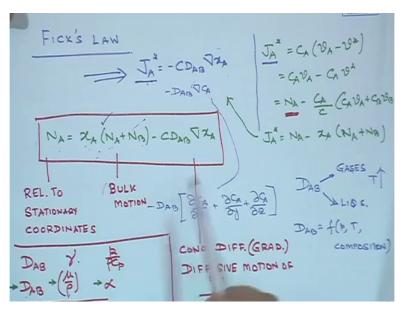
## Transport Phenomena. Professor Sunando Dasgupta. Department of Chemical Engineering. Indian Institute of Technology, Kharagpur. Lecture-44. Mass Transfer (Continued).

So we have discussed about the basics of mass transfer, the 1<sup>st</sup> introduction to mass transfer which we have understood from our discussion that it has 2 components. Any mass flux of a species A has 2 ways by which it can it can get transported from one point to the other. The 1<sup>st</sup> one as due to the convection, if there is any bulk convective flow present in the system, then species A would get transferred from one point to the other. Apart from that if there is a concentration gradient present, then a diffusive flux of A will also result in.

So therefore the total mass flux of component A in any system in the algebraic sum of convective flow and diffusive flow. So we have also seen how to express convective, how to express diffusive flux in the form of Fick's law. And we have defined 2 relative velocities, one when we have, when what is the mass average velocity and the 2<sup>nd</sup> is the molar average velocity. So the diffusion velocities were defined either with respect to the mass average velocity with respect to the molar average velocity.

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So when we saw the expanded full form of Fick's law, this is what we have seen in the, in the last class, where the mass flux is essentially proportional to the gradient of concentration if C is constant, you can bring the C inside, so this Dell of CA, so the gradient of concentration, well, this would give rise to the mass flux vector. And the definition of mass flux with

respect, since we have a star which denotes in terms of the molar average velocity is, so this is the definition of molar flux which we have the mass flux due to diffusion and due to convection.

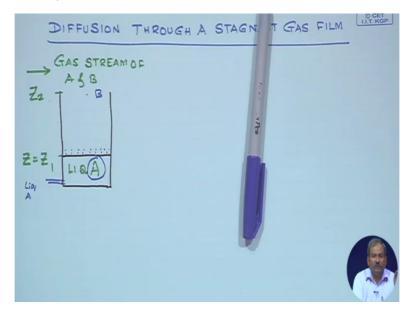
So when we substitute this in here, what we get is the relative to stationary coordinates, whatever be the molar flux, this is a result of a bulk motion which is denoted by XA where XA is the mole fraction of A, NA + NB are the molar fluxes of component A and that of component B. And we also have a concentration difference or concentration gradient imposed diffusive motion of NA here. And the diffusion coefficient which has units of meters per second and it is analogous to the other numbers are for example mu by rho which is the kinematic viscosity having the same unit and the thermal diffusivity K by rho CP again having the same unit as metre square per second.

So therefore one, in order to model such a system, one can express the flux, be it on mass basis or moles basis as the algebraic sum of diffusive flux and convective flux. So with this we are in a position right now to start modelling a system, simple system in which there is maybe a convective flow, there may be a diffusive flow and we may have reactions taking place in the diffusing mixture. So these A and B, the 2 components that are present in this, they may react with each other, they may react with each other throughout the, throughout the volume of the mass in which case it is called a homogeneous reaction or they may react, A may react to form B and B will diffuse back on, let us say on a catalyst surface.

So a reaction which takes place everywhere in the system, homogeneous reaction, then the source or sink term in the form of generation of B or depletion of A will appear in the governing equation itself. On the other hand if A gets transformed into B on the catalyst surface, then this heterogeneous reaction will appear as a boundary condition to the governing equation. So these 2 things will have to be kept in mind, where does the where does the effect of heterogeneous or how does the fact of heterogeneous homogeneous reaction included in either in the governing equation or in the form of boundary conditions.

So the 1<sup>st</sup> problem that we will try to model in this class is about at tube which is partially filled with the liquid that is volatile. So initially the liquid is kept separated, it was not allowed allowed to evaporate by some sort of a cover. And the tube, since it contains a small quantity of the of the of the liquid, volatile liquid, the moment you remove the so-called protective covering, the liquid will start to evaporate, the vapours are going to form and those vapour molecules will then travel through the tube up towards its mouth.

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So that is what I have drawn in here and we are looking at the problem of diffusion through a stagnant gas film where we have a liquid A and I have B present in here, so initially everything was filled with B and the moment you remove the cover of the volatile liquid A, A molecules will start evaporating and the A molecules because of the concentration gradient between the surface of the liquid and at the mouth of the tube, they would start moving in the + Z direction. And we will say that by means of supplying liquiDABy an external agency, the levels of liquid A is always kept at Z1. So in order to maintain steady-state you have to supply, you have to replenish the amount of liquid which has evaporated from the pool inside the tube.

And at Z equal to Z2, which is the mouth of the tube, this is a gas stream which is flowing with a constant concentration of A and B. So the concentration at A and B at this location may be assumed to be constant and initially it was mostly B which is there in the tube. So what happens is when you have the evaporation starting, then molecules of A will form near the gas liquid interface and the molecules will try to move towards that top due to the presence of a concentration difference.

Since it is a rapidly volatile mixture, volatile component A which is present in here, so the amount of A molecules which are going to generate over here and their number density is going to be too large and as we move towards this, the density, the number density will decrease, the concentration of A will decrease and come to a constant concentration at the mouth of the tube. So these upward moving molecules of A will try to push the B molecules out of the tube creating or imposing a concentration gradient of B in the process.

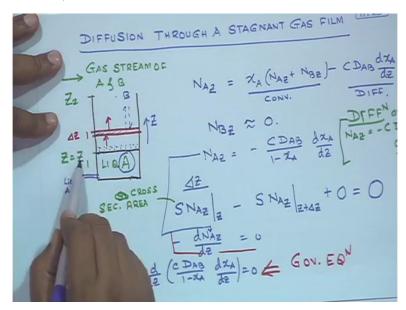
I will go through it once again just to make sure that we understand the process is that the molecules of A which are formed near the liquid air, liquid vapour interface since they are moving out rapidly through the tube towards its mouth, the concentration of B which was initially uniform to begin with will start to become skewed. The concentration of B near the liquid air, liquid vapour interface will be reduced and most of the B molecules will be pushed out of the tube.

And as the concentration difference of B between the surface of the liquid and the top of the tube is increased as the difference is increased, there would be, there would be diffusive motion of B which will start in the reverse direction. So B molecules because of the imposed concentration gradient created by the rapidly diffusing A molecules, B molecules will try to come towards the liquid surface from the top of the tube. So A molecules will move away from this surface, whereas B molecules will try to move towards the surface since the diffusing A molecules has created a concentration gradient of B.

Now this process will go on till the time when the motion of the B molecules out of the tube due to the motion of A molecules is exactly counterbalanced by the diffusion of B molecules in the reverse direction. So the B going out, B being pushed out and the B being diffusing, B diffusing towards the surface, these 2 will balance each other at certain point. And that, the moment that happens, what you will have is the dynamic state in which the net movement of B molecules will be 0. And that is what I call as stagnant B, stagnant gas film. There is a movement of B upwards and the moment of B downwards for 2 different reasons but they cancel each other.

Since they cancel each other to the diffusing A molecules, the B molecules will appear as stagnant. So the problem that we are trying to handle, trying to model in this case is diffusion of one species through a stagnant film of another species. So that is what we are going to do over here.

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So therefore the flux of NA in the Z direction, this is the Z direction is from the previous expression which you have which you have seen is going to be NAZ + NBZ where they say the convective flux - C, DAB times DXA by DZ. We also understand that the concentration of Z is a function, concentration of letter as a function of only Z and that is why I can use the ordinary differential sign, differential sign. It is not a function of any of the coordinates, so that is why it is simply DXA DZ. So this is the convective or the bulk motion and this is the diffusive or molecular motion.

According to whatever we have discussed so far NBZ is essentially stagnant, so NBZ is equal to 0, do not have any net motion of B in the Z direction, the moment of B is exactly counterbalanced by the convective moment of B out of the tube. So since N BZ equals 0, your NAZ would simply be equals - C DAB 1 - XA Times DXA DZ. Just a quick observation here is a mentioned that A, liquid A is not highly volatile, let us say it is just water and the water molecules when they change phase, the number density of water molecules would not be large and therefore the entire motion of the air molecules towards the mouth of the tube will be governed by diffusion by only diffusion.

So there would be no conviction created by the rapport evaporation of A for those liquids which have higher vapour, higher , higher temperature of boiling. So if it boils at a higher temperature than the number of molecules there would be less, there would be no convection, so for a diffusion-only case, the form of this equation would simply be equals NAZ is equal to - C DAB DXA by DZ. So this is the case for diffusion only and it has to be mentioned

specifically in the problem that it is a diffusion only case. Otherwise if it is a stagnant medium of component B then this is the form that we are going to take.

Exactly like before we have now have to think of a think of a small shell across which we are going to make a species balance. Since the concentration is changing in the Z direction, this is going to be my shell of some thickness Delta Z and across which I am going to make a balance of A. So for an incremental height delta Z, Delta Z is the incremental height, rate of A in the Z direction coming in at Z must be equal to the rate of A in the Z direction which is going out of Z + delta Z. So this is simply the cross-sectional area, cross-sectional area of the tube.

So S area multiplied by flux gives you the moles per unit time of A coming in and moles per unit time of A going out. Since A and B are non-reacting, there is no reaction between A and B, so A will travel through this without reacting with B and there is no source or sink term, so that is why I put 0. And at steady-state this must be equal to 0 as well. So rate of rate of number of molecules of A coming in to this imaginary shell and what is going out once we equal. So this can then be divided by Delta Z and what you have then is D NAZ by DZ to be equal to 0, and I put the expression of NAZ from here at this point, so what you get is D DZ of C DAB by 1 - XA Times DXA DZ to be equal to 0.

So the governing, this is going to be your governing equation for this, even this is the expanded form of the governing equation for this process. Once I have that, then I should be able to integrate it without any problem and effort is ideal gas mixture, C is not going to be a function, the concentration is not going to be function of Z and therefore C can be cancelled out and the modified form of governing equation would simply be 1 by 1 - XA Times and DAB is also a constant, so D XA balloted DZ is equal to 0. So this becomes why new governing equations since C and DAB for this system are constants.

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( I-XA dxA)= 0 GON. EQN  $h(1-Z_{A}) = C_{1}Z + C_{2}$ BC (20/2B)

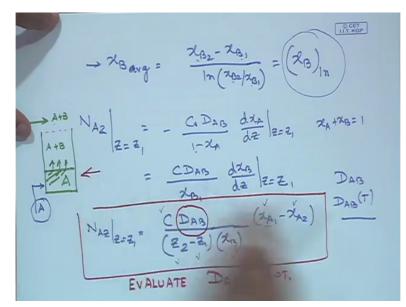
So once you integrate this, the form of the variation in the bowl fraction of component A is going to be C1 Z + C2, C1 and C2 are constants of integration which can be evaluated through the boundary condition that at Z equal to Z1, XA, the mole fraction of A is some known value XA1 and at Z equal to Z2, XA is equal to XA 2. So the concentration of A and these 2 locations at Z equal to Z1 and Z equals Z2 are known. So with the help of these boundary conditions, the constants of integration can be evaluated. So when you evaluate this and substitute it back into the expression, what you get is 1 - XA by 1 - XA1 is equal to 1 - XA 2 by 1 - XA 1 to the power Z - Z1 divided by Z2 - Z1.

So this is the variation of concentration of, variation of small fraction of A as a function of Z. And we know that one is XA for a binary mixture is simply going to be equal to XB where XB is the mole fraction of B. So this equation can also be written as XB by X B1 is equal to XB 2 by XB1 and then the whole thing Z - Z1 by Z2 - Z1. So these 2 equivalent forms of distribution of either A or B can be expressed as a function of Z, that is distance from the surface from the liquid surface. It is not only the concentration at any specific point, sometimes we would like to find out what is the average concentration of let us say B in this case.

So average concentration of B which is XB average in the dimensionless form XB1 would simply be equals Z2, sorry from Z1 to Z2 XB by X B1 DZ multiplied, sorry divided by Z1 to Z2 times DZ. So this is the XB average is nothing but the length average value of the mole fraction of B along the entire length of the tube. So this is and you can plug this in here, perform the integration and what you would see is what we would expect at that XB average

is simply going to be X B2 - X B1 by LN of X B2 divided by XB1 and we know that this kind of difference, this kind of difference has a special name which is known as the log mean temperature difference.

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Log mean temperature difference, so this time it is going to be log mean concentration difference. So it is, whatever be the concentration at the mouth, the concentration at the surface, divided by the logarithmic logarithm of XB 2 by XB 1, so this is the log mean concentration difference of B on the surface. Another thing of interest is what is the mass flux, from this profile can you obtain what is the mass flux at Z equal to Z1, which from my previous analysis we understand that the mass flux is - C and DAB 1 - XA DXA by DZ. So if I have to find out what is the mass flux at this point, I simply have to evaluate this mass flux at Z equals Z1.

I also understand that since it is a non-reacting system, D DZ of, D DZ of NAZ as a 0 which only shows that NAZ is not going to be a function of Z. So N AZ is simply going to be constant, so whatever gets evaporated, the entire amount at steady-state will travel through the tube to the outside. But in order to obtain NAZ at Z equal to Z 1, understand that it is going to be the same at any value of Z is - C DAB by 1 - X A times D XA DZ at Z equals Z1. Which when expanded could be written as DAB by XB 1 times DXB by DZ at Z equals Z1, simply using the definition that 1 - XA is XB.

And since XA + XB is equal to 1, so D XA DZ is going to be equal to - D XB DZ, so the - sign disappears from here and if we use the previous regulation NAZ at Z equals Z1 would

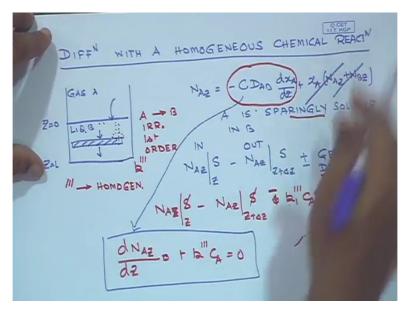
simply be equals C DAB by Z2 - Z1 and over here you get XB logarithmic concentration difference XA 1 - XA 2. This equation is important. Why it is important, because you can think of an experiment around this expression. What you have here is a, what you have here is A + B and what flows above it is also A + B. Since the evaporation is taking place, in order to maintain the level at this level, you need to supply some amount of liquid A to maintain the level.

The amount of liquid A that you need to supply to maintain the level can be measured, since you are supplying it from outside. So if you know what is the amount, then the amount is essentially S Times N AZ. N AZ is the flux of A which is going out of the tube. So this N AZ when multiplied by the by the cross-sectional area would simply give you in KG per seconds or in KG moles per seconds the amount of A which is evaporating per-unit time. So you can precisely calculate experimentally, manage experimentally what is the amount of A in moles per unit time that is evaporating as a result of diffusion and imposed convection taken together for the highly volatile liquid.

And if you see the right-hand side of the expression, everything in this expression is known to you. The total concentration is known to you, the geometric values of Z1 and Z2 are known to you, you can measure what is XA 2, the concentration of A that you are maintaining at the mouth, you can make an estimate of XA 1 which could be just the mighty interface the concentration is going to be the saturation concentration of A in B. So XA 1 is also known to you and XB LN is known to you. So everything in this expression is known, except DA B. So if you experimentally measure this, experimentally measure what is the Z2 and Z1 and you know the values of XA 1 and XA 2, then it is, you can evaluate, evaluate the diffusion coefficient of A in B experimentally.

So this is a nice experiment, clean experiment by which you can measure the diffusion coefficient of A in B and since it is so easy to control, the temperature of a tube containing some small quantities of a liquid, you can measure DAB at different temperatures as well. So not only you measure the diffusion coefficient of A in B but you also find out how does the diffusion coefficient change with the temperature. So this is a nice experiment to obtain DA B and to obtain DA B as a function of temperature. Okay. So for experimental determination of diffusivities used in film theories of mass transfer and so on, we have this type of arrangement, this type of experiment has been extensively used.

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The next problem that we are going to look at is diffusion with a homogeneous chemical reaction. So this is a case in which a chemical reaction takes place throughout the, throughout the body of the liquid. So we have a liquid which is B, liquid B and you have a gas A. So A as soluble, A gets solubilised, A is solubilised in the system and then A starts to diffuse in B. As A starts to diffuse in B, this, there is going to be a reaction which is taking place. So A gets converted, A undergoes an irreversible first-order reaction. So it is an irreversible first-order reaction from A to B, okay.

So if that is the case, then we have to find out what is what is going to be the concentration distribution of A in the liquid B. So  $1^{st}$  of all since we understand, let us say this is at Z equals 0 and this is at Z equals L, this could be a beaker, it could be container, it could even be a concrete tank where a gas gets dissolved in B and it starts to diffuse towards the bottom and as it travels, it reacts with B through an irreversible first-order reaction. So the  $1^{st}$  statement,  $1^{st}$  thing that we are going to do is we are going to see what is going to be my NAZ as before DAB times D XA by DZ + X A Times NA in the Z direction + NB in the Z direction.

Now if A is sparingly soluble, soluble in B, then XA is going to be very small and the entire motion of A in the Z direction in B is going to be as a result of rates diffusive flow. The A molecules small in number due to the sparingly soluble nature of A and B, the small numbers of A molecules which are getting dissolved in B, they cannot form, they cannot create a convective current. So the entire process therefore can be treated as a diffusion only process, there is no convection, it is only a diffusion only process.

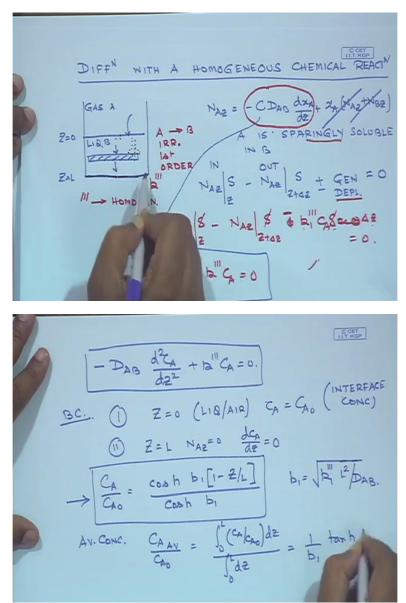
So if it is a diffusion only process, then the diffuse, then the convective term of the species equation, species equation means this N AZ, this equation, the  $2^{nd}$  term can be neglected and your N AZ is simply going to be - C DAB DXA DZ. As before this is going to be a shell across which I am going to make a species balance, A coming in, A going out through a diffusive process only. So N AZ times S evaluated at Z - N AZ at Z + Delta Z multiplied by the same cross-sectional area. So this is going to be the in, this is going to be the out and then you are going to have + or -, generation or depletion, at steady-state this is going to be 0.

Now what is the generation and depletion? Since it is a first-order reaction, let us assume the reaction rate constant as K1 triple prime as I mentioned triple prime refers to homogeneous reaction whereas double prime refers to heterogenous reaction. So this, the equation, governing equation would therefore be N AZ times S evaluated at Z - N AZ at Z + Delta Z, again the area and then we have + K1, sorry, - K1 triple prime CA. First-order reaction where A gets converted to B, since A gets converted to B, it is not a generation, it is a depletion and that is why we have the - sign.

And since in order to make it the uniform, the units to be the same, it is also multiplied with the cross-sectional area. So what you do in and this is taking place over a thickness Delta Z, so this is equal to 0. So N AZ times S, N AZ times S and the reaction that is taking place by first-order and you multiply it with the volume. So this is the volumetric, this is the reaction rate constant in volumetric basis, so we multiply it with the volume to make it into moles per unit time.

So you divide both sides by S Delta Z and what you get is the governing equation to, governing equation to be a D D DZ of N AZ + K1 triple prime CA to be equal to 0. So this is your governing equation and for N AZ you are only going to plug this in, without the 2<sup>nd</sup> term which is not relevant for such cases.

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So when you do that, the equation takes the form - DAB D2 CA by DZ square + K1 triple Prime CA to be equal to 0. So this equation will now have to be integrated, before integration we need to find out what are the boundary conditions. The boundary conditions could be at Z equal to 0, that means that the liquid air level, the concentration of A is maintained at C is 0 and this could be just the interface concentration. So if it is the interface concentration, it could be the solubility of A in the liquid. So here Z equal to 0 refers to this and when the diffusing molecules come to Z equals L, this essentially behaves like an insulated surface in heat transfer.

No diffusing A molecules can penetrate beyond Z equals 0. So if you are making if you have a reaction taking place in a beaker, then when the diffusing molecules reach the bottom of the

beaker, it cannot penetrate, it cannot go beyond that surface. So at Z equal to 0, the physical boundary condition which you are going to use is that the mass flux, mass ormolarflux of A at that point is going to be equal to 0. And we know the mass flux is simply, or molar flux is simply going to be - DAB times that D CA DZ at Z equals L.

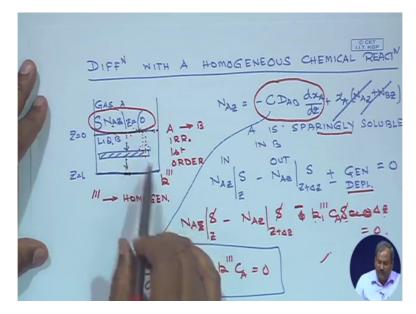
So this entire quantity which denotes the molar flux of A beyond at Z equals Z, the bottom of the plate, if that is 0, and C DAB is constant, it would simply give you that DCA by DZ at Z equals L is going to be equal to 0. So this is an impervious boundary condition, impervious wall boundary condition that is commonly used in mass transfer so as to denote that there cannot be any flow of mass beyond a certain boundary which is a rigid impermeable barrier and which behaves like the insulated wall in heat transfer giving the concentration gradient at that point will have to be 0, the same way the temperature gradient was 0 on the insulated surface.

So the 2<sup>nd</sup> boundary condition for this specific problem would simply be at Z equal L, N AZ, since N AZ is 0, what you are going to get is DCA DZ to be equal to 0. So this is boundary condition one and this is boundary condition 2 and when you use this boundary condition, the concentration profile which is available in your text, you would see that it is going to be 1 - Z by L and here it is cos hyperbolic B1 wells B1 is simply equals K1 triple prime, the reaction rate constant times L square, the depth of the film divided by DAB.

So this is the profile of A and since you have the profile of A in here, the average concentration can also be calculated. So CA average by CA0 would be equal to, from 0 to L, same definition as before, CA by CA0 times DZ divided by 0 to L DZ. And if you work this out, it would be 1 by B1 tan hyperbolic B1. So this is a clean example of a homogeneous reaction taking place in a medium and using a shell balance, using the situation or taking the situation to be diffusion only process you would be able to obtain a clean expression, a clear expression, a concise expression for the concentration distribution of A inside this.

And we understand that since A cannot go beyond a certain levels because of the presence of the solid impervious wall, so we are going to use the condition that D CA D Z at that point to be equal to 0. And the solution to this is provided and you can, once you have the concentration difference, concentration expression there, you can use it to obtain what is going to be the average concentration and you can also find out what is going to be the amount of dissolution of A in the liquid.

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The amount of dissolution of A in the liquid if you again look at this figure, it is going to be, whatever be the flux of A at Z equal is 0 multiplied by the surface area. So S Times N AZ at Z equal to 0 is going to be the amount of A which is getting dissolved in A. But remember unlike the last problem, N AZ is now a function of Z. N AZ D DZ of N AZ is not equal to 0, there is a sink term present, so value of N AZ, the value of N AZ will reduce till it comes to a value equal to 0 and this. So in order to obtain the dissolution of A in the liquid, you have to find out what is the value of N AZ at Z equal to 0.

One more point is that whatever I am showing, whatever I have showed in the previous class, the problem that I have discussed today are all there in the textbook of Bird, Stuart and Lightfoot. So the initial part of the mass transfer study, I have taken completely from Bird, Stuart and Lightfoot, if you follow that book, then you would see all these problems are provided, given in the text and you would be able to follow.

At some point I will start giving examples from other books or other resources and I will tell you exactly where I am taking those from so that you would be able to read them but it is mostly going to be Bird, Stuart and Lightfoot as far as mass transfer is concerned. Once we have a few more problems on mass transfer solved, then we will move to the last part of this course which is establishing the analogy between heat, mass and momentum transfer.