Transport Phenomena.
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Lecture-37.
Transient Condition (Continued).

So we are going to solve a problem on transient conduction. The problem that we are going to solve, it consists of a thin cylindrical wire and the cylindrical wire has some resistance, electrical resistance. It is submerged in an oil bath whose temperature is lower than that of the wire and while the current flows through the wire the the convection coefficient for this specific case is provided.

So a a wire whose electrical resistance is provided, the amount of current that flows through it is given, it is submerged in a liquid bath, oil bath whose convection coefficient for this case is known. What we have to find out is what is the steady state temperature of this electrical wire and how long does it take for the wire to reach within 1 centigrade, 1 degree centigrade of the steady-state value. So I have written down the problem in this way.

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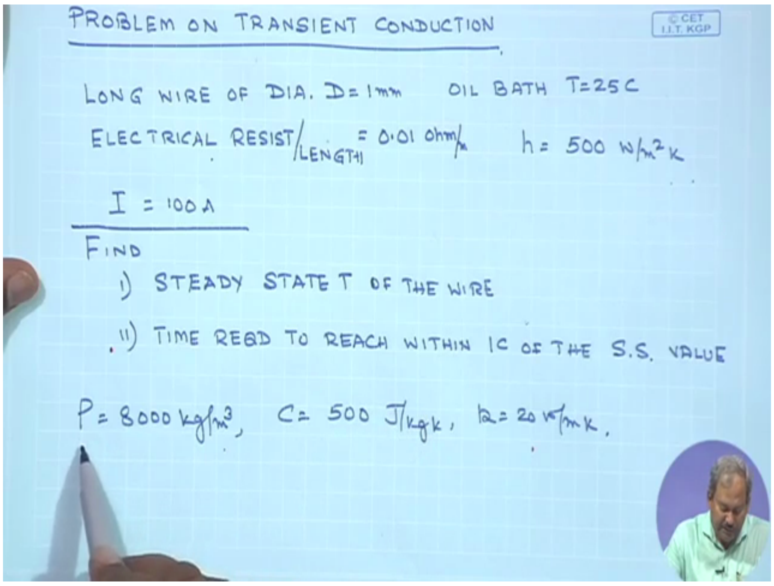

What I have then is, we have a long wire of diameter 1 millimetre, the electrical resistance per unit length of this wire is 0.01 ohms per metre, it is it is submerged in an oil bath whose temperature is 25 degrees centigrade and the entire process is governed by convective heat transfer coefficient of 500 watts per metre square Kelvin. And the current which flows through this wire is 100 angstrom, sorry 100 ampere.

What you have to find this what is the steady-state temperature of the wire and what time is needed in order for the wire to reach a temperature which is going to be within 1 degree centigrade of the steady-state value which you have calculated in the 1st part. The density of the solid material of the wire, material of construction of the wire is 8000 watts per metre cube, 8000 KG per metre cube, the heat capacity is 500 Joules KG Kelvin and the thermal conductivity of the solid material is 20 watts per metre per Kelvin.

So these are the 2 things which we have to find out. But before we start this problem, one 1st have to find out what is going to be the Biot number in this specific case such that we would be able to determine whether or not lumped capacitance model is valid. So Biot number is, this is a cylindrical system, so ideally we should be R0 by 2 which is a characteristic length but as if mentioned before, in order to be on the conservative side for calculating the Biot number and to decide about the applicability of lumped capacitance model, generally the characteristic length is taken to be the length scale across which you get the maximum change in temperature.

So obviously when a thin wire is placed in, in an oil bath, the maximum temperature is going to be at the centre line of the wire and as we go towards the Periphery, the temperature will decrease and right at the Periphery of the solid cylindrical wire, the temperature is going to be the least before it starts convection to the cold oil bath. So the maximum difference in temperature, maximum drop in temperature is taking place over a length scale which should be equal to the radius of the cylindrical wire.

So therefore the characteristic length for this specific case in order to remain conservative is taken to be equal to R0 which is the radius of the cylindrical wire and Biot number is calculated based on this value of R0 and not R0 by 2 which was predicted, which was predicted by the formula for the for the characteristic length.

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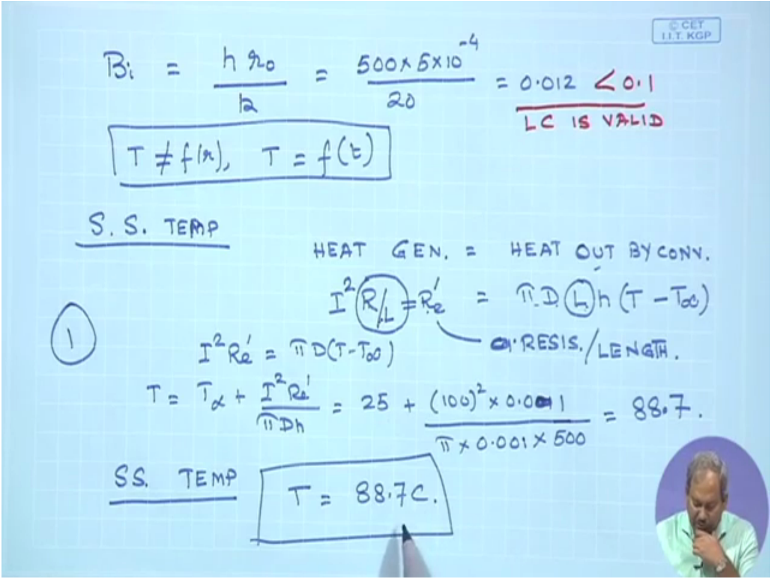

So Biot number is HR0 by K and when you plug in numbers be equal to 500 into 5 millimetres, so 5 into 10 to the power - 4 by K which has been given as 20 is 0.012 and it is less than 0.1, so which shows that LC, the lumped capacitance model is valid. If the lumped capacitance model is valid, then T is not going to be a function of R and T is going to be function only of time. That is what essentially lumped capacitance is.

Now when we write the, when we think about the steady-state temperature, the steady-state at at the steady-state, whatever heat that is produced, heat generated must be equal to heat that goes out, heat out by convection. So heat out by convection would simply be equals pie D L H temperature of the of the solid which is a function of time but not of space, T times T infinity. And the amount of heat that is produced is I square R square where R is the resist, R is the resistance of the wire.

So I bring this L to this side and make it as R by L. So when I make it R by L, this is nothing but the resistance per unit length of the wire and this is how I denote it. So resistance per unit length of the wire. So initially I started with I square R equals pie DL, the surface area times H T - T infinity brought the L to this side and make R by L equal to RE prime which is the resistance per unit length of the wire. So I have the relation as I square RE prime to be equals pie D times T - T infinity.

When you plug in the numbers, you simply get T equals T infinity + I square RE prime by pie DH and you put the value of T infinity, put the value of A to be 100 ampere square, RE prime is 0.0, sorry 0.001 ohms per metre and pie, the diameter is 0.001 and the value of the conductive heat transfer coefficient is H which gives, which would give the temperature to be, the steady-state temperature to be a function, to be equal to 88.7. So the steady-state temperature of the wire by a simple heat balance would be equals 88.7 degrees centigrade. This is the part one of the problem which we are dealing with.

So in the next part we have to find out how much time is needed for the wire to reach within one degrees centigrade of the steady-state value, or in other words, what are the temperatures, what are the time that is needed in order for the wire to reach a temperature of 87.7 degrees centigrade.

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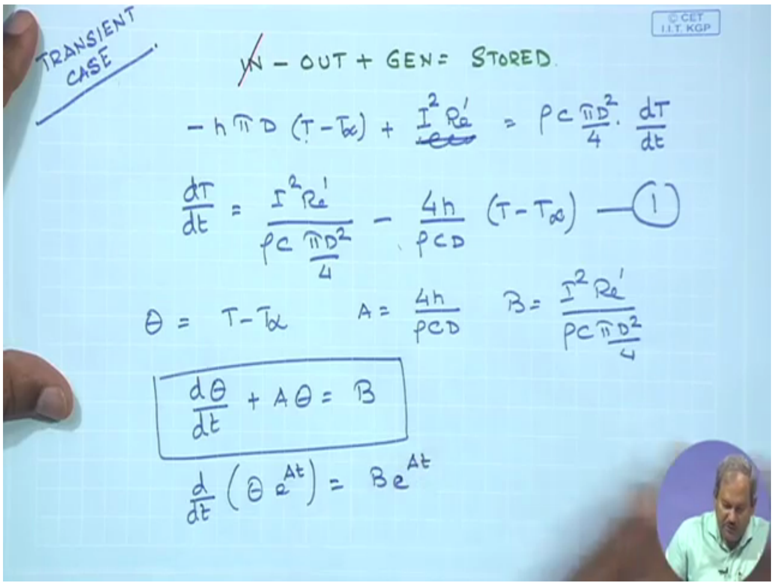

In order to do that, we 1st have to realise that what is going to be my governing equation for the transition, so this here we are talking about the transient case. And in the transient case, the fundamental equation in - out + generation is equal to stored, all these are rates. So rates of energy in - rates of energy out + generation rate is equal to the rate at which energy is stored in the system. Of course nothing comes in to the solid but there is a convection in which heat is lost to the surroundings but at the same time since we have a current which is flowing through the wire, some amount of heat is going to be generated.

As a result of which the energy would be would be different. If you if you remember the development that we have done so far, out was equal to the amount of energy, rate of energy stored, there was no generation term previously in our expression. But each problem we have to be careful before using any formula, just to make sure that the problem at hand exactly conforms to the assumptions or descriptions of the problem, textbook problem which based on which some relation or a correlation was developed.

So I cannot directly write the expression which was obtained in the last class for transient conduction because in that development there was nothing like, nothing called a heat generation term. But here I have a distinct heat generation term while it is undergoing transient conduction. So it is better to start from the fundamentals, fundamental rate equation for heat transfer and see which are the terms that are not going to be present.

And if it conforms to the textbook problem only then, textbook situation, only then you will be allowed, you can use the expression derived under such conditions. But here we clearly see that we have a heat generation terms that makes this specific problem distinct from what we have analysed previously. Therefore we need to use, you need to start from the fundamentals, from the basics and derive an expression for the temperature distribution for the time variation of temperature on our own. And use that that expression to obtain what would be the time needed for the for the wire to reach a temperature within one centigrade of the final, that is steady-state temperature.

So we start with this equation. The equation, the heat that goes the heat that goes out would be a times pie D, temperature which is a function of time - T infinity. Look here that I did not include an L term, the length scale term, the characteristic length in here, so all, what I am doing this heat out terms is is on a per unit length basis. So the heat out is on a per-unit length basis, so therefore it will be for me to write the generation which is simply Joules Heating times RE prime.

I can use RE prime which is resistance per unit, resistance per unit length because I brought this L down at the denominator of the resistance and make it a distance per-unit length diverted by, sorry I square RE. And on the stored side I have rho C, the density, the heat capacity and the volume pie D square by 4 in into L, however everything is expressed in per-unit length basis. So L will not appear in here, so rho times pie D square by 4 times L would give me the mass, mass times C, so M CP and the change in temperature with time.

So that is the time rate of change of energy stored in the solid on a per-unit length basis. So this is heat out, convective heat out of the solid on a per-unit length basis where the temperature is instantaneous temperature. And since the value of Biot number is less than 0.1, this is only a function of temperature, only a function of time and not of any special, any space coordinates. T infinity is the fixed temperature of the of the liquid, I square RE is the heat generation per-unit length that this is the rate of energy stored per-unit length in the solid.

So a slight rearrangement would give you as I square RE prime rho C pie D square by 4 - 4 H by rho CD times T - T infinity, this is the governing equation which I need to integrate in order to be noted to solve for the time needed for the temperature to reach a specific value. So I define Theta is equals T - T infinity and all these numbers, all these big expressions are put into, in the form of new constants as A and B where they are defined, nothing but what we have here RE prime by rho C pie D square by 4.

And then this expression would simply be equals D Theta by DT + A Theta is equal to B. So this is the compact form of the governing equation where this is B and this is A and T - T infinity is Theta. So it is D Theta DT + T Theta equals B. It can be solved with, this ordinary differential equation can be solved using an integrating factor which is A, e to the power At.

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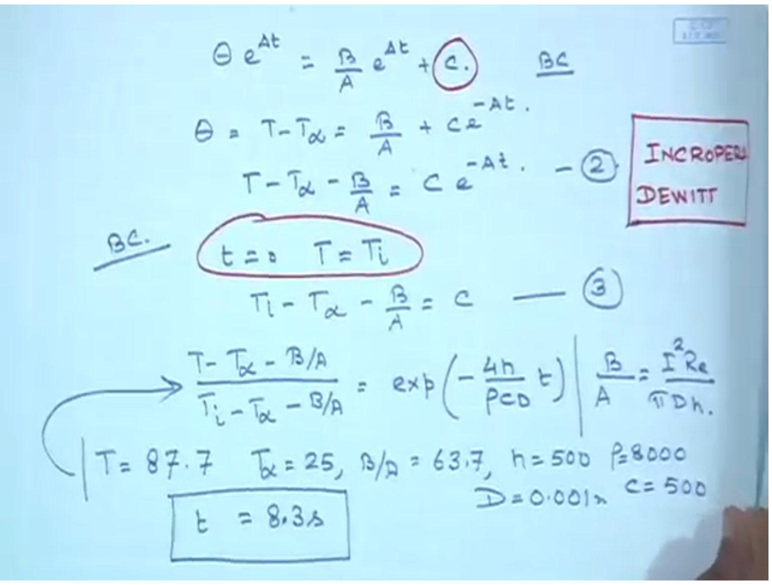

So you can integrate it once to obtain the final form as Theta e to the power At is equal to B by A e to the power At + C. So therefore Theta which is T - T infinity is equal to B by AC e to the power - At. This C is the constant of integration which needs to be evaluated through the use of boundary condition. But once again what we have here is that T - T infinity - B by A is equal to C, E to the power - At.

The boundary condition that is, that can be used even at t equals to 0, the temperature is, the initial temperature of the initial temperature of the solid wire, so therefore T I - T infinity - B by A is equal to C and this is that T equals 0. So this is my equation 2 and this is my question 3. So this is the boundary condition that has been used in order to evaluate C. So what I do is I divide equation 2 by equation 3 and what we get is T - T infinity - B by A, B and A, I would constants by T I - T infinity - B by A is exponential if I put back the value of A in here - 4H by rho CD times t.

And from the definition of B and A, we know that B by A is, you can figure out that B by A is I square RE by Pi DH. So your temperature is given as one centigrade within one centigrade of the steady-state temperature. The steady-state temperature was 88.7, so a final temperature of 1 degree within that temperature, steady-state temperature would be 88.7, the T infinity is 25. The B by A, when you plug-in the numbers in here, you would see that the number is 63.7, the H is 500, I am not using the units, you can put the correct units corresponding to each of these terms.

Rho is 8000 KG per metre cube, the C is 500 and the diameter is provided as 0.001 metre. So when you plug these numbers, + all these numbers in this, you would find out the unknown time is to be equal to 8.3 seconds. So this is, this example tells you how to solve transient conduction problem using understanding the physics of the problem which may have a different form than that present in your textbook development of transient conduction or any development for that matter.

So you should always be on the lookout for cases which differ from the standard cases and you, if you describe the physics of the process, you would be able to incorporate that, incorporate those changes in your development process so as to obtain an expression which will be valid for the specific case that you are handling. So this concludes our part of the transient conduction process, there are many more things in transient conduction that I did not touch upon in this course but I think you will have a fair idea of that when you go through any textbook on the transient, on heat transfer in the chapter transient conduction.

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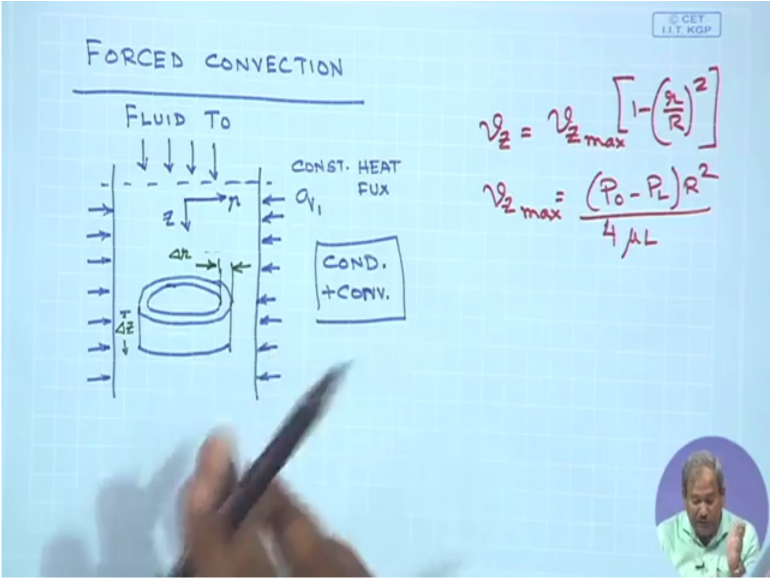

This specific part I have used from the book to use is Incropera and Dewitt, the entire part, the transient conduction part, you should be able to find in the book of Incropera and Dewitt. Now next we move into a problem which is slightly more involved and which will continue in the next class since we probably do not have enough time today is the case of forced convection. So what I would do is I will simply introduce the problem in this and identify the different mechanisms by which energy gets transported in here.

So this is a pipe through which a fluid at some given temperature T0 is coming into the pipe. And both the, it is a cylindrical pipe where a constant heat is being applied through the side walls of the pipe. Okay, so this is a constant heat which is being applied to the pipe and let us call this constant heat flux. The constant heat flux that is added to the pipe is K1. And as the fluid is moving, it is going to receive some amount of energy from in the R direction by conduction and we also realise that the temperature, the temperature of the fluid will increase as a result of convection as well.

So we have a system in which both conduction and convection are present, there is a constant heat flux which is being supplied through the side walls as the fluid is moving. So as the fluid is moving, it is going to gain more and more energy which would be manifested by an increase in temperature of the fluid. But unlike the cases that we have dealt with before, the temperature profile of the fluid is not only going to be function of axial position which let us call it as Z, it is also going to be a function of R, where the fluid particles are located with respect to the side walls.

So this is a case in which both conduction and convection are present, it is going to be, the temperature profile will vary with the axial position and at a fixed axial position the temperature will also vary with the radius, with the radial position. So looking back at that figure again, the fluid is moving, it is a constant heat flux which is being added and you have both conduction + convection which are present in the system.

Now whenever you have convection present in the system, you are dealing with a velocity of the fluid. So if you are, if you are talking about the velocity of the fluid, the convection is induced by the velocity of the fluid and we understand that when the fluid flows in a pipe on, upon the action of gravity as well as the imposed pressure gradient, in laminar flow the velocity profile that you would expect is going to be parabolic in nature, which we have done in our treatment of the fluid mechanics for momentum transfer when we saw that the profile is going to be, going to be laminar, it is going to be parabolic and the velocity is expressed with the following formula.

That VZ is equal to V Z Max times 1 - r by R whole square. So this is the parabolic distribution of velocity and the VZ Max, the maximum velocity which obviously takes place at the centreline is expressed in terms of the pressure and if you remember correctly, this P0 contains the effects of both, the imposed pressure on the gravity. So P0 - PL is the total effect of the gravity force, the body force and the surface force.

So P0, if, I would advise you to go back at the beginning of this course and see the derivation of the Huggle Pouso equation or flow through a pipe in presence of a pressure gradient and when the body force, effect of the body force is important. If I do that, we would see that P0 contains, P contains both the pressure, imposed pressure as well as the effect of gravity. So it is important, it is prerequisite in order to solve the heat transfer, heat transfer problem in which there is slow and the walls of the pipe are receiving a constant heat flux, one needs, since it involves convection, one needs to know what is the velocity distribution.

So to solve heat transfer problem, it is imperative, there is a prerequisite that we solve the fluid mechanics part of the problem 1st in order to obtain the velocity profile. And this velocities profile can then subsequently be used in order to find out what is the convection, what is going to make the convective, convective flow of heat in such a system. So solution of fluid mechanics is prerequisite for the solution of heat transfer and as I mentioned previously, there is a coupling between momentum transfer and heat transfer and the coupling will always be one directional, that is fluid, then heat and not the other way round, provided the physical properties remain constant.

If the physical properties of rho, CP, mu, etc. do not remain constant, then there is going to be a two-way coupling and the simultaneous solution of heat and momentum transfer has to be done to arrive at the expression for velocity as well as to find the expression of temperature. So looking back at this figure once again, since it is a case of conduction and convection and the temperature varies with Z. This is my R direction and this is the Z direction. So here we see that the temperature varies with R and the temperature varies with Z.

So if I have to assume a shell for our balance, the shell will be something like this, this shell is going to have a length equals Delta Z, since the temperature varies with Z. And since the temperature varies with R, the other dimension of the shell, imaginary shell across which we are going to make a heat balance must be equal to Delta R. So see the problem that we are facing right now. So far we were visualising only one smaller dimension and making a heat balance.

But now since the temperature is a function both of R and Z, the shell that we have to think of and the shell across which all heat, all heat have to be balanced has 2 smaller dimensions, one is Delta Z and the other is delta. And through the top surface, through the annular top surface which has an area of equal to twice pie are delta, you are going to have convective flow due to the motion of the fluid as well as conductive flow since the temperature varies with the axial position.

So T is a function of Z, so variation in T at different values of Z will initiate, will initiate a conductive heat flow in the Z direction which would enter into the control volume through the annular area pie R delta R, and I have a convective flow. On the side walls on the side walls of my imaginary shell, I am going to have conductive flow of heat coming, conductive flow of heat. But since the flow is one-dimensional, the flow is only in the Z direction, there is not going to be any convective flow through the surface that I call as Delta Z.

So through one boundary of the control volume I have both conduction and convection, through the other boundary of the, through the other boundary of the control volume I have only conduction. So in order to balance, in order to write our balance equation we need to think, we need to express conduction in the Z direction, convection in the Z direction, conduction in the X, conduction in the R direction. So all those will have to be taken into account to derive a governing equation of this.

And the treatment of this will underscore the utility of having a generalised equation which like the Naviar Stokes equations that we have discussed before will make will make it possible not to use this shell heat balance for complicated geometries but have a generally equation in which all the terms which are not relevant can be cancelled to obtain the final form of the energy equation, the final form of the heat transfer equation.

So we would go to some extent in solving this problem using shell balance. But from next class onwards or the class after that, we will switch to the formulation of a generalised energy equation and from that point onwards, all problems of heat transfer, be it convection or conduction will be handled by looking at the right component of the energy equation cancel the terms and arrive at the final governing equation.