and to see under what conditions we can use the correlation developed for one type of flow can be used interchangeably as the relation for transport in another type of flow. For example can heat transfer equations, if they are derived can they be used for mass transfer as well and vice versa. So we would see that it is possible under some special cases and if we can add some approximations to it, then this type of analogy will remain valid over a large range of operating conditions to be characterised by the grouping of them together leading to the formation of the dimensionless groups.

So some of the dimensionless groups would be relevant for heat transfer, for momentum transfer and for the mass transfer process. So the 1st job of would be to identify what these different dimensionless groups can be and we should start fundamentally such that these dimensionless groups would appear automatically looking at the governing equation for transport, for any type of transport. Now whenever we talk about transport, we also know from our study so far that all these transport phenomena are going to be located, going to be going to be prevalent in a very thin layer close to the solid surface where we have a solid fluid interface.

So these boundary layers the heat, mass and momentum transfer are going to take place within this thin boundary layer and outside of this boundary layer the flow can be treated as inviscid where the temperature will remain constant or whether concentration will be equal to the bulk concentration and therefore it is going to remain constant. So we are going to concentrate more on the thin boundary layer that forms close to a solid surface. And write the equations which are valid inside the thin boundary layer and working with these equations, can we identify the relevant dimensionless groups and under what conditions the relations of let us say velocity or of temperature can be used as a relation which would give rise to an expression for concentration distribution .

So the 3 major parameters which will be of relevance here are the velocity inside the boundary layer, the temperature or its variation inside the boundary layer and the concentration variation inside the mass transfer boundary layer. We also realise that the thicknesses of these 3 layers are going to be different. So whether the thickness of the hydrodynamic boundary layer is going to be equal or different from that of thermal boundary layer would be covered by again a parameter that should automatically appear in the equation describing the growth of the hydrodynamic boundary layer or the growth of the thermal boundary layer.

Of these 3 we have extensively studied the behaviour of the momentum boundary layer or hydrodynamic boundary layer. That we could do that because the hydrodynamic boundary layer, it only has the velocity in X and Y direction and variation of velocity with either X or Y. We are only dealing with two-dimensional steady flow, so the study was a function of X and Y. So the Naviar Stokes equation which in its special form for use inside the boundary layer will therefore contain VX, Dell VX Dell X, VY, Dell VX Dell Y and kinematic viscosity nu times Dell square VX Dell Y square.

This was the starting point for the Blassius solution of hydrodynamic boundary layer under laminar flow condition. So this partial differentiation equation along with the continuity equation which is simply Dell VX Dell X + Dell VY Dell Y equal to 0, so these 2 equations were solved simultaneously by the introduction of a stream function, of a dimensionless stream function. So the entire equation was not converted in the dimensionless form and instead of the 2 independent variables X and Y, we introduce the combination variable and by an order of magnitude analysis we could get what would be the approximate form of this combination variable which we have denoted as Eta.

So with the help of of this combination variable, the partial differential equation describing flow inside a hydrodynamic boundary layer can be converted, could be converted to an ordinary differential equation, higher order nonlinear ordinary differential equation. This higher order nonlinear ordinary differential equation was then solved numerically and we obtained a table containing the value of the dimensionless variable and the value of the stream function or its gradient in terms of the defined dimensionless independent parameter eta.

So with the use of this table, the numerical results present in this table, we could obtain what is the expression for Delta in terms of Reynolds number and X where X is the axial distance and we could also obtain what is the shear stress exerted by the fluid on the solid plate. So this type of analysis even though complicated was possible only for the simplest situation, that is laminar flow of a Newtonian fluid over a flat plate in absence of any pressure gradient such that the approach velocity is equal to the free stream velocity which is the velocity outside of the boundary layer.

We then used an approximate method, since any, any other situation, for example introduction of turbulence in the system could not be handled by an analytic solution or even a numerical solution because it is so complicated and it is so difficult, it is almost impossible to obtain an universal velocity profile which will be valid near the wall, far away from the wall or in the intermediate region or the transition region between the the faraway part by the flow can be treated as almost inviscid and to the point where the flow is very close to the solid surface and it can be treated as if it is terminated by viscous forces only.

And the transport of momentum in turbulent flow will not only be due to the velocity gradient, there would be the formation of eddies which will carry, which will cause additional transport of momentum between layers of fluids. So these additional complicacies of turbulent flow prompted us to use an approximate solution method, approximate solution for hydrodynamic boundary layer which was momentum integral equation, which we have discussed again in detail. And there we saw that we need, we, what the approach, the integral equation, the momentum integral equation would give rise to an ordinary differential equation, the only requirement is we have to provide, we have to suggest a form of the variation of velocity with distance from the walls.

So the dimensionless velocity in the X direction, VX by capital U where capital U is the phrase velocity. So that VX by U is the dimensionless velocity is expressed as a function, could be polynomial, for example A + BX + CX square, A + BY + CY square where Y is the distance from the solid wall. The constants A, B and C are to be evaluated with the use of appropriate boundary conditions. So the boundary conditions were no slip at the liquid solid, at the fluid solid interface and as we move far from the, from the from the plate, what you would see that that the edge of the boundary layer, the velocity, the axial velocity V would be equal to the free stream velocity and the velocity profile inside the boundary layer would approach this free stream velocity asymptotically with , that means the velocity gradient would disappear at the, at the edge of the boundary layer.

So dVX dY at Y equal to delta would be 0. So with the use of these boundary conditions which could open the profile and then we have seen that one can use one 7^{th} power law which is completely empirical to express the velocity in turbulent flow. But we realised that what are the shortcomings of use of, use of the one 7^{th} power law that it cannot be used to obtain the shear stress at the liquid solid interface since it gives you an infinite velocity gradient at the solid liquid interface.

So we handle that part of the momentum integral equation using Blassius correlation which we have obtained from our definition of friction factor and so on. So the left-hand side of the momentum integral equation was solved using Blassius correlation friction factor and so on, the right-hand side which involves integration of the velocity profile over the entire thickness of the boundary layer, there we could substitute the one 7^{th} power law for the velocity, for the velocity in the X direction.

So with these approximations and assumptions we finally obtain the variation of delta, that is the thickness of turbulent boundary layer as a function of operational parameters, for example what is the imposed velocity and property imposed velocity, the geometry of the system in terms of the length of the plate over which this flow takes place and relevant physical properties which are mu, the viscosity and rho, the density. So as in the case of laminar boundary layer, the boundary layer growth in turbulent boundary layer was also expressed in terms of Reynolds number.

But it was shown that the growth of the turbulent boundary layer is going to be much more faster than that of that laminar boundary layer. And with this knowledge of the growth of the boundary layer we were also out able to obtain what is going to be the shear stress, what event to be the friction coefficient for the case of turbulent flow and now we have the laminar flow results and the turbulent flow results and we could see the variation of these parameters, relevant engineering parameters, for example the friction factor in laminar flow and turbulent flow and the growth of the turbulent, boundary layers in laminar flow and in turbulent flow.

But in the case of the development of turbulent flow it has been assumed that the flow is turbulent from the very beginning. That means at the beginning of the plate, the flow starts as tablet which we know that it does not happen. You really have to cross certain length, length which corresponds to a Reynolds number to be equal to 5 into 10 to the power 5, so L times V where V is the velocity outside the boundary layer, L times V times rho divided by mu, where mu is the viscosity is equal to 5 into 10 to the power 5 and the value of L that you get by this equality, that Reynolds number is equal to 5 into 10 to the power 5 is the length over which the flow will remain laminar and beyond that line the flow is going to be turbulent.

But our analysis of turbulent flow has assumed that the flow is going to be turbulent from the very beginning of the plate. Therefore to account for mixed flow cases where the flow is initially going to be laminar, followed by turbulence after you reach a certain value of Reynolds number, the relations of friction factor that were obtained for turbulent flow were modified and we got relations for mixed flow which we have discussed previously as well. We have also understood that the parameters of interest in many cases is not only the friction factor but it is the drag coefficient.

And the drag is of 2 types, one is friction drag and the 2^{nd} is pressure drag. So we have also obtain the relations from the expression of CF, we have obtained the expressions for Cd which is the drag coefficient. Then we have proceeded to solve for the laminar flow heat transfer cases and there we saw the presence, the emergence of the coupling between the momentum transfer and the heat transfer by the appearance of velocity in the thermal energy equation. So the thermal energy equation was VX dell T dell X + VY dell T dell Y is equal to the thermal diffusivity which is mu by rho CP times dell 2T by dell Y square in absence of any heat generation.

So these, in order to solve for this equation, one must know VX, etc., B, the velocity components which are present inside the boundary layer. So the solution, the hydrodynamic part of the boundary layer has to be solved up Priory before you even start to solve the thermal boundary layer equation. And there where we nondimensionalized it, like previously, we saw that we have, we saw the emergence of Prandlt number which would appear in the hydrodynamic boundary layer equation.

So once a value, a specific value of Prandlt number was chosen and with the help of the data that we have in terms of, from the momentum boundary layer, momentum boundary layer solution for one specific value of Prandlt number, the temperature profile, more importantly the temperature gradient at the solid liquid interface could be obtained. So you have a value of the dimensionless temperature L of dimensionless temperature by dell Eta where Eta is the dimensionless variable at Eta equals 0, which signifies that it is on the solid surface, this engineering important parameter, the temperature gradient, the dimensionless temperature gradient at the solid liquid interface could be expressed as a function of Prandlt number.

So dell T star by dell Eta at Eta equals 0 where Eta is the combination variable was expressed in the form of Prandlt number to the power one 3^{rd} and a constant associated with it. So working with this fitted value of the solution, one would be able to obtain what is the expression for Nusselt number which is the, which is the most important parameter that one use this in, one refers to in heat transfer, the Nusselt number for the case of laminar flow heat transfer in a thermal boundary layer. So what we obtained was a Nusselt number relation which equal to some constant into Reynolds, function of Reynolds number and a function of Prandlt number.

So this we have obtained within, within certain ranges of Prandlt number, as well as range range of Reynolds number because the relation which we have obtained so far is only valid for laminar flow heat transfer process. But how do we extend that to turbulent flow? The same way we have done for heat transfer, similarly what should be able to obtain what is the expression for the engineering parameter which is convective mass after coefficient and the associated dimensionless group is the Sherwood number. So what Nusselt number is to heat transfer, Sherwood number is to mass transfer.

So if we have an expression for Nusselt number, using the same logic, same methodology, I should be able to obtain an expression for Sherwood number for the case of laminar flow mass transfer in the thin boundary layer from the nearby, it is the interaction of the fluid with a solid surface. And as before the relation thus obtained would be valid only within a certain range of a parameter that is equivalent to Prandlt number in heat transfer. So the number, the dimensionless number which would see how they appear, the dimensionless number which is equal into Prandlt number in heat transfer, so Prandlt number in heat transfer and the corresponding number in mass transfer would be the Schmidt number.

So the Schmidt number is mu, where mu is the viscosity by rho, density and d, diffusivity. So Prandlt number and Schmidt number are same in terms of concepts, as the same way the Nusselt number and Sherwood number are identical in terms of conceptual development. So, so far what I have described to you is whatever we have done in the treatment of hydrodynamic, thermal and concentration boundary layer. We were successful in obtaining a solution, not a closed form solution better numerical solution for the case of hydrodynamic boundary layer under laminar flow and an approximation would give us the value of the, of those parameters for the case of turbulent flow.

Extension of the laminar flow solution gave us the Nusselt number relation for heat transfer and Sherwood number relation for mass transfer, for the case of heat transfer and mass transfer, for the case of laminar heat transfer, laminar flow heat transfer and laminar flow mass transfer. But we were not successful, we still do not know how to converse, how to extend the relations of laminar flow heat transfer and mass transfer to turbulent flow situations.

Because in turbulent flow situation, the appearance of the eddies would make the situation much more difficult to handle and therefore the way we have started, we have simplified the situation in the, with the use of momentum integral equation, it is, it will not be easy or other there must be a better way, looking at the similarities between these processes, can we derive relations, or can we suggest proposed relations by simply looking at the relation, what we have, that we have for the case of hydrodynamic boundary layer.

So the CF, the friction coefficient or the Cd, the drag coefficient, the relations for which way we have some confidence in the relations of CF and Cd, both in laminar and turbulent flow as well as mixed flow in the case of hydrodynamic boundary layer, is there a way extend these relations without excellent going into the complex mathematical treatment of turbulence in heat transfer or in mass transfer? Is there a way? So the remaining classes will therefore be devoted to find this link, find the way by which we could connect all these processes.

And once that is done, then we are going to solve certain interesting problems of heat transfer results being applied to mass transfer and vice versa. So as, from our discussion so far, we understand that the convection transfer equations inside thermal hydrodynamic and concentration boundary layers, they play a very important role in deciding about the similarities between these transfer processes. So the 1^{st}, our 1^{st} job would be to examine these equations, to examine the development of these convective transport equations for all these 3 transport operations in greater detail and to try to see where we can start our similarity exercise.

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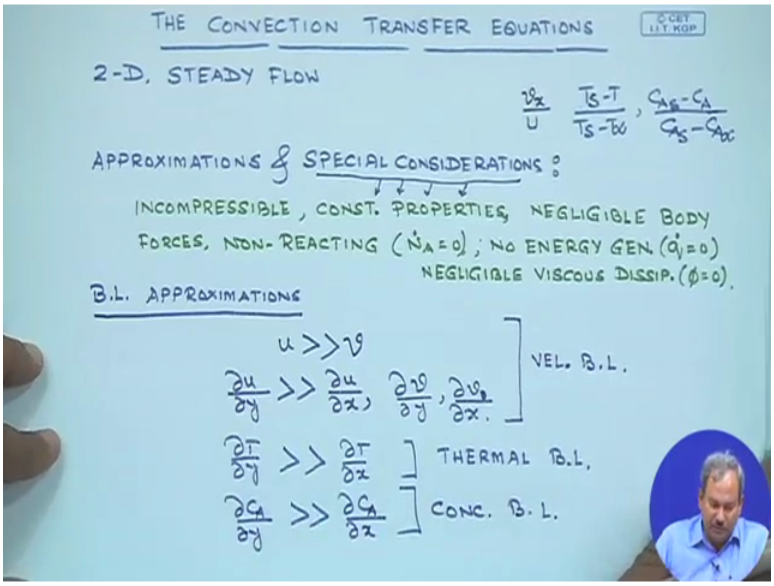

So the 1st thing that we are going to do here is the convection transfer equations that we see over here. So, we are going to start with two-dimensional steady flow situations and we know the dimensionless velocity is defined as VX or VY divided by U where it is, U is the free stream velocity, this is the dimensionless temperature, TS being the temperature of the solid substrate, T infinity being the temperature at a point far from the solid wall or it is the bulk temperature, CAS is the concentration of the component A on the surface and CA infinity is the concentration of component A at a point far from the surface. And CAT and VX are the concentration, the temperature and the velocity which are functions of both X and Y square X denotes the axial distance and Y denotes the distance perpendicular to that of the solid plate.

So let us see what are the approximations and special considerations we are going to invoke in order to solve, in order to see these analogies, 1st is we are going to assume that it is an incompressible fluid, the fluid properties are going to remain constant, there is, there is the presence of a negligible body force, so the effect of gravity, etc. would be unimportant. It is a non-reacting system, so therefore there is not going to be any production, any generation or depletion of the component A by reaction and it is also there is going to be no energy generation, so the Q dot, the rate of production of energy is going to be equal to 0.

And it is going to be a relatively, the flow velocities are going to be moderate and therefore we are going to have negligible viscous dissipation or the viscous dissipation function Phi would be equal to 0. So these are the, these are the special considerations which we are going to use in order to tackle the problem of simultaneous heat, mass and momentum transfer and our starting point would be the convective transfer equations in the form of equations inside the boundary layer. So, what, what we what we then do is, we are going to see what are the approximations, boundary layer of vaccinations that we have used so far, we are, we have already used that U, the velocity in the axial direction is going to be very large, in terms, in comparison to the component of velocity which is perpendicular to the direction of flow.

And variation of axial velocity with Y is going to be significantly larger than variation of the velocity with respect to X, so dell U dell Y would be much more significantly larger than dell U dell X and it is also going to be very large compared to dell V dell Y and delta V dell X. So the X component of velocity, the axial velocity, the change of rate of, rate of change of axial velocity with distance from the solid wall is going to be significantly higher than dell U dell X, dell V dell Y and dell V dell X and collectively together, these are known as the boundary layer, boundary layer approximations for velocity boundary layer.

The 2^{nd} one is that we have is dell T dell Y is going to be quite large, very large compared to dell T dell X. So T the temperature, so the variation of temperature with respect to the distance, the variation in temperature over the thickness of the boundary layer, dell T dell Y would be significantly higher than the axial temperature, temperature temperature gradient and this is the approximation with we are going to use, which we have used before, the thermal boundary layer, which is,all these assumptions are reasonable since the thickness of each type of boundary layer is very small.

So since the velocity boundary layer is small, the variation in velocity over this variation in velocity from a value equal to 0 due to no slip condition, on the solid and its velocity which is a constant velocity, which is U throughout the rest of the boundary, throughout the rest of the region outside of the boundary layer, this variation from 0 to capital U takes place over everything region. So dell, so therefore the variation in velocity with respect to Y will be significantly more than the variation in velocity of, of the variation in velocity with respect to X.

So vertical direction velocity gradient of the X component of velocity would be much more as compared to the gradient in the axial direction. Using the same logic, same understanding, the temperature varies from TS, which is a sub, temperature of the solid substrate to T infinity, which is the temperature outside of the thermal boundary layer, this changeover takes place over a very thin boundary layer. Therefore this gradient is going to be very large as compared to dell T dell X. Using the same logic, one can write that dell CA dell Y is going to be very large compared to dell CA dell X.

So this is for the concentration boundary layer. So these are the approximations which we have used and which we are going to use in our subsequent studies of the analogy. Some special considerations here, one must say that, there will be situations in which, let say a species transport is taking place from a solid surface, so would the species transport affect the transfer inside the boundary layer? So let us say you have sublimation taking place from a solid surface into air. So would this sublimation, that means therefore the velocity may not be equal to 0 on the solid plate since you have a free stream, sublimation taking place.

So if you have situations like that, is it reasonable to assume that the transport operations are going to be unaffected of by the presence of this, by the absence of the no slip condition at the solid surface. What has been shown, what has been observed is unless you are talking about a significantly high transport taking place, transformation taking place at the solid liquid interface, all the concentrations that we had discussed so far will still prevail. So only in the case where steam is condensing on the solid surface, there in the additional heat transfer process is going to affect the growth of the velocity boundary layer or the growth of the concentration boundary layer.

But for most of the practical considerations, the effect of species transport can be neglected while developing these boundary conditions.

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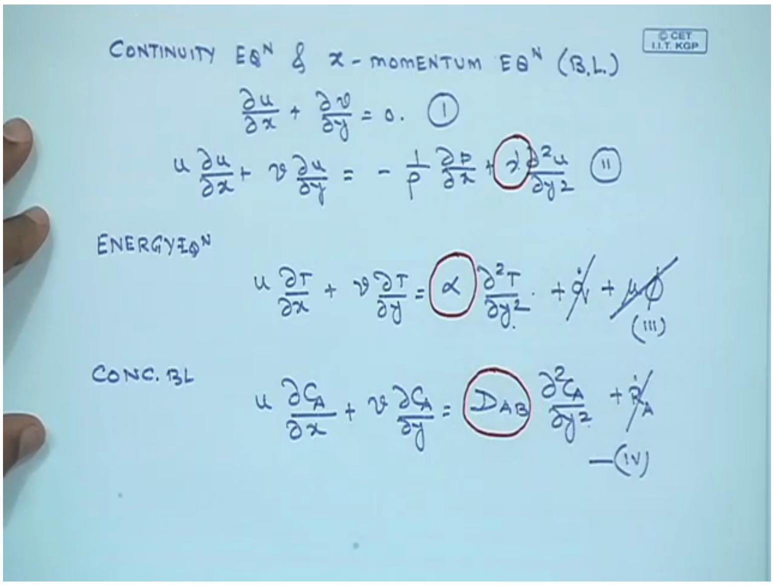

So now I think we can start the equations, the, I would simply write these equations for this class and then we will discuss about their implications in the next class. So for the case of velocity boundary layer, we know that the, we have the continuity equation and the X momentum equation inside for the situation inside the boundary layer and this is the continuity equation and what we have in terms of the X momentum equation is U dell U dell X + V dell U dell Y, this is the X component of the Naviar Stokes equation. In utilising the approximations boundary layers that I have discussed so far, the -1 by rho, dell P dell X + nu del square U by dell Y square.

So this is my equation number 1, this is the equation number 2. Now if you see carefully that I have kept this dell P dell X comes in the X momentum equation signifying that I am not restricting myself to the case of flow over a flat plate only. So as long as I have this dP d X in here, my situation is therefore equally applicable for flow over curved surfaces as well. So the equations in these forms refer to laminar flow over a solid surface that may be flat in which case dP dX may be 0 or it is going to, it could be also for a curved surface with dP dX which may not be equal to 0.

Similarly, so the energy equation would be equal to U times dell T dell X, U being the velocity + V times dell T dell Y is equal to Alpha which is, which is the thermal diffusivity, which is equal to K by rho CP dell 2T by dell Y square. And we have discussed already Y dell 2T dell X square can be neglected in this case, since temperature varies sharply with Y, therefore this, since, this is valid, dell T dell Y is very much larger, large larger in comparison to dell T dell X, therefore the transport, this, the right-hand side of the equation refers to conductive flow of heat and the conduction in the Y direction will far outshadow the conduction in the X direction.

Therefore there is going to be only one term. I do not have a Q dot which is heat generation per unit volume, I do not have this term, as well as I do not have this term which is mu times Phi where Phi is the viscous dissipation term. So these 2 terms are not present in the, under the conditions that I am I am describing right now. So this is my 3rd equation which is about the energy equation inside the boundary layer and then for concentration boundary layer, what we have then is U times dell CA dell X + V times dell CA dell Y and the transport, relevant transport coefficient here is going to be DAB times dell square CA by dell Y square.

So as before I am not considering any reaction which is taking place inside the boundary layer, it is non-reacting system. Therefore the effect of reaction would be, would be 0. And the transport of species A through diffusion in the Y direction will be significantly more than the diffusion of A in the X direction so that dell square CA by dell X square term is neglected, the same way I have neglected dell square T by dell Y square. So conduction and diffusion in the Y direction predominates over conduction and diffusion in the X direction.

And if you again notice that the transport coefficient here is kinematic viscosity which is mu by rho, this is thermal diffusivity which is mu by rho CP and this is simply the mass diffusivity. So these transport coefficients essentially will dictate how the diffusive transport process is taking place. So these equations can then be used to identify key boundary layer parameters, the similarity parameters as well as the important analogies between heat transfer, mass transfer and momentum transfer.

So in the next class what we are going to do is we will start with these 3 equations, expressed in dimensionless form, express them again, all the boundary conditions are also expressed in dimensionless form and then we will try to see under what special conditions, the governing equations will look identical, the boundary condition will be identical and if we have the same type of form of governing equation and same form of dimensionless boundary conditions, then the 2 systems, one having let say heat transfer and the one having mass transfer, these 2 systems will become dynamically similar.

So in order to become dynamically similar, the governing equations should be the same, the boundary conditions in dimensionless form should be the same and when the 2 systems become dynamically similar, the expression for one type of process can be used as the expression for another type of process. In other words the heat transfer equations can be used as mass transfer equations, as long as and this is important, as long as we change the relevant dimensionless parameters of heat transfer by the equivalent relevant parameters for the case of mass transfer.

So whatever be the parameters of heat transfer, that we have to identify 1st and that for mass transfer and for momentum transfer. And then we will have to see mathematically when these 3 different types of, when the 3 equations, these 3 systems along with their boundary conditions become dynamically similar. So that is what we would do in the next class.