Transport Phenomena.
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Lecture-48.
Mass Transfer (Continued).

So we will continue with what I was doing in the last class, that is the derivation of the equation of change for multicomponent systems. And we have seen that with the, with by assuming a volume element fixed in place and identifying the ways by which mass of species A can enter the control, can enter the volume element, the reaction which may result in the generation or depletion of A. Together with these 2, the concentration, the mass concentration of A inside the volume element may change. So we are, what we are doing essentially is writing the species mass balance equation for a volume element fixed in space.

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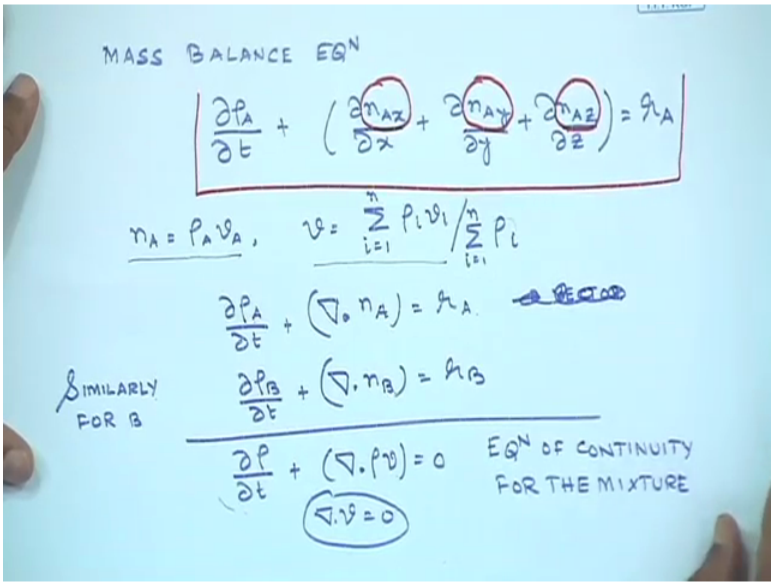

MASS BALANCE EQ^N

$$\frac{\partial \rho_A}{\partial t} + \left(\frac{\partial n_{Ax}}{\partial x} + \frac{\partial n_{Ay}}{\partial y} + \frac{\partial n_{Az}}{\partial z}\right) = r_A$$

$n_A = \rho_A v_A$, $v = \sum_{i=1}^{n} \rho_i v_i \Big/ \sum_{i=1}^{n} \rho_i$

$$\frac{\partial \rho_A}{\partial t} + (\nabla \cdot n_A) = r_A$$

SIMILARLY FOR B

$$\frac{\partial \rho_B}{\partial t} + (\nabla \cdot n_B) = r_B$$

$$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho v) = 0$$

EQ^N OF CONTINUITY FOR THE MIXTURE

$\boxed{\nabla \cdot v = 0}$

And what we are, we have obtained this expression, the mass balance equation, this is rho A is the mass concentration of A, the NA X, that the NA Y and NA Z are the components of the mass flux vector for component A in the X direction, in the Y direction and in the Z direction. And this R A is simply the rate at which A is produced in the, in the volume element as a result of reaction. And we stress once again that this NA contains both the diffusive flux as well as the convective flux which we have to substitute at some point in future. These are some of the basic relations of which you are aware of.

That is the mass flux is simply the mass concentration times the velocity and the velocity is expressed, the mass average velocity, the formula for that is this. And you similarly have a formula for molar average velocity where the rhos are going to be substituted by C, the molar concentration. So if I express this equation in vector form, what I would get is dell rho by dell T + Dell dot NA, the mass flux vector is simply going to be equal to the reaction rate. And if I write, so this is for component A and I can write the same expression for component B as well where only this rho A is going to be replaced by rho B and NA is going to be replaced by NB and RA by RB.

Once again I mention that these reactions are expressed, the reaction rates are expressed in terms of mass and not for this moment, not in terms of moles. So these are the reaction rates in masses. So if you if you if you add these 2 equations together, what you are going to get is the mass concentration of A and the mass concentration of B, so dell dell T of rho A + rho B is simply going to be the mass concentration of the solution which consists of 2 components A and B. So rho A I have used this formula, I have used the relation rho A + rho B is equal to rho. So the 1st term becomes this and secondly the sum of mass flux, that is NA and NB of 2 species NA and NB, if you look at the expression of NA and what is the definition of V, then you can clearly see that NA + NB is simply equal to rho V.

So rho A VA + rho B VB is simply equal to rho, the denominator is rho, so rho times V, so the equation that we would get is dell rho dell T + delta dot rho V to be equal to 0. And you can you can identify this equation, you can identify this equation to be that of the continuity which we have written for a pure fluid. So the equation of continuity which we did right for a pure fluid and what we are doing here is we will have normal to component system consisting of components A and B, so when we write the species balance equation for component 1 and that for component 2 and add them together, what I get is the conservation of mass for the entire solution consisting of 2 components A and B.

So it should and it did revert to the equation of continuity which we are familiar with. So what you get is then dell rho dell T + dell of, dell dot rho V is equal to 0 and if you are dealing with a fluid which is, in which the density is constant, it is not varying, then simply this term would be, can be equated to 0 and what you would get is dell dot V to be equal to 0. So this is another form of equation of continuity that we have used extensively in our in our discussion with fluid mechanics.

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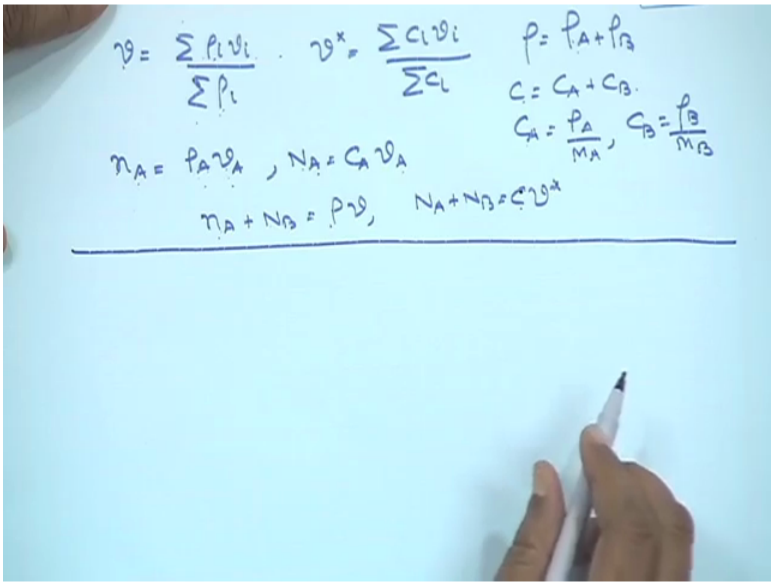

$$v = \frac{\sum \rho_i v_i}{\sum \rho_i} \cdot \quad v^* = \frac{\sum c_i v_i}{\sum c_i} \qquad \rho = \rho_A + \rho_B$$

$$c = c_A + c_B.$$

$$c_A = \frac{\rho_A}{M_A}, \quad c_B = \frac{\rho_B}{M_B}$$

$$n_A = \rho_A v_A \quad , N_A = c_A v_A$$

$$n_A + N_B = \rho v, \quad N_A + N_B = c v^*$$

Here some of the relations that once again write for reference, so this is the mass average velocity, so rho I V I, I equals 1 to N, equals 1 to N, rho I V I by summation rho I. The molar velocity, molar average velocity C, C is the molar concentration, C I VI times CI, the mass concentration, total mass concentration is rho A + rho B, the total molar concentration is CA + CB when we are talking about the binary system. And CA, the concentration, the molar concentration of A is simply rho A by MA, MA being the molecular weight of component A and CB, the molar concentration of B is simply rho B by MB.

Again continue with the basic definitions, NV is equal to the mass flux of component A is equal to rho H times VA, the molar flux of component A is CA times VA and therefore NA + NB NA + NB would give me the mass flux of the entire solution, so which would be rho A VA + rho B VB and if you look at this expression, rho A VA + rho B VB would simply be equal to V times rho, so rho times V, so this follows directly from the definition of V. And similar, in similar way if you find out that NA + NB, it is going to be C, the total concentration, total concentration in molar terms multiplied by the molar average velocity.

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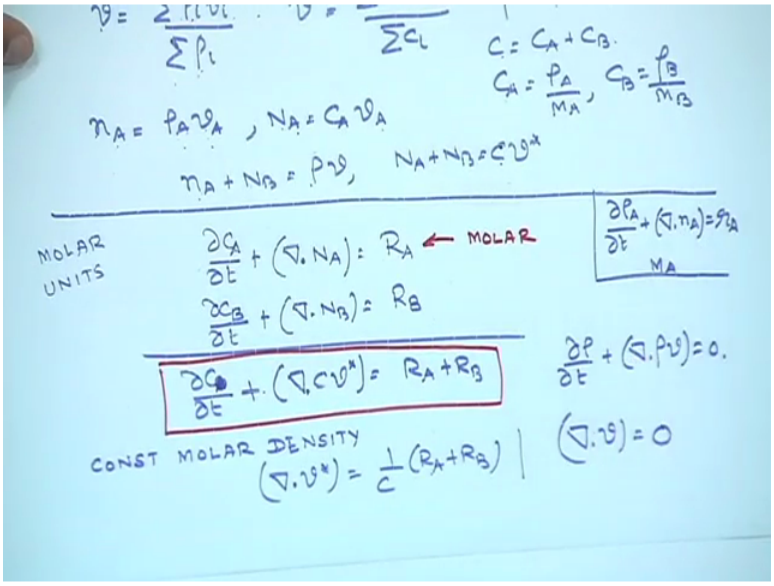

$v = \frac{\sum \rho_i v_i}{\sum \rho_i}$

$\sum c_i$

$C = C_A + C_B$

$C_A = \frac{\rho_A}{M_A}, \quad C_B = \frac{\rho_B}{M_B}$

$n_A = \rho_A v_A, \quad N_A = C_A v_A$

$n_A + n_B = \rho v, \quad N_A + N_B = C v^*$

$\frac{\partial \rho_A}{\partial t} + (\nabla . n_A) = r_A$

M_A

MOLAR UNITS

$\frac{\partial C_A}{\partial t} + (\nabla . N_A) = R_A \leftarrow$ MOLAR

$\frac{\partial C_B}{\partial t} + (\nabla . N_B) = R_B$

$\boxed{\frac{\partial C}{\partial t} + (\nabla . C v^*) = R_A + R_B}$

$\frac{\partial \rho}{\partial t} + (\nabla . \rho v) = 0.$

CONST MOLAR DENSITY

$(\nabla . v^*) = \frac{1}{C}(R_A + R_B)$

$(\nabla . v) = 0$

So this, these are standard forms which we will use subsequently but I thought it is important that we have this presented once again. So we, if we write the expression which we have obtained before which is dell rho A dell T + delta dot NA is equal to RA, all are expressed in terms of, all are expressed in terms of mass. And if I divide this equation with the molecular weight of A, that is if I divide both sides of the equation by MA which is the molecular weight of A, then this equation in mass terms can be converted to molar units. So in terms of molar units, this equation would simply be equal to dell CA by dell T + Dell dot NA is equal to RA, note that this RA and everything acts, in this expression is in terms of molar units.

I can write this for component B as well, so dell dot NB is equal to RB which is again in terms of moles, so when you sum them together, dell CA by dell T, sorry dell C by dell T + dell of C V star is equal to RA + RB. So this is the equation of continuity in terms, in molar terms. Now the, compare the difference between the 2 equations, when we express this in mass terms, this was equal to 0. This is the expression which we have obtained. So if it is a 2 components system, so for mass of A produced must be equal to the mass of B which is consumed. So in mass terms of the amount produced by reaction and the amount consumed by reaction must be equal.

So mass will always be conserved, conserved but we cannot say that for in molar terms. So not in all cases one mole of A is going to give rise to one mole of B which will depend on the stoichiometry of the equation. So therefore capital RA and capital RB, the sum of these 2 may or may not be equal, may not be equal and opposite. So their sum of RA and RB in terms of moles may not be 0. So in order to maintain the general nature of the equation, of the relation, underscoring the special nature of capital RA + capital RB, that being it may or may not be 0 and like the case of mass expressed, rates expressed in terms of mass where they always will be 0 and keeping this RA + RB in the solution as well.

So if we have a system of constant molar density, so this is, this may not be equal to 0, constant molar density, that means C is going to be a constant, so therefore you are going to get is dell dot V star is equal to1 by RA + RB. Since C is a constant, this is going to be 0, this can be taken outside and dell dot V star is this, compare that with what we have obtained when the mass concentration is 0.

So this is the same thing, these 2 equations are identical, one is expressed in terms of moles, molar average velocity, molar rate of reaction, here the expressions are in terms of mass, so they are similar in concept but different in form. Next comes the, what is going to happen to NA. We have said that NA contains both a convective contribution and a conductive contribution, same applies for capital NA as well.

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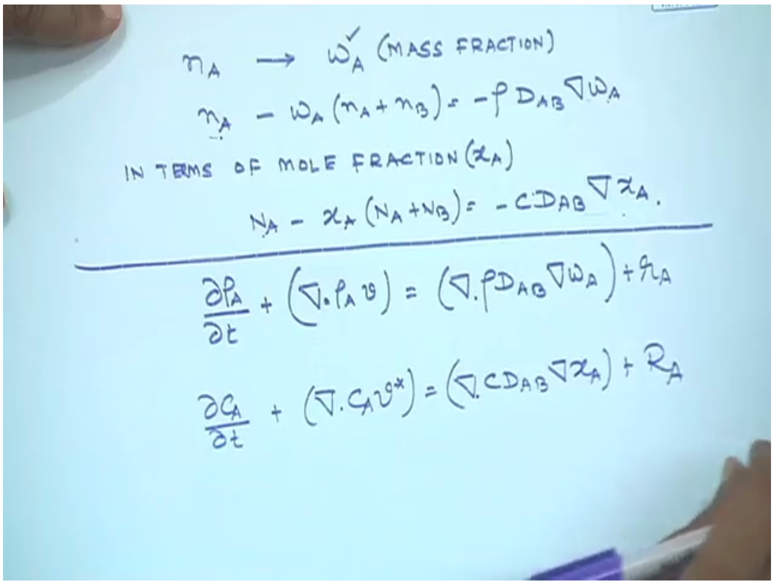

$n_A \rightarrow \overset{\checkmark}{\omega}_A$ (MASS FRACTION)

$$n_A - \omega_A(n_A + n_B) = -\rho D_{AB} \nabla \omega_A$$

IN TERMS OF MOLE FRACTION (x_A)

$$N_A - x_A(N_A + N_B) = -C D_{AB} \nabla x_A.$$

$$\frac{\partial \rho_A}{\partial t} + (\nabla \cdot \rho_A v) = (\nabla \cdot \rho D_{AB} \nabla \omega_A) + r_A$$

$$\frac{\partial C_A}{\partial t} + (\nabla \cdot C_A v^*) = (\nabla \cdot C D_{AB} \nabla x_A) + R_A$$

So if we if we write NA in terms of mass fraction WA, then the Fick’s law would be WA times NA + NB is equal to - rho DAB times dell of omega A. So this is again, this is mass fraction, so we are writing this equation in Fick’s law, the generalised Fick’s law in terms of mass fraction, this is mass flux, mass fraction, mass flux, constant density, DAB and mass fraction again. And if you write in terms of mole fraction, in terms, if I write in terms of mole fraction, we get the familiar equation, so mole fraction we denote by X A and the family is expression is X A times N A + NB equals - C DAB dell of XA.

So again these 2 equations are identical, we have so far used this form of the Fick's law where the molar, where the diffusive flux, where we have the diffusive flux and this is the bulk flow term. So it is the same as this except the, except the difference between mass and moles. So the 2 equations which we have in here, this equation in terms of mass and this equation in terms of moles and what we are going to do is we are going to substitute small na and capital NA in these equations, small na over here and NA over here, these 2 equations in that and we would obtain 2 identical equations, completely equality equations, not identical, equivalent equations, one is in terms of mass, this is in terms of mass and mass fraction, in terms of mass and mass fraction and in terms of moles.

So fundamentally there is no difference between these 2, one as I said in terms of mass, the other is in terms of moles. This is mass average velocity, this is molar average velocity. This is rate of generation of A in mass per unit time, this is rate of generation of A in moles per unit time. So these are identical equations but we should also point out other certain limitations, certain things that we have assumed which must be mentioned. We said that the diffusion takes place only when there is a concentration gradient present in the system but there are other ways by which diffusion can take place.

If we have temperatures different between 2 points in an otherwise concentrate, otherwise , in a solution where the concentrations are the same but the temperatures at 2 points are different, that could also create a diffusion. If you have a system, if you have a, if you have other parameters are different in a perfectly, in a solution which is, which has the same concentration everywhere, then also you can have diffusion, because diffusion, if we use the term strictly is not dependent on concentration gradient alone. Diffusion is going to result if there is a chemical potential difference between 2 points in a solution.

The true definition, the true cause of diffusion is not only concentration difference, though that is the most common cause of diffusion, mass diffusion, actually it is the concentration potential, the chemical potential difference which causes a species to move from one point to the other. And chemical potential is a complex function not only of concentration but also of pressure and temperature. So if concentration being equal at every point in a solution, if the pressure and temperature vary in between 2 points in a solution, even if, even if the concentrations are same, we can still have diffusion. But in our analysis, we have, what we have presented we did not consider the diffusion induced by temperature or by pressure.

So the expression that we have obtained is only for concentration difference induced diffusion only. So this restriction or this limitation of the equation should have to be kept in mind, though it is going to be the diffusion, the concentration difference is the most common cause of diffusion, of movement of species from one point to another.

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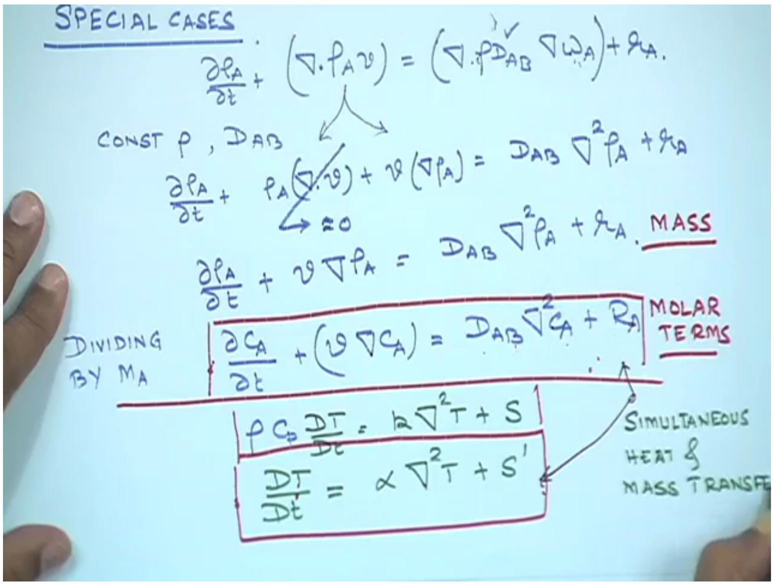

SPECIAL CASES:

$$\frac{\partial \rho_A}{\partial t} + (\nabla \cdot \rho_A v) = (\nabla \cdot \rho D_{AB} \nabla \omega_A) + r_A.$$

CONST ρ, D_{AB}

$$\frac{\partial \rho_A}{\partial t} + \rho_A \underbrace{\cancel{(\nabla \cdot v)}}_{=0} + v(\nabla \rho_A) = D_{AB} \nabla^2 \rho_A + r_A$$

$$\frac{\partial \rho_A}{\partial t} + v \nabla \rho_A = D_{AB} \nabla^2 \rho_A + r_A.$$ MASS

DIVIDING BY M_A

$$\boxed{\frac{\partial C_A}{\partial t} + (v \nabla C_A) = D_{AB} \nabla^2 C_A + R_A}$$ MOLAR TERMS

$$\rho C_p \frac{DT}{Dt} = k \nabla^2 T + S$$

$$\boxed{\frac{DT}{Dt} = \alpha \nabla^2 T + S'}$$

SIMULTANEOUS HEAT & MASS TRANSFE

So these are the 2, 2 equations and I am going to write these expressions once again. These are called the special cases, special cases that we encounter. The 1st equation which is dell rho A, the 1st equation that we have written + dell dot rho AV is equal to dell dot rho DAB times the dell of the mass fraction + RA. So if I just expand this term, what I expand this term and assume that DAB is a constant, so for the case of constant total mass concentration of the solution and constant DAB, this expression would be simply dell rho A by dell T + rho A times dell V + V times rho A and this since DAB and rho, both are constants, so it can be taken outside.

So DAB is taken outside and rho is taken inside, this is mass fraction, multiplied, mass fraction multiplied by the total density, so this would simply be equals dell square times rho A + R A will remain unchanged. So I expand this into these 2 terms, to DAB outside and took rho inside. So the mass fraction would simply be equal to the mass density, mass fraction multiplied by the total density would simply be equal to the mass density of component A. And we know that for a constant rho system, this is equal to, this is equal to 0 from our equation of continuity. So what we have then is dell dell rho A dell T + B times dell rho A is equal to DAB times dell square rho A + RA.

And when you divide this, divide by the molecular weight of component A, if I divide this term, this equation by the molecular weight of component A, what I am going to get is dell CA dell T + V times , V times dell CA is equal to DAB dell square CA + RA. So these are in mass terms and this is in molar terms. So these are identical equations, conceptually they are the same but one is in mass concentration, this is in molar concentration, molar concentration, molar concentration, reaction rate expressed in terms of moles rather than molar concentration expressed in terms of masses.

So if you look at this expression a bit more carefully and if you see what is this, this is nothing but the, this is nothing but the substantial derivative of concentration of A. So what I have, if I would like to compare this with the equation which we have obtained and I hope you remember this as dT by d, d temperature by d time equals K dell square T + some sort of a generation term, if you divide both sides by rho CP, what you get is dT by d time is equal to K by rho CP and K by rho CP is nothing but Alpha times dell square T + S prime where S prime is simply S by rho CP.

So when you compare these 2 terms, these 2 equations or rather these 2 equations, what you have here is the substantial derivative of concentration, what you have here is the substantial derivative of temperature, what is on this side is the transfer of mass A, species A as a result of diffusion. This is transfer of energy through conduction because of difference in temperature. So this and this are identical, this is a generation term through reaction and this is energy generation, it could be of various means a nuclear source, a current source which is resulting in heat and so on. So the 2 equations fundamentally, conceptually are then identical. And this is the beginning of simultaneous identical treatment of mass transfer and heat transfer process.

So the mass transfer and heat transfer process, the form of the governing equation, the form, the forms are absolutely the same, conceptually one is the substantial derivative of concentration, this is the substantial derivative of temperature, this is the diffusive transport of mass, this is the diffusive transport of energy, this is the source term you to reaction, this is a source term due to by various means that is nuclear, ohmic and so on. So if the governing equations, the same form of governing equations underscore the fact that it is possible to treat mass transfer and heat transfer together.

So in many cases, in many situations where a simultaneous heat and mass transfer is taking place, the form of the equations governing the heat and mass transfer will be the same. And we will see special cases where not only the forms are the same but the boundary conditions expressed in dimensionless forms will also be the same. So if those, if that situation happens, that is the governing equations are the same and the boundary conditions in dimensionless forms are the same, then a significant simplification of the entire process can be obtained, which we will study in when we are going to, when we are going to explore the analogy between heat, mass and momentum transfer.

But it would be sufficient here to mention or to underscore, underline the same form of the equation, same form of the equation of heat transfer and equation of mass transfer in these cases. So these 2 equations, these 2 equations are to be kept in mind, the similarities between them when we are going to solve for the problem. So this is a starting point for simultaneous heat and mass transfer. So whenever we deal with simultaneous heat and mass transfer, we have to think in terms of the normal nature of the equations.

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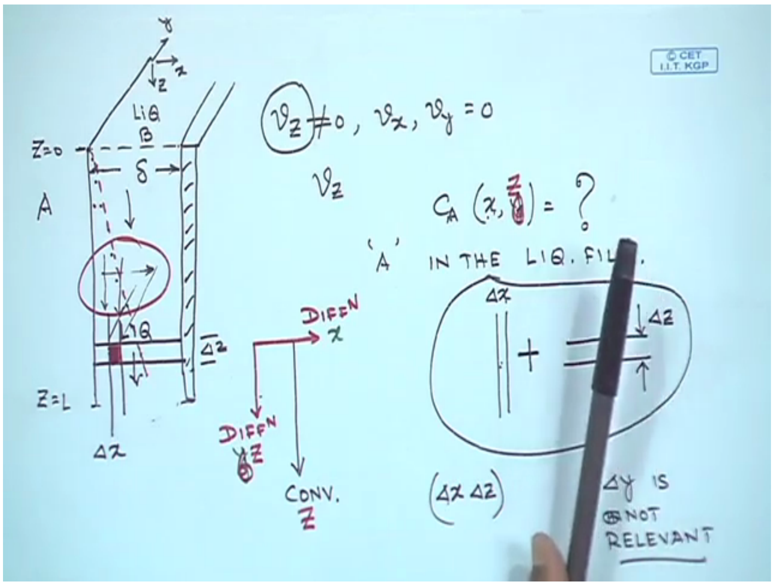

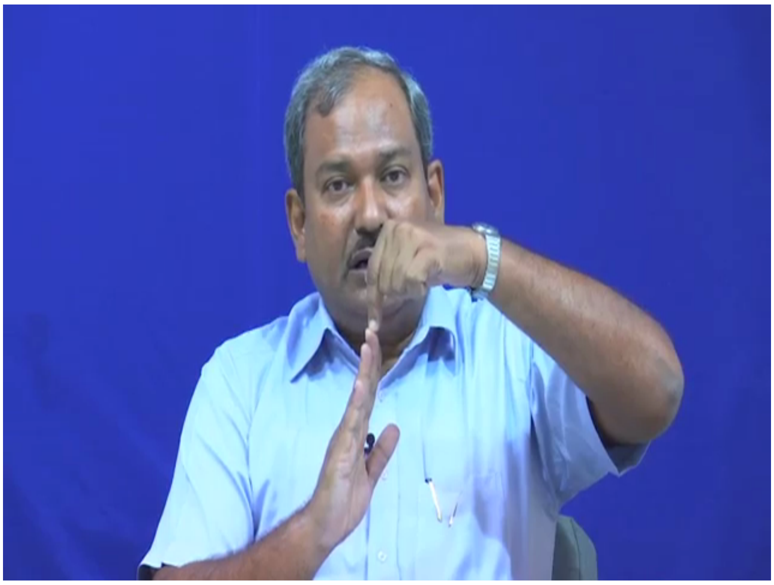

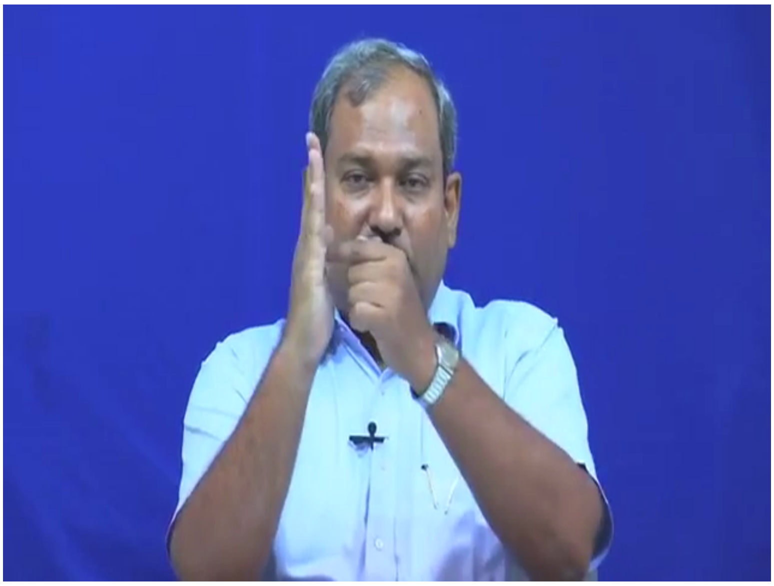

So coming back to the equation for mass transfer, what we have then is dell CA dell T + V times dell CA is equal to DAB del square CA + a reaction term when everything in terms of moles. So this expression, this equation has, this equation can therefore be found, the form of the equation in Cartesian coordinates, in cylindrical coordinates and in spherical coordinate. So we choose equation and its form and cancel the terms which are irrelevant, what you get is your governing equation. So in the next part, in the next part of the, next class, what I am going to do is I will show you a problem which is difficult to visualize when you try to do a shell momentum shell mass balance.

But the problem, and getting to the governing equation is a lengthy exercise. But if you do that with this equation, the generalised species transport equation, how easy it would be to get to the final form of the, final form of the governing equation. So in the remaining 2 or 3 minutes that I have for this class, I will simply introduce the problem and show you what kind of a shell one has to assume in order to obtain the governing equation. And then in the next class I will show you how to use the species balance equation to solve for the problem.

So the one that we are, I am going to do right now is a solid wall and a liquid film is falling on the solid wall, okay. And there is a species, so this is liquid B and liquid A gets dissolved, so at this point, from this point onwards, the falling film of B sees the component A. So if we talk about this being the Z direction, this being the X direction and this being the Y direction, then at Z equals to 0, so this is Z equal to 0 and we have Z equals L. And we will assume that the film is wide enough, such that none of the parameters are going to be function of Y. So as the film falls, it sees component A only at Z equal to 0 up to Z equal to L.

And this component B a gets dissolved in the liquid B and it is, let us assume I do not make any comment on whether A is sparingly soluble or highly soluble. The moment an A molecules gets dissolved, it starts its downward journey. But there is a difference in concentration in this direction, so A must diffuse in this direction and A is going to go in this direction by convection. And we are going to assume that I have only one component of velocity VZ which is not equal to 0 but both V X and VY are equal to 0. So it is the one-dimensional motion of the liquid that after A gets dissolved carries it in this direction.

And this distance, the thickness of the film is delta. So we have no A present in it, some A present up to this and the penetration of A inside the liquid film will keep on increasing as we move in this direction. So some sort of a profile of A is going to be felt, it will be developed in here. I would like to find out how much A is going to get dissolved in liquid B when it falls with, so A has a velocity VZ, I would like to find out the variation of CA and I understand the concentration of A going to be function of X, it is also going to be function of Y. So it is going to be function of X and it is going to be function of Y.

So I would like to find out the profile A in the liquid film. So A, profile of A the liquid film. And we understand there is a diffusion in this process, diffusion in this process and convection in this process. So if I, this is my diffusion in the X direction, I have diffusion in the Y direction and I have convection, sorry, Z direction, in the Z direction, CA I am sorry CA has to be function of X and Z. So these 3 are taking place, the question then comes, how am I going to take a shell, because we generally take a shell, is it going to be of size delta X since CA is a function of X. What is it going to be of shell delta Z, since CA is a function of Z, so depend to be my shell, of delta X thickness or of delta Z thickness?

And you can clearly see, since CA is both the function of X and Z, the shell that you are going to design is going to be a function both of X and of Z. So therefore this, the shell that you are going to do is going to be a shell like this which is delta Z and a shell like this which is going to be delta X. So this is the shell with across which you going to be all the balance. So your shell is going to be of area dell X dell Z. And it could be of any Y, it does not matter, all the, all the parameters are going to be function of dell X and delta Z, dell Y is not relevant. Now you see the problem, you are imagining, so far you are dealing with a shell with one dimension, now we have a shell in which the smaller dimension could be 2.

So instead of a thin page type shell like this in dell X or in dell Z, now you have a shell which has some dell X and some dell Z, how do you how do you do that? You can still do it, you can still do it but you have to maintain and visualize through this face I am having conduction and convection both, through this face, since there is no velocity, this is my film, through this face, since I do not have any velocity, any bulk velocity, I am only having conduction. So the top faces of the imagined shell will be exposed both to conduction as well as to convection, the side face of the shell is only exposed to diffusion.

And in this way, in the Y direction, it is not going to be, it is not relevant since nothing, the concentration is not a function of Y. So for a two-dimensional system your entire shell momentum balance starts to break apart. Now consider this to be a situation in which the transient effects are added to it, how the complexity increases when you start having a velocity or a concentration which is having a which is dependent both on X and Y and as well as T. If you look at your textbook Bird, Stuart, Lighfoot, this specific problem has been done using a shell balance, Shell species balance.

So it is still he would be able to visualize it. But since now you know the species balance equation, what am I going to do in the next class is solve the same problem but start with the species balance equation and cancel the terms and you would see what has required one-page of the text and some visualising, imagining of the different transport processes on an imaginary shell, how easily it can be done when you pick the right component of the equation and cancelling terms which are not relevant in this specific case. So this is the one which we are going to see in the next class.