Transport Phenomena.
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Lecture-38.
Forced Convection.

So we would start our analysis of forced convection and we assume that when we have forced convection in a tube with a constant heat flux that is supplied through the walls, so there can be 2 different types of conditions, one is constant temperature condition where the tube walls are maintained at a constant temp at the 2nd, more common is which is a constant heat flux condition, where through an external agency, constant heat flux is provided through the tubes, so that you walls.

So therefore as the liquid starts to move in the tube, its temperature will keep on changing, will keep on increasing. And since we have flow and the flow, we will assume it to be in laminar region and the flow is taking place as a result of imposed pressure gradient as well as gravity. So we can think of a vertical tube in which the liquid is flowing downwards because of imposed pressure gradient, higher pressure the top as compared to the lowermost point and the gravity is also acting in this direction, therefore the entire flow, even though it is in laminar region, it will be under the effect of surface force which is pressure and body force which is gravity.

We know from our study of fluid mechanics that this type of situation gives rise to a parabolic velocity profile, where the velocity starts to vary as as a function of radius, as a function of radial distance from the centre line. So this parabolic velocity profile needs to be known a priory, that means whatever be the flow condition, we should be able to solve it to obtain profile of the velocity distribution inside the conduit in order to use, in order to use that velocity profile in our analysis of the energy balance of the system.

So the 1st job is to obtain is to decide about the shell across which we are going to the going to find out what is the rate of heat in, rate of heat out, + if there is any generation of heat inside the shell which is not there in this, for this specific problem. And as a result of all these, there would be energy stored, changing, rate of change of energy stored in the system that would be the form of the equation. So heat in - heat out + generation is equal to the rate of storage.

When we talk about heat in into that shell, we appreciate that if this is the shell, then there is flow in this direction, so with flow comes some energy which we call as the convective transport of energy. And since there is a temperature gradient with the axial position during flow, since the tube walls are heated, there would be temperature gradient in this direction which would also create a flow, this time conductive heat flow in the Z direction where the Z is the axial direction.

I have a tube wall like this and the temperature would obviously be maximum near the tube and as I move in this direction, that temperature would progressively decrease. So there is a gradient in the R direction, gradient in temperature in the R direction as well. So there would be conductive flow of heat from a value of higher R towards the value of lower R. That means from the tube wall, there would be flow of heat towards the centre line.

If the system is different, that means a hot fluid is flowing and the tube wall is being cooled, then the direction of this heat would simply be in the opposite direction. We have we are deciding about the smaller dimension of shell based on which is the direction in which the temperature is changing. So we realise that the temperature in this case is changing with Z as well as with R. So any shell that we assume must contain 2 dimensions, to smaller dimensions delta and delta Z.

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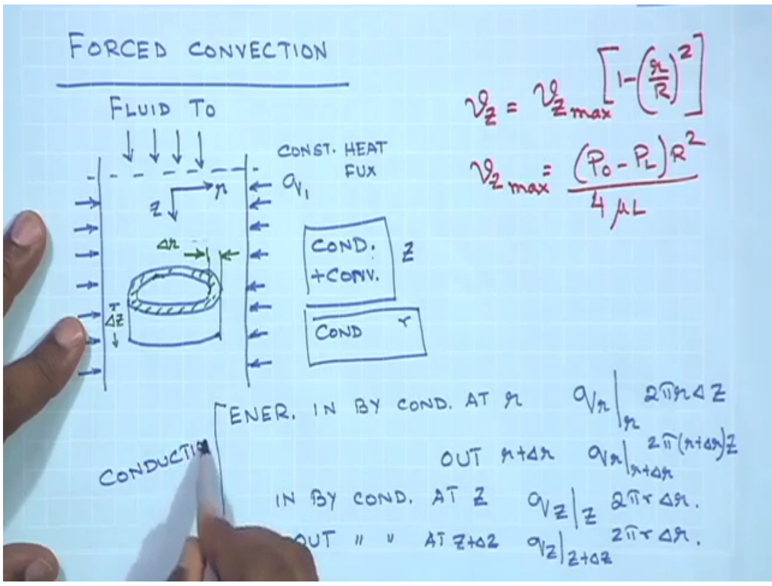

So in order to tackle this type of problem in which the temperature can be a function both of R and of Z, the shell, the assumed shell across which we are going to going to make the energy balance will probably look something like this, which we have drawn in the previous class. So we have a fluid which is coming in and some amount of heat, constant heat flux Q1 being added to the side walls. So it is a case, it is it is a problem of conduction and conviction, especially if you consider the Z direction in which, since the temperature is changing and I have, we have flow.

So we will have both conduction and convection, wherever, whereas, since we are assuming that it is a 1-D flow, there is no VR component. So if there is no VR component of velocity for the flowing fluid, then there cannot be any convection in the R direction. So the velocity is 0 in the R direction and therefore no convection. However the temperature is changing in the in the R direction, so therefore the there would be always conduction in the R direction.

So the conduction + convection in the Z direction and only conduction is going to be there in the R direction. So this is a situation and therefore the shell that we are assuming is of length delta Z and it is a radius, the change in radius, the cylinder, the annular area of the cylinder, the radius, difference in radius is delta R. So we are considering a annular cylinder, annular shaped cylinder inside the flowing fluid and we are going to, we are going to find out what is the flow of heat into this control volume, into this shell due to conduction and convection.

So we are 1st going to write the 1st condition which is energy in by conduction at any R. And we assume that energy flows in this direction from R to R + delta, that is the usual convention that we always use, in is always at the lower value, so N is always going to be at R and the out is always going to be at R + delta R. And once we use this specific convention, then stick to it, then the profile will automatically adjust itself based on the boundary conditions that are provided.

So the energy in by conduction at r, if I take this the flux to be QR which is evaluated at R and it is, it has to be multiplied by the area, so it is going to be inside area of the annulus, inside area of the annulus would simply be equal to 2 pie R times delta Z. So think of it in this way that when you have, when you have the annular area and you are talking about flow of heat in this direction, so the area that it faces will simply be equal to twice pie R times delta Z, so where delta Z is the length of the Shell that is assumed, R is the radius, inner radius, so therefore twice pie R times delta R would give you the area through which the heat conductive heat in the R direction is entering the imagined shell.

So the in term would be twice pie R times delta and the out term would obviously be at R + delta which will be QR evaluated at R + delta R times twice pie R + delta R times Z. So that is the out term at Z. Similarly in by conduction at Z would be equal to, which is Q times Z evaluated at Z multiplied by twice pie R times delta R. So twice pie times delta R is essentially the top annular area. So the top annular area multiplied by the heat flux, conductive heat flux in the Z direction would be provided, would can be written as QZ at Z times twice pie R times delta R.

And therefore out by conduction at Z + delta Z, that is at this point would be QZ evaluated at Z + delta Z times twice pie R times delta R. So these are all for conductive, these are all due to conduction.

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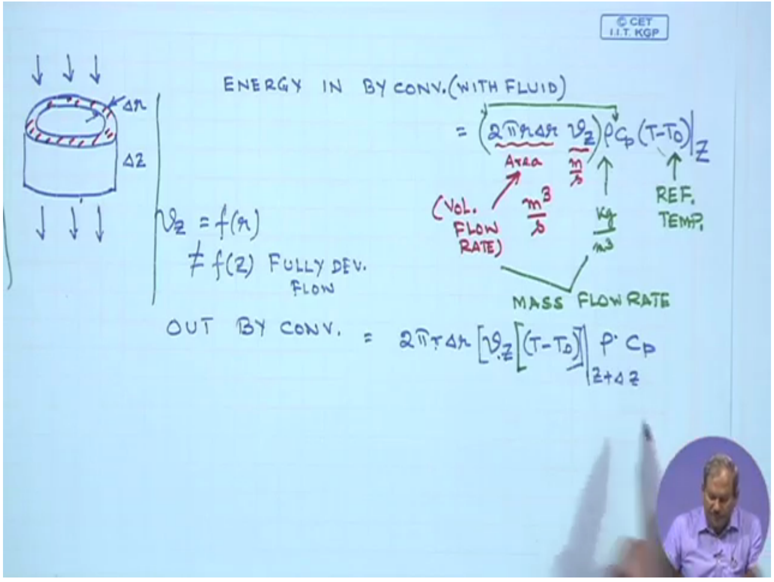

When we think about convection, I am, I am drawing the figure once again. I have flow, therefore convection, and flow which goes out, this is delta Z and this is delta R. So the heat that energy which is coming in by convection, that means with the fluid would be equals twice pie R delta R times VZ. I will mark this as one part of it times rho CP time T - T0 at Z. Let me explain this once again but I think you all will remember from your study of fluid mechanics that twice pie times delta R essentially is this area.

So when you multiply the annular area with the velocity which is in meters per second, so essentially these 2 together would give you in metre cube per 2nd which is nothing but the volumetric flow rate. So this entire part is the volumetric flow rate. When you multiply it with rho which is the density, that is in KG per metre cube, so this together would be the mass flow rate. And the mass flow rate, so it is M dot, this whole thing together up to this point is M dot which is the mass flow rate times CP times delta T.

So this essentially and this T0 is some reference temperature because we cannot have energy, we cannot express energy in explicit form, it is always expressed in relative and therefore this T0 is some reference temperature. You can define any temperature as a reference temperature as long as you are consistent and you use the same value of temperature everywhere.

So it is M dot CP delta tee evaluated at Z, that is energy in by convection and therefore the energy out by convection which is at this point would be simply was the same area twice pie times delta R, this velocity which is VZ, the temperature difference which is T - T0, all these, the entire thing is evaluated at Z + delta Z and we have Rho and CP. So twice pie R delta R would be the mass flow rate, would be the volumetric flow rate multiplied with rho you get the mass flow rate and then CP delta T.

However this T - T0 is evaluated at Z + delta Z. This VZ is also evaluated, the velocity is also evaluated at this point but from our study of fluid mechanics we understand that VZ is a function of R only and VZ is not a function of Z. So if VZ is not a function of Z for fully developed flow which we have assumed in order to obtain our expression for the velocity profile, therefore this VZ can be taken out of this this sign and therefore your T - T0 which, where the T is changing with Z as you move in this direction, the temperature will change.

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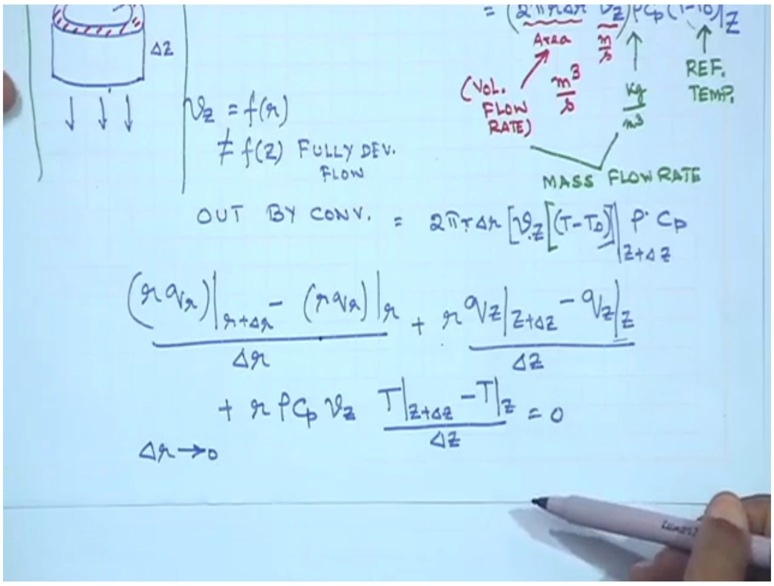

So T is a function of Z but VZ is not a function of Z, so we can take the VZ out of this. And T0 is just the datum temperature. So we add the all the 4 terms that we have, all the 4 terms that is 4 terms for conduction in and out, and 2 terms for convection in and out. So when we do that and express and divide both sides are twice pie R delta R, what we get is R QR evaluated at R + delta R - R Q R at R divided by dell R. This is essentially the net addition of heat by conduction +, this is the net addition of heat by conduction in the Z direction.

And also we have the convective term VZ, this is T at Z + delta Z - T at Z divided by delta Z is equal to 0. So this is the conduction in the R direction, net addition of heat by conduction for heat flow in the R direction. This is net, this part is net addition of heat by conduction in the Z direction and this is the amount of convective heat flow into this control volume by convection.

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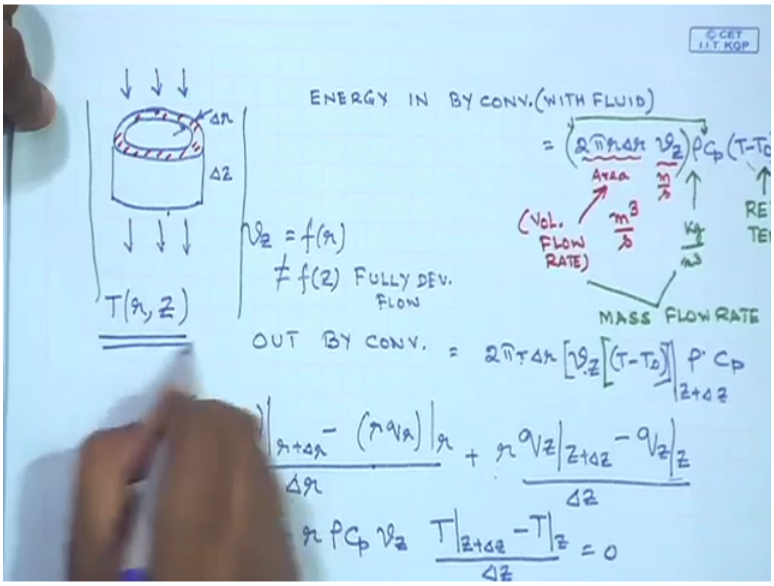

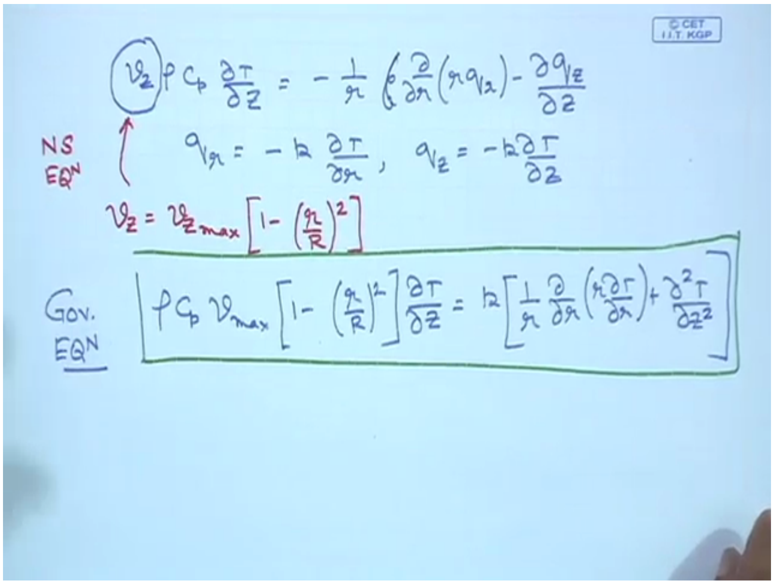

So when the limit you take, when the limit you take delta R tends to 0, you convert this difference equation into a differential equation and the differential equation would be rho CP dell temperature by dell Z is equal to -1 by R dell dell R of R QR - dell QZ by dell Z. So the right-hand side refers to the conductive heat flow, the left-hand side refers to the convective heat flow. And when you put QR using Fourier’s law as -, these are all flux, so QR is dell T dell R and QZ is equal to - K dell T dell Z, variation in temperature with R and variation in temperature with Z.

And we understand that in this system the temperature will be a function of Z as I move in this direction the temperature will increase and the temperature will also be a function of R. As I move closer to the wall, since the walls are heated, the temperature will be more. So T is a function of Z and T is a function of both R and Z. So this is something we have to keep in mind and therefore I write this equation, put the Fourier’s law, put the Fourier’s law in here.

I am sorry I missed a VZ in here, this VZ should also come in here. So we put the equation of qR and QZ on the right-hand side and for VZ, from our study of fluid mechanics we know that VZ is equal to VZ Max times 1 - small r by capital R where capital R is the radius of the tube. We plug this in from the Naviar Stokes equation, solution of Naviar Stokes equation, then what you would obtain as the final form of the solution as rho CP V max which will obviously be at the centre line.

1 - small r by capital R whole square times dell T dell Z is equal to K times 1 by R dell dell R of R dell T del R + dell 2 T by dell Z square. So this therefore now becomes my governing equation. And this governing equation will have to be solved with appropriate boundary conditions which we will discuss later on. The point that I would like to make here is that cannot to obtain this governing equation, we have to go through a complicated process of finding out what is going to be my smaller dimension for the assumed shell.

And since velocity, since the temperature is a function both of R and Z, therefore for this kind of two-dimensional temperature variation, the formulation of the entire problem becomes (()) (20:05) magnitude more difficult than the case where the temperature is a function only of one direction. So for a very simple geometry of flow through a tube when the tube walls are heated, we see that we are having problems in visualising this shell and makes, we are we are we have to ensure that we are putting the values correctly, the expressions correctly and finally we arrive at a solution, a complicated, arrive at a complicated governing equation starting with the fundamentals.

We are, we cannot expect to do this every time we come across a problem. So anytime we see a two-dimensional or even a three-dimensional problem, if you have to go to this process, then that is repetitive, that is unnecessary and there must be a more general method to solve for situations like this which would necessitate the formulation, the development of a generalised equation which can be used for any geometry steady on an steady and so on.

So this is the background, this underscores the requirement of a generalised statement which I am going to start now. And I will come back to this problem, I will come back to this problem and show you that from the generalised equation of energy conservation in a system where flow in and out, where, it is an open system where we allow the fluid to come in and leave. So if we have that kind of a generally questions, it can very quickly be resolved to obtain the governing questions like this which we have obtain after a long series of analysis, thinking and so on.

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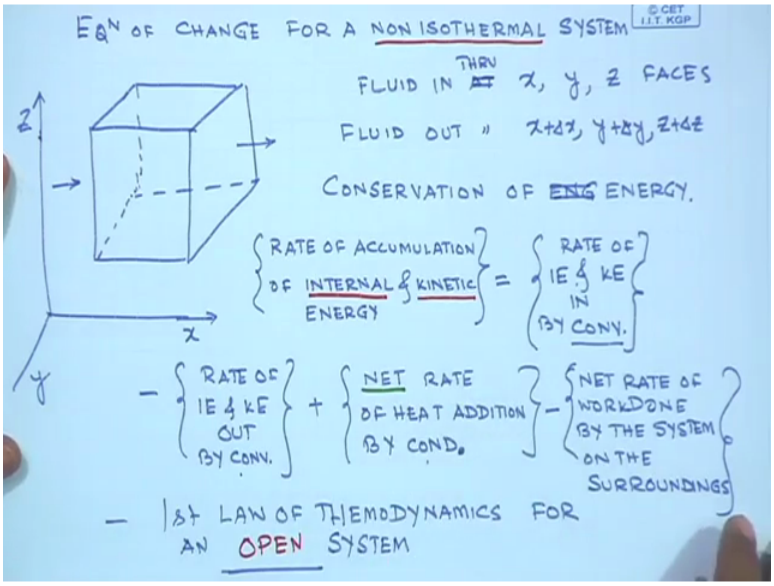

So we start our next next assignment but I will come back to this equation, next assignment where we are going to derive at least conceptually the equation of change for a non-isothermal system. So this is the keyword, it is a non-isothermal, non-isothermal system and as before we assume a stationary volume of as I said before, so this is stationary with respect to X, Y and Z. And fluid is allowed to come in and go out, so fluid in at X, Y and Z faces, through X, Y and Z faces.

And fluid out through faces at X + delta X, Y + delta Y and Z + delta Z. So this is, this is how the fluids can come in and go out of this control volume. So if I write the equation, energy equation, the total, the conservation of energy, if I write the conservation of energy, energy for such a system, for such a system, then one can write as rate of accumulation of internal and kinetic energy.

So this is the rate of accumulation of energy and look that I not only have taken into account into energy, I also, I am also going to consider the kinetic energy of the system. So this is the total energy of the system and this is going to be equal to the algebraic sum of rate of internal energy and kinetic energy in by convection - the rate of internal energy and kinetic energy out by convection.

So since we have a flow, then with the flow, some amount of internal energy and because of the flow, the amount of kinetic energy is entering the control volume and also going out of the control volume. So this is the net addition of internal and kinetic energy to the control volume because of convection. Then I can do the same thing here for the conduction and in the, in the case of conduction I am not breaking it into 2 parts, I am simply using the word net which essentially tells me it is the difference between in and out.

So the net rate of heat addition by conduction, this is by conduction. However there is an extra term which we are putting in here is the net rate of work done by the system on the surrounding. So let us see slowly what we have done. I am writing the energy conservation equation, which simply says that the rate of in, energy in and by in I understand it is going to be convection and it is going to be conduction - rate of energy out by convection and by conduction, so these terms together, it is essentially the net rate of internal and kinetic energy which has added to the control volume.

And the means, the mechanisms by which this net addition is going to take place is a combination of both convection and conduction. So depending on the situation at hand, I can have both present or I can have just one present in the system. So I can never stop conduction if there is a temperature difference, so therefore it is going to be either conduction + convection or only conduction if you are talking about a solid system. So thus, if the if the system does work, I am talking about the internal and kinetic energy, so if the system does work or the work, work is done on the system by some agent in the surrounding, then its internal energy or the total energy of the system will change.

So if the system does work, then its energy gets reduced, if work is done on the system, its energy will increase. So when we talk about the net, the rate of accumulation of energy, both internal and kinetic energy, net, the rate of accumulation of energy inside the control volume, it must be equal to the net rate of heat additions, energy, I should not say energy, heat, the net rate of energy additions by means of conduction and convection and an additional term which will tell us, which will give us the rate, the rate of work done by the system or on the system.

If it is by the system, it will be negative, if it is on the system, it is going to be positive. So this equation what you see over here is nothing but the 1st law of thermodynamics for an open system. And when we talk about open system, that means we are allowing fluid to come in and go out of this. So from this generalised energy equation we can they can subtract the mechanical energy equation and we can obtain the more commonly used thermal energy equation.

So the equation that I have presented, that I have shown you is for the total energy, which contains both the thermal energy part as well as the mechanical energy part. So from this equation I am going to subtract the equation for mechanical energy and what I would obtain out is a more commonly used equation of thermal energy. So the thermal energy balance equation considering both conduction, convection as well as the work done which is nothing but the 1st law of thermodynamics for an open system is something, is the generalised treatment of energy transfer in a system which should give rise to the equation that we are looking after, a general equation for energy transfer.

So this is the one which I am going to continue in the next class. And we will revert to the problem of forced convection in a tube with constant heat flux. And we will see how quickly one would be able to obtain the governing equation by choosing the right component of energy equation and cancelling the terms which are not relevant for the problem at hand, the same way we have done for the Naviar Stokes equation.