Transport Phenomena.
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Lecture-40.
Energy Equation.

So we would continue with our understanding of the energy equation that we have derived in the last class. And also we would see how using the energy equation a generalised treatment for heat transfer, both convective and conductive heat transfer can be taken into account and at the same time if there is any heat generation in the system or if the total internal energy of the system changes as a result of work done by the system or on the system. So when we started with our derivation of the energy equation, we started with the 1st Law of thermodynamics for an open system.

And we have taken into account all the energy that comes in to the control volume as a result of conduction and convection. We have also identified the forces which are acting on the system, so the rate of work done would simply be equal to force times the velocity in the appropriate direction. And the force, the work will also be done against surface forces, against the other surface force which is the viscous force and the work done against the viscous force will manifest itself in the form of rate, in the form of an elevated temperature, the same way we have seen for the case of a solid, for the case of solid friction.

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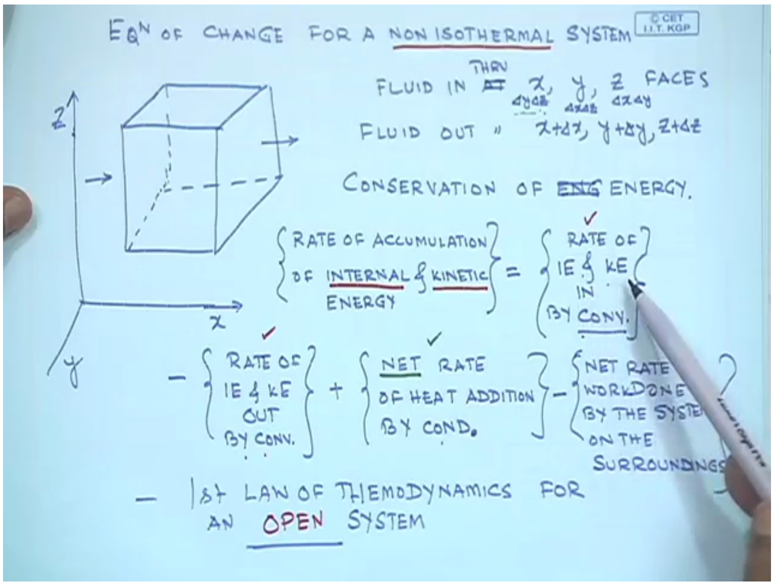

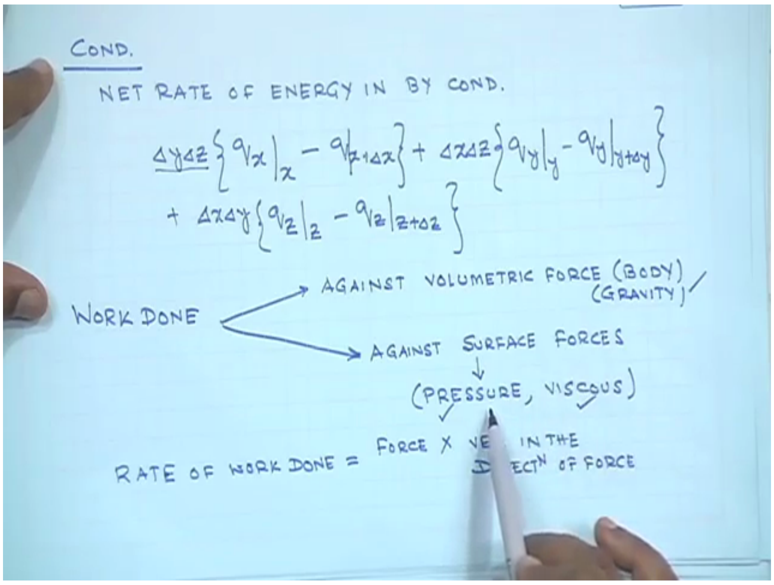

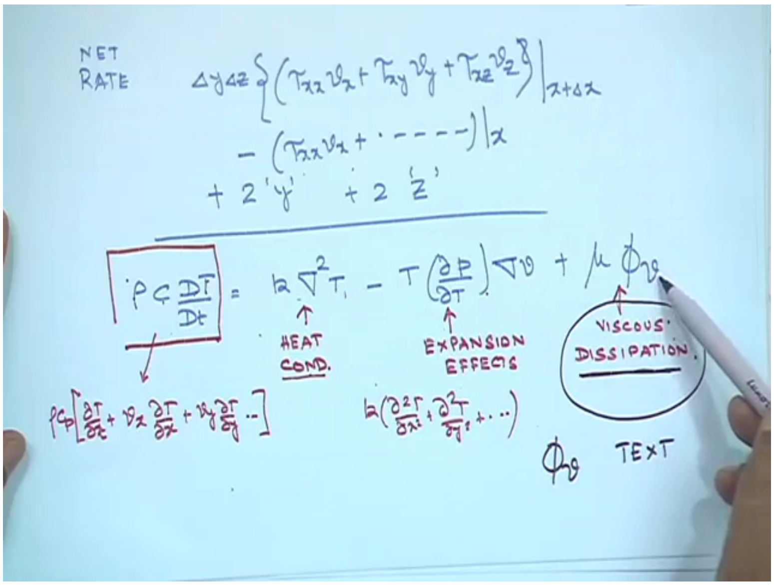

So we started with the analysis of the 1st Law of thermodynamics for an open system using the figure that I have drawn over here. So rate of accumulation of internal and kinetic energy must be equal to the rate of internal and kinetic energy in by convection, out by convection, net rate of heat addition by conduction and net rate of work done by the system on the surroundings. So from this generalised conservation of energy equation, the equation for mechanical energy is subtracted and what we obtained was the general form for the energy equation taking into consideration the work done as well as heat generation.

The work done against volumetric forces like gravity, against forces as I said pressure and viscous forces, where the rate of work done would simply be equal to force times velocity in the direction of force. So with this after we simplify this, we have obtained the equation of energy for a system in which both convection which is embedded into the substantial derivative in this form, the conduction, heat conduction, the expansion effects and a function Phi, Phi sub V which is known as the viscous dissipation which takes into account all the forces, all the all the work done against the viscous forces.

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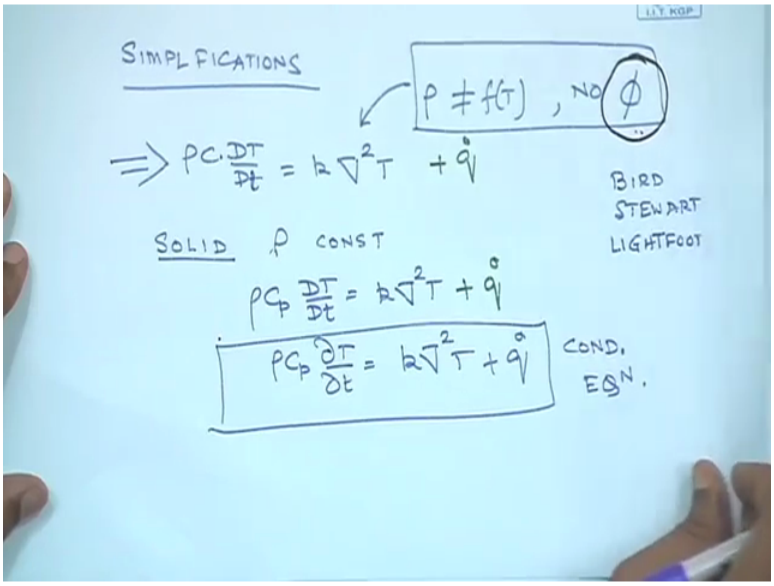

The simplification of this equation which I have shown before results in the following equation where the Phi, which is the viscous heat generation is usually small unless it is a special situation like high-speed flow, flow through a narrow conduit or a very high viscous fluid, very high viscous fluid flowing with a high velocity. So this term is generally dropped and if we drop the viscous dissipation some, then what we have is the left-hand side which is, which has the transit effect as well as the convection effect, this is the conduction effect and this is the rate of heat generation.

So this is the conduction equation which we have obtained in vector tensor notation. And if you refer to your textbook, what you would see here is the equation of energy is given in a tabular form with 2 colors. The 1st colour refers to energy, the convective heat in the form of QX, QY and QZ in rectangular coordinates. Whereas on the right right-hand side of the table, the Fourier's law has been substituted in here and if we assume the thermal conductivity K to be independent of position, then this would be the form of the equation of energy for a rectangular coordinate system.

Notice the term, this entire term with viscosity mu in front of it. So if you have viscosity, this term refers to the viscous heat generation or in other words the phi, the function Phi that I have introduced as before. So if we neglect this term, what you get is simply the energy equation which for a rectangular coordinate system would look like this. So rho CP, the substantial derivative is equal to K, the thermal conductivity times Delta 2T Dell X square Dell Y square and Dell Z square.

And if there is a heat generation over here, a Q dot term which is a volumetric heat generation must be added to the right-hand side. The viscous dissipation is generally small in most of the cases and therefore is neglected. The same equation has been written for the cylindrical coordinates in terms of R, Theta and Z where Z denotes the axial direction of flow. And again if you have to consider the viscous heat generation, then this entire term needs to be added to the energy equation but for most of the normal cases this term would be 0 and therefore does is going to be my governing equation for cylindrical coordinates.

Similarly for spherical coordinates, when we express the everything in terms of R, Theta and Phi, the entire equation except the term containing mu, which as before is small in most of the cases from here till here, this is the energy equation, the thermal energy equation for the spherical coordinate systems. So starting with the appropriate value of the energy equation they would see how easy it would become to arrive at the governing equations, the governing equation for any particular situation and for a specific system.

With the use of appropriate boundary conditions that equation can then be solved to obtain the temperature profile as a function of X, Y, Z or a function of time as well if it is a transient case. So the problem that we have introduced in the last class and we have used a complex methodology in order to obtain what is the, what is going to be the temperature distribution, what is going to be the governing equation, I refer again to the problem of force convection that I have introduced in the previous class.

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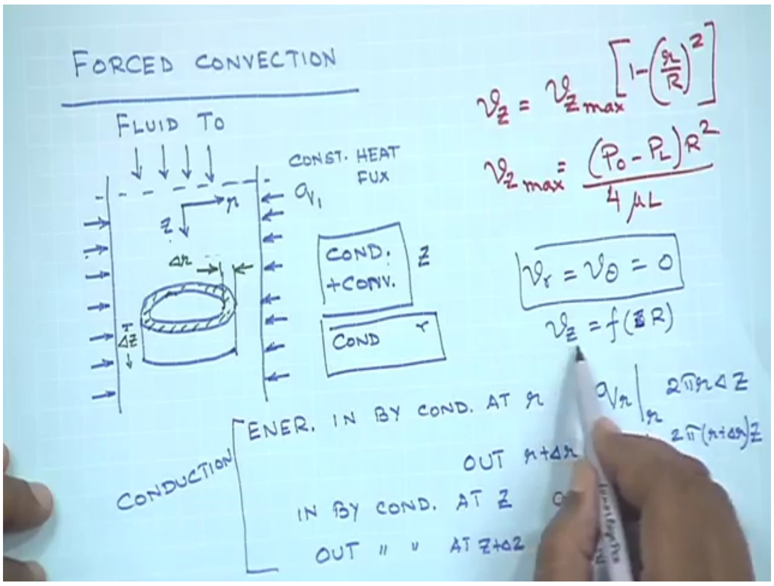

In this case I had a tube through the sides of which a constant heat flux Q1 is added to a flowing fluid, the fluid starts at a uniform temperature of T0 but as we move inside the temperature is going to be a function both of R, that means where the fluid element is located with respect to the wall and at what is going to be its axial position. So as I understand that as the fluid moves to, more and more into the tube, the temperature of it is going to increase. So T is therefore a function of both X and, both Z as well as R.

And there would be conductive heat transfer in the R direction, there is going to be convective heat transfer in the Z direction and there is going to be conductive heat in the Z direction as well since the temperature is going to be a function of Z. However it is a one-dimensional flow situation, that means we have only R VZ present in the system, there is no VR or there is no V Theta. So VR and V Theta, both are going to be equal to 0, only what we have is VZ is going to be as we know from our study of fluid mechanics, it is going to be a function of, function of the radial position.

With the nonzero VZ present in the system, we know from fluid mechanics that the velocity is going to be a function of whatever be the imposed pressure gradient present in the system and this form of imposed pressure gradient also includes the effects of body force if any that is present in the system. This R is the radius of the tube, and this r is simply the radial location. So it is going to be a parabolic velocity distribution which would depend on the imposed pressure gradient, on the geometry of the system as well as on the Thermo physical property mu, the viscosity of the system.

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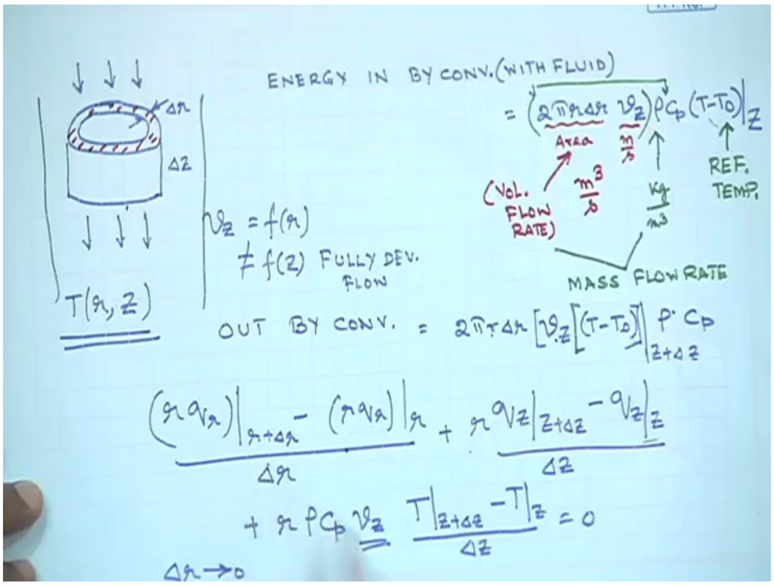

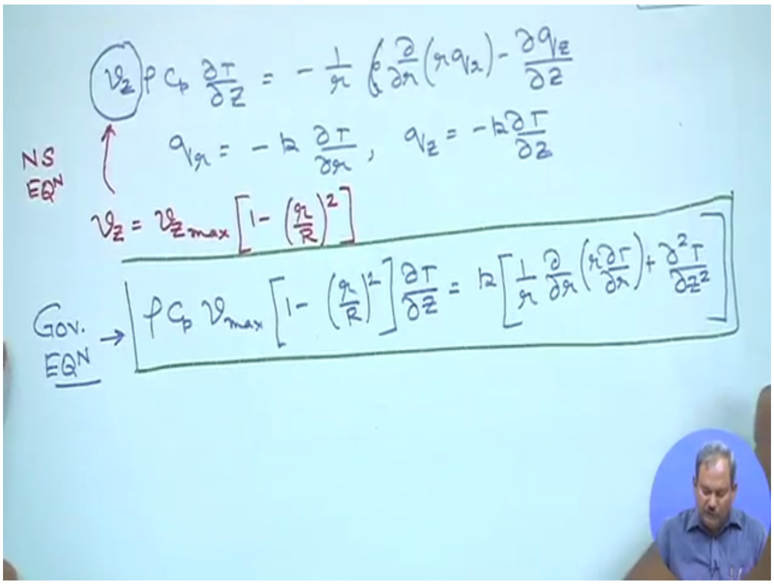

So we have assumed a complex shell of size Delta R and Delta Z and made a balance of all the energy that go, coming in and going out of the system taking into account the area through which the conductive heat is coming in, the area through which the the the convective heat is going out, the amount of heat addition by conduction and so on. So with, at the end of the complex process, after plugging in the value of VZ, the axial velocity, the radial heat flux in the R direction and radial heat flux in the Z direction, we have obtained a governing equation in this specific form.

So since we now have the energy equation at hand, it, we do not need to go through all these complex labourious steps in order to obtain the governing equation. So what I am going to do 1st in this class is right Z component of the energy equation, since the principal and then try to find out what is going to be the governing equation. So 1st I write the energy equation without taking into account the viscous dissipation term which is neglected in this specific case.

So we start today in this case is, is trying to see if we can obtain the same governing equation by writing the energy equation in cylindrical components and cancelling the terms which are not relevant. So my goal is to obtain the same equation through the use of energy equation in cylindrical coordinates.

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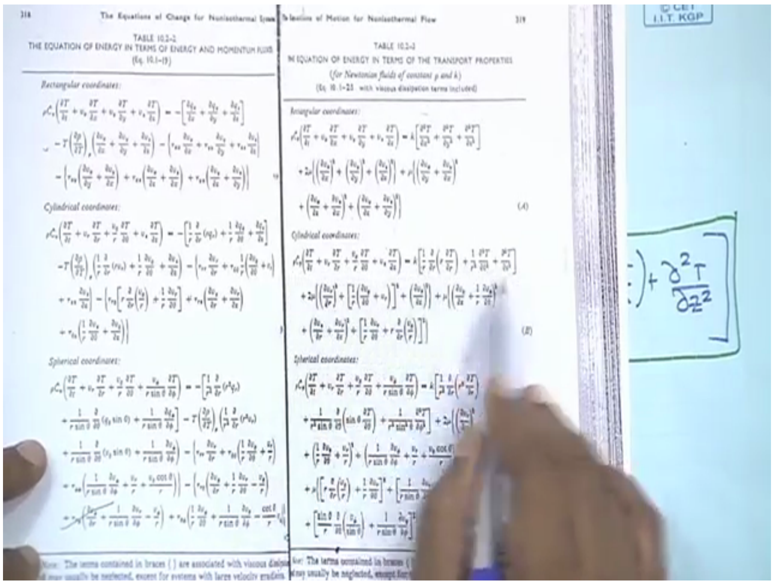

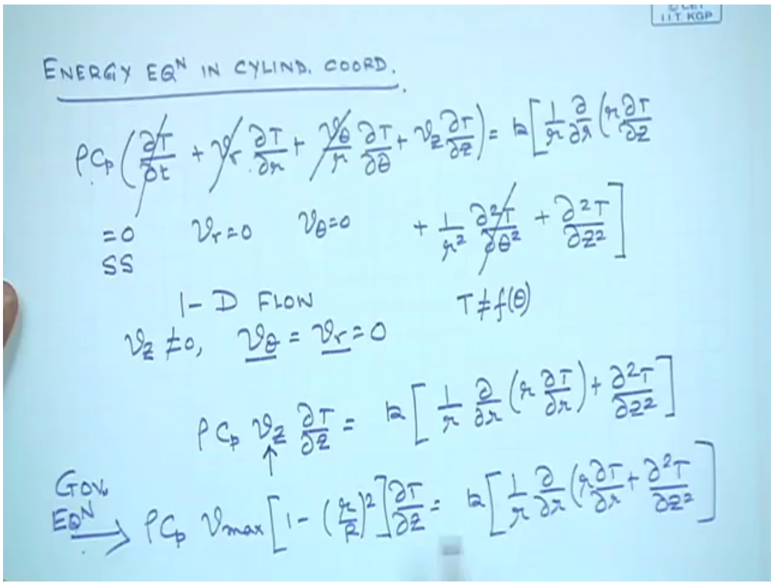

So I will write this equation, try to see if we get the same expression. So the 1st thing is I write the energy equation in cylindrical coordinate system. So this equation would simply be rho CP times Dell temperature by Dell time + VR times Dell T Dell R + V Theta by R Dell T by Dell Theta + VZ Dell T Dell Z is equal to K times 1 by R Dell Dell R of R, Dell T Dell Z + 1 by R square Delta 2 T by Dell Theta square + Delta 2T by Dell Z square.

One point I would like to stress, mention here is that you are not supposed to remember any of these equations. For any problem that will be asked to you, the forms of energy equations, Naviar Stokes equations, the species balance equations, all these equations would be provided to you. So you just have to choose the right equation in the appropriate direction for the case of Naviar Stokes equation in the right coordinate system and then using logic, clear logic you have to cancel the term.

So you do not think that you remember this equation for your test or for anything else. So we start with this, the 1st is I am going to say this is 0 since this is a steady-state situation. The 2nd one is we, we know that V Theta is equal to 0, there is no flow in, no flow in the Theta direction, so this part would be equal to 0. And T is not a function of Theta, so therefore this term would also be equal to 0. So what I end up with is the equation where rho CP Dell T Dell R VZ, Dell T dell R and all these terms.

Now remember, if you remember, we have assumed this is a one-dimensional flow, so by one-dimensional flow as I mentioned before, VZ is not equal to 0 but both V Theta and VR are equal to 0. So since V Theta and VR are of both equal to 0, this term would be also equal to 0. So if you simply by following the flow situation and the heat transfer that we have in here, so what we have then here is rho CP times VZ Dell T Dell Z is equal to K times 1 by R Dell Dell R of R Dell T dell R + Delta 2T by Dell Z square.

This is my governing equation and I simply substitute the axial velocity from my knowledge of fluid mechanics as rho CPV max 1 - r by R whole square times Dell T Dell Z is equal to K times 1 by r Dell Dell r of R Dell T dell R + Dell 2T Dell Z square. So this is my governing equation. Now let us see the governing equation which I have obtained in just 2 steps, is it identical to the governing equation which I have obtained through a shell energy balance.

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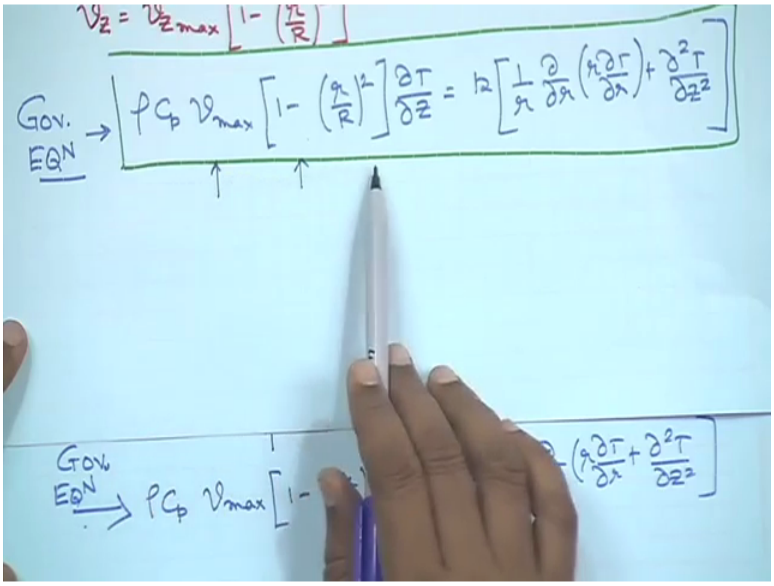

So if you compare these 2 equations, what you are going to see is they are exactly identical. So the expression, the governing equation that you have obtained by complex shell energy balance method, you can arrive at the same equation by simply following these 3 steps. So this underscores the utility of energy equation in dealing with any problem of heat transfer, energy transfer, work done in any process. I have neglected any viscous dissipation in the formulation and I have also assumed that there is no heat generation in the fluid, in the liquid itself.

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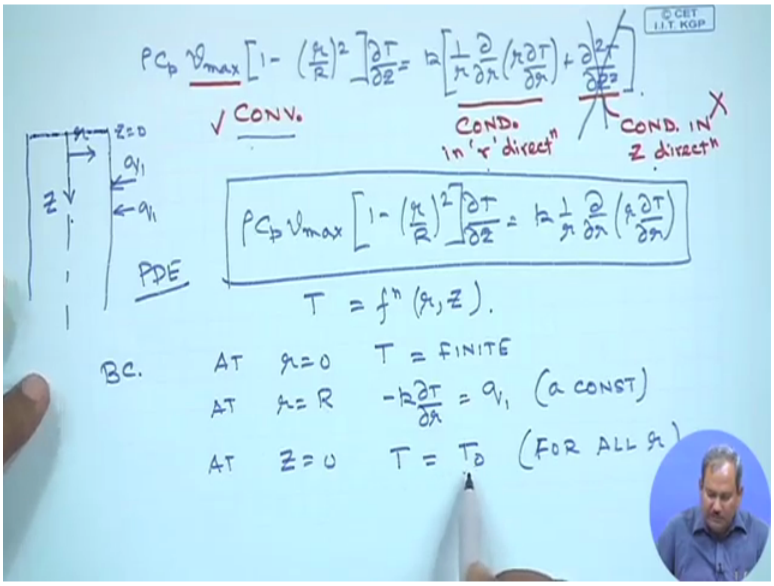

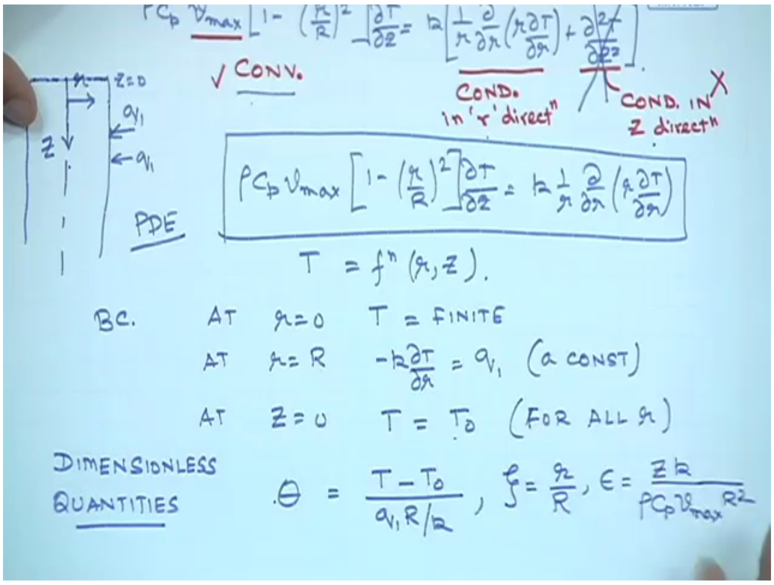

So this is my governing equation which I now need to solve with appropriate boundary conditions. So let us see my 1 equation, how it can be solved or what are the simplifications I can make in order to solve this equation. If you consider the significance of each of these terms, the 1st time on the left-hand side, since it contains the velocity, it must be taking into account of convection. This and this, it has a thermal, it has a thermal conductivity K in it and it has been obtained by substituting Fourier's law, so this is conduction in R direction, this is conduction in Z direction.

Now if you consider the effect of these terms in energy transport process, you would see that there is going to be a significant variation in temperature in the R direction in comparison to the temperature variation that you would get in the Z direction. Or in other words the conductive heat transfer in the Z direction will contribute less, significantly less in comparison to the conductive heat transfer that he would get in the R direction. So there is a sharp temperature gradient from the wall to the centreline but as the fluid moves in the Z direction, the principal reason for heat transfer in the Z direction is convection and not the conduction.

So since the principal reason of heat transfer in the Z direction is by convection, I should be able to safely neglect without introducing appreciable error any condition that takes place in the Z direction. So I if you follow this one more time, I will say in this that the motion in the Z direction contributes to the heat transfer process. Since its motion is in the Z direction, it is convection, whereas the temperature gradient in the R direction is sharp and therefore the heat transfer in the R direction is principally governed by conduction.

Whereas the effect of conduction in the Z direction is small as compared to the convection in the Z direction. So my reduced governing equation now takes the form V max times 1 - r by R whole square times Dell T Dell Z is equal to K times 1 by R Dell Dell R of R Dell T dell R. This is equation that I need to solve in order to obtain the temperature profile and other parameters. But it is still a partial differential equation. My T is a function of both R and Z.

So what are the boundary conditions? The boundary conditions I can write at R equals 0, that means at the centreline, T has to be finite and at r equals capital R - K Dell T dell R which is the conductive heat flux must be equal to Q1, Q1 being the heat flux which is supplied that the wall. So therefore the the conductive heat, the conductive heat at R equals, at R equals capital R must be equal to Q1 which is a constant, constant heat is being supplied.

And at Z equals 0 that means at the beginning of this T, the temperature is going to be a function, going to be a constant, let us call it T0 and this is for all R. So at the entry point, the temperature here is uniform and if equal to a constant T0, it does not depend on R because it is coming at a constant temperature and it is entering that tube which is heated by the introduction of a heat flux through the sides. So at Z equal to 0, this is the Z equal to 0 plane, at Z equal to 0, the temperature is simply going to be equal to T0.

Next, for any equations of these types, dimensionless quantities are introduced which allows us to make the equation more compact and in some cases as we would see later on that this would give rise to certain numbers which will have some physical significance. The dimensionless quantities at some, many a times would give rise to dimensionless groups. And the dimensionless groups together would tell us about the importance of one type of process to the other.

It may tell us something about the nature of the flow, it may tell us something about the dimensionless groups which should automatically appear in any relation involving in this case for example forced heat convective heat transfer under forced convection condition. So nondimensionalizing a system has several advantages. So we are going to nondimensionalize this equation and see how it helps us. So for that I would 1st design a dimensionless temperature Theta as temperature at any location - the constant temperature with which it is entering and Q 1 is the constant heat flux which is added by K.

So this is the dimensionless temperature, a dimensionless location, radial location is defined as zeta equals r by R, so it varies between 0 to 1 and a value of Epsilon which is the dimensionless axial location denoted by Z K by rho CP V max times R square.

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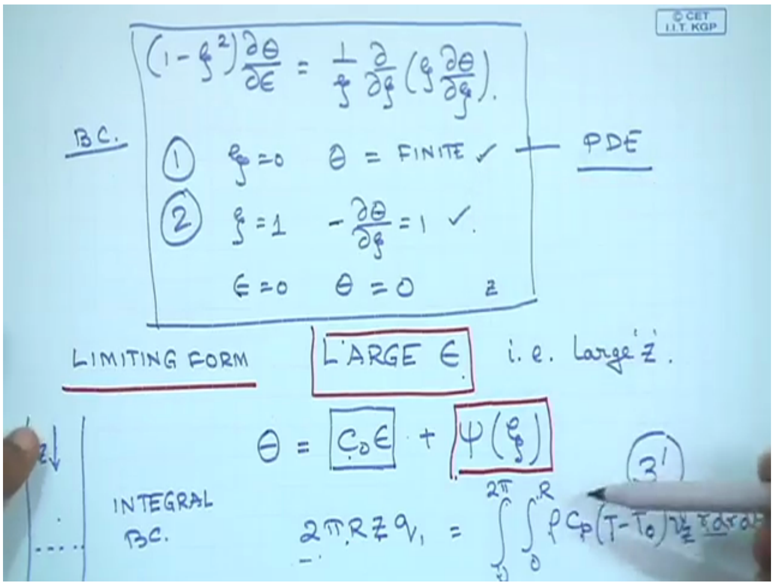

So these are the dimensionless term which have been introduced and upon introducing these dimensionless numbers and putting them into this equation, the governing equation takes the compact form as 1 - + zeta square times Dell Theta by Dell Epsilon is equal to 1 by zeta which is the R coordinate, dimensionless R coordinate, Dell Dell zeta times zeta dell Theta by Dell zeta. And the changed boundary conditions are at zeta equals 0, that means at the centreline, your temperature, the dimensionless temperature is finite.

So please refer to this, at R equals 0, T is finite, the dimensionless form of R is zeta, so at zeta equal to 0, T, the dimensionless form of which is Theta, Theta must be finite and that is what I have written over here, at zeta equal to 0, Theta is finite. And at zeta equals 1, which is at the sidewall - Dell Theta by Dell zeta is equal to 1. Again if you compare with the physical boundary condition, at r equals R, - K Dell T dell R is Q1, the convective heat is equal to the heat which is added through the side walls and since it is expressed in dimensionless form, it is going to be - Dell temperature, dimensionless temperature by Dell zeta is equal to1.

And at Epsilon equal to 0, that means at Z equal to 0, Theta is equal to 0. Let us see what it is over here, at Z equal to 0, that means that Epsilon equal to 0, T is equal to T0, if T is equal to T0, Theta must be equal to 0. So the, conceptually the governing, the boundary conditions for the dimensional equation has now been changed to the dimensionless form. It makes the equation and the boundary conditions compact but this is still a PDE and since it is still a PDE, it cannot be solved under analytically.

There is only a limiting form of the equation which is valid for large Epsilon, that means for large values of Z. So this is the limiting form of the solution for large Epsilon, that is large Z. It can be assumed that means when the fluid has really travelled far into, this is the Z direction, for into the fluid, it is safe to assume that the temperature, dimensionless temperature is going to be a linear function of Epsilon, that means Z, + psi, a function of zeta.

This is a condition which needs some explosion. So I will try to explain that to you. As the fluid starts to move in the Z direction, its temperature will progressively increase. But after a certain distance, after the fluid has travelled a certain distance, the shape of the profile which is given by the function, by its functional dependence on R or in dimensionless form zeta, the shape of the profile will remain unchanged, only the values will keep on increasing and that too linearly with Z.

So that is a condition of fully developed flow, thermally fully developed flow in which the shape of the profile which is denoted by this function, the shape of the profile does not change but the values of temperature is going to be a linear function of Z, of Epsilon, that is Z and C0 is simply a constant. So my dimensionless temperature profile is simply going to be a linear function of Z and that any, any at any axial, at any axial point, the functional form will remain unchanged.

For the 1st 2, 1st 2 boundary condition will still be valid and the 3rd boundary condition needs to be replaced by an integral boundary condition which simply says that the amount of heat that you have added over a distance from 0 to Z. So this is the amount of heat, that is twice pie R is the area twice pie R Times Z is the area, Q1 he is the heat flux, so this is the total amount of heat which you have added to the system must be equal to the amount of energy, amount of increase in energy that is there in the system.

This part I think is clear, the amount of heat added to the volume of the liquid from 0 to Z. And this equation essentially tells you that the total mass of fluid which is rho times R dR d Theta integrated over the appropriate limits, CP time T - T0, where T0 is the initial temperature and T the temperature at any Z location. So the double integral here denotes the, in order to get whatever be the volume of the system multiplied by rho which gives you the mass of the system, so it is the M dot CP and T - T0 where T0 is the reference temperature.

Whatever heat that is added through the side walls must be manifested by a change in the internal energy content of the flowing fluid. So this is the 3rd boundary condition that we are using, going to use, this is my 1st boundary condition, this is my 2nd boundary condition this is the 3rd condition, 3rd boundary condition, because the 3rd condition which I have written over here does not conform with the form of Theta that I have written in the limiting case for large value of Epsilon.

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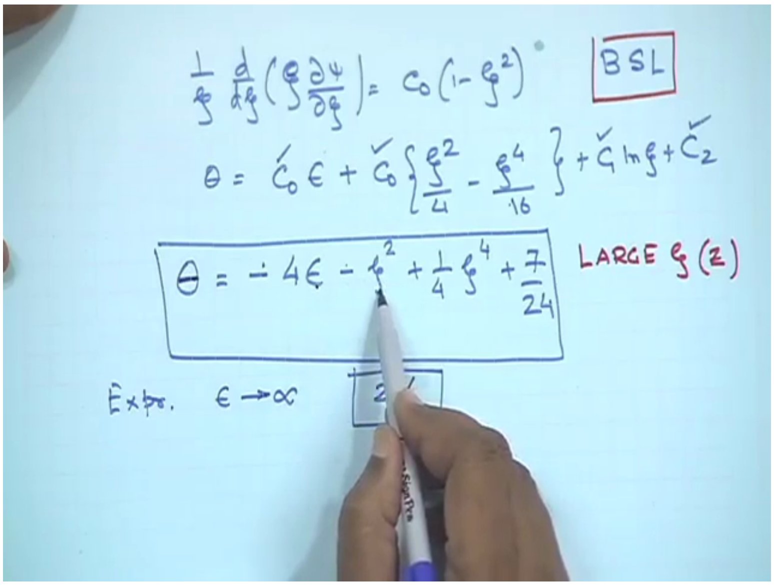

So when you use these 3 conditions and evaluate the boundary conditions, evaluate the, evaluate the constants, C and this psi over here, you would get the integration, this this one often substitute of Theta in here would simply be written as one by zeta d d zeta times psi dell psi by dell zeta equals C0 1 - zeta square. So a partial differential equation like this can be transformed into an ordinary differential equation by substituting an assumed expression with proper justification of the fully developed flow.

And once you substitute this in here, what you get is an ordinary differential equation. This equation can then be simply integrated to obtain the form of the, form of the temperature profile, this is you can do on your own, it is also given in detail in Bird, Stewart, Lightfoot, in the textbook Bird, Stewart, Lightfoot, so you can see how C0, C0, C1, C2 are evaluated with the use of the boundary condition and what you get is Theta equal to - 4 Epsilon, - zeta square + 1 by 4 zeta to the power 4 + 7 by 24.

So this is the approximate solution of the temperature profile for large zeta that means large Z when you assume thermally fully developed flow. And this equation is surprisingly, this expression that you would get is surprisingly accurate, is accurate when Epsilon tends to infinity and the deviation is generally only about 2 percent for most of the cases that we handle. So the equation essentially gives me an expression for the temperature, dimensionless temperature is a function of the R location and as a function of the Z location.

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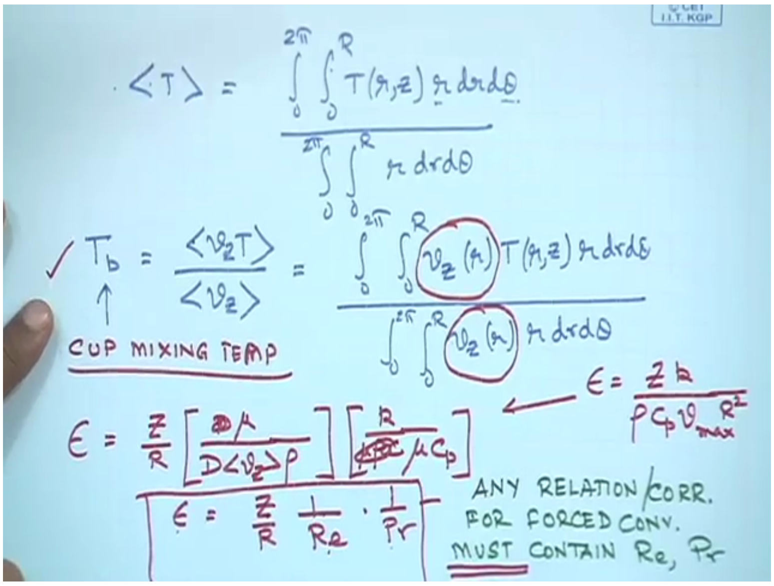

One more quick thing before I conclude this is you can define an average temperature by simply making an area average T r, Z, R dR d Theta divided by the flow area, the same way we do for average velocity which is R dR d Theta. So this is the area average temperature where it it is simply averaged over the area. But the more common one which is denoted by, let us say TB which is VZ T divided by VCZ. And as the expression suggests it is going to be 0 to 2 pie, 0 to R VZ which is a function of R times T r Z R dR d Theta divided by 0 to 2 pie, 0 to R VZ, which is a function of R, R dR d Theta.

So this is the area average temperature and this one is called the cup mixing temperature, which is most commonly used in heat transfer. Since I have multiplied velocity, brought in velocity in here, the cup mixing temperature is the temperature, let us say you are, you have the tube which is heated from the sides, the flow is taking place and suddenly you decide at some location you are collecting the fluid.

As you are collecting the fluid and mixing it, there is a variation in temperature with R but since you are mixing it together, that means you are getting a temperature which is the mixed state or you mixed all the incoming streams together, at the end of the pipe the temperature that you get is known as the cup mixing temperature. And by definition, this is the definition of cup mixing temperature where the difference is the incorporation of VZ in here.

So this is going to be important in many of the fluid mechanics, in many of the heat transfer study that you are going to deal with later on. If I work with this Epsilon which is defined as Z by R and I simply introduce some of the velocities, rearrange the terms, you remember the Epsilon that we had used before, Epsilon was defined as Z K by rho CP V max R square. So from here I simply added a mu, added a diameter and it is simply rearranged in this way.

So your Epsilon is simply going to be Z by R, this is 1 by Reynolds number and this is 1 by Prandlt number. So what this tells me is something interesting, it tells me that the Epsilon, the radial coordinate, the axial coordinate can be expressed in terms of Reynolds number and in terms of Prandlt number with an axial location and this is the total, this is the radius of the tube itself.

So, one would expect that any relation or correlation for forced convection must contain Reynolds number and Prandlt number. So through a simplified analysis we have shown how the temperature profile for flow through a tube with uniform heat flux at the boundaries, at the tube walls can be expressed, the governing equation through the use of energy equation. We identified the boundary conditions, we nondimensionalized the equation and we have seen that a closed form solution can be obtained if we use a large axial location assumption.

At the large axial location, the temperature profile can be thought of as a function of Z and as a function of R. We assume, which has been supported by experiments that the shape of the profile will no longer change, the dimensionless shape will remain unchanged. But the values are going to be different based on its actual location. So the shape of the profile as it moves down with the flow in the tube that is heated will remain same but the values, individual values are going to be different, that is the fully developed condition.

With that assumption one can convert the PDE to an ODE and using boundary conditions one should be able to solve the boundary, the governing, the integration constants. Once you solve that, you simply get a closed form solution of the dimensionless temperature distribution, which is fairly accurate. That is an amazing achievement that with these many assumptions, we still get a temperature profile which is very close to the original experimental profile.

Then 2 types of average temperatures are introduced, one is the area average temperature that we normally use and the 2nd one is the mixing cup temperature where you simply cut the tube at a specific point and mix the fluid at that location and find out what is the mixing cup temperature, cup mixing temperature. That is the temperature which most of the cases are used in heat transfer correlations. And finally what we saw is that the Z location, Epsilon can be, the terms, the definition of Epsilon, the terms there can be reorganised and 2 dimensionless groups will emerge out of that exercise.

One is Reynolds number and the other is Prandlt number. It tells you that if you are dealing with a forced convection situation and if you cannot solve analytically or by other means, the temperature profile, at least you should try to express your data, the dimensionless temperature or other parameters for example Nusselt number, etc. in terms of these 2 parameters. So your experimental data can be fitted with as a function of Reynolds number times Prandlt number.

Now you try to recall all those correlations, relations that you have used for the forced convection. All of them without fail had the Reynolds number and Prandlt numbers into those correlations. And this example shows you not only the use of energy equation but it explains why these 2 dimensionless numbers automatically appear in any relation of forced convection heat transfer.