Transport Phenomena.
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Lecture-36.
Transient Condition.

We will continue with our treatment of transient conduction in this part of the class. What we have discussed previously is that in transient conduction the temperature of a solid object can be a function of the space coordinates, for example X, Y and Z and it is it is also a function of time. So when you have a hot solid object being cold in a stream of cold air, depending on the dimensions, properties and other conditions of the solid block, the temperature inside the block can be function of X, Y and Z.

And since the, since with time it is going to get cooled, it is going to be function of time as well. Handling a situation in which the dependent variable which is temperature in this case is a function of 3 space coordinates, as well as time is difficult and that would give rise to a partial differential equation which would require a special technique for their solution. However one can make an assumption which would simplify the problem significantly.

And one of those assumptions which I have mentioned in the last class is if we assume that the temperature in the solid object is a function only of time but it is not a function of the space coordinates, then what we would get out of the physical statement of the problem is an ordinary differential equation, which then can be solved using appropriate boundary conditions and you would get the temperature profile, the temperature variation of the solid as a function of time only.

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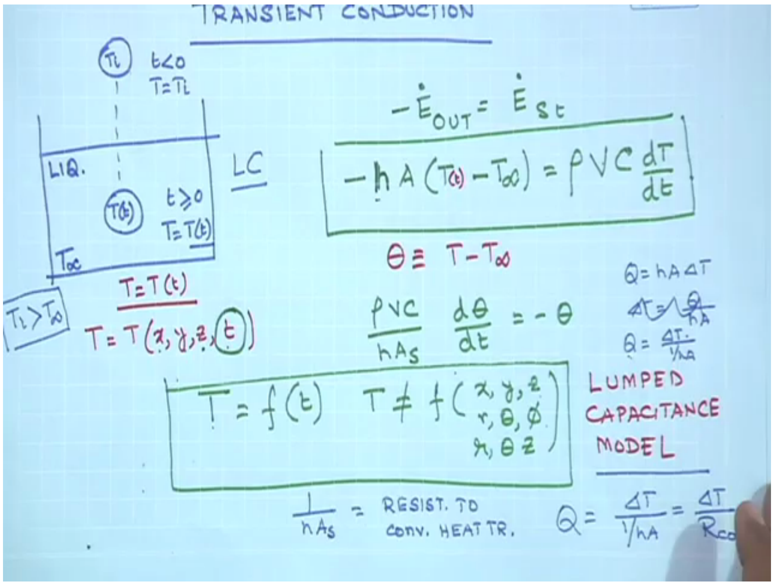

So that is what we have done in, we have started in the previous class, our introduction to transient conduction. The reason that I have included introduction of transient conduction in this in this course is to ensure, is to show you how the physical concepts can be used in modelling a process which would play a major role in all our future, in all our future development of model equations of a system of which we would like to express the physics in terms of a differential equation.

So here we have the case of a transient conduction in which there is a liquid whose temperature is T infinity and the temperature of the object initially TI is greater than T infinity. So this is essentially a quenching process in which the temperature of the object would change with time, would reduce with time because of convective heat transfer from the walls of the solid to the liquid. So the temperature inside the solid is going to be a function only of time if we assume that slumped capacitance model is valid.

So if the lumped capacitance model is valid, then the temperature is a function of time but it is not a function of the space coordinates, for example X, Y, Z, R, Theta, Phi or R, Theta and Z depending on the coordinate system we are using. If we write the governing equation E dot, that is the rate of energy in - rate of energy out + any generation in the system must be equal to the energy stored in the control volume.

However in this case there is no energy which comes in to the control volume since its temperature is higher than that of the surrounding fluid, so energy will go out of the solid object through a convection process. I have written the convection process in terms of a convective heat transfer coefficient, the surface area which is interacting with the liquid, the temperature at any given instant and T infinity is the temperature of the fluid which is far from the solid.

And we would also assume that the thermal capacity of the liquid or the amount of liquid is really large and therefore quenching one solid, one solid particle or a solid sphere would not alter the alter the temperature of the liquid. So T infinity is essentially a constant which does not vary with time. So this is going to be the energy out out of this. And there is no generation, so as a result of the energy leaving this solid object, the thermal energy storage term will also change with time.

So the amount of energy change, stored energy change inside the control volume would be simply in rho times VC which is the thermal, which is the heat capacity of the solid material times change in temperature with time. So if I define a dimension less temperature as T - T infinity, then this equation would simply, would simply reduced to rho VC by HA S. Now if you see this rho V C, it is some sort of the thermal capacity of the system.

So rho times V is mass, mass times heat capacity, so this is the thermal capacity of the solid in terms of the temperature, in terms of the energy that it can store. Whereas 1 by HAS, 1 by HAS is a resistance because from Newton's law we know that Q is equal to HA time delta T. And therefore delta T is equal to Q by HA or I would rather say, I would express it as Q is equal to delta T by 1 by HA. So this is the effect, the heat flow and this is the cause.

So if I express Q as delta T by 1 by H A, so this can be termed as delta T by resistance due to convection. So the 1 by HA, 1 by HAS, where AS denotes the surface area, 1 by HAS is some sort of a resistance to heat transfer, convective heat transfer. So therefore if you look at this system, I have a capacity term and a resistance term. And the differential equation that relates the change in temperature with time, this data is defined simply as T - T infinity in this specific form if I use the lumped capacitance model.

So when I use this lumped capacitance model, the question would obviously come is, when we can say that the lumped capacitance model is valid? What exactly happens in the case of lumped capacitance when it would be prudent to use the lumped capacitance model?

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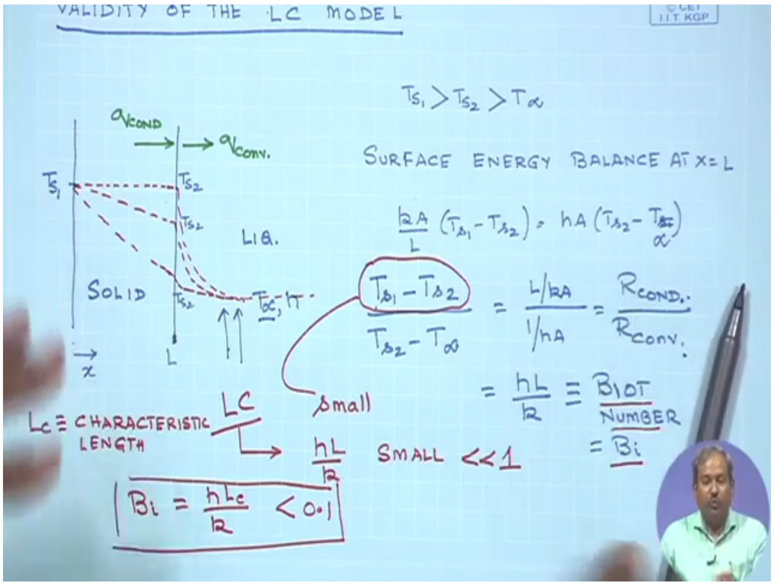

In order to address this question, what we have done is , I I am I have, I am looking into the validity of the LC model, the lumped capacitance model and I think, let us say this is a solid object which is in contact with a liquid, the temperature of the solid at X equal to 0, at this location is arbitrarily denoted by TS1.

Where are the temperature at the other end of the solid which is in contact with the liquid is entered TS 2 and the temperature of the liquid far from the solid wall is T infinity. There is a flow of a fluid past the solid object which would carry heat away from the solid because of convection. So the this boundary X equals L boundary of the solid is experiencing a convective heat transfer process due to its interaction with the fluid whose temperature is T infinity and the flow of the liquid maintains a heat transfer coefficient, convective heat transfer coefficient denoted by H.

And it is further assumed that TS1, the temperature at this location is more than TS2, the temperature at the interface and it is greater than T infinity. So of course the heat is going to flow from X equals 0 to X equals L and then by convection to beyond. So if I do this surface energy balance at X equals L, so what I can say is that at steady-state, the amount of heat which comes from the location X equals 0 at a temperature TS 1 to a location X equal to L which could be a temperature of TS2.

And the convective flow of heat would simply be K times E A which is a cross-sectional area, L is the distance between these 2, TS 1 - TS 2. So TS 1 - TS 2 divided by L essentially gives you the temperature gradient, the temperature gradient multiplied by the thermal conductivity and it would give you the heat flow from X equal to X equals L. At the other side of the interface, this heat is going to be convected it out by the Newton's law of cooling where H is of, H is the convective heat transfer coefficient, and sorry, this should be T infinity.

And this temperature is going to be equal to T infinity, TS 2 - T infinity. So if I take the ratio of these temperature drops, what I can write, TS 1 - TS 2 divided by TS2 - T infinity is equal to L by KA and 1 by HA. If you recall your, from the studies of conductive heat transfer, you know that L by KA is simply the conduction resistance, the resistance due to conduction and I have just described that one by HA is nothing but the resistance due to convection.

And if you simplify this, it is going to be equal to HL by K. Where H is the convective heat transfer coefficient, L is the length scale, L is the length of the, length of the solid object and K is the thermal conductivity of the solid. And this is known as, this is defined as Biot number. And the Biot number is the dimensionless number, is expressed as BI. So HL by K is termed as Biot number. So therefore you can see in order to have less temperature gradient between in the solid, that is between X equal to 0 and X equals L, the value of the Biot number must be small.

So if your conduction resistance is quite small in comparison with the convective resistance, the resistance due to, the resistance to convection, then you temperature drop in the solid, TS 1 - TS 2 is going to be small. In order for LC model to be valid, you would like this to be small, the temperature drop in the solid. So for validation, for L, for validation of LC. Therefore in order for LC to be valid, your HL by K should be small.

So a small value of Biot number essentially tells us that we can use, we can simplify the system using the lumped capacitance model. So Biot number is therefore an indicator, that is your number has to be quite small in comparison to 1 and the small value of your number is an indication of whether or not we can used lumped capacitance model. Normally the Biot number, the Biot number which is defined as HL by K and this L I would write, I would now substitute with LC.

So LC is nothing but the characteristics length, characteristic length of the solid, of the solid. So this Biot number should be less than 0.1. So experimental values suggest that one can safely use lumped capacitance model if the Biot number based on the combination of convective heat transfer coefficient, the characteristic length of the system and K, the thermal connectivity of the solid is less than 0.1. So while solving, while solving, while trying to solve any transient conduction problem, the 1st thing one should do is to is to check what is the value of the Biot number.

If it is less than 0.1, then the radiation of temperature inside the solid object or the spatial distribution of temperature inside the solid can be neglected. And therefore at any instant of time the solid can be treated as space wise isothermal. At every point the temperature would be the same if I fix the time. But obviously with time the temperature of the solid will change and depending on whether or not it is exposed a hot environment or a cooler environment, the temperature may reduce or may increase, however it will remain is isothermal as long as Biot number is less than 0.1.

Now again from the studies of heat transfer, you probably have seen a dimensionless number which is HL by K. Normally HL by K, we term it, the more common one is the Nusselt number. So what is the difference between Nusselt number which is also explained by HL by K and Biot number, again expressed as HL by K. The difference between the 2 is that for Nusselt number, the K that you have in the denominator of the expression refers to that of the fluid, whereas in the Biot number the K that you have is the thermal conductivity of the solid.

So depending on what you use, whether it is the thermal conductivity of the solid or the thermal conductivity of the surrounding liquid, you either have Biot number or you have Nusselt number. But Biot number therefore plays a very important role in transient conduction and the value of which would let you, would let you know whether or not lumped capacitance model is valid and can be used. Now only talk about this characteristic length if I expand this characteristic length , how do I know what is the characteristic length?

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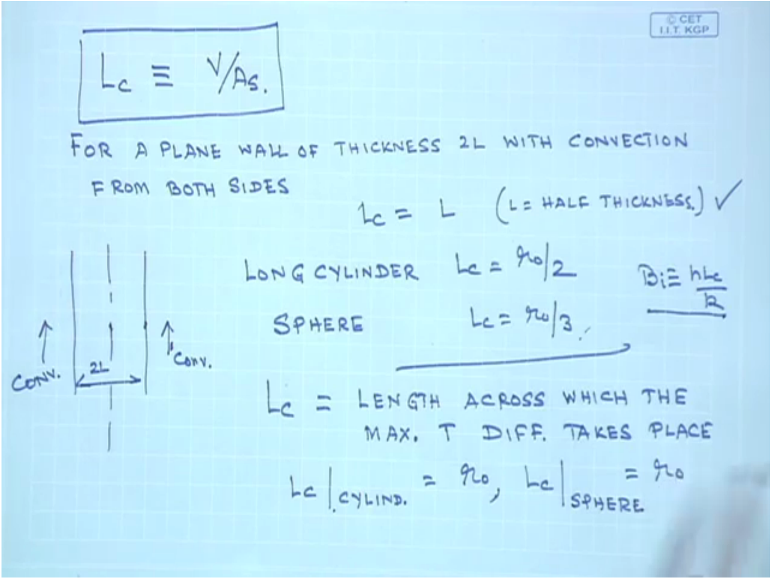

That is the characteristic length LC is defined as the volume divided by AS, where this V is the volume of the solid and AS is the surface area. Therefore a simple geometrical, simple geometry would tell you that for a plain wall of thickness 2L with convection from both sides, the figure that I have drawn initially from both sides. So we have a system like this, the total thickness of the solid is 2L and you have convection here as well as here. This LC will turn out to be equal to L where L is simply the half of thickness.

And if you take a long cylinder, this LC would turn out to be R0 by 2 where R0 is the radius of the cylinder. And for the case of a sphere, this LC is going to be equal to R0 by 3. So by definition one should choose the characteristic length while calculating the boundary Biot number for a system, the length, the Biot number defined as H LC by K, this LC should be this.

But the common practice is to make your calculation to make your assumption more conservative, the length scale is chosen, the length across which the maximum temperature difference takes place. So this is a conservative estimate of what would be the characteristic length. So therefore you can see the maximum temperature difference takes place over L and therefore this is correct. So for plain wall we can take LC equals L but if we think of a long cylinder, the maximum temperature difference would take place between the centreline and the outside.

Therefore for this case, LC for a cylinder, the conservative estimate would take the LC to be equal to R0 and similarly LC for a sphere, spherical system is taken to be equal to ask you again. So though the mathematical definition of LC is this, the conservative estimate for to calculate Biot number is taken in this, this is the dimension across which you get maximum temperature difference.

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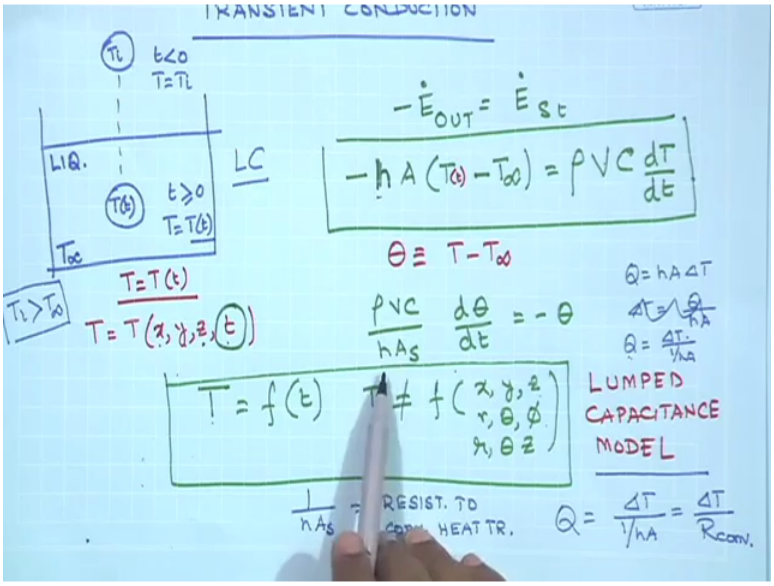

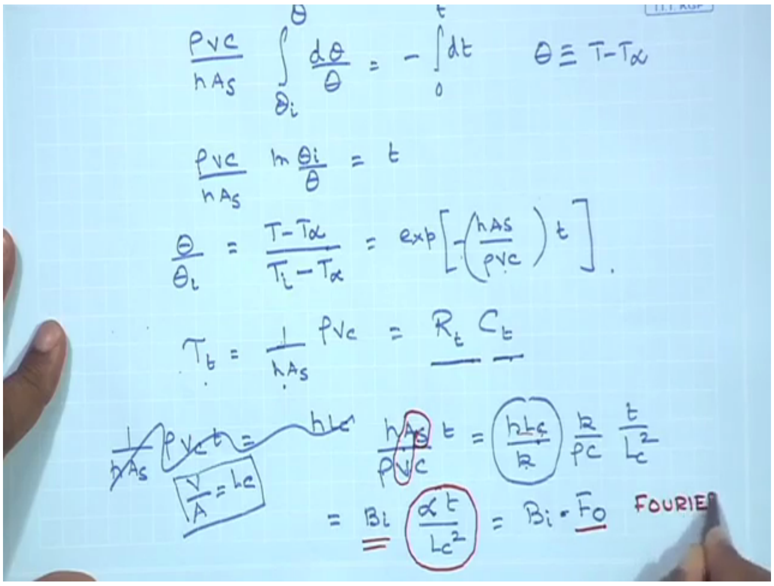

Now I go back to this formulation where we have, we know that rho VC, this is the this is the governing equation which we have we have written. This is essentially E dot out - E dot stored and you get this expression. So if you integrate this expression, what you would get is simply rho VC by HAS, integration from Theta I to Theta d Theta by theta is equal to 0 to T dT, remember that theta has been defined as T - T infinity. So when you when you integrate this, what you would get is rho VC by HAS, LN theta I by theta is equal to time.

Therefore Theta by theta I would be equal to T - T infinity divided by TI which is the initial temperature of the solid before it is exposed to the convection environment is equal to exponential HAS by rho VC - HAS rho VC times T. Okay. And looking at, therefore the temperature of the object would change in exponential fashion with time and the time constant of the process is simply the inverse of what I have written inside the bracket.

So this is the convective heat transfer coefficient, this is the surface area rho, volume and the heat capacity. So the thermal, the time constant of the process can be written as 1 by HAS times rho VC. And as I mentioned before, 1 by HAS is the resistance due to conduction and rho VC is some sort of thermal capacitance of the system. So the thermal constant of the process of a process of quenching or of the process of changing the temperature of a solid when it is exposed to a convection environment would be, this thermal constant would depend upon the resistance to convection and the thermal capacity of the system.

So the behaviour of the variation of temperature with time is analogous to the voltage decay that occurs when a capacitor is discharged, discharged through resistance in an electrical circuit. So of course, the higher the value of Tao, the system will respond slowly to any, slowly in terms of the change in temperature with time. But this is, this gives you the very simple way to treat the conduction in solid, the transient conduction in solid, provided you can use the lumped capacitance model.

So next we will quickly see how the, how this constant, the Tao T that I have referred to in terms of HAS by rho V, how it can be combined to give you some more insight into the process. So I am going to start with the expression for Tao T which is 1 by HAS times rho VC. And therefore this can be written, this and I also have the time present over here, so I am talking about the entire thing inside the exponential sign it can be expressed as H LC, I am sorry, the reverse way.

HAS by rho VC times T, this can be written as H, bringing a length scale by K, then K by rho C and t by LC square. So the purpose of my doing this is to bring in the concept of Biot number. So I have brought in an LC and I put an LC over here and this essentially are the same. So this is the Biot number and what I have over here is K by rho C is the thermal, the thermal diffusivity of the system, I have a t and LC square. So if I rearrange the terms, what I would get is H LC by K, you remember that B by A, B by A is the characteristic length of the system.

Which I have plugged in here and since I have added an LC, here I have LC square. So let A by V or rather V by A in here is substituted by LC. So H LC by rho times T, so H LC, H times rho LC. So I brought in another LC over there. And I have this formula. If this is dimensionless, this number which contains time must also be dimensionless. So this dimensionless number of is called Fourier number and expressed by the, by this Fo, so this is Fourier number.

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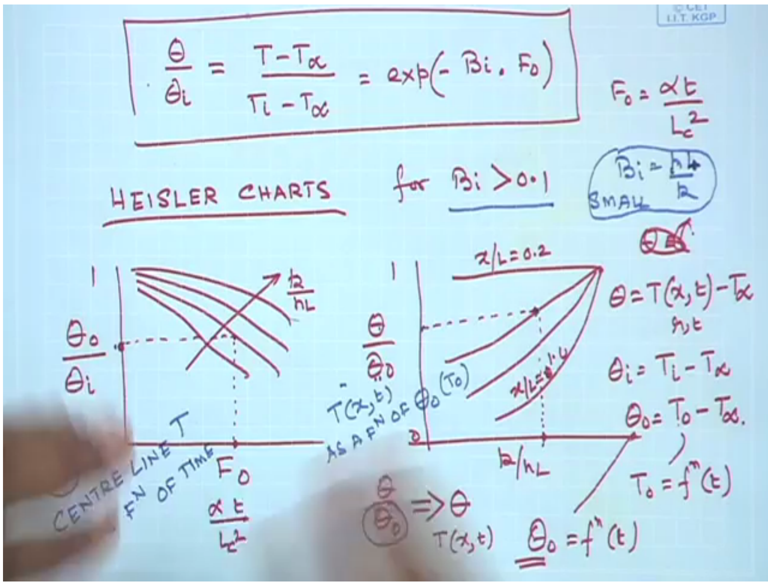

So the change in, change in temperature of the solid object as a result in convection outside and conduction inside and if lumped capacitance model is valid, is expressed in terms of 2 dimensionless quantities, one is the Biot number and the other is a Fourier number. So the Fourier number compares a characteristic, when Fourier number is Alpha t by LC square. So the Fourier number compares a characteristic body dimension with an appropriate temperature wave that penetrates, that gives you how fast the temperature wave will penetrate into the depths of the solid.

So this (())(27:44) completes our treatment of LC, our treatment of transient conduction which is required in transport phenomena. But there is one thing which I would very quickly touch upon into 2 or 3 minutes, is there would be conditions in which the lumped capacitance model validation statement, that is Biot number has to be less than 0.1 will not be made. So if Biot number is greater than 0.1, then there is no straightforward way to calculate what is the temperature.

So you have to, you have to write the equation will be partial differential equation and you have to solve it numerically. Fortunately for practising engineers, the solutions of transient cases when the Biot number is greater than 0.1 is present is available in all textbooks of heat transfer in the form of charts which are called Heisler charts.

So if you look at Heisler charts, you will be able to find out, looking at the Heisler charts, looking at the, knowing the dimensions of the system, the properties of the system, the convection environment and so on, you would be able to obtain what is a surface temperature, what is the centreline temperature, how much heat is lost in a given amount of time and so on. So Heisler charts I would write it over here, the Heisler charts, the Heisler charts, these are for the case when Biot number is greater than 0.1, they are available.

And essentially what they, it looks like is Theta 0, that is that 0eth time by Theta I as a function of Fourier number, this is between 0 to 1. And you get a family of curves like this for different values of K by HL. And similarly you get Theta by Theta 0 as a function of K by HL, so these are X by L to be equal to let say 0.2 to X by L to be equal to 1. So knowing the Fourier number, you would be able to, where Theta is defined, sorry Theta is defined as temperature as a function of space and time - T infinity or R and t - T infinity.

Theta I is the initial difference in temperature and Theta 0 is the centreline temperature - T infinity. Now you understand, we realise that T0 is essentially a function of time. This is this is the temperature how it, the centreline temperature how it varies. So Theta 0 is going to be a function of time. So therefore Fourier number contains time, since it is equal to Alpha t by LC square. We know what is LC, so at different values of time I can read, I can calculate what is the Fourier number and I go corresponding to this that A K by HL, the numbers of which are known to me and I come over here and I get the value of theta 0 which is T0 - T infinity.

So this 1st curve gives me the temperature of the centreline of the object as a function of time. So once I find out what is theta 0 at a given time, then I come to this 2nd curve. The 2nd curve to me theta which is temperature at any location divided by the temperature of the centreline as a function of K by HL. Since K by HL is known to me, I read that value, I evaluate that while and go to the point corresponding to the X that I desire. So let us say I desire X at this point and then come to this side to find out what is the value of theta by theta 0.

Since theta 0 is known to me, I would be able to, Theta 0 is known to me from the 1st figure, I would be able to obtain what is Theta, that means what is T as a function of both X and time. So the 1st curve, curve 1 gives me centreline temperature as function of time. The 2nd curve gives me temperature at any location and at any point of time as a function of theta 0. That is at the function of the centreline temperature.

So the 1st curve, since I know that at any given point of time, at any combination, the value of this combination gives me the centreline temperature, the 2nd one, these lines are 4 different locations at the desired location what is the Theta, that is the temperature as a function of the centreline temperature. So this combination using, using these combinations together, these 2 graphs together, I would be able to find out what is the temperature of a solid object at a given location and at a given time.

Remember the Heisler chart is only to, only used when the Biot number is greater than 0.1 and lumped capacitance model cannot be used. But in most, in many of the practical cases he would be able to use your number less than 0.1 and use LC and obtain an ordinary differential equation which you can then integrate in order to obtain the temperature is a function of time. The space do not, does not appear there since it is space wise isothermal as per the lumped capacitance model.

So Biot number less than 0.1 and Biot number is KL by H, so Biot number would be small if K is if the Biot number would be, you have to look at the values, the value of the Biot number which is HL by K, Biot number is HL by K, so in order for Biot number to be, Biot number to be small, the value of H has to be small, the length has to be small or the thermal conductivity has to be large. So these situations arise if it is open to natural convection, if the length scale is too small or if you are working with a material which has a very high conductivity.

So this property combination and the geometric observations, geometric parameters would give you the value of Biot number. So that is more or less that I wanted to cover in the transient conduction problem. We would solve one problem on this in the next class just to give you an idea of how to model a process, how to model a process in which the temperature is changing time and we would this would give, this would be a useful exercise to know more about modelling the transport processes around a solid sphere which is losing heat to the surrounding fluid.