Transport Phenomena.
Professor Sunando Dasgupta.
Department of Chemical Engineering.
Indian Institute of Technology, Kharagpur.
Lecture-41.
Free Convection.

So in previous classes we have seen the use of the energy equation, how it can quickly and in a more convenient fashion give us the governing equation for any heat transfer process. Be it conduction, convection, a source where heat can be generated, transient effects and most importantly how to take care of the viscous heat dissipation. So the dissipation function Phi that has also been included in the development of the energy equation. Even though we understand that it is going to be irrelevant in most of the cases, unless some very special cases in which there will be a large velocity gradient or we are dealing with high viscosity and so on.

Then we proceeded to obtain the governing equation for forced convection heat transfer in a tube where the side walls, through the side walls, a constant heat is being added to the system. And then we have seen that how easily we could obtain the governing equation through the use of the energy equation. So we took the energy equation in the radial coordinate system and cancelled out the terms which were not relevant. So what we obtained is the governing equation, still a PDE and with we could also identify the boundary conditions.

Then we, we have nondimensionalized the equation, and we took the special case, a special limiting case in which the fluid has traversed a significant distance in the tube such that we can assume that the, that the profile of the fluid temperature will not change with the radial will not change anymore, the shape of the profile will remain unchanged, however a constant amount is going to be added as the fluid front moves more and more into the comment to the tube.

So this assumption of that we could separate the R dependence of the temperature profile and Z independence of the temperature profile as the sum of 2 distinct functions, enable us to convert the PDE to an ODE, so we got a limiting solution which is truly valid when eta, dimensionless variable denoting the axial distance when eta tends to infinity, what it has been shown that we get to a reasonably good approximation of the temperature profile, even for smaller, lower values of Epsilon, the axial distance as well.

And then we have also seen that 2 dimensionless numbers, namely the Reynolds number and Prandlt number appear automatically in the governing equation. So if we have to express or if we have to represent the data of force convection for which, let us say for a situation for which no standard relations can be obtained, so it would be customary, it would be expected that the data should be fitted with some function of Reynolds number and Prandlt number.

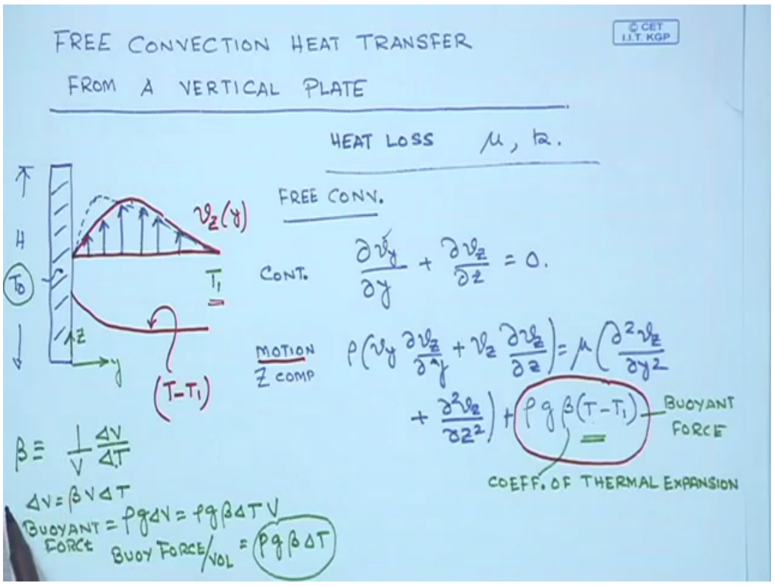
So the utility, the importance of dimensionless groups has been underscored by this, by the analysis where we could identify the important terms, the important dimensionless numbers in the description of the entire process. Since we have done, we have worked with forced convection in the last class, let us try to see what would be the case when we have free convection or natural convection. So forced natural convection is there everywhere, okay, it is the most common form of heat transfer. Even if you do not have any forced flow of fluid over a surface which is hot or cold, you will always have natural convection.

So the process is characterised by a change in buoyancy, so the change in buoyancy as a function of temperature will create lighter fluid near the top and heavy fluid near the bottom, so if this is an object, whose temperature is different, higher than that of the surrounding fluid, then the fluid closest to the surface, closest to the hot surface, its buoyancy will be changed and therefore it would start to rise and as it rises, its temperature will progressively increase and therefore and then it will go to the bulk. And whatever is rising from here is going to be repressed by cooler fluid from the side.

So therefore a cycle will start in which the cooler air will extract heat from the hot surface and this process, even though the heat transfer coefficient is significantly lower than that of force convection, but the ubiquitous nature of the free convection would ensure that there would be substantial heat losses, even in the absence of imposed flow. And in many of the conditions you need to take into account the heat transfer, both due to forced convection as well as free convection. So mixed condition most of the times exists when heat transfer is taking place from a hard surface.

So the free convection is characterised by no imposed flow, a change in buoyancy resulting in upward for the case of hot surface, upwards flow of the a, upwards flow of the fluid to be replaced by the cooler fluid from the surrounding and thereby it has thereby the heat transfer process continues as long as there is a temperature difference between the hot surface and the fluid surrounding it.

(Refer Slide Time: 6:49)



So we are going to 1st start with a free convection heat transfer from the simplest possible geometry, that is that of a vertical plate. So let us assume that we have a vertical plate which is in contact with air, so this is a vertical solid plate whose length is H and whose temperature is maintained at a constant temperature of T0. So T0 is the, T0 is the temperature of the solid and we have the coordinate system as this is Z direction and this is the Y direction and we will assume that in the X direction, it is wide enough such that the process is not going to be governed, is not going to be dependent on whatever happens in the Z direction.

So then what you would have is the air over here has a temperature, the temperature which is different from the temperature of the solid and let us assume that the temperature here is at T 1. So if you draw the picture profile for such a case, it would probably look something like this, when it would asymptotically reach T1. So I am plotting T the temperature of the air near the wall - T1. So T - T1 would slowly approach 0 as we move away from the plate.

So this is the temperature profile or rather a temperature difference profile which one would expect for the case of a hot plate, hot plate in contact with the cooler fluid. So what happens is the fluid near the walls, its buoyant force would force it to move in the upwards direction and cooler fluid from the surrounding is going to come and replace it. So is going to be a motion in this direction and a filling up of the void left by the upwards moving fluid by that from the surrounding.

So if I draw the velocity profile, it would probably be, this, because of no slip condition, it will start with velocity equal to 0 and then it would rise and then slowly decrease till it comes to be equal to 0. So this is going to be the velocity profile, however my, in the drawing it is greatly exaggerated and this profile would be skewed towards the solid plate. That means ideally it would probably look like something like this. Okay, because of clarity, in order to bring in clarity I have drawn it in this way but the peak is going to be towards the solid plate and that is what the profile would be.

So this VZ, the velocity in the Z direction is definitely going to be a function of Y. Now the problem statement therefore is that a flat plate heated to a temperature T0 is suspended in a large motionless body of fluid which is at a temperature T1. And we need to find out the heat loss, what would be the heat loss from such a system in terms of the velocity, in terms of the physical properties, for example mu and K and so on. So, in this case the convective, convection, free convection, this is a case of free convection and the equation of motion that I am going to write, the equation of continuity which is Dell VY by Dell Y + Dell V by Dell Z is going to be equal to 0.

There would be some VY, otherwise if there is no VY, then there would not be any replacement of the hot air that has, that has left this space. So VY, even small, it would still be present in the governing equation. So this is equation of continuity and then I am writing the equation of motion, which is rho VY Dell VZ Dell Y, this is the Z component because most of the most of the motion is in the Z direction. So I am writing the Z component of equation of motion + VZ dell VZ + Dell Z is equal to mu times, these are the viscous transport terms.

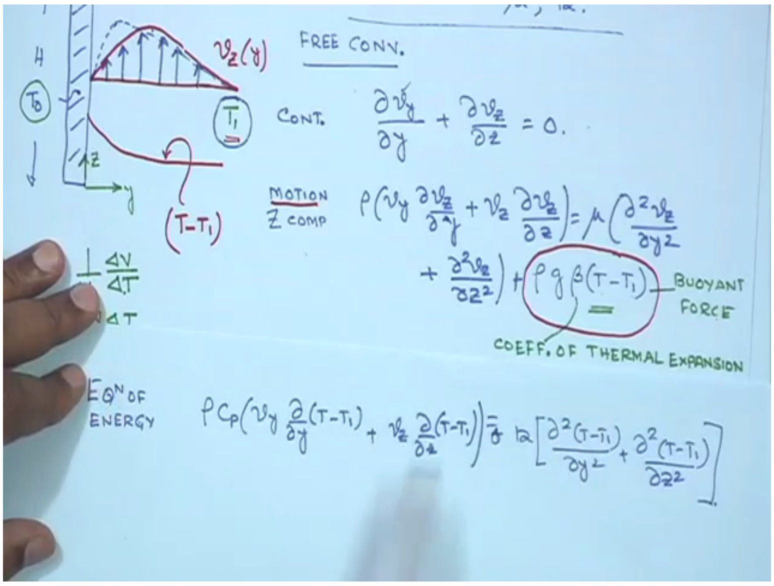
So this is the additional term which appears in the equation of motion, Z component of equation of motion, so this refers to, this essentially tells us the buoyant force, the expression for the buoyant force and the beta is the coefficient of thermal expansion. So if beta is the coefficient of thermal expansion, so beta is simply defined as the change in volume with a change in, with the change in temperature, nondimensionalized by the volume. So this is the change in volume with respect to the original volume as a result of change in temperature.

So I tried the change in volume as beta times V times Delta T and therefore the buoyant force, since this is the change in volume, the buoyant force would simply be rho G Delta V which is rho G beta Delta T and the buoyant force per unit volume which everything in equation of motion is expressed in per-unit volume, so the buoyant force per unit volume would simply be equal to rho G beta Delta T. So this is the form of the buoyant force per unit volume which is used in here.

Because unlike gravity, since motion here is sustained by a change in temperature. So everything is expressed in terms of the thermal expansion coefficient which is an experimental parameter, which is measured experimentally for most of the gases over a wide range of temperature. So it is customary to express the buoyant force in terms of the thermal expansion coefficient. So utilising the definition of thermal, thermal expansion coefficient, I would be able to obtain what is the change in volume as a result of change in temperature, what is the force, the buoyant force to the change, due to that change in volume and therefore we can find out what is the buoyant force per unit volume which can then be used in equation of motion as the prevalent body force.

So fundamentally I am not doing anything new, I am simply substituting, I am simply separating the expression of buoyant force in the body as the body force term in the equation of motion. So now with this equation of motion in place, I think we are now in a position to write the equation of energy as well.

(Refer Slide Time: 14:57)



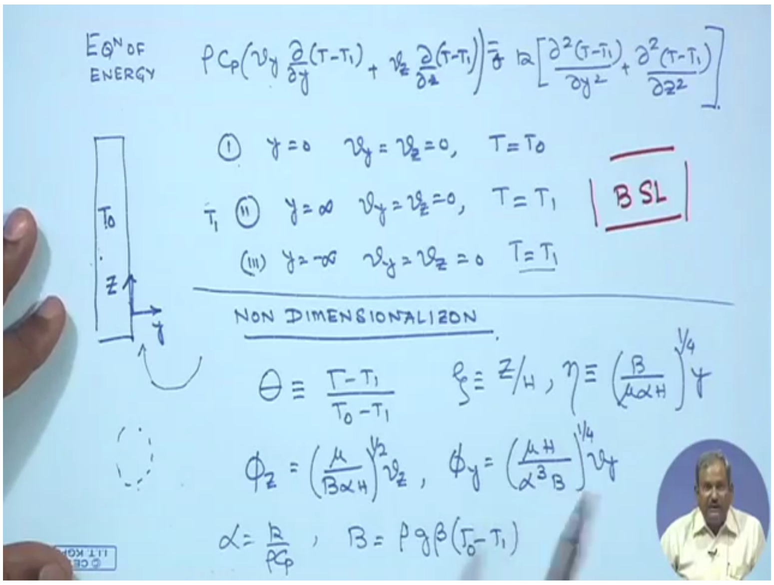
So the equation of energy in this case would simply be rho CP V Y Dell Dell Y of T - T1 where T is the temperature at any point in the rising film of the fluid and T1 is a constant temperature, reference temperature we take this reference a preacher to be the temperature far from the wall. So this is the constant fluid temperature that you would get when you, when you move away from the plate. So this is simply a constant. However the temperature in here is going to be a function of both of Z and of Y.

So that is why I have to use a partial sign and the 2nd term would simply be VZ, Dell Dell Z, again T - T1, and the, so this is the convective flow term. And the 2nd term that the right-hand side what I have is K times Dell square T - T1 divided by Dell Y square + Dell square T - T1 by Dell Z square. So in all these cases the left-hand side refers to convective heat transfer, right-hand side refers to conductive heat transfer and there is no, there is no heat generation term as well as no viscous dissipation since the flow is very small, very low velocity flow.

So what I have is then the 3 equations that I need to solve, equation of continuity, equation of motion and the equation of motion I express it in terms of the free convection form of the equation of motion and finally what I have is equation of energy. Now if you look closely into these 3 equations, the temperature rise appears in equation of energy, it also appears equation of motion. Velocity, the components of velocity present in the equation of motion and also in equation of energy.

However it is the presence of the temperature term in both the equation couples these 2 equations in a more comprehensive way, in a way that we have not seen before. So the presence of temperature term makes it a two-way coupling, the energy equation and the equation of motion are coupled both ways. You will not be able to solve any of equations without solving the other one or in other words both the energy equations and the equation of motion will have to be solved simultaneously, so that makes it more complicated.

(Refer Slide Time: 18:10)



So before we do that, before we attempt to do that, let us 1st find out what are the boundary conditions for such a case. So the boundary conditions that you would get for a vertical plate where this is the Z direction and this is the Y direction, the boundary conditions, the 1st one is that at Y equals 0, due to no slip condition, both VY and VZ will be 0 and temperature is going to be equal to T0, where T0 is the temperature of the solid plate and T1 is the temperature of the plate at a point far from it and therefore it is a constant temperature. So both these temperatures are constant temperatures.

And Y equals infinity, that means at a point far from it, again VY and VZ, both will be 0 and T would be simply equal to T1. So at a point over here, there is no effect of the solid plate and therefore no motion is induced because of the presence of the solid plate and therefore all of them are going to be equal to 0. And the 3rd condition is at Y equals - infinity, VY equals VZ equal to 0. So Y equals - infinity refers to the point over here. So any motion of the, any motion of this is going to be like this, so therefore this region remains unaffected by the presence of the solid plate.

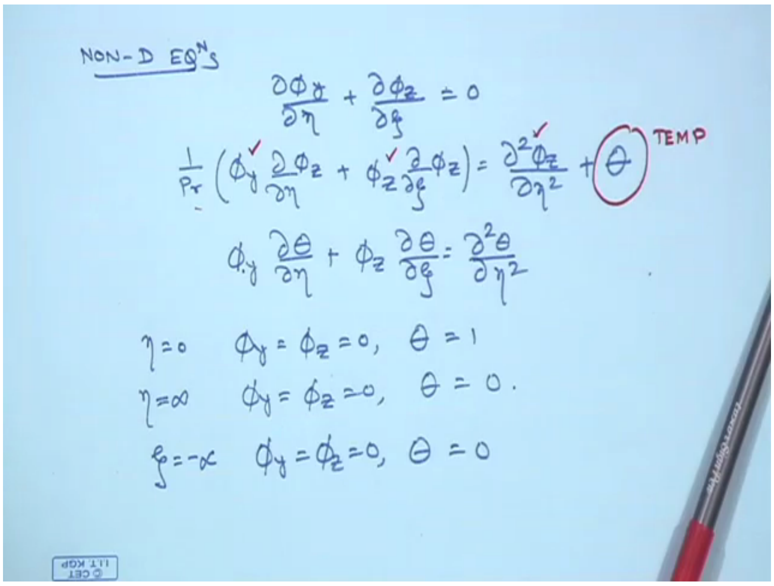
So Y equals - infinity is at a point far from, far from the plate and below it and therefore at Y equals - infinity, VY is equal to VZ equal to 0 and T is again going to be equal to T1 which is the temperature, which is the temperature of the air surrounding it. The next, so the equation, these 2 equations will have to be solved with these 3 boundary conditions. As we have seen so many times, it would be much better if we can nondimensionalize the entire thing.

And in order to do that, I define Theta the temperature as T - T1, dimensionless temperature is 0 - T1, the Zita is defined as Z by H, so this is the dimensionless Z component and what I have Eta, which is the dimensionless Y, which is defined, all these are defined as B by mu Alpha H to the power one fourth times Y. The phi Z, this is the velocity in the Z direction is mu B Alpha H times to the power half times VZ. So this is the dimensionless Z velocity and Phi Y is the dimensionless Y velocity which is defined as mu H by Alpha cube B to the power one 4th into VY.

And Alpha is KY rho CP is the thermal diffusivity as we have seen before and B is simply the buoyant force which is present in the system and these are the different ways by which, these are the different nondimensionalization parameters which are introduced in it. Again you can see the derivation and detailed treatment of this equation in your textbook of Bird, Stuart and Lightfoot. I would only discuss about the salient features of the solution and the physical concepts involved. So it is not important for you to memorise any of these, they are going to be, they are in their texts.

If ever a problem comes in which you have to nondimensionalize, what could be the nondimensional parameters, they would be specified in the problem. So you do not have to invent how to nondimensionalized a specific variable in order to make the equation more compact. So do not try ever to memorise this, just try to see the pattern and be rest assured that it would be provided to you in any exam, which and this is given in detail in Bird, Stuart and Lightfoot.

(Refer Slide Time: 22:42)



So when you put these nondimensionalizing parameters into the governing equations, what you get is the nondimensional equations, nondimensional equation becomes Dell Phi Y by Dell Eta + Dell Phi Z by Dell Zita, this is the equation of continuity and the equation of motion becomes phi Y Dell Dell Eta of phi Z + phi Z Dell Dell Zita of Phi Z is equal to Dell square phi Z by Dell Eta square + Theta where this Theta is the temperature term. These are all values of velocity, whereas this Theta is nothing but the temperature term.

And the equation of energy becomes phi Y Dell Theta by Dell Eta + phi Z Dell Theta by Dell Zita equals Dell square Theta by Dell eta square. So this is the convective momentum transport, convective momentum transport, diffusion momentum transport or conductive momentum transport, this is a buoyant body force present in the system. This one is the convective transport of energy, Phi Y, Phi Z refers to the velocity in the Y and Z direction, this Theta is the temperature change with respect to Eta and temperature change with respect to Zita.

And what you would get here then is, this is the conductive term. So it, when you convert these 2, convert the boundary condition, it becomes that at Eta equals 0, at Eta equals 0 simply tells you that at Y equals 0, that means on the plate there would be no slip condition and the temperature would simply be equal to T0. So at Eta equal to 0, both the velocities Phi Y and Phi Z would be equals 0 and the temperature, dimensionless temperature would be equal to1. And Eta equals infinity at a point far from it there would be no velocity and the temperature is going to be equal to T1. And if the temperature is going to be equal to T1, then Theta is going to be equal to 0.

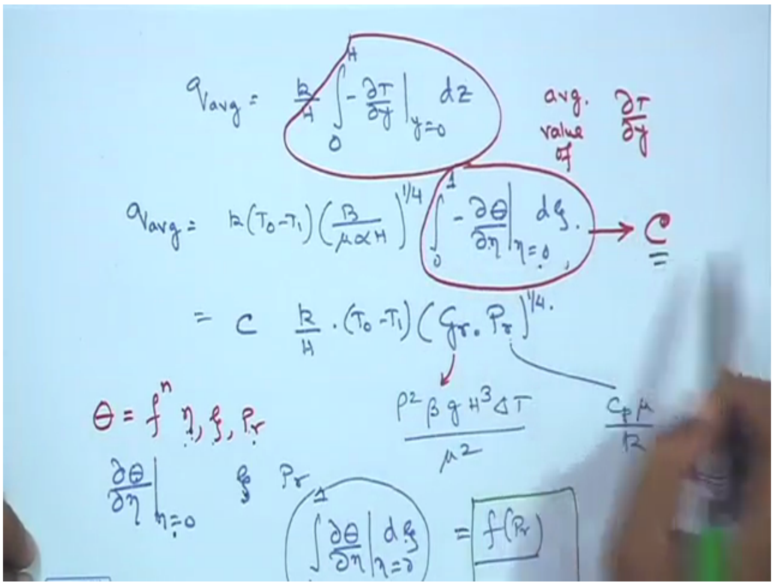
So with this geometric condition I again write phi Y is equal to Phi Z equals to 0 and Theta equals 0 and at Zita equals - infinity, that means at the point where Z equal - infinity at really at this point that would be no velocity and the temperature would again be equal to T1, so Theta would be equal to 0. So Phi Y equal to Phi Z equal to 0 and your Theta would again be 0. So these 3 questions would have to be solved this with these following boundary conditions.

However an analytic solution to this is still not possible, but we can make a heuristic, heuristic type of solution and then try to see is it possible to reduce the problem in such a way that a, that we can make a solution out of such a complicated system. 1st of all the left, if you remember the left-hand side of the equation of motion refers to convective transport of momentum. So if you see the dimensionless form of this equation of motion in here, what you would see is this left-hand side where you have the Prandlt number, this refers to convective transport of momentum.

But the convective transport of momentum is strongly dependent on the velocities, velocity in the Y direction in velocity in the Z direction. Rather convective passport is dependent on velocity. So save the velocities are very small, then the effect of the left-hand side of the Naviar Stokes equation, there is the effect of convective transport would be small. In the limiting case when you have creeping flow, you remember we have studied creeping flow before. So if we have creeping flow, then this entire left-hand side can be made equal to 0.

But if it is not, here the problem may not be equal to creeping flow but it is still a slow flow, so the effect of the left-hand side of the Naviar Stokes equation on the final form of the solution should be significantly smaller. So that is the assumption that we are making. Since free convection flow is characterised by slow upward movement of the fluid, so therefore we do not expect the effect of convection to be significant on the final solution of the problem.

(Refer Slide Time: 28:10)



So it, we are not saying it is 0, such in the case of creeping flow but we are seeing its effect is going to be small. Now let us see how we can get the average heat flux which is Q average which can simply be equals K by H - Dell T Dell Y at Y equals 0. So this is average heat flux from the, average heat flux from the walls to the outside is, if you forget about this K, then this would essentially, this part is the average value of Dell T Dell Y. So this average value of Dell T Dell Y which by definition is this, multiplied by K with a - sign would give you the conductive heat lost by convection, the heat lost by conduction combination methodology from the solid plate to the atmosphere.

So this is nothing but the average of Dell T Dell Y. So if you nondimensionalize this, you would simply see that it is going to be T0 - T1 B by mu Alpha H to the power one fourth and instead of temperature gradient I am going to write dimensionless temperature gradient which is going to be - Dell Theta by Dell Eta at Eta equals 0 times D Zita. So these are constants as we have defined before and we are going to call this, the entire thing by a constant, by, say we denote C for this entire term, because the rest of the things you can, you can calculate based on the geometry and property.

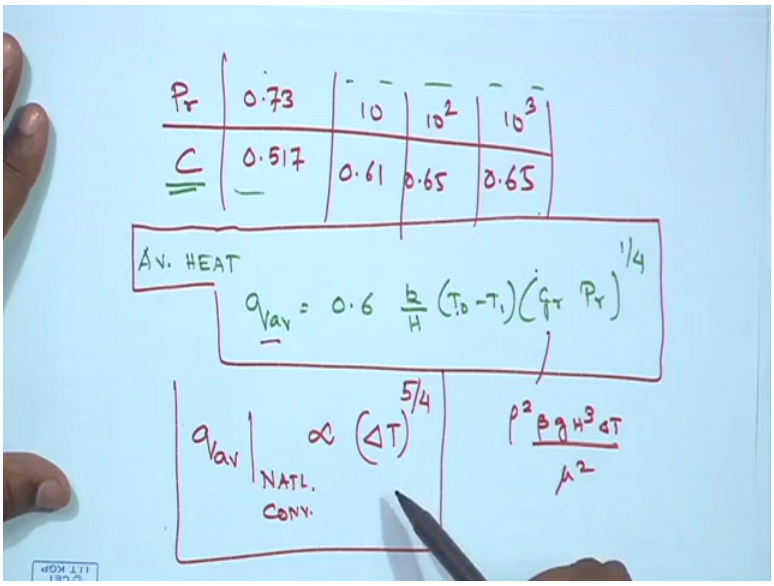
So it is going to be C K by H times T0 - T1 Grashof number, Prandlt number to the power one 4^{th}. So what is Grashof number, Grashof number is defined as, which appears automatically is rho square beta G HQ Delta T by mu square. And Prandlt number we know that it is simply mu by K. So what we have then is if we could evaluate this, then we have a next equation for the average heat loss from the solid surface. Now what is this, we understand that this Theta is going to be a function of Eta, Zita and Prandlt number. If we look at the governing equation, then this Theta would simply be a function of Theta, Zita and the constant Prandlt number.

So if Zita is a function of Eta, Zita Prandlt number, Dell Theta by Dell Eta at Eta equals 0 will only be a function of Zita and Prandlt number. So if Theta is a function of Theta, Zita and Prandlt number, then tell Theta Dell Eta evaluated at a specific value of Zita will no longer be a function of Eta, it is only going to be a function of Prandlt number. What we are doing is we are integrating, it is a definite integrals from 0 to 1 over D Zita. Since we are taking a definite integral of the gradient over Zita, it is a definite integral, so this entire thing cannot be a function of Zita, it can only be a function of Prandlt number.

So what this simple heuristic logic tells us is that this term which we in order to evaluate this we have to solve the partial differential equations. But we do not need to solve it if we look carefully and think about the functional form of Theta to be a function of Eta, Zita and Prandlt number. Since I am taking a derivative of that at a specific value of Eta, it does not remain a function of Eta, it is a function of Zita and Prandlt number. So if I, since I am taking a definite integral over Zita, so it no longer remains a function of Zita, it is only a function of Prandlt number.

So the entire problem boils down to finding out this as a function of Prandlt number. And we also know that even though it is a function of Prandlt number, it must be a slow or a weak function of Prandlt number because we have earlier said since the flow rate is slow in convective heat transfer at the limiting case it approaches the creeping flow. But this is not the case of creeping flow, we have still very slow flow. Since it is very slow flow, the dependence of the solution on the values of the Prandlt number will be small, will be very small. So we do not need to solve equations, we just need to find out what is the value of C.

(Refer Slide Time: 33:25)



And when experimentally the values of C were calculated for different values of Prandlt number for Prandlt number equals 0.7, this value is 0.517, for 10 it is 0.61, for 10 square it is 0.65 and for 10 cube it is again 0.65. So we can see that the value of this C experimentally obtained does not depend strongly on the value of Prandlt number. So the average heat flux, the average heat flux that from a heat from a vertical plate would simply be equals 0.6 times K by H T0 - T1 Grashof number, Prandlt number to the power one 4^{th}.

So this is an expression which you have obtained by looking at the governing equation, looking at the significance of each of these terms and recognising that you do not need to solve the problem, you just, you identify the correct figure of the equation saying that it is a slow flow, so the effect of convective heat or mass transfer, convective, sorry convective heat or momentum transfer would be small. So the integral which you need to calculate from the solution of the coupled PDEs, you do not need to do that, the integral is going to be a function only of Prandlt number.

And it is very weak function of Prandlt number, so if anyone can tell, anyone can tell you what is that value of the function for one value of Prandlt number, you are safe to use it over a wide range of Prandlt number, wide range of fluids experiencing different conditions and you will still be within the limits, still be able to predict what would be the total heat loss from a vertical plate which is placed in a stagnant body of cold fluid. So your analysis makes the solutions, makes the requirement of the solution of the PDEs redundant. So here is an example where an analysis, an understanding of transport phenomena can create, can reduce your workload greatly in a minute in a significant manner.

And this is one of the beauties of transport phenomena, study of transport phenomena which from 1st principles tells you how to deal with a system from 1st principles and arrive at a solution without actually solving the governing equations. So what we see over here is, what we have obtained over here is a formula for the average heat lost by the solid surface as a function of K, Grashof number and Prandlt number, which are all Thermo physical, combinations of Thermo physical properties, H is the geometry and T0 - T1 is the imposed temperature gradient.

And since by definition your Grashof number contains, Grashof number is, contains Delta T in this form. So this Delta T and this Delta T essentially tells you that the Q in natural convection is proportional to Delta 2T to the power 5 by 4. So this is also a relation which directly follows from this expression and which has been widely used, widely cited for the analysis of natural convection.