Transport Phenomena.
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Lecture-39.
Energy Equation.

So we are going to continue with our treatment of the energy equation or the development of an equation of change for a non-isothermal system. And in order to do that, what we have written down is the 1st law of thermodynamics for an open system where the net accumulation of energy inside the system is expressed as the algebraic sum of net heat addition to the system both by convection and conduction. As well as we consider the effect of work done by the system or on the system.

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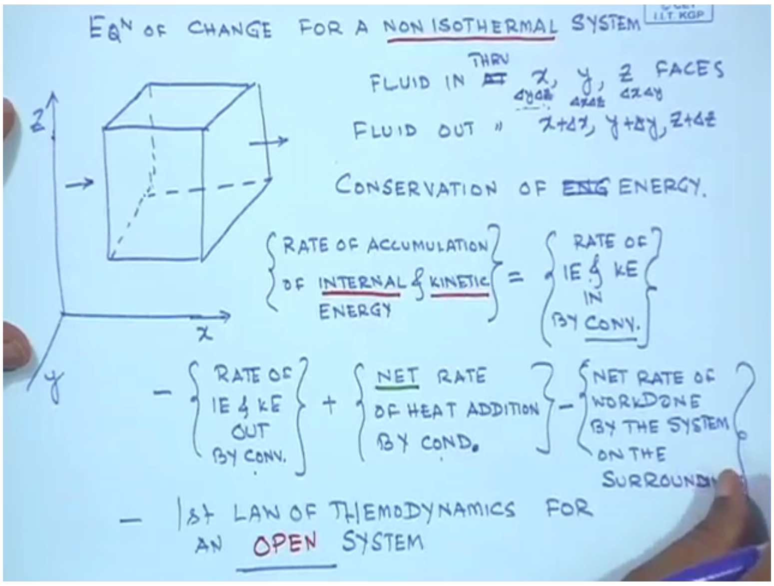

So this is equation which we have written for open system and I am going to continue evaluating each of these terms considering that all in are at X, Y and Z faces. X face having an area equal to Delta X, sorry Delta Y Delta Z. Y face has an area equal to Delta X Delta Z and the Z face is going to have area which is Delta X times Delta Y. So through all these faces energy is going to come in by conduction and by convection. So when we talk about convection, we simply have to multiply the area with the corresponding corresponding expression for, corresponding component of velocity.

So if we have to find out what is the total energy that comes in by convection through the X face, I simply have to multiply the area of the X face with the velocity VX which is a component of velocity in the X direction, multiply that with rho to make it the mass flow rate multiply it with CP and Delta T. So that would essentially give me the total amount of convective heat being added to the control volume because of flow with a component of VX through the X face.

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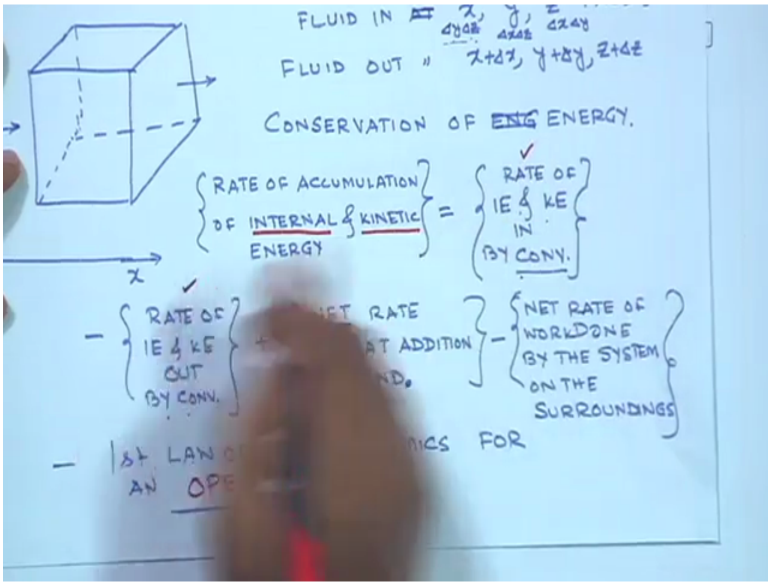

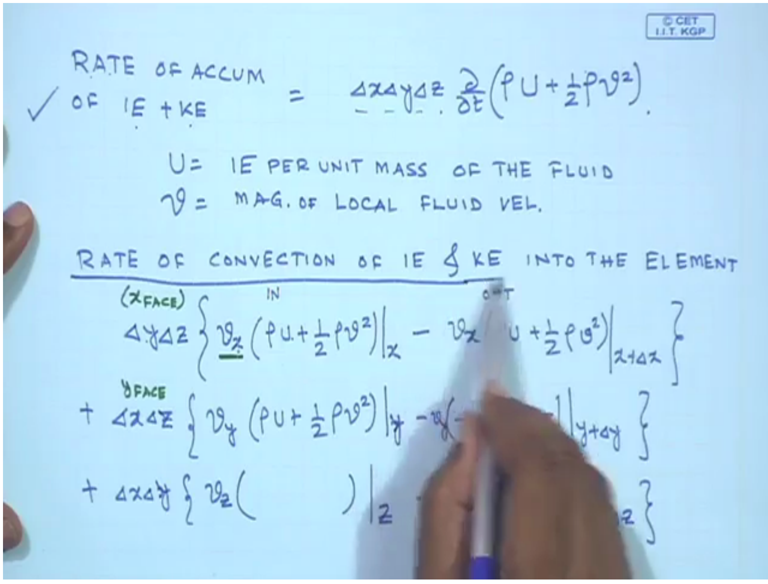

So we are going to, we are going to do that but before let us 1st evaluate the left-hand side of this equation which is the rate of accumulation of internal and kinetic energy present in the system. So the 1st one is rate of accumulation of internal energy and kinetic energy of the system which is the volume Dell X Dell Y Dell Z and the time rate of rho times U + half rho V square. So what is U, U is the internal energy per unit mass of the fluid.

And obviously internal energy as we all know would depend among other things on the temperature of the fluid. And V is the magnitude of local fluid velocity. So if it is a magnitude of local fluid velocity, then half rho V square would be the kinetic energy per unit mass of the fluid. This kinetic energy is going to be the kinetic energy and when you multiply this with Dell X Dell Y Dell Z, you simply get the time rate of change, time rate of accumulation of internal and kinetic energy present in the system.

So my left-hand side of the equation is evaluated. So when we talk about the next term, that is the rate of internal and kinetic energy in by convection and out by convection. So now I am going to evaluate these 2 terms. So the next step is rate of convective, convection of internal energy and kinetic energy into the system into the element that we have chosen of volume Dell X Dell Y Dell Z. So it would simply be equal to Dell Y times Dell Z.

So what I am doing here is this is the X face, in the X face the volume is, the area is Dell Y Dell Z. And I have the velocity expression, the velocity at that point, so Dell Y Dell Y Dell Z VX would give me the volumetric rate of fluid which is entering through the X face. And it carries with it the internal energy and the kinetic energy as we have seen before. So this is metre cube per second, this is KG per metre cube and the U is energy per unit mass.

So what, when you take all of them together, what you are, what you find out if the rate of energy, both the in, both the internal energy and kinetic energy that you are adding to the system because of flow, because of this VX which is the velocity through the X face. Once again just try to think what is the area of the X face which is Dell Y Dell that. What is the velocity through the velocity, that is VX. So Dell Y Dell Z times VX would give you the volumetric rate of flow in the X direction through the X face.

So this volumetric flow will carry some amount of internal and kinetic energy. So U is the internal energy but unit mass, this is metre cube per second, this is KG per metre cube, this is energy per unit mass. So what you get out of this is the rate of energy that is being added, that is being accumulated in the system. So this is the so-called in term and the out term, whatever be the velocity at location X + Delta X, this is the out term and I simply have rho U + half rho, half rho V square where everything is evaluated at X + Delta X.

So this is the net addition of heat due to convection through the X phase. So if this is the end, this is the net addition of heat through the X face, I can simply would be able to write what are the, what is the net addition, this is my Y face and since we are talking about the Y face, the volume, the velocity, velocity component of velocity would be simply VY. And this part will remain unchanged at Y - the same thing evaluated at Y + Delta Y.

So this would be the addition of heat by convection through the Y face and I can write for the Z face as well, where the Z face, this velocity would simply be VZ, this part will remain same and it would be VZ, this part will remain same, except that it is evaluated at Z + Delta Z. So these terms together would be the rate of convection of internal and kinetic energy into the volume element.

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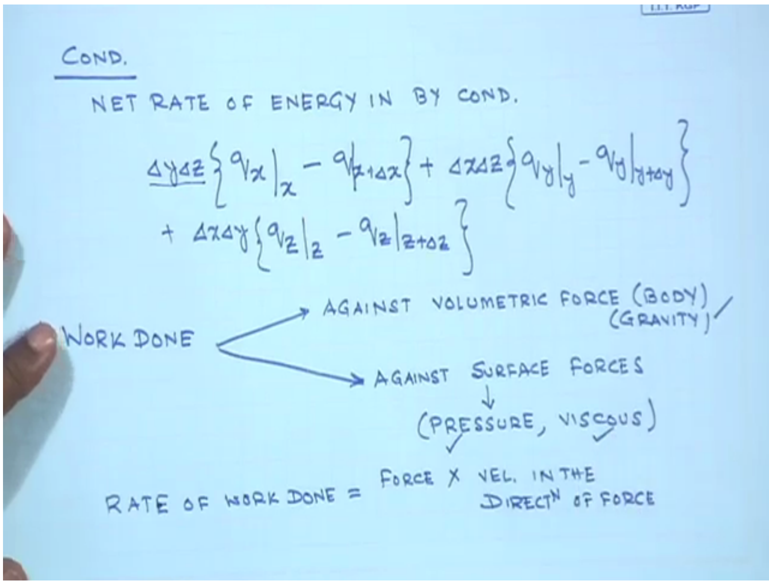

When we talk about conduction, net rate of energy in by conduction would simply be equal to the volume Dell Y Dell Z, the flow of energy by conduction in the X direction evaluated at X - the flux of conductive flow of energy or the conductive energy flux evaluated at X + Delta X multiplied by the area of the X face + Y face having area Dell X Dell Z, here we are going to write the heat flux qY at Y - qY at Y + Delta Y and so on. The other 3rd missing term would be Dell X Dell Y, this is the Z face, qZ at Z - qZ at Z + Delta Z.

For the moment you write 1, you should be able to write the other 2. So the only remaining part is essentially the work done which we have to in this case, so we have evaluated the rate of accumulation of internal and kinetic energy, the net rate of heat addition by conduction, so all these terms are evaluated, except the work done by the system on the surrounding. So the work done can be against 2 types of forces, against volumetric forces which we are also known as body forces, the example of which could be gravity and against surface forces.

And the surface forces are obviously going to be 2 types of surface forces that we will consider, one is pressure which is the surface force and the other type of surface force as we have seen before is a viscous force. So to, the work done by, work done by or on the system is therefore the against body forces and against surface forces. The surface forces are of 2 types, which is, which could be pressure and the viscous forces. Now we are talking about rate of work done and what is work done switches force times distance.

And the rate of work done, that is time rate of work done would be force into distance by time, so essentially it is force into velocity. So the rate of work done which we have to incorporate into the equation can be expressed as force times velocity in the direction of the force, that is important, in the direction of the force. So what I write then is that rate of work done, I identify that rate of work done is equal to force times velocity in the direction of force.

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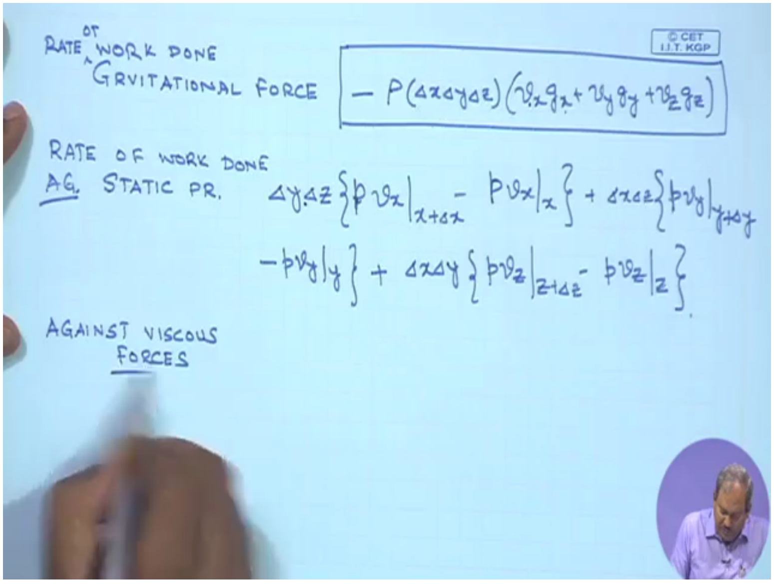

And we are going to write this expression for gravity, for pressure and for viscous. The gravity part is straightforward, the rate of doing work against gravitational forces, so when we talk about gravitational forces, the rate of work done would simply be equals - rho times Dell X Dell Y Dell Z times VX gX + VY gY + VZ gZ. So this is essentially force times the velocity and the - sign is used because the work is done against gravity where V and g are opposed.

So the component of the force is multiplied, component of the force because rho times Dell X Dell Y Dell Z is simply the mass. So mass times gX is the force in the X direction, so force in the X direction is multiplied by the velocity in the X direction. Force in the Y direction is multiplied by the velocity in the Y direction, so therefore their product gives us the gravitational force, the work done against the gravitational force or rather rate of work done against the gravitational force which is given by this.

When we again find out what is the rate of work done against static pressure and the static pressure I have to write what is the area on which I am going to evaluate the effects of this force due to static pressure multiplied by rho VX at X + Dell X - rho VX at X, this is the net force due to static pressure multiplied by the component of velocity, component of velocity, so this is P, pressure. So this P is the, P is the pressure multiplied by the area, so this is the force, force multiplied by the velocity, velocity in the direction of force and this is going to give me the work, the rate of work done against static pressure.

So this is for the X component and I can similarly write for the Y component which is PVY at Y - PVY at Y and the 3rd one is for the Z face, PVZ at Z + Delta Z - P VZ at Z. So these 3 these 6 terms together would give me the rate of work done against static pressure.

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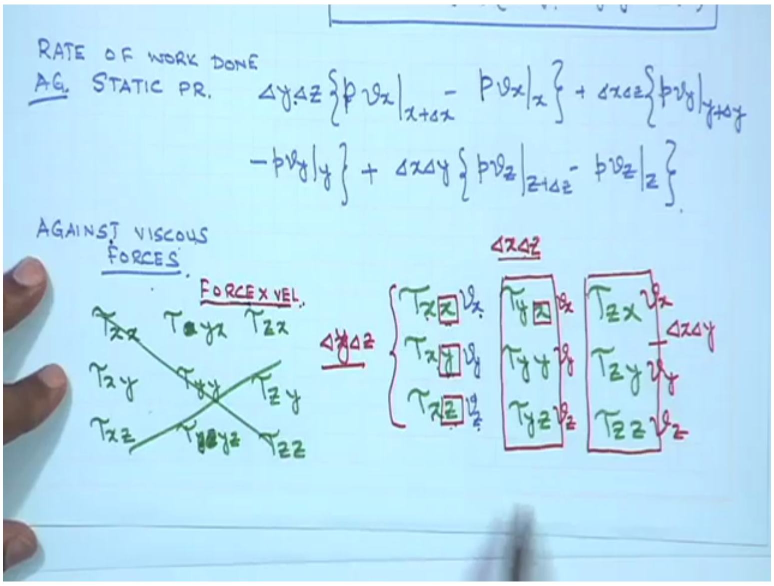

What is going to be against viscous forces? This is slightly more involved and would require careful consideration of the components since Tao is a tensor, the components of Tao are going to be Tao XX, Tao XY, Tao XZ, Tao XY, Tao YY, Tao ZY, Tao ZX, Tao ZY and Tao ZZ. So if you, these are the 9 components Tao XX, XY, XZ, sorry this should be YX, YY, YZ, ZX, Z by, I will write it once again. 9 components are Tao XX, Tao XY, Tao XZ.

The Y Y, on the Y face what they are having is Tao YX, Tao YY, Tao YZ, Tao ZX, Tao ZY and Tao ZZ. These are the 9 components of the shear stress tensor. As I know that force, the work done against viscous forces would simply be force multiplied by velocity in the direction of the force. Now when you went you see in this, case this is X component of momentum being transported in the X direction. So this is nothing but the normal stress. This is Y component of the momentum getting transported in the X direction and Z component of momentum being transported in the X direction.

So when we, the direction of momentum is the 2nd subscript. This is the direction of momentum in these cases. So these are also the directions of momentums. So if we need to find out what is the work done against viscous forces, since all of them are acting on the X face, these 3 are to be the area on which they are acting is Dell Y Dell Z. So the area on which these forces, these shear stress is acting is Dell Y Dell Z, the area on which these shear stresses are acting in the Y face, so it is Dell X Dell Z, the area on which these 3 components of the shear stress tensor is acting would be Dell X Dell Y.

I think it is clear, but I will still go through it once again. This is X momentum being carried in the X direction, Y momentum in X direction, Z momentum in X direction, so all of them are acting on the X face, the X face having an area equal to Delta Y Delta Z. But similarly all of them are acting on the Y face which has area with areas as Dell X Dell Z, all these 3 components act on the Z direction and Z face, so the Z face having area Dell X Dell Y.

Now since we are talking about X component of momentum in this, the velocity would simply be equal to in this specific case would be VX which is the component of velocity in the direction of momentum change, in the direction of force. Similarly for this case the corresponding velocity component would be VY and for this the corresponding velocity component would be VZ. Because you are talking about Z momentum, so the force is in the, forces in the Z direction, therefore the, in order to obtain the force due to shear, due to the component of shear, I need to multiply that with the velocity in order to obtain the work done, the rate of work done against viscous forces.

So 1st identify the 3 components of shear which are acting on the X face, then we realise that the momentum, the force in the X and Y and in the Z direction, since the forces are the X, Y and Z direction, I must multiply that with the component of velocity in the X direction, component of velocity in the Y direction and the component of velocity in the Z direction. When we when we come to this part, here again the deciding factor is X, that is the X component, so I multiply it with V and I multiply this with VY and I multiply this with VZ.

So these are the forces in 3 different directions due to the components of shear acting on the Y face and therefore this is straightforward now, this is VX, VY and VZ. So this is essentially giving me the total amount of force due to shear acting on all the 3 faces, all the faces of the assumed, assumed control volume.

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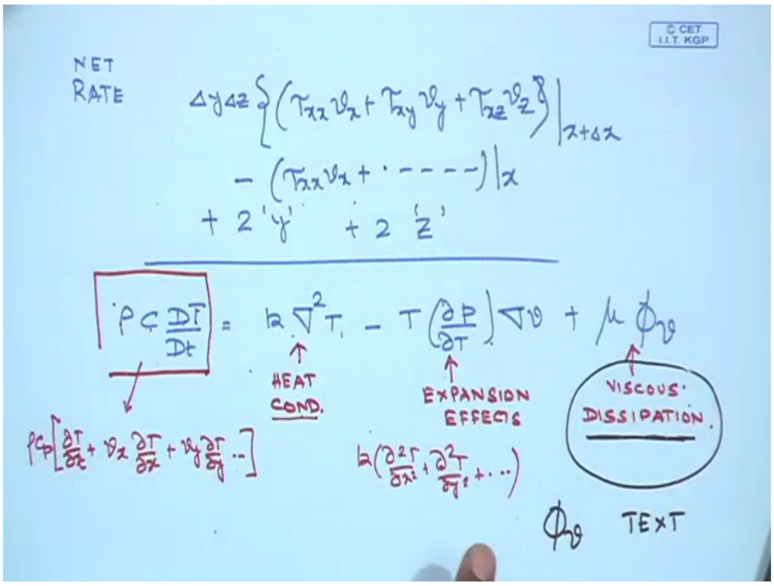

So the rate when we talk about the net rate, it would simply be Delta Y Delta Z Tao XX VX + Tao XY VY + Tao XZ VZ, this is one part evaluated at X + Delta X - Tao XX VX + all these terms are evaluated at X. And similarly I have 2 more terms for Y and 2 more terms for the Z direction, so essentially the one which I have explained before. When you put, so now I have, now I have all the, all the terms required to write the energy equation.

I know the left-hand side, what is the, what is the net rate of accumulation of internal and kinetic energy inside the system, the net rate by which energy is added, both internal and kinetic energy is added by conduction and by convection. And I also find out what is the net rate of work done by the system or on the system. The work is done against body forces which is gravity, expression for which we have written, it is also against pressure and against shear.

Okay, so these are the 6 different components of the entire 1^{st} law of thermodynamics written for an open system. This derivation is given in detail in your textbook of word, Bird, Stewart and Lighfoot transport phenomena textbook. So you can see the different, each of the each and every step in there. But I am not going to write all those steps, I have just given the fundamental concepts and with the fundamental concepts, after simplification you can write the energy equation in a more compact vector tensor notation.

So based on the concepts I directly jump to the final result and in between steps, no new concepts are involved, it is just an algebraic simplification of the complex expression that we have obtained and subtraction of the mechanical energy equation from that, from the total equation to obtain the more commonly used energy equation. So once again I reiterate that I am not doing all the steps which are there in your textbook but no new fundamental concepts are involved from the point where I identified all these terms to the point where the final equation is written.

So I am going to write the final energy question for an open system. So this equation essentially tells that the temperature of a moving fluid element changes because of heat conduction, because of expansion affects which and because of viscous dissipation. Here this capital D refers to the substantial derivative, the one which we have we have seen before. So the velocity of the fluid is embedded in this expression. So if you if you expand this, you are going to have VX times Dell T by Dell X + VY Times Dell T Dell Y and so on, multiplied by rho CP times Dell temperature by Dell time + all these terms in there.

So the convective effect is incorporated in the left-hand side of this equation where rho is the density, C is a heat capacity and D DT is a substantial derivative of temperature. So this is going to be, this change is an effect of heat conduction, net heat conduction to the system and we have identified this from your, from the heat transfer the study. So this is nothing but, when you add Fourier’s Law, when you incorporate Fourier’s law, so this for a Cartesian coordinate would simply be K times Dell 2T by Dell X square + Delta 2T by Dell Y square and Dell 2T by Dell Z square.

This is the expansion effect and this is important, which is known as the viscous dissipation. The expression for phi V is, can be found in your textbook for cylindrical systems, Cartesian coordinate systems and for spherical system. This is generally not important, I have discussed when viscous dissipation is important, only when you have high-speed flow through a narrow conduit of a viscous material, the effect of heat generation by liquid friction or viscous dissipation is important.

So only, for those cases in which you have flow, high-speed flow maybe through a narrow conduit and of a material which is highly viscous, this phi V, the dissipation function needs to be incorporated, included in the energy equation. So for all practical purposes, we do not need to consider the last term on the right-hand side, that is the viscous dissipation term. And if there is an heat generation term, it simply can be added to the right-hand side of the of the energy equation that I have written.

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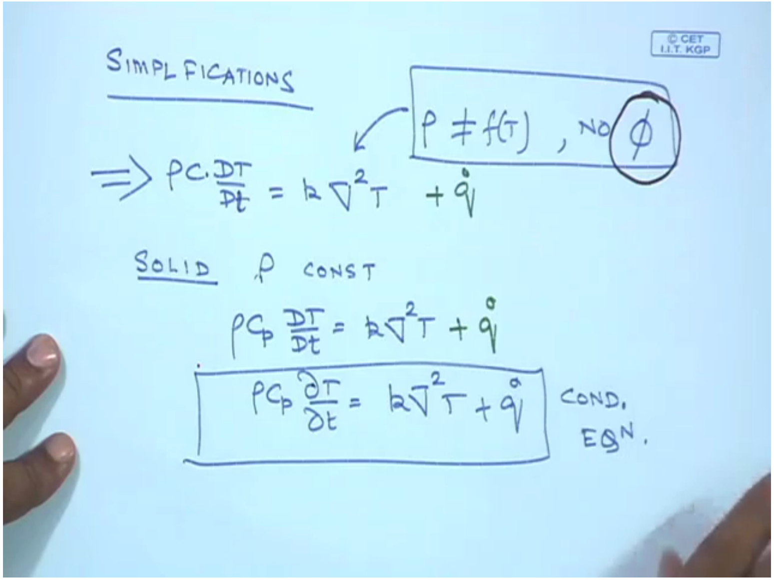

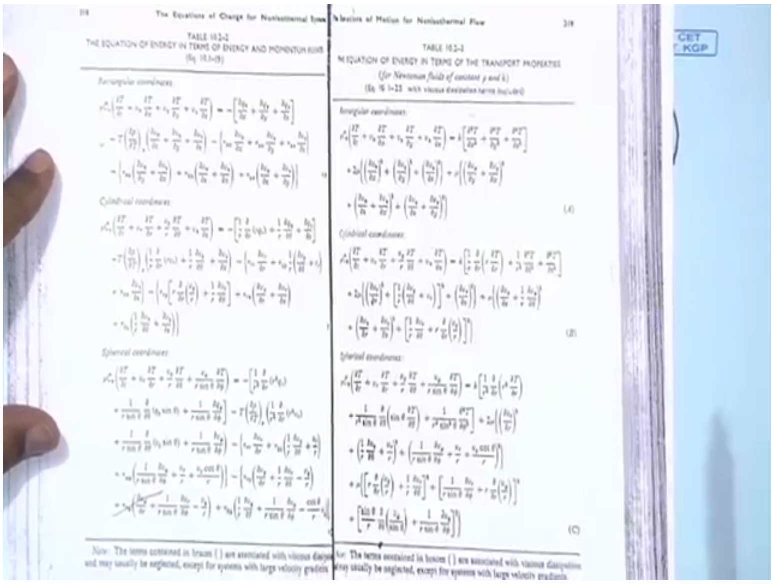

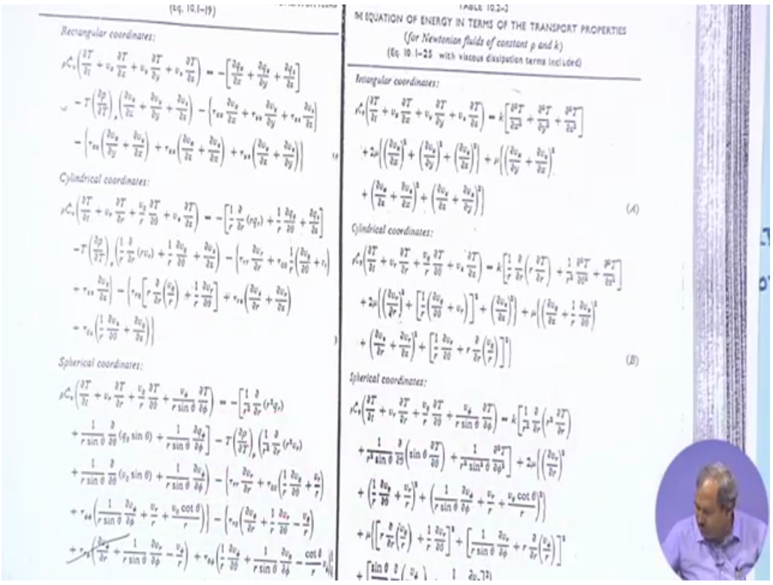

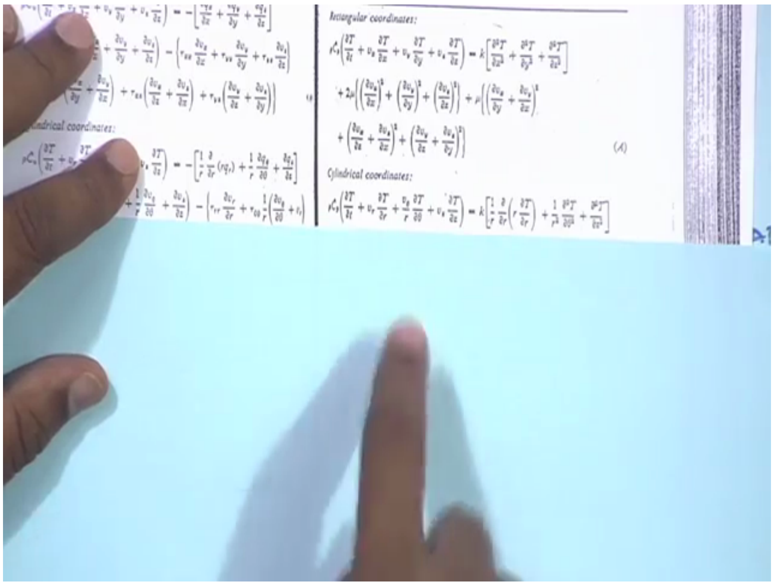

So this expression therefore is the most, most common form, most general form of the equation of energy and some simplifications of this equation can be obtained, so the simplifications that one would see, the simplifications that one would see of this equation would consist of rho C D temperature by D time and if you have a fluid where rho is not a function of T, if rho is not a function of T, then what you would get is simply the 2nd term of this would be 0.

And you have rho C is equal to K times Dell square T. There is no phi, no viscous dissipation and there is no effect of, effect of temperature or density. So if that is these 2 conditions are met, then what you get is this expression. So on the left-hand side you have the substantial derivative which includes the velocity, on the right-hand side you only have the conductive flow and if it is a heat generating systems, you simply add q dot, which is heat generated per unit volume to it.

So that is one way of, that is that is most, that is one of the common forms of the energy equation that are used. And if it is, if you are applying it for a solid, if you are applying it for a solid, then rho is usually a constant and if rho is usually a constant, and we can set velocity to be equal to 0, so it is like going to be rho CP D temperature by D time is equal to K Dell square T. And if you have a heat generation, you have q dot in here. But since the velocity is not there, so for a solid, so what I can simply write then is tell temperature by Dell time is equal to K Dell square T + q dot.

So this is the same as the conduction equation that we have derived before. So from the general energy equation, the special cases can be obtained and in all these cases we have neglected phi, that is the viscous dissipation, which may be included in these equations for special cases where you have high-speed flow of a viscous material through a narrow conduit giving rise to large values of velocity gradient. So the expressions of phi are there in your texts and you can also see the different forms.

This is from the text, all, whatever I am teaching in this part is from Bird, Stewart and Lighfoot and the energy equation in its full form for rectangular coordinates, cylindrical coordinates or spherical coordinates are given in the textbook that you would see any term that contains mu, that over here, the term over here and the term at this point, this entire part is due to viscous dissipation. So if for a system viscous dissipation is unimportant, you can simply drop this part of the energy equation.

So you are only dealing with these 2, these 2 combination of terms. So any, any these terms contain mu are for viscous dissipation which generally we neglect. And if we, if we do neglect this, then the equation is exactly same to the conduction equation which we have which we have obtained, which we have obtained before. And if it is for solid, then there is no question of VX, VY and VZ, so we will only have the 1st on the left-hand side which is rho CP Dell temperature by Dell time and these 3 terms on the right-hand side + q dot term which if there is an heat generating system.

So this is what we have for rectangular coordinates, what we have, not taking into account the viscous dissipation term for a cylindrical coordinate and this is for spherical coordinates, again these terms are not to be taken into account as they are for viscous dissipation. So for the cylindrical system you simply start with this equation, cancel the term which are not there and you would see that you would obtain the same form, the same expression for forced convection that we were dealing with at the beginning of this class.

But I will pick that up in the next class and we will show you that starting with this equation, the cylindrical coordinate energy equation how quickly and how easily we can get to the governing equation for forced convection in a tube. And once we have this tool, the rest of, for rest of the problems, at least coming to the or obtaining the governing equation.

From the physical understanding of the problem you should be able to write the boundary conditions and then we have to see whether it is possible to obtain analytic solution based on the form of the governing equation or we have to take recourse of specialised solution techniques and or numerical techniques if that is required.

But the governing equation part should not be any problem from this point onwards. And that was one of the objectives of the course transport phenomena, is that you should be able to obtain the model equation of a process without any problem.