

**Transport Phenomena.**  
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**Lecture-49.**  
**Mass Transfer (Continued).**

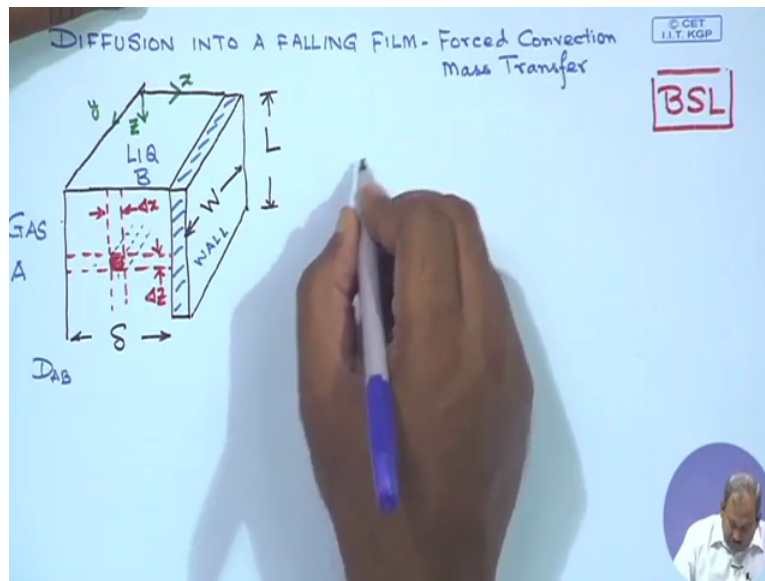
We would be discussing about the application of species balance equation which is nothing but the conservation of mass for species A and we have seen how the conservation of species A, B, C etc. when they have, when they are added together, it would give the continuity equation that we are familiar with. So when we see these species balance equations, there is a temporal term which denotes the variation in concentration of one species at a with respect to time and there are other terms which would talk about the convective mass transport of, convective transport of species A. And on the right-hand side we have the diffusive transport of species A because of the existence of a concentration gradient.

And the last term that we have included, that we have identified to be something which would result in a change in concentration of the species A as a function of time and position is a generation term. So the most common generation term of the depletion term that we can think of in mass transfer is through reactions. So the species balance equation is the, is a statement of conservation, is a statement of conservation of mass, it has temporal terms, it has terms containing velocities which denote the convective transport of mass which is due to the flow and so on. So it would also be useful to start with a problem where we would see the increasing difficulty of expressing the species balance through a shell mass balance approach.

So far the shell balance approach was very successful but as we have seen before with the case of momentum transfer and with the case of heat transfer, the momentum geometry becomes a bit more complicated and where we have to take into account the variation in the measurable quantity which in this case is concentration is a function of more than one space coordinates or if it is a function of time as well, then expressing that or expressing that in terms of an imaginary shell becomes increasingly difficult. So we would start with the problem of diffusion in a falling film, so we have a wall and across along the side of the wall a film is falling vertically.

So let us say that liquid film, we denote that, the solvent, the liquid solvent here as B and there is the gas present in contact with the falling film, let us say A. So A is the gas which is going to be absorbed, which is going to be absorbed in the liquid which is B and then the species of A will travel, will start its downward journey along with the film.

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And we would try to find out what is the concentration distribution of A in the falling film or more importantly what is the rate of absorption of A into the falling film and pictorially it can be represented in this way. Where we have this as the solid wall, so this is a solid wall of a width  $W$  and length  $L$ , then I have a liquid film which is falling, this wall is vertical, the liquid film is coming down as there is a gas A which is going to be absorbed at the liquid, liquid gas interface. And as the A molecules get into the falling liquid, it is going to start its downward journey.

But as it traverses, as it starts its onward journey, there exists a strong concentration gradient on this side, so it will start to, start its convective motion downwards and there is going to be a diffusive motion of the species in the X direction as well. So in the Z direction I have the presence of both convection and since the concentration of A will vary depending on where we are in the Z location. There could also be a diffusive motion of A molecules in the Z direction may also be setup.

So the system is then a gas being absorbed in the liquid and the gas starts its diffusion convection motion in the falling film and we need to find out what is the concentration, what is the concentration profile of A in the film and how much of A is going to get absorbed in the falling film as a function of parameters which could be the geometric parameters like the thickness, the width of the wall, the length of the wall, the diffusivity of A in the liquid B and several other factors. So if we start with our shell balance approach, we have always assumed shell to be the smaller dimension of the shell was the direction in which the concentration or the temperature or the velocity is changing.

But if you if you think of the figure, the figure that I have just described to you, the concentration is changing out only with Z but it is also changing with Y, not only with along the film but across the film as well. So if I have to choose a shell, it is going to be, it is going to be of size  $\Delta Z \Delta X$  and it could be any Y because none of the parameters would depend on Y. So coming back to the figure once again, so this is my  $\Delta Z$ , since my concentration changes with Z, so this is going to be one of the smaller dimensions, it is a function of X since we have a concentration gradient in this direction. So  $\Delta X$  is going to be another one, so what I would get is some sort of a rodlike structure in which this is  $\Delta Z$ , this is  $\Delta X$  and it could be any let Y, you can take that to be Y as well.

And across these faces, there is going to be convection and diffusive motion. So you can see how complicated, I mean it is becoming, it is coming to a point where it is easy to make a mistake in identifying what would be the mechanism by which the mass A, the species A enters the control volume or leaves the control volume and so on. So this problem has been shown, has been presented in Bird, Stuart and Lightfoot using this shell species balance. So you can take a look at that but what we are going to do, since we know what is species balance equations in, what we are going to do now is we are going to simply write the equation, the species balance equation in the rectangular, in XYZ coordinate and in Cartesian coordinate system and we will simply cancel the terms which are not relevant.

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DIFFUSION INTO A FALLING FILM - Forced Convection  
Mass Transfer

$D_{AB}$

BSL

$$\checkmark \frac{\partial c_A}{\partial t} + v \nabla c_A = D_{AB} \nabla^2 c_A + R_A$$

ASSUMPTIONS

- i) NO REACTN
- ii) STEADY STATE
- iii) A IS SLIGHTLY SOLUBLE IN B ( $\mu$  CONST)
- iv) DIFFN OF A INTO B IS A SLOW PROCESS
- v) LAMINAR, 1-D FLOW. ( $v_z \neq 0$ )

$$\checkmark \frac{\partial c_A}{\partial t} + v_x \frac{\partial c_A}{\partial x} + v_y \frac{\partial c_A}{\partial y} + v_z \frac{\partial c_A}{\partial z} = D_{AB} \left( \frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2} + \frac{\partial^2 c_A}{\partial z^2} \right)$$

$\checkmark v_z \frac{\partial c_A}{\partial z} = \text{CONV. IN Z}$ 
 $\checkmark \frac{\partial^2 c_A}{\partial z^2} = \text{DIFF. IN Z}$

So we will skip the requirement of defining a shell, identifying the mechanisms by which the species A enters into the shell and arrive at the governing equation. So this is going to be the case, so I know that in Cartesian coordinate system, the species balance equation which is also there in any of your textbooks is, this is the one which we have discussed in the last class. So this is the concentration of species A as a function of time and since it has velocity in it, so it is going to be the convective term, this has the diffusion coefficient, so this is the diffusive term and we have the rate of reaction which consumes or produces A.

So 1<sup>st</sup> of all we are going to make certain assumptions, 1<sup>st</sup> off there is no reaction, that means that A does not react with B, so A and B are non-reacting to each other and we have a steady-state which is, which is reached in this case. So we are dealing, we are handling, we are solving the problem only at steady-state and the 3<sup>rd</sup> one is A is slightly soluble in B. That means the  $\mu$  is remaining a constant, so  $\mu$  is the constant and the 4<sup>th</sup> one is diffusion of A into B is a slow process. Then what this simply tells us is that the penetration of A into the B is taking place over a period of time, it is not a very fast, it is not a very fast penetration.

So the and the other one is it is laminar 1-D flow such that we have velocity, so we have only  $v_z$  which is not equal to 0, then the velocity in the Y direction and no velocity in the X direction. So now I am going to expand this, if I expand this, it is simply going to be  $\frac{d c_A}{d t} + v_x \frac{d c_A}{d x} + v_y \frac{d c_A}{d y} + v_z \frac{d c_A}{d z} = D_{AB} \left( \frac{d^2 c_A}{d x^2} + \frac{d^2 c_A}{d y^2} + \frac{d^2 c_A}{d z^2} \right)$ . I am not writing the  $R_A$  term since I have already assumed that there is no reaction

which is taking place, so there is no question of generation or depletion of A in the liquid gas mixture when the gas gets absorbed in the liquid.

So the reaction term is dropped. This would be 0 because of 2, the assumption 2, so these are my assumptions. So because of assumption 2, it is a steady-state and this would be 0 because of my assumption 5 which says it is a one-dimensional flow, so  $VZ$  is simply going to be, is the only nonzero term, so this would be equal to 0. For the same logic I am going to get the  $VY$  to be 0 as well since again it is laminar 1-D flow. But I understand that  $VZ$  is not equal to 0 and  $CA$  is going to be a function of  $Z$ . As I move in this direction, what I am going to see is that at a fixed  $Z$ , the concentration is going to change with  $Z$ , it is going to increase with  $Z$  since it is being absorbed by the liquid.

So the entire left-hand side then I am going to have is  $VZ \frac{dC_A}{dZ}$  is equal to, then I am going to think about the right-hand side. So in the right-hand side what I have then is  $DAB$  will obviously be there and  $\frac{d^2 C_A}{dX^2}$  because  $CA$  is a function of  $X$ , so this term must be there +  $\frac{d^2 C_A}{dZ^2}$ .  $CA$  is not a function of  $Y$  because it is independent, it is independent of  $Y$ , so the  $Y$  dependence of  $CA$ , this term can be cancelled and what we have is this as the governing equation. So let us see what is the, what are the significance of each term.

So this is convection in  $Z$ , this one multiplied by  $DAB$  is diffusion in  $X$  and this multiplied by  $DAB$  is diffusion in  $Z$ . So at this point I think we can make a judgement which is going to be important in this case. There is substantial convection in the falling film because of the because of the velocity, predominant velocity  $V$  that in the  $Z$  direction. So molecules of species A which are being absorbed in the liquid will travel towards the bottom at a fast, at a at a high rate, so convection in the  $Z$  direction can never be neglected. So the 1<sup>st</sup> term in here must remain in my governing equation.

$CA$  as a function of  $X$ , the concentration of A at this point is probably the highest which could be the interface solubility of A in B. And as I move in this direction, the concentration will change very rapidly since A is slightly soluble in B, so the concentration of A will change rapidly and therefore this term, the diffusive term, the diffusion of A in the  $X$  direction is going to be, is going to be significant and cannot be neglected. However when we talk about the diffusion in  $Z$ , the transport of A in the  $Z$  direction is principally by convection. There would be slight change in concentration as we move in  $Z$  direction but the concentration

change is going to be insignificant as compared to the concentration change which you are going to see in the X direction.

So concentration change in Z direction is going to be small as compared to the concentration change which you are going to see in the X direction. So if that is the case, then this term can be neglected as compared to this term. So the principal the summary of this is that the principal reason for A to get transported in the Z direction is by convection and the way A can get transported in the X direction is by diffusion and since convection predominates over diffusion in the Z direction, therefore the contribution of diffusive species transport in the Z direction can safely be neglected in comparison to convection in the Z direction or diffusion in the X direction.

So if you are comfortable with this, then we can proceed to solve it, we can try to solve it. But it is important to understand what is important in any transport process. So which, in which direction the convection is important, in which direction the diffusion is important, you will always have to keep that in mind and try to use a sound logic to cancel, if possible the contribution of a term with respect to relative to other terms present in the equation. So a simple analysis, picturisation, understanding of the transport of the species in the X direction and in the Z direction would ensure that we realise in the Z direction it is because of flow, the motion in the X direction is because of concentration gradient.

Concentration gradient present in the Z direction is very small in comparison to the concentration gradient which is present in the X direction. So diffusive transport prevails in the species transport in X direction and convective transport overshadows any diffusive effect, any diffusive transport of the species in the Z direction. So this would result in the compact governing equation which you would have obtained if you have used the shell species balance but as I, as we have seen it is going to be complicated.

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NS EQN  
 $x=L, v_z=0$   
 $x=0, \frac{\partial v_z}{\partial x}=0$   
 $\tau = \frac{\partial v_z}{\partial x}$

$\rho \frac{\partial v_z}{\partial t} = \rho g - \mu \frac{\partial^2 v_z}{\partial x^2}$  - Gov.

$v_z = v_{max} \left[ 1 - \left( \frac{x}{\delta} \right)^2 \right]$

$\rho v_{max} \left[ 1 - \left( \frac{x}{\delta} \right)^2 \right] \frac{\partial C_A}{\partial z} = D_{AB} \frac{\partial^2 C_A}{\partial x^2}$  Gov.

$z=0, C_A=0$  (LIQ FILM BEGINS AS PURE LIQ)  
 $x=0, C_A=C_{A0}$  (INTERFACE CONC./SOLUBILITY OF A IN B)  
 $x=\delta, \frac{\partial C_A}{\partial x}=0$ .

LIMITING SOL<sup>N</sup> → SHORT CONTACT TIME

It is much better to start with the species balance equation in the right coordinate system and then cancel the terms from that point on to arrive at the final solution, final governing equation which would then be equals  $v_z$  times  $\frac{dC_A}{dz}$  is equal to  $D_{AB} \frac{d^2C_A}{dx^2}$ . So this is going to be my governing equation but I would like to draw your attention to this to the dependence of the species transport equation on the solution of the momentum transfer equation. So in order to solve the problem, the prerequisite is that you have an idea of the expression of  $v_z$  from Navier Stokes equation, from the solution of Navier Stokes equation.

Because you understand in this figure, the  $v_z$ , the variation, the velocity in the  $z$  direction is going to be a strong function of position on the  $v_z$  which is in this direction. So this is your  $v_z$ ,  $v_z$  is going to be 0 due to no slip condition on the wall and since we have a liquid gas interface at  $z$ , at  $x$  equals 0, so at  $x$  equals 0, the shear stress is going to be 0. So the conditions that were used to obtain the, obtain the solution of  $v_z$  which is from Navier Stokes equation, so you write the Navier Stokes equation and when you write the Navier Stokes equation in the  $z$  component and it is a free flow, so it is only true to gravity and the boundary conditions that we have used at  $x$  equal to  $L$ ,  $v_z$  is equal to 0 and at  $x$  equals to 0  $\frac{dv_z}{dx}$  is 0 which in other words is saying that  $\tau$  is going to be equal to 0.

So this is no slip condition at the liquid solid interface, this is no shear condition at the liquid gas interface. So no slip and no shear at the liquid solid and at the liquid vapour interface. So when you do that, this shows the coupling between the mass transfer, the species transport equation and the momentum transport equation unless you solve the momentum transfer

equation in order to obtain an expression for velocity you will not be able to solve the species balance equation. So this appearance of velocity in the governing equation of mass transfer couples the mass transport process with the momentum transport process.

So as I mentioned many times before, there is a one-way coupling between momentum transfer and mass transfer as long as the properties remain constant. So  $\mu$  is not going to be a function of position,  $\mu$  is not going to be function of concentration since the gas A is sparingly soluble in B, this ensure that  $\mu$  is a constant. And therefore the solution of Navier Stokes equation is decoupled from the species transport equation and it can be solved, the Navier Stokes equation can be solved independently to obtain the expression of velocity in the Z direction.

Had it not been the case, then both the momentum transport and the species transport equations will have to be solved simultaneously creating enough complexities, creating complexities which would probably require a solution by numerical techniques only. So we know what is this VZ going to be, we have an idea that VZ for such a case is going to be equal to V Max, it is going to be parabolic distribution  $1 - X$  by  $\delta$  whole square and this V Max is the maximum velocity which is there in, obviously in at this point.

So the expression for V Max is provided, I will not going to that once again. So your final form is going to be V Max which is a constant times  $1 - X$  by  $\delta$  whole square times  $\frac{dC_A}{dZ}$  equals  $\frac{D_{AB}}{\delta^2} \frac{d^2 C_A}{dX^2}$ . Okay. So this is the governing equation, the final governing equation after you bring in the expression of VZ. So the boundary conditions which are required in here is that at Z equal to 0, this was the figure, this is the wall and you have the film of liquid which is falling, so this is your liquid and this is X, Y and Z, the same figure that I have drawn over here, this is  $\delta$ , this is W and this is L okay.

So at Z equal to 0, that means at the beginning, at the top the liquid starts as a pure liquid, so liquid film begins as pure liquid. So no A is present in the liquid when it comes to when it comes to in contact with the wall. So at Z equal to 0, that means in this plane, the liquid is pure liquid as it travels, starts to travel downwards, it is going to absorb and so on. And then at X equal to 0,  $C_A$  is equal to  $C_{A0}$ , so this  $C_{A0}$ ,  $C_{A0}$  is the interface concentration of A in B and in many cases it is the solubility of A in B. So what you get here is that at X equal to 0, that means on this plane at X equal to 0, for any value of Z and of course any value of Y, the



concentration of A remains constant which is equal to the equal to the interface concentration or the solubility of A in B.

At the other end which is at  $X$  equal to  $\delta$ , what we have is then a solid wall. At  $X$  equal to  $\delta$ , we have the solid wall, so no diffusing A can penetrate this solid wall, so therefore the solid wall will behave as if it is an impermeable wall as far as A is concerned. So we understand the, this is going to be  $\frac{dC_A}{dX}$  to be equal to 0. So these are the 3 boundary conditions which one which one can, which one must use in order to solve for it. In this form an analytic solution for this is not possible, so we are going for a limiting solution.

So what is a limiting solution? We will assume that we are going to solve this equation for a very short contact time. So what, what is the implication of a short contact time between the liquid film and the gas? So if we have a short contact time, then if this is the liquid film, then a species A which gets absorbed at the liquid vapour interface, it does not have enough time to penetrate deep into the liquid. Before it penetrates deep into the liquid, it reaches L, that is the end of the film end of the contacting process and therefore during this contacting process, during the time when A is in contact with B, it cannot penetrate much into the, it cannot penetrate much into the falling film of B.

So if and we understand that what is a velocity profile that is going to be, so the velocity profile would probably look like, the velocity profile is definitely a parabolic velocity profile, so it is something like this. So the velocities are going to be this, a parabolic velocity profile. So if this is a parabolic velocity profile and if this is the A molecule which will only penetrate a little bit into the falling film, so what velocity of the falling film does this diffusing A molecule will sense? If you are the A molecule and you jump into a liquid stream where the velocity is parabolic in nature, where you meet the stream the velocity has reached its the plateau, the top of the parabola, what you would expect is that for you, for the diffusing A molecule it is as if the entire film is moving with  $V_{Max}$ .

So coming back to this picture once again, if the A molecule cannot penetrate far into the falling film, in that case for all the A molecules which have gone into the liquid B, it would seem that there is no variation in the velocity of the falling film, it is because of the nature of the velocity profile. And as if the entire film is falling with a constant  $V_{Max}$  and the variation of  $V$  with  $X$  to the diffusing molecules, this part is as good as absent. So the diffusing A molecules will not sense that a velocity profile exists in such a case, what it

would see is that as if the whole film, the entire film is coming down with a constant velocity and that velocity is equal to  $V_{max}$ .

So if that is the case, for a short contact time when the penetration depth is small, the modified form of the equation would then be that this part would not be present and it is as if  $V_{max}$ ,  $VZ$  is going to be equal to  $V_{max}$ .

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$x=L \quad v_z=0$   
 $x=0 \quad \frac{dv_z}{dx}=0$   
 $\tau$

$v_z \frac{dC_A}{dz} = D_{AB} \frac{d^2C_A}{dz^2} \quad \text{Gov.}$   
 $v_z = v_{max} \left[ 1 - \left( \frac{z}{\delta} \right)^2 \right]$   
 $v_{max} \left[ 1 - \left( \frac{z}{\delta} \right)^2 \right] \frac{dC_A}{dz} = D_{AB} \frac{d^2C_A}{dz^2} \quad \text{Gov.}$

$z=0 \quad C_A=0 \quad (\text{LIQ FILM BEGINS AS PURE LIQ})$   
 $x=0 \quad C_A=C_{A0} \quad (\text{INTERFACE CONC./SOLUBILITY OF A IN B})$   
 $x=\delta \quad \frac{dC_A}{dx}=0.$

LIMITING SOL<sup>N</sup>  $\rightarrow$  SHORT CONTACT TIME  
 $v_{max} \frac{dC_A}{dz} = D_{AB} \frac{d^2C_A}{dz^2}$

$v_{max} \frac{dC_A}{dz} = D_{AB} \frac{d^2C_A}{dz^2}$

PREV.  $z=0 \quad C_A=0$   
 $x=0 \quad C_A=C_{A0}$   
 $x=\delta \quad \frac{dC_A}{dx}=0.$

NOW  $z=0 \quad C_A=0.$   
 $x=0 \quad C_A=C_{A0}$   
 $x=\delta \quad C_A=0.$

LIMITING SOL<sup>N</sup>  $\rightarrow \frac{C_A}{C_{A0}} = 1 - \text{erf} \left( \frac{x}{\sqrt{4D_{AB}z/v_{max}}} \right)$

BSL  $\rightarrow$  PLATE SUDDENLY SET IN MOTION  
 vii)  $T_i$

So the modified equation for a short contact time, I can write it here itself is going to be  $V_{max} \frac{dC_A}{dz} = D_{AB} \frac{d^2C_A}{dz^2}$ . So this dependence of velocity on  $X$  is no longer relevant for the case of short contact time. So I would start with the my modified equation again, this modified equation is now  $V_{max} \frac{dC_A}{dz} = D_{AB} \frac{d^2C_A}{dz^2}$ . So at  $Z$  equals to 0,  $C_A$  is 0, this was what we have

used in the previous case and now I can use the same that at  $Z$  equal to 0,  $C_A$  is 0. And at  $X$  equal to 0,  $C_A$  is  $C_{A0}$ , I use the same over here, that is at the interface the concentration is going to be equal to some interface concentration which is which is could be the solubility of A in B.

And the 3<sup>rd</sup> condition that I have used if the impermeable wall condition, that is at  $X$  equal to  $\delta$ ,  $\frac{dC_A}{dX}$  is equal to 0. But what happens in here, in this case is if this is a falling film, this is the wall, this is the liquid, this is the gas and the velocity profile is something like this and we understand that the A molecules can only diffuse, only up to a small distance. So if it only goes into the film up to a short distance, then for the diffusing A molecules,  $\delta$  which is the thickness of the film is as good as situated at an infinite distance. So the A molecules, since it cannot go far into the film, for the diffusing molecules this  $\delta$  is at, is close to  $X$  equals infinity.

So  $\delta$  is essentially then infinity to the diffusing A molecules. And what is the condition that we should use for  $X$  equal to  $X$  equal to  $\delta$ ? This question is, this equation is no longer valid, it is meaningless since no exists at that point. So you will we will use the same condition as we have done for boundary layers and I would simply say that  $C_A$  is going to be equal to 0, which is relevant, which is true. Because if this is the thickness of the film and A molecules are only car very close to this point because it cannot penetrate much, so whatever happens over here, at a point far from the interface, this is essentially mathematically speaking is infinite, at an infinite distance as far as A molecules are concerned.

And what is the concentration of A in here, nothing, no A exists at that point since A exists only up to this point. So in order to have some more mathematical handle on this equation, I would put the condition, I will modify the condition not as the impermeable wall but as  $X$  tends to infinity, the  $C_A$  is equal to 0. So if you look at this partial differential equation, this is my, this is my IC and these, these 2 are the boundary conditions. So this is the initial condition, initial condition and these are the boundary conditions.

Now if we compare that with the standard solutions which we have obtained for the case of a plate with liquid on top of it, initially it is 0 but suddenly at  $T$  equal to 0 it is set with a velocity equal to  $V$ . And we are trying to find out how would the velocity, velocity profile exists, starts to go into the deeper of the liquid as a function of time. And any form of equation with these set of boundary conditions and initial conditions is a prime example of combination, method of combination of variables, this I have done in the case of while

solving the problem where a plate is suddenly set into motion and I am trying to find out velocity as a function of distance, let us say  $Z$  and as a function of time and then we have seen these are going to give rise to an error function solution.

I will not do this since I have already done it in the class. What you can once again check how it is done in by looking at the text book which is Bird, Stuart and Lightfoot. So the type of situations which would give you an error function solution are when a plate suddenly set in motion. The 2<sup>nd</sup> could be when you have let say, this is a solid object, temperature is initially uniform, at time  $T$  equal to 0, one of the boundary temperature is changed to a new value. So it was  $T_1$  to begin with and suddenly it has been changed to  $T_0$ . And how would the temperature front penetrate in here?

You will get the same of boundary, same type of governing equation, same type of, same type of initial condition, same type of boundary conditions and the solution would simply be an error function. So this error function solution is quite common in many fields of transport phenomena. If you get the equation in this form, you can directly write, if your initial condition and boundary conditions are like this, one is at  $Z$  equal to 0, one is at  $X$  equal to 0 and  $X$  equal to infinity, if these are the forms, then the dimensionless concentration can be written in the form of error functions.

And you can simply cite any one of these 2 which is solved in detail in Bird, Stuart and Lightfoot and you can check how you are going to get this which I will not use any more. So once you, so this is a limiting solution that you get for the case of concentration distribution. In the next part of the class I would show you how to use this limiting solution to obtain the total amount of mass transfer from the gas to the film and what are the parameters on which it would depend on.