Transport Phenomena.
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Lecture-42.
Thermal Boundary Layer.

In this class we are going to shift to something which we have done for the case of momentum transfer. We are going to look at the transport processes which are taking place at a point very close to a hot surface. Let us say a flat plate is in contact with a hot fluid and there is going to be simultaneous free and forced convection and let say the fluid is moving over the hotplate with some velocity. So a hydrodynamic boundary layer is going to form as well as a thermal pond layer is going to form on the plate.

So if you start at the beginning, the temperature of the approaching air and the temperature of the and the velocity of the approaching fluid will have some value. The moment it starts to flow on the hotplate, the velocity is going to be equal to the velocity of the hotplate, so that means the velocity, if it is stationary plate, the velocity will be 0 which is in no slip condition and the temperature of the fluid in contact, in direct contact with the solid plate will be that of the will be the temperature of the solid plate.

So from that point onwards the velocity would start to grow as we move deeper and deeper into the fluid till it reaches the free stream velocity. Since we are dealing with a flat plate, the approach and the free stream velocities are going to be equal. Similarly as we move away from the hotplate, the temperature of the fluid will decrease and will slowly approach the temperature of the free stream. So therefore there is going to be a gradual change in the value of the velocity as well as in the value of temperature, both will asymptotically much to the free stream.

So it is we understand that the momentum transfer, most of the momentum transfer as well as the heat transfer is taking place in a layer very close, in a very thin layer close to the surface of the solid. And we have worked with this in our analysis of hydrodynamic boundary layer to obtain the velocity profile as well as to obtain the expression of the important parameter in engineering which is the expression for the friction coefficient. Such that we can find out what is the drag force, what is the frictional drag force exerted by the moving fluid on the stationary solid.

So our objective was twofold to reduce, to simplify the equation of motion such, in such a way that we get a compact partial differential equation and then try different methods, the differential approach which has led to the Blassius solution or an integral approach which has turned out to be very successful in very handy in order to get an ODE out of the system. And both these approaches go parallel and they give more or less identical results within, for the case of the integral approach within errors, within acceptable errors. And therefore the ease of use of integral approach makes it a better alternative as compared to the differential approach.

So we are going to do the same for the growth of the thermal boundary layer. But in thermal boundary layer we are interested in finding out what is the temperature in the boundary layer at every point, but we are more interested to find the engineering parameters just like friction coefficient in the case of hydrodynamic boundary layer, we would like to find out what would be the expression for the convective heat transfer coefficient generally denoted by small h.

So it is the part of the, it is our job in this part of the class to obtain an expression for h, the convective heat transfer coefficient for flow of a fluid over a flat plate where the temperature of the fluid and that of the solid plate are different, these 2 temperatures are different. So we will be having heat transfer, what is the value, what is the expression for the heat transfer coefficient. So that is what we are going to do in this class. So in to do that 1st we will start with what is the convection transfer problem. So we have a, let us say we have an object whose area is AS and it is at a constant temperature of TS.

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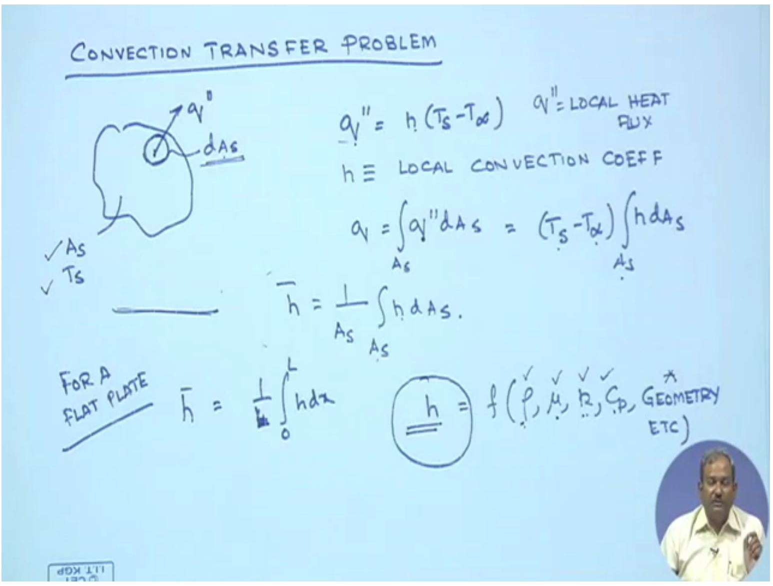

And it is losing some amount of heat, convective heat out of this and the area from which the heat loss is taking place is dAS. So you know from Newton's law of cooling, the local heat flux is provided by h where h is the, h is the coefficient, h is the local convection coefficient in this differential area. So if we try to find out what is the total heat loss from the entire object, I simply have to integrate this local coefficient, local heat flux over the entire area and if I put the expression of the local heat flux in has, since TS - T infinity is constant, it comes out of this and I have h averaged over the entire surface area.

So if I define an average value of heat transfer coefficient as h bar, then h bar by definition, if you look at here is simply going to be 1 by AS which is the total surface area and integration of h dAS over the entire surface area. This, this expression, this problem becomes slightly more straightforward for the case of a flat plate. And we know that for the case of a flat plate, if this is a flat plate, the width over here is very large. So it does not play any part in the overall convection process, so the heat transfer, average heat transfer coefficient is simply going to be one by L where L is the length of the hot plate, 0 to L h dX.

And this, its value for the heat transfer coefficient is going to be a function of properties like rho, mu, K, CP, it is going to be a function of the geometry among other things. So essentially the job of the convective heat transfer in the goal of convective, analysing convective heat transfer is to obtain an expression for h in terms of these parameters or in terms of dimensionless groups which will arise by the combination of these parameters. So that is what we are going to do, we are going to obtain the expression for H, the convective heat transfer coefficient in terms of all these parameters.

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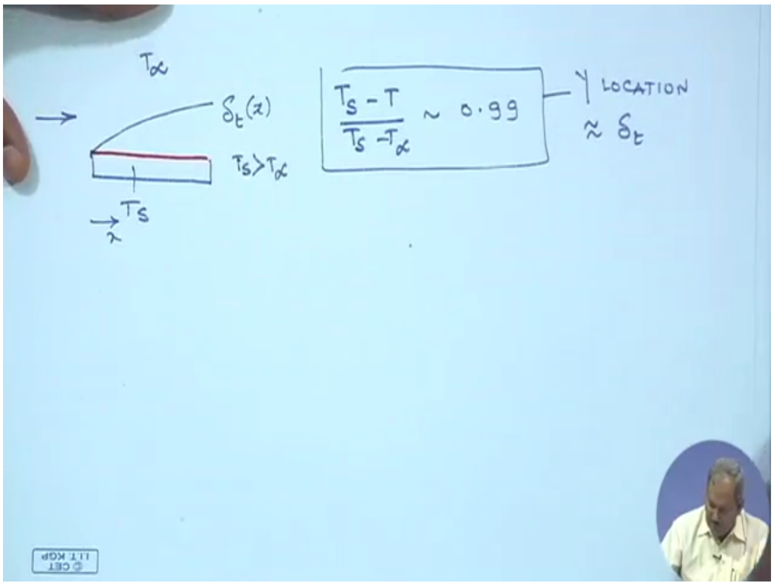

So we would start 1st with the case of, this is a solid plate and you have a flow, the temperatures, the 2 temperatures are different, this temperature is that TS at the temperature over here is going to be at T infinity, so this is the exaggerated view of the boundary layer thickness, this is the thermal boundary layer which is going to be, the thickness of which is going to be a function of X. As we move in the X direction, obviously the thickness of the boundary layer will keep on increasing and the boundary layer thickness is defined as before where the dimensionless temperature difference and here we assume that TS is greater than T infinity where TS - T by TS - T infinity is equal to 0.99.

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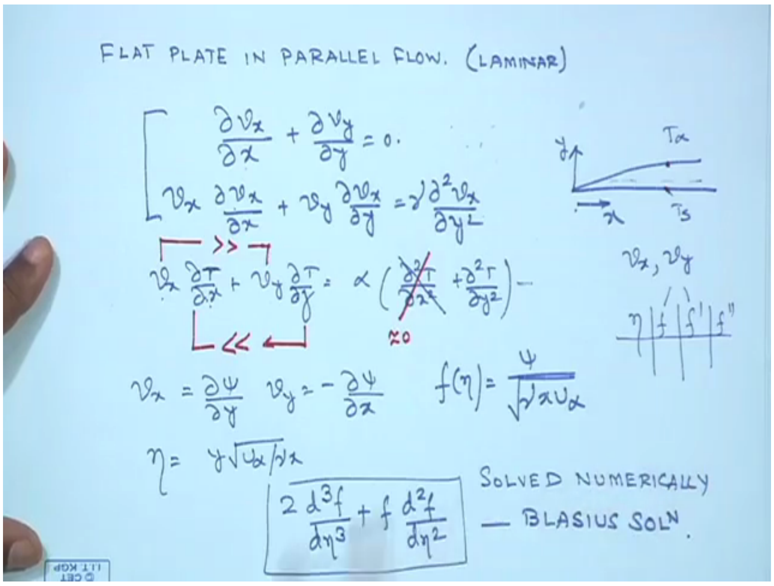

So the location, the Y location where we have reached this condition, so the Y location corresponds to that, corresponds to these conditions is called delta T, that is the thermal boundary layer thickness. So we understand how this be, what would be the equations so we would start our part of the analysis of flat plate in parallel flow. So if it is a flat plate in parallel flow and we are going to consider 1st laminar flow only, if it is laminar flow over a flat plate in which there is a temperature difference between the solid and the liquid, the governing equations, equation of continuity, equation of motion, these 2 we have seen before.

And the equation of energy would simply be dell T dell X + VY dell T dell Y Alpha which is K by rho CP dell 2T by dell X square + dell 2T dell Y square. And we assume there is no heat generation, etc. The same way we have decided about the importance of each of these terms, here we see that this is very large compared to VY, however dell T dell Y is large compared to dell T dell X. So you have this over here, this is my Y and this is the X, so there is going to be principal motion in in the X direction, so VX is large compared to VY.

However the temperature changes to TS from TS T infinity over a very small Y. So dell T dell X, the temperature does not change that much in the X direction, so this is going to be small compared to this, so none of the terms on the left-hand side can be neglected. On the right-hand side, these 2 terms refer to conductive heat transfer which are strong, which is a strong function of the temperature gradient. Now if you see the temperature gradient in the Y direction, it is going to be much more, it is going to be very large in comparison to the temperature gradients in the X direction. The same logic as over here.

So since the temperature gradient in this direction is significantly larger than the temperature gradient in this direction, this term is going, can be neglected in comparison to this term. So the governing equation for thermal boundary layer, for flow inside, heat transfer inside a thermal boundary layer can be expressed by this form of the energy equation where the 1st term on the right-hand side which signifies conduction in the X direction is neglected. And if you look at these 2 equations, the 1st 2 equations are uncoupled from the 3rd equation.

But the 3rd equation is coupled because it contains VX, VY, etc. in it, however these 2 equations they do not contain any temperature term. So the 1st 2 equations are uncoupled from the 3rd equation but the 3rd equation, the energy equation is coupled to this. So it is a prerequisite that we need, we need to solve these 2 equations 1st, obtain expression for VX and VY, plug them in here and then only we should be, we should be able to attempt to solve the temperature profile, that is temperature as a function of X and Y.

One more time, these 2 equations are independent of the 3rd, these 2 equations should can and should be solved 1st to obtain expression for VX and VY in terms of X, Y, nu, etc. Once VX and VY are obtained, they now can be put into this equation and after, only after we obtain an expression for VX and VY, put, incorporate into this energy equation, we should proceed to solve for T. So we need to have expressions for VX and VY we already have from the Blassius solution. This is the stream function if you remember this is psi is the stream function.

So this is, by definition the expression for VX and VY, we have also defined a dimensionless stream function as nu X U infinity, we have defined a dimensionless distance Y root over U infinity by nu X. And with this we have converted these 2 equations into a partial differential equation, into an ordinary differential equation non-linear but ordinary differential equation which was then solved numerically, solved numerically and it is essentially the Blassius solution. The Blassius solution as we have seen has given us for different values of Eta what is the value of F, f Prime, f double Prime, etc. and we know that from f and f Prime we could often what is VX and VY.

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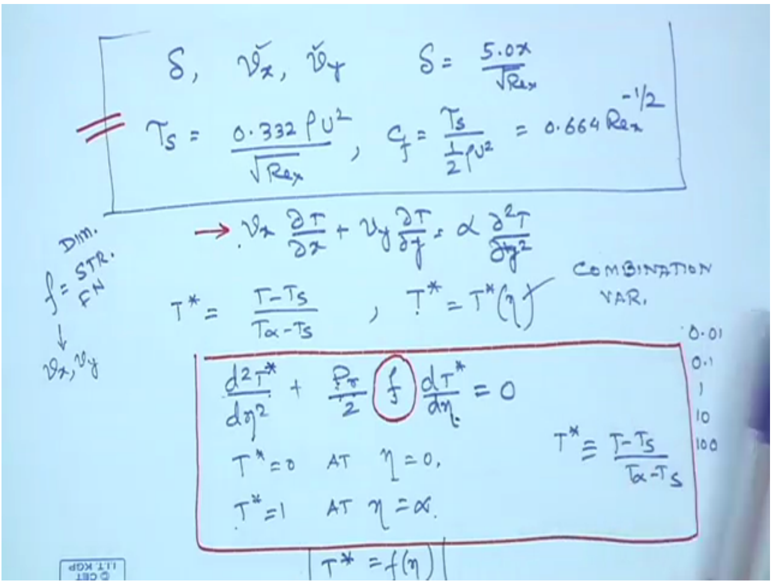

So we have obtained, we have solved this equation, obtained the, obtained the expression for delta, we know what is going to be VX, BY and so on. So with this we obtained delta to be equals 5.0 X by root over R E X and we obtained the shear stress to be equals 0.332 rho U square by root over REX. And the local friction coefficient was defined as Tao S divided by half rho U square which is 0.664 REX to the power - half. So all this we have done before.

Now since VX, VY, everything is known to me, I should be able to solve, I am in a position to attempt to solve the energy equation.

This is the energy equation inside the boundary layer, we can try to solve the equation now. So this is a requirement for starting the solution over here. So 1^{st} what we do is we define a dimensionless temperature as T - TS by T infinity - T S and as before we say that this is going to be function of Eta which is a combination variable only. The same way we have done, we should be able to convert this equation and equation that we would get… Please look carefully into what I have done over here.

What we have obtained as from the partial differential equation, that is for heat transfer inside a thermal boundary layer, we have substituted dimensionless temperature, we have identified that this dimensionless temperature is going to be a function only of Eta where Eta is the combination variable, where Eta is the combination variable. With this you are able to convert the equation from an ODE to a PDE. But look here, the appearance of f in the energy equation, if you remember f is the stream function, dimensionless stream function, so f essentially contains VX, VY, etc. So the presence of f in the energy equation couples it with the with couples it with the momentum equation.

So the momentum equation inside the boundary layer must be solved up priory before you even attempt to solve this. Now this equation cannot be solved and these are the conditions, that this term is 0 at on the plate and this term is equal to 1 when Eta tends to infinity. So this is essentially Y equal to 0 and Y equal to infinity. At Y equal to 0, T is equal to TS, so therefore T star is 0. At Y equal to infinity, T is essentially T infinity, so T star becomes equal to1. So even at this condition, simplification, you will not be able to solve the problem analytically.

So what can be done as this, we can define, we can assume different values of Prandlt number, let us say starting at 0.01, 0.1, 1, 10, 100, or anything, any numbers in between. So we define different values of realistic values of Prandlt number, what we said, what we get is a series of equations for different values of Prandlt number. Once I have these values of Prandlt number, then I should be able to solve this equation, since my value of f is known from my previous analysis of momentum equation. I will I will discuss it one more time.

I get an equation which I should be able to, which I should be able to evaluate, the only problem is I have Prandlt number present in there and f present in there. The method, solution methodology as, since f is known to me from the solution of the momentum, momentum boundary layer. Therefore if I choose the value of Prandlt number to be something, let us say 1, I get an equation T 2T, d Eta square + Prandlt number equals 1 f times d d T star d Eta is equal to 0. So what I need to do, I will bring your attention to this equation once again.

Prandlt number is 1, so this becomes half, I choose a value of Eta, the moment I choose a value of Eta from my analysis of the Blassius solution, if I choose the value of Eta, the values of f, f Prime, f triple Prime, double Prime, etc., all are known to me. So choice of Eta would give me the value of f from the momentum part of the momentum boundary layer. So when we start the thermal boundary layer, I choose a value of Eta and I get the value of f if I know the Prandlt number.

So the moment I choose the value of Eta and I get the value of F, I should be able to obtain, I should be able to numerically solve this equation. In other words, for each value of Prandlt number I will be able to solve this equation for different values of Eta provided I know the corresponding value of f which I already know from this table. So the steps would be, assume the value of Prandlt number, whatever be the value of Prandlt number, then start solving the energy equation, for each value of Eta you get the value of F, for each value of Eta you know the value of F, the value of Prandlt number is already known to me, so I should be able to solve to obtain T star as a function of Eta.

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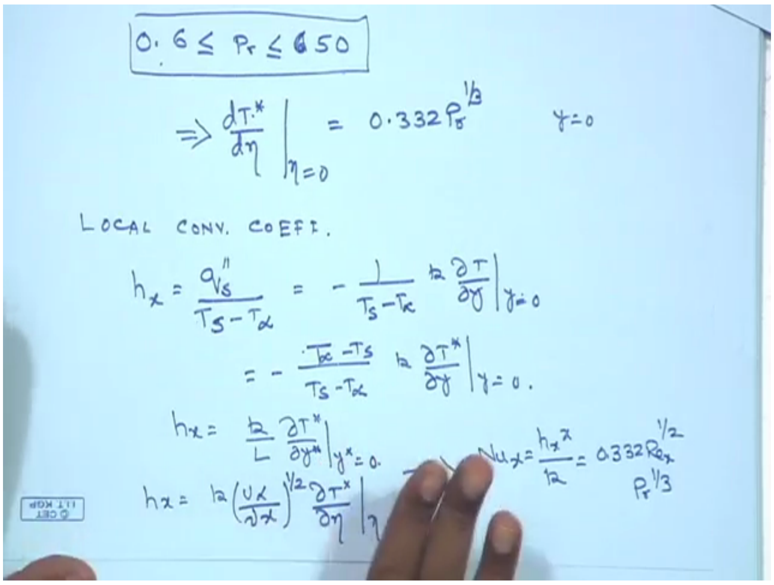

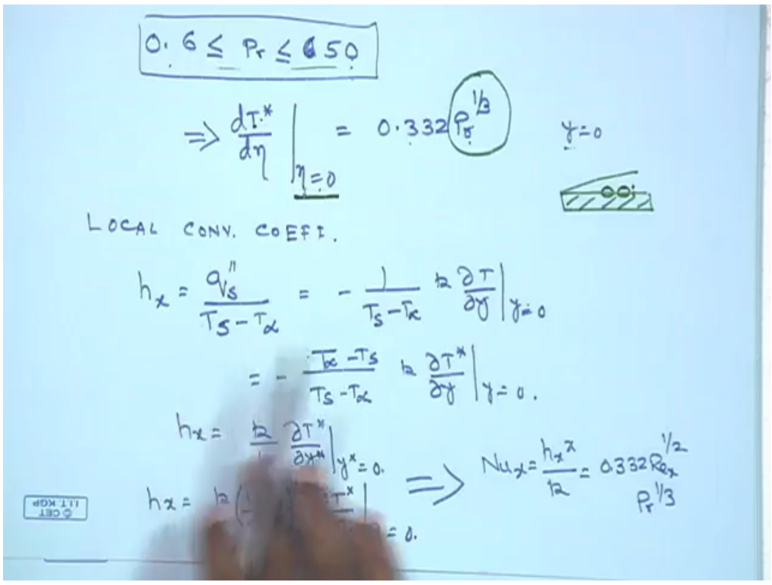

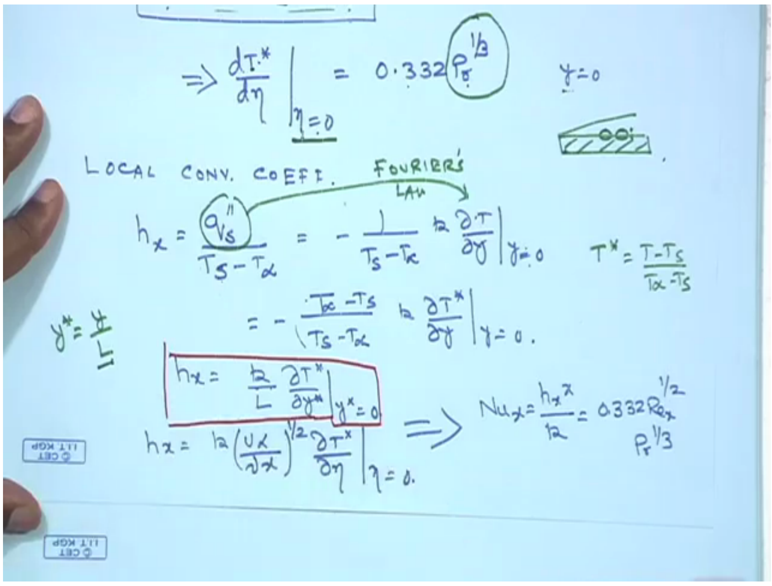

So this is what I am going to do next. What has been shown when that solution was done, it has been shown that for the value of Prandlt number between 0.6 and 50, this is the range in which most of the liquid, most of the fluid is that we deal with on routine basis, they do lie in this range of Prandlt number. So when the solution was done, it was found that d T star d Eta, at Eta equals 0 can be expressed as 0.332 Prandlt to the power one 3rd. And this for a value of Prandlt number between the range 0.6 to 50, the temperature gradient at Eta equals 0, that means at Y equals 0 can be fitted to this form Prandlt to the power one 3rd.

So with this we now proceed, with this experimental and numerical observation we now find out what is the local convection coefficient, which is h suffix X is a local value of the convective heat flux Q suffix S TS - T infinity, Fourier's law, simple substitution. So I have written so many things, let me go slowly over this and try to explain it. For a value of Prandlt number between this range, it have been found from the solution that the temperature, dimensionless temperature gradient at Eta equals 0 which corresponds to Y equals 0, that means on the solid plate can be expressed as a function of Prandlt number.

So with this, the equation, the governing equation, this is the governing equation has been solved and the table similar to that, what we have done for the case of hydrodynamic boundary layers, table of that were obtained where the value of T star, the values of f, the values of dT star, d Eta, these were obtained. Once the values of the dimensionless temperature gradient at Eta equals 0, that means the solid plate are obtained and are analysed carefully, they are found to be fitted, they are found to fit very well with this type of an expression where Prandlt number to the power one 3^{rd} multiplied by a constant.

So this has been obtained numerically and then fitting the value of gradient at Eta equals 0 to Prandlt number. Eta equal to 0 is significant because if you see, this is your solid plate, you as an engineer, you are interested in what is happening, what kind of heat transfer situation you have at the solid liquid interface. So this signifies the solid liquid interface and this is the temperature gradient in dimensionless form, so the temperature difference in dimensionless form is expressed in terms of Prandlt number.

With this knowledge we now proceed to obtain what is the local convection coefficient. The local convection coefficient, this is nothing but Newton's law, this is the local heat flux, this is the difference in temperature TS - T infinity and let say HX is the convection coefficient. So this is by definition the expression for heat transfer coefficient. So here I bring in the TS - T infinity to the outside and for QS I use Fourier's law, Fourier's law, substitute in there - K dell T dell Y at Y equals 0.

I then proceed to nondimensionalize T star as you remember T star is defined as the T - TS by T infinity - TS. So if we bring in the nondimensional form, this comes out, it is again at Y equals 0 and we define Y star, the dimensionless Y position as Y by L where L is the length of this plate. So I have a K by L which is outside, dell T star by dell Y at Y star equal to 0. I will come back to this expression to clarify something later on. See we are getting more and more compact that HX is K by L dell T dell Y. I have an expression what is dT star d Eta at Eta equals 0. What I have here as dell T dell Y, dell T star dell Y star at Y star equal to 0.

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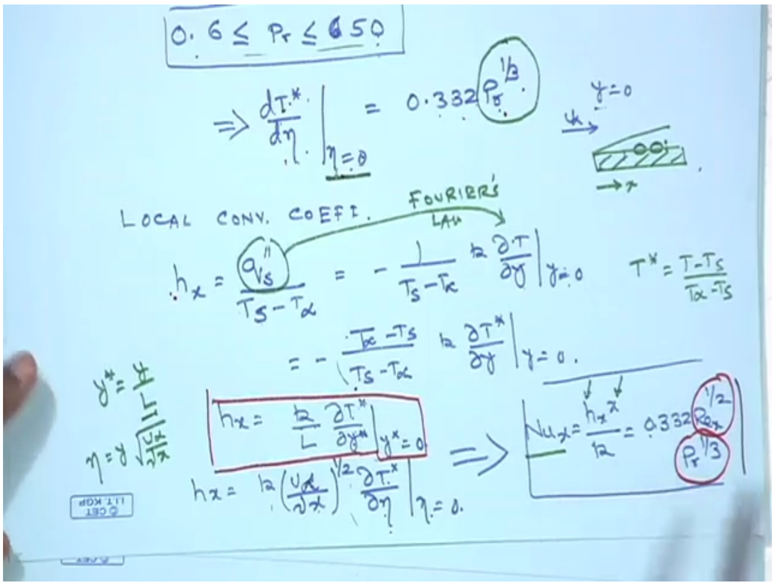

So the next step is to convert this Y to Eta such that this expression can be used. And that is what we do next if you remember that my Eta has been defined before as Y root over U infinity by nu X in the hydrodynamic boundary layer treatment. So my HX is delta T star dell Eta at it equals 0 and these terms, this is U infinity, U infinity is the free stream velocity, this is U infinity, so free stream velocity. Now I could be able to substitute dell T star dell Eta at Eta equals 0 from over here and if I bring my X to the other side, the Nusselt number, local value of Nusselt number denoted by NU suffix X is the local value of the heat transfer coefficient HX, the distance from the leading edge which is, so this is my X, at any value of X, thermal conductivity which is equal to 0.332 REX to the power half times PR to the power one 3^{rd}.

So the entire expression can be obtained, this Nusselt number expression can now be obtained as a function of Reynolds number, as a function of Prandlt number. So what you then see is you need to use numerical techniques to solve the governing equation. But in order to solve for the equation, you need 2 things, the value of Prandlt number and the values of f or gradients of f at a different values of Eta. The values of f at different values of Eta were obtained from the hydrodynamic boundary layer solution. So if you assume Prandlt number and if you start solving it putting the values of f obtained previously, you get a series of solution.

If you, when you look at the series of such solutions, you see that the temperature gradient at Eta equals 0 can be fitted to a function of Prandlt number. The fitting equation is this, 0.332 Prandlt to the power one 3^{rd}. Then using the definition of HX, the local convection coefficient and by converting the temperature and the distance to dimensionless temperature and dimensionless distance, you get a compact expression for Nusselt number which is, which I'm sure you have seen many times before.

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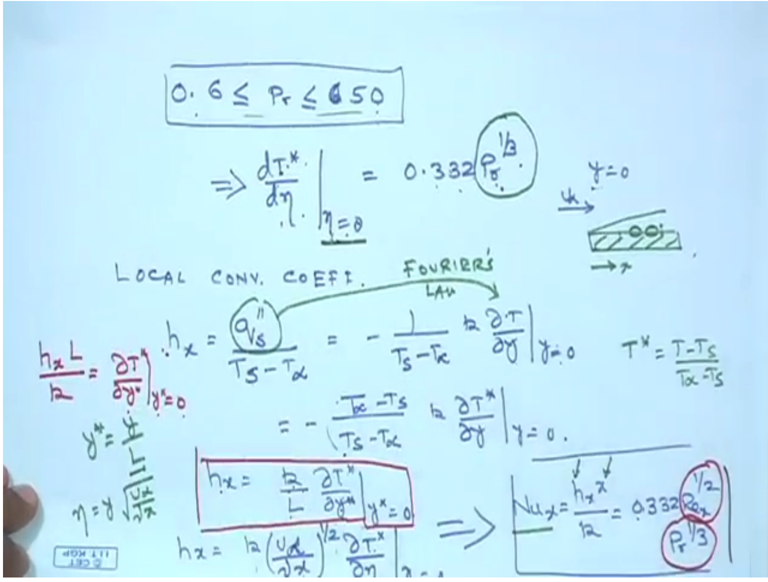

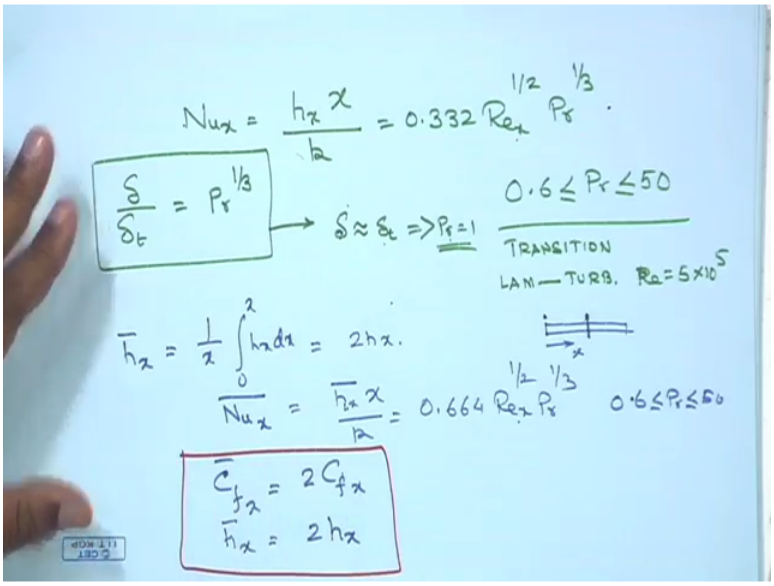

Which is NUX, local Nusselt number X, K 0.332 local, local value of Reynolds number and Prandlt to the power one 3rd. But remember that this expression is only valid within this Prandlt number range. We cannot use the expression beyond these values of the Prandlt number. But before I close, I will tell you take a look at this. Which says that local convection coefficient is equal to K by L dell T by dell Y at Y equals 0. I bring L and K on this side, so what I get is h X L by K is dell T star by dell Y star at Y star equal to 0.

What is h X L by K, this is nothing but all of you remember, realise that this is nothing but the Nusselt number. So the true definition of Nusselt number is equal to, if you look at the expression once again, if you look at this expression, the definition of Nusselt number as, it is that dimensionless temperature gradient at the solid liquid interface. So the scientific definition of Nusselt number is therefore the dimensionless temperature gradient at the solid liquid interface.

But anyway we are here where we have obtained in Nusselt number expression and you would be also able to see that if you find out what is delta, the thickness of the momentum boundary layer by the thickness of the thermal boundary layer, this is Prandlt, this is Prandlt to the power 1 by 3 and therefore of course it follows that delta would be equal to delta T when Prandlt number equal to1. At that does not happen in most of the cases, so therefore you either have the hydrodynamic boundary layer to be thicker and the thermal boundary layer and so on.

And as before the transition from laminar to turbulent, the cut-off number is Reynolds number to be equal to 5 into 10 to the power 5. So the value of Reynolds number is 5 into 10 to the power 5, it gets converted from laminar to turbulent. So if HX, expression for HX is known, so if I find the average value of the heat transfer coefficient 0 to X HX dX, this would find out to be 2 HX, so Nusselt number average value over the entire X is HX average times X by K is equal to 0.664 REX to the power half Prandlt to the power half. So this value of HX is the local value of the heat transfer coefficient, this h bar X is the average value of heat transfer coefficient from 0 to X.

So if this is X for the solid plate, this expression gives you what is the value of heat transfer coefficient at this point. Whereas this expression would give you what is the average value of heat transfer from X equals to 0 to X equals some specific X. So this is for the local value and this is for the average value and as before it is valid for a Prandlt number range between 0.6 to 50. So we can take a corollary of this with the, with friction coefficient which we have obtained for the case of hydrodynamic boundary layer which was found to be 2Cf X.

So the average value of the friction coefficient is twice the local value of, local value of the friction coefficient. Similarly h bar X, the average value of the heat transfer coefficient, convective heat transfer coefficient is twice the value of the local heat transfer coefficient. So this is more or less what I wanted to cover in the laminar, the treatment of the laminar boundary layer where heat convection is taking place. And we have seen how our understanding and analysis of the hydrodynamic boundary layer helps, helped us in obtaining a final form of the Nusselt number, the engineering parameter of interest, the Nusselt number, the average value of the heat transfer coefficient in a in a much more quicker way.

And now we understand what are the physical significance of each of these terms and why it is imperative that we need to have the solution of the hydrodynamic boundary layer in place before we even start solving the thermal boundary layer. And in solving the thermal boundary layer we assume a specific value of Prandlt lumber, we know what is f at different values of eta from our previous solution and then I can proceed and find out what is, how does T temperature, the dimensionless temperature varies with Eta the dimensionless distance at a specific value of Prandlt number.

So if we try to make a generalised solution out of this which would be valid over a large range of Prandlt number, we look at the solution and we see that the, the temperature gradient at the interface can be expressed as a function of Prandlt number, empirically fitted function of Prandlt number. So this would extend the validity of the entire analysis and we obtained a compact expression of Nusselt number in terms of Prandlt member and in terms of Reynolds number. These dimensionless groups must appear as I mentioned before in any situation, in any expression of forced convection heat transfer, so we get an expression of that.

And the same way we have seen for the case of hydrodynamic boundary layer, the local value of the friction coefficient is or twice the local value of the friction coefficient is equal to the average value of the friction coefficient. Twice the local value of the convective heat transfer coefficient is equal to the average value of the convective heat transfer coefficient. So those are the similarities between these processes, between momentum transfer and heat transfer.

So in next subsequent classes we will look at mass transfer process and then bring in the concept of concentration boundary layer and then we relate the hydrodynamic boundary layer, the thermal boundary layer and the concentration boundary layer together to get at different similarities, the similarities between these different processes. That was the goal of transport phenomena course which we would address towards the end of this course.