Transport Phenomena.
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Lecture-50.
Mass Transfer (Continued).

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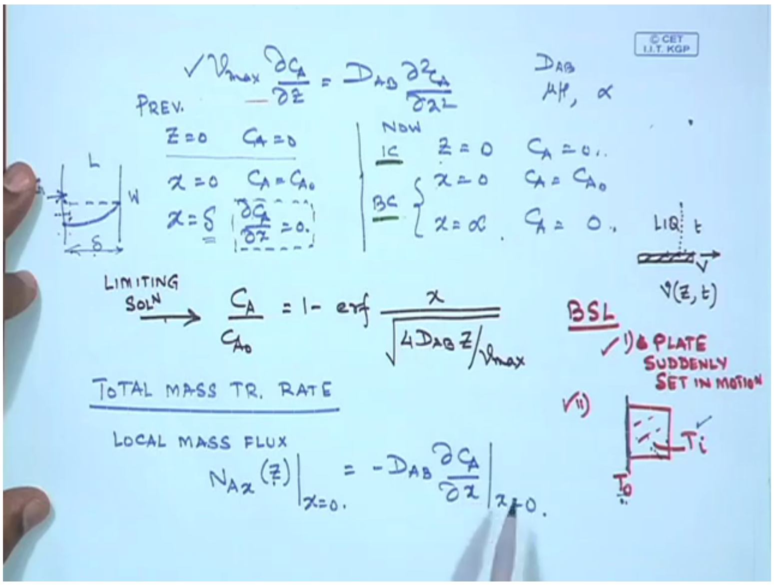

So we are going to continue with what we have done, what you are doing, so we have obtained a limiting solution for the special case when the penetration depth, when the penetration of A in the liquid is not too high, too large, which will be the case if it is, if the contact time between A and the liquid B is short. So this is the error function solution and the examples of error function solutions can also be obtained when it is about the velocity when a plate in contact with the velocity, with the liquid is suddenly set in motion or you can see it in heat transfer as well when a block which is maintained at a uniform temperature, one end of which is suddenly exposed to a different temperature and you are trying to find out how the temperature inside the solid object change as a function of distance from the surface and as a function of time.

In all cases the form of the governing equation would look like this. So and the initial and the boundary conditions are also, so the initial condition for these 2 cases would be at T equal to 0, that means at time equal to 0, the velocity is 0, the velocity is 0 at any Y. And the 2 boundary conditions which are which are relevant over here as you can see that at Y equal to 0, no matter whatever be the time, the velocity is simply going to be equal to the concept velocity V with which this rod is being pulled, this plate is being pulled. So at Y equal to 0, for this case, if this is Y, Y equal to 0, the velocity is going to be equal to capital V.

And if you go to a distance far from the solid plate, there the fluid still does not know that there is a plate and it has started moving. So at a point far from the wall which is this, the velocity, the concentration, the equivalent of concentration in that case would be the velocity, the velocity would still be 0. So this is identical with what we have written here in terms of the boundary conditions and when you think of the temperature, when one its end is suddenly changed, we know that at Z equal to 0, the equal it of that in this case is time T equal to 0, the dimensionless temperature, the way we defined it, the dimensionless temperature is 0, so T equal to T I, so T - T I is going to be equal to 0. At X equals to 0, that means at this point the temperature is going to be equal to T0, the new changed temperature which remains constant.

So at X equal to 0, that means on this plate at which the temperature has changed, it is a constant and since if you move far from the wall, from the face at which the temperature is changed, the temperature there will still remain equal to TI. So the dimensionless temperature would be equal to 0. So I would request, I would advise you to read, to see these 3 problems together and see what is the similarity between the governing equations, the coefficients, important is the coefficient, is the transport coefficients which arise automatically in the governing equations, look at the similarities between the initial condition than boundary conditions and for one of them follow the solution methodology.

So once you have the same form of governing equation, same initial and boundary conditions, then the solution obtained in one can obviously be written as a solution of the one which you are dealing with provided you change accordingly. For example velocity, dimensionless velocity would be replaced by a dimensionless concentration, the time would be replaced by X, Z in this case. Time would be replaced by Z in this case and the boundary conditions are at X equal to 0 and X equal to infinity. And the transport course which is which is DAB for the case of mass transfer, when you do it for plate suddenly set in motion, you would see that it is going to be mu by rho or the kinematic viscosity.

And when you deal, when you write it for the, when you look at the expression, when you look at the governing equation for heat transfer, you would see that it is going to be Alpha or mu by rho CP, that is the thermal diffusivity. So these 3 equations, these 3 situations, these 3 equations will not only give you identical conditions, boundary conditions and initial conditions, it would also identify the transport coefficients of interest of relevance in these problems, namely the diffusion coefficient, the kinematic viscosity and the thermal diffusivity, all having units of metre square per second.

And any one solutions you follow, you should be able to write the solution for the case at hand using the same steps. So that is why I am not doing, not solving the problem, simply writing the final form of the solution based on our knowledge of the previous cases. So here we have the limiting solution and we would like to find out what is the total mass transfer rate that is the important one which we wanted to find out, mass transfer rate. And the total mass transfer rate, in order to do that, I would 1st find out what is the local mass flux.

So this is N of A, component A, it is in the X direction which is a function of Z, we are evaluated at X equal to 0. So I am trying to find out what is NAX that enters the film which is a function of Z, at different points this NAX would be different but I am going to do that at one plane. So coming to the larger picture, this NAX, it varies with Z but I am evaluating this NAX at X equal to 0. So I am trying to find out what is the mass flux across the liquid vapour interface. And therefore this would simply be equal to - DAB dell CA by dell X at X equal to 0. So the flux of A in the X direction at X equal to 0 would simply be equal to - DAB dell CA dell X at X equal to 0.

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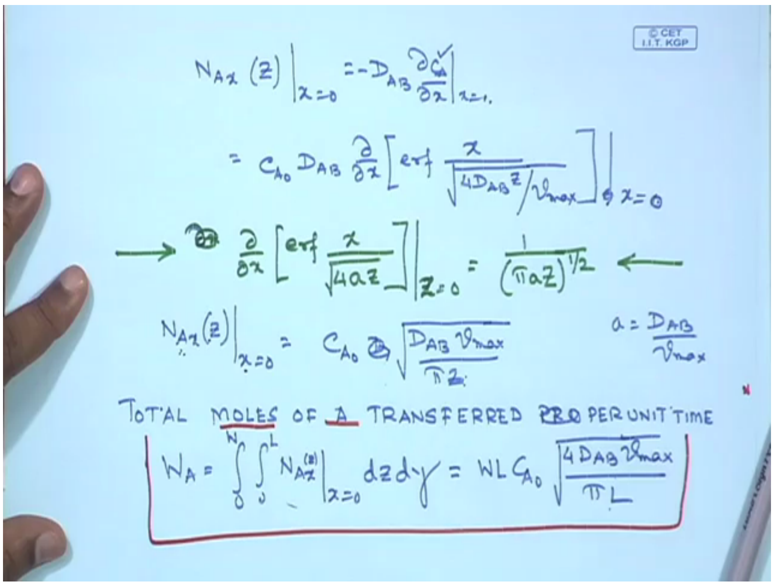

So that is straightforward from the Fick’s law from the definition of diffusive mass transport. So I will write it again, NAX at which is a function of Z, evaluated the specific X which is the liquid vapour interface would be - DAB dell CA by dell X at X equal to 0. So this would, when I substitute CA, when I substitute CA from this are function form, what I am going to get is equal to CA0 DAB dell dell X of error function X by root over 4 DAB Z by V Max at X equal to 0. So this directly form, directly is formed by putting the expression of CA from here to this point and then performing the differentiation, so this normal one would go away and you would simply have dell dell X of error function of this evaluated at X equal to 0 at that for diverted over here.

Now one of the properties of error function is, one of the properties of error function is dell dell X of error function X by root over 4 AZ at Z, at X equal to 0 is equal to1 by pi AZ to the power half. So this is one of the relations of error function which are available, so I am going to use this relation in this. So my N A X at Z, at X equal to 0 would simply be equal to CA0 times DAB V Max by pie times Z. So when I do this, so my A is simply, if I compare between this and this, my A is simply equal to DAB by V Max. So this with this formula which is provided I can write my, the local mass flux at X equal to 0 which is a function of Z and this is expressed in this form.

So the total moles of A transferred per-unit time is simply if I express it as WA is 0 to W, 0 to L NAX at X equal to 0, this is the same thing, X is a function of Z, at X equal to 0 times dZ dY. So this is the mass flux, mass flux of A, species A, this is the flux of, sorry moles of moles of A getting into the X equal to 0 plane and we understand that this could be a function of Z which is shown over here. In order to obtain the total moles which are transferred, I am going to integrate it over the entire area of, entire area of this face, entire area of the X face which is from Y, the integration of Y and integration of Z. So the Y integration is from 0 to W and the Z integration limits are going to be from 0 to 0 to L, 0 to L.

So that is why it is integrated over 0 to L dZ and 0 to Y, 0 to W dY and we understand that this N a X is a function of Z but it is not a function of, it is not a function of Y. So when you perform this equation you would see this is going to be CA0 root over 4 DABV Max by pie L. So this is then the number of moles of A which gets transferred from the gas to a falling film of liquid but with some assumptions. So what are the assumptions, let us recapitulate once again. The assumptions that we have used our it is one-dimensional laminar flow, the contact time is small, so A does not penetrate to a greater distance, to a large distance into B and it is falling, it is a steady-state process and the diffusion of A in B is a slow process and therefore we could make certain simplifications of the governing equation based on what is important in which direction.

So we understand that diffusion is the only way by which mass gets transported in the X direction, we have both diffusion and convection in the Z direction but the convection overshadows the conductive or diffusive mass transport in the Z direction. Then we have obtained the governing equation and identify the boundary conditions. But we have, if we have used the condition of very small contact time, then what you would see is that the A does not penetrate much into B, so for the diffusing A molecules, the other end, the solid wall is as if it stays, it is at infinite, it is at infinity.

And since it is only diffusing by a small distance inside the liquid, to all diffusing A molecules it would seem as if the entire liquid film is falling with a velocity equal to V Max. So a further simplification of the governing equation and a further simplification of the boundary conditions are possible which would make them identical in form and in governing, in boundary and in initial conditions, 2 situations for which analytic solutions are available.

So we get an error function solution and using the error function solution I can obtain what is the flux of A molecules onto the surface of the liquid and integrating this flux over this entire area, it is from 0 to L and from 0 to W, I can obtain the total number of A molecules, total number of molecules of A which gets transported, which gets absorbed by the falling film of liquid. So this expression which we have obtained finally is relevant in many falling film devices where a gas, a gas is going to be absorbed on the liquid film. And it is a nice example to establish the utility of using the species balance equation rather than the shell species balance. So from now on I will think you will be more comfortable while using this equation for all your subsequent analysis.

So what I would do next is a very quick problem, very quick tutorial problem on mass transfer and which would again which would which would give you some more ideas about the use of error function, the difference is enough under function and how we can use mass balance in a valid engineering problem, in an engineering problem where you are going to find out the rate of decrease of a layer of, of a slab of salt which is kept in contact with water. So I have a slab of salt and a body of water above it but they are separated from each other by another plate, another impervious plate.

So what is going to happen at T equal to 0 is this plate which separates the salt slab and water, it is removed. So now the solid slab is in contact with the water. So initially the water does not contain any salt but as time progresses, the salt from the interface is going to get into the water and therefore the concentration of salt in water is going to increase with time, if you fix the location, or the concentration is going to be a function of this direction. That means as you move far from the slab, the concentration of salt will progressively decrease. So it is a case of concentration varying with time and with position.

So a salt slab suddenly brought in contact with a deep pool of liquid and we have to find out what is the, how does the concentration, salt concentration changes in the liquid and what is the recession rate, that means as time progresses the salt slab is going to get thinner and thinner because it is getting dissolved in water. So what is the rate of recession that is d, if L is the thickness of the salt slab, what is dL dT? That is rate of recession of the salt slab as a function of time and other parameters.

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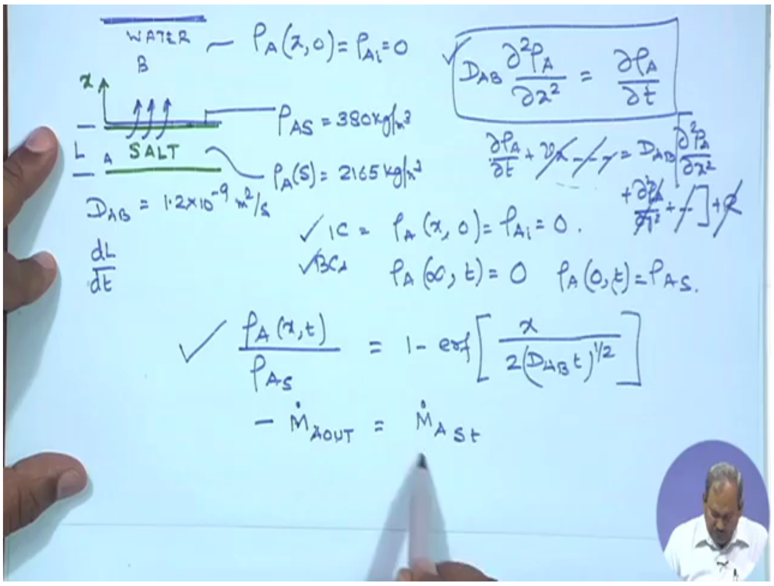

So very quickly what we have then is, this is my salt slab and we have water present on top of it, the concentration of salt in water at 0 time is, we call it as rho A I which is equal to 0. The concentration of salt at the interface, at this point is equal to 380 KG per metre cube which could be the solubility of salt in water. And the solid salt, solid salt slab, the rho A of the salt, solid, this is the salt slab is equal to 2165 KG per metre cube, the dAB, that is the diffusion coefficient of salt in water is 1.2 into 10 to the power - 9 metre square per second. So this is water B and salt is A. Okay. And what we have to find out is how does the concentration of salt varies in water and what is the rate of recession, so this is initially the thickness is L, we will assume that this is my X direction and I would like to find out what is the recession rate that is dL dT.

How does the thickness of the salt slab change with time? So as before I can clearly write that the governing equation is going to be equal to, so DAB is equal to dell rho A dell T. So this, if you see is identical in sense to this equation, okay. So change in salt concentration, if you start with the equation, that is dell rho A dell T + VX and all these terms is equal to DAB times dell square rho A dell X square + same thing dell square rho A by dell Y square + Z square + the reaction term. So there is no question of velocity in here, so the entire convective term would be 0, the rho A, the salt mass concentration of salt is a function only of X, it does not depend on Y, it does not depend on Z and there is no reaction term. So the governing equation would simply be dell rho by dell T is equal to DAB times dell square rho A by dell X square.

So quickly you can obtain what is your governing equation. And the initial condition is this, that is rho A at any value of X at 0 time, would be equal, if I call it as rho AI, this is going to be equal to 0. And the boundary conditions that you would get is rho A at infinite X, that means at a far, point far from the interface, for any value of time would be equal to 0. So there is no salt present in this water and rho A at X equal to 0 for anytime X on this surface but at anytime must be equal to rho AS which is the concentration of salt in the liquid, so this is equal to rho AS. So in terms of IC, BC and the form of the equation, this is the same, this is the same as this equation. Okay.

Only thing is this V Max here is 1, so if you compare between the forms of these 2 equations, they are same. The initial conditions, the boundary conditions, the initial condition that the boundary conditions, they are the same. So you can directly write, for such problems you can directly write what is going to be rho A as a function of X and T by rho AS which is the one which you using to nondimensionalize the mass concentration is 1 - are function X by the 2 DAB times T to the power half, previous solution that we have obtained for this case.

This is, part A is done, now we are trying to find out what is the recession rate. So we make a mass balance of the salt, some of the salt is going to get dissolved as a result of which the amount of salt present in the slab is going to get decreased. So if I express it in terms of an equation, the rate of mass, the rate, the rate of mass of A which is going out must be equal to the M dot A which is stored in the system. So we would not have any salt coming in, the salt is going out, as a result of salt is going out, the amount of mass of salt content in the slab is going to be reduced.

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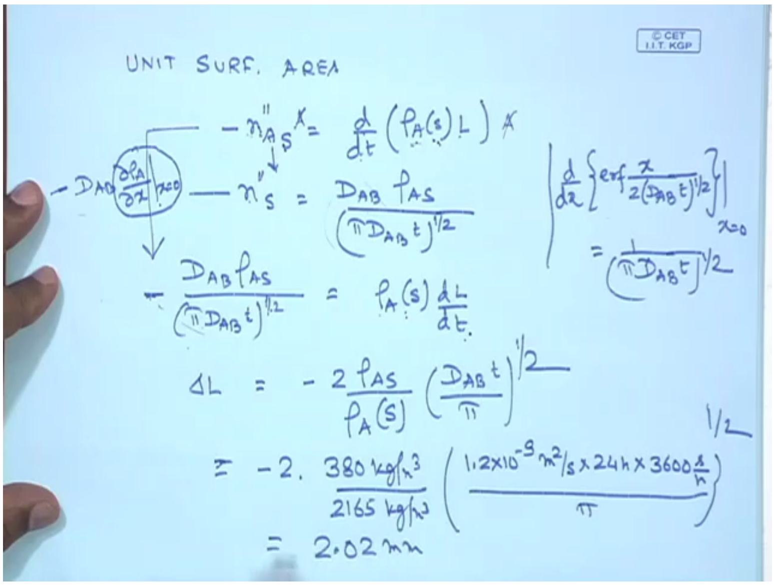

So the equation in this form is M dot out is equal to M dot stored, A refers to salt. So out is equal to stored, so which we, if we take a unit surface area, so mass flux of A of the salt is equal to d dt of rho AS times L. This is the species mass flux at the surface and what you would then get is, when you perform this d dT and we know what is the expression of rho A from here and I know how to differentiate the error function and find out the value at X equal to 0, then had X equal to 0, then you would you can simply directly write as NA double prime S is equal to DAB rho AS by pie DAB times t to the power half.

I have used the formula that d dX of error function of X by 2 DAB t to the power half evaluated at X equal to 0 is equal to 1 by pie DAB t to the power half, standard formula that I have used, so this is the formula. And here I have evaluated the left-hand side. And when we go to the right-hand side, when we go to the right-hand side, I simply get DAB rho AS by pie DAB t to the power half is equal to rho AS comes outside since it is a constant times dL dT. Okay. So this with a - since I started with a -, expression of NS is this and therefore this equation now becomes, this is equal to rho AS on the outside times dL dT.

And you can quickly integrate this, so you get delta L, over time you get delta L is equal to - 2 rho AS by rho H of the solid salt times DAB T by pie to the power half. So it is it is given that, when you plug-in the values, 2 will remain as it is, rho AS is 380 KG per metre cube, rho A of the salt is 2165 KG per metre cube, the value of DAB is 1.2 into 10 to the power - 9 metre square per second, time is 24 hours into 3600 second per hour divided by pie to the power half. And what you would get is 2.02 millimetre. So over a 24-hour period, the salt slab will be will be reduced by the size or thickness of the salt slab will get reduced by 2 milli, about about 2 millimetres.

So let me quickly go through it once again to show what I have done. On a unit surface area-based, 1st of all I have obtained an expression for the concentration which is an error function. Then I have made a mass balance across the salt slab, the one that goes out, it goes out by dissolution into water. This is going to make a change in the stored amount of salt in this, so this is my governing equation. When I express in terms of unit surface area, so the area will cancel, area will cancel from both sides, so what I have then is the mass flux of A which is at the surface is equal to ddT of rho A of the solid salt times L and the area simply gets cancelled from both sides.

So my mass flux can be written as DAB times dell rho A by dell X at X equal to 0 with a - sign. So this dell rho A by dell X at X equal to 0, I already have the expressions for rho A in terms of error function, so I am try to find out what is dell rho A by dell X at X equal to 0 and I use the formula which I have shown you in the previous example. So I have a compact expression for this as well. So the flux, the mass flux is evaluated as using this formula has this. So my - NA double prime S becomes - this DAB rho S by pie DAB t to the power half. Please do it yourself and verify that you are getting the same answer following the example which I have shown you before.

And this differentiation, since rho S, rho AS is a constant, it comes outside, so it is dL dT. And then you integrate, integrate it once and what you would get is delta L, from, so you integrate from 0 to T and from 0 to L, this is the expression which you would get and when you plug-in the numbers you would see that the, that the thickness of the slab decreases by about 2 millimetres over a 24-hour period.

So what we have done in these 2 classes, showing the utility of the use of species balance equation, identifying that are function appears in many problems of heat, mass and momentum transfer, utilising the property, the mathematical property of the differentiation of error function at X equal to 0, at equal to 0, at some specific point we can obtain, in the last problem we have obtained the surface recession rate of a salt slab. So through theory, examples, we have established mass transfer, the base model mass transfer, the use of species equation and complicated situations in mass transfer.

So now I think we are in a position to get into the analogy part, the last remaining part of transport phenomena. Can we compare mathematically utilising the similar nature of the governing equations and similar nature of the boundary conditions, the coefficients, the relations of heat transfer in mass transfer, what needs to be done, that is the topic, that will be the topic in the remaining classes of this course.