Transport Phenomena.
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Lecture-35.
Viscous Dissipation.

In this class we are going to solve another problem which is quite common in many of the bearing systems. So let us say we have 2 cylindrical elements, coaxial, one inside the other and there is a very thin gap in between. One of them, let us say the outer one is rotating at a higher angular velocity while the inner one is stationary, so the stationary inner cylinder and the rotating outer cylinder.

It is it is a very common occurrence in many applications, in order to reduce the friction in between the 2 cylinders, the most, one of the most common ways to reduce the friction is to fill the gap in between the 2 cylinders by a properly chosen lubricant. So the lubricant essentially reduces the friction between the 2 cylinders. The choice of the lubricant in this is very important. And let us also assume that the outer and the inner cylinders are maintained at 2 different temperatures.

So suppose the 2 cylinders, none of the cylinders is moving, then simply I am going to have a temperature of the outer cylinder and a temperature of the inner cylinder. The gap between the 2 cylinders is too small and we have seen before while working with the problems of momentum transfer that if the separation between the 2 surfaces is very small in comparison to the radius or in other words, if the curvature of the system is not too small, then you can convert a radial system into a planar system.

So a radial system, the cylindrical system where we talk about 2 cylinders and the gap between them is extremely small, then it is, we can simplify the system by simply opening up the cylinder and making it as if it is a system of 2 parallel plates separated by the small distance. So this is how we have done some of the problems of the bearings, cylindrical bearings and if one of the one of the cylinders is moving, then a couette type flow will be established in the intervening space between the 2 cylinders.

So 1st of all if the radius, if the radius is the large, if the radius of the cylindrical system is large in comparison to the gap in between the 2 cylinders, then I can simply cut open the cylinders and make them as if they are 2 parallel plates with one plate moving with some velocity, the other stationary. And any liquid in between, in between in the intervening space is simply going to going to have a Couette type flow because of the motion of the top plate.

So in this specific case we can also treat the system as if it is the flow, it is a case of flow between 2 parallel plates, a lubricant which is placed between 2 parallel plates, one plate is moving with some velocity and the 2 temperatures, the temperatures of the 2 plates are different. So the difference in this problem as compared to the problem that we have done in fluid mechanics is that here is the 2 temperatures, here the temperatures of the 2 plates are different. If the temperatures of the 2 plates are different and if the gap in between them is very small, then viscous forces will ensure that there is going to be a negligible effect of convective heat transfer and most of the heat transfer between the 2 plates due to the difference in temperature will be due to conduction.

So this is a situation in which conduction will prevail and the entire problem can be thought of as if it is a flow between 2 parallel, it is it is it is 2 parallel plates which are maintained at 2 different temperatures with a liquid in between where there is no convection. So it is a conduction problem. So if it is a conduction problem, then we know that in absence of any heat generation in the in the fluid, in the liquid in between, it is simply going to be linear distribution of temperature.

And the linear distribution because in this case there is no variation of temperature with Z, no variation with Y, only with respect to X the temperature will change. So if I think of the equation that describes conductive heat transport at steady-state in absence of any heat generation, we simply have K d2t dX square is equal to 0. And K d2T dX square equals 0 would give rise to a linear temperature profile and the 2 concepts of the profile can simply be evaluated by through the use of boundary conditions that at one point, at one plate the temperature is T0 and on the other plate the temperature is T1.

So there would be the profile of the temperature would be linear between T0 and T1, that is in absence of any heat generation. Now whenever we have a fluid, lubricant which is placed in between 2 rotating substances, 2 rotating surfaces and the gap is small, what you see is that the fluid layers would be, there would be a very strong velocity gradient present in the system. A velocity is 0 over here and the other plate, the top plate moves at a very high velocity and the gap in between them is very small.

So the velocity gradient, which is in this case V - 0 divided by X - 0, so it is V by X, V is large and X is small, so the value of the velocity gradient would be very large. If the value of the velocity gradient is large, then the adjacent layers would start to slip past one another, move past one another with a very high relative velocity difference. Now whenever if you think of a solid object which is being pulled over another solid surface with some velocity, you are going to raise the temperature of the solid block due to friction.

So friction will ensure that the frictional losses will manifest itself into a temperature rise of both this, both the solids. So the surface, the inner surface, these 2 surfaces are going to have an increase in temperature due to friction, due to solid friction. The same thing we can also or may also take place for the case of liquids when the layers in laminar flow slip past one another at a very high velocity. So this is something which is, which can loosely be called as, loosely be called as liquid friction. Okay.

And this type of frictional heat generation is quite common. It is a volumetric heat generation, it is due to friction, so this kind of volumetric heat generation should have to, will have to be taken into account whenever you have the gradient present, the velocity gradient present in the system is very high. So we do not see the heating effect, the thermal effect, the heat generation effect in during the flow of the liquid quite often.

But only in very special cases, for example in the flow of the lubricant or when a spacecraft re-enters the Earth’s atmosphere, the velocity gradient is so high that you get extremely high, substantially high generation of heat and the entire spacecraft will glow red due to the temperature change temperature increase. So in this specific case we understand that the temperature profile is going to be linear but due to the volumetric heat generation due to viscosity it is a linear nature of the distribution will no longer remain linear.

So we have to call, the in the governing equation itself we cannot now neglect Q dot which is the heat generation per unit volume. So k d2t dX square will not be equal to 0 for a system in which we have viscous heat generation. The heat generation which is due to viscosity, the heat generation due to the property of the fluid which resists the motion of adjustment layers. So that is why it is called the viscous heat generation. So K times d2 T dEX square + Q dot would be equal to 0. We cannot neglect heat generation due to frictional forces.

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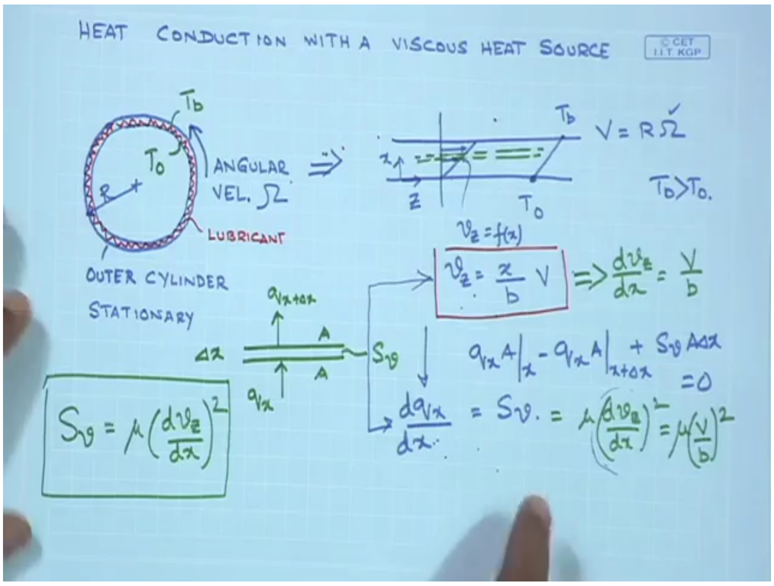

So we would start with the governing equation which is simply would be, let us say 1st of all I convert this to a system in which the top plate is moving with a velocity, this was the angular velocity, so the top plate is moving with a velocity equal to R omega and the temperature over here is T0 and the temperature over here is TB. And TB is greater than T0. So in absence of any viscous heat generation, the profile will simply look, the temperature profile look, simply look like this.

And we know that since the top plate is moving with a constant velocity, the velocity profile would also be linear where if this is my X and this is the Z direction, then the velocity profile would simply be a function of X and VZ would simply be equal to X by the separation between, X by the separation between the 2 plates multiplied by V. So this is the velocity, axial velocity profile imposed by the motion of the top plate, X by B times V square B is the separation between the 2 plates.

Now, next is you have to think of shell, since the velocity is varying in the X direction, my shell is going to be of size delta X, it could be any area A, heat is going to come in, it is a convective, conduction only process, no convection is to be taken into account. And this is Q X + delta X and due to friction, let us say some amount of volumetric heat generation is present which is denoted by FV. So QX multiplied by A evaluated at X - QX A at X + delta X + heat generation which is A times delta X at steady-state would be equal to 0.

So this expression, this equation can now be expressed as in terms of differential equation dQX by dX is equal to SV. Now this, the expression for SV, the volumetric generation of heat can be expressed as the velocity gradient square. I will not be able to explain why this is so unless and until we derive equation of energy. Until and unless energy equation is introduced, the form of the viscous heat generation due to the presence of a velocity gradient would not be clear to you.

So right now please accept that the volumetric heat generation and the form as viscosity times velocity gradient square. But in the subsequent classes we would see why, how such a form can be prescribed for viscous heat generation. So for the time being we are assuming that the volumetric heat generation is simply equal to mu times velocity gradient square. So SV is mu times, the only velocity gradient that exists is in the X direction, only velocity that we have in the system is in in the Z direction.

So VZ is the nonzero component of velocity which varies with X. So the volumetric heat generation is mu times d VZ dX square. So if we assume that SV so be equal to mu times dVZ dX whole square so I am simply going to write this as a dVZ dX square. And this is an example and if you see the expression over here, then your dVZ by dX would simply be equals V by B. So this would be mu times V by B whole square. So the presence of the heat generation, the viscous heat generation essentially couples these 2 equations.

The momentum transfer equation and the heat transfer equation gets coupled because of the presence of velocity or velocity gradient in the energy equation, in the equation for energy. So momentum equation and energy equation are now coupled to do the presence of the velocity gradient. So which also means that you have to, you have to solve for the velocity profile, you have to solve the momentum equation 1st before you can attempt to solve the energy equation.

So the coupling that we see here is one-way coupling, that is energy equation is coupled to the momentum equation but if you look at the momentum equation, the momentum equation is not coupled with the energy equation. So in most of the cases, you would see the presence of one-way coupling. You have to solve for the momentum equation 1st, get the velocity profile and then, then derive the energy equation where a velocity expression would arise either because of the presence of convection which we are not considering at this moment or due to the coupling could be due to its presence in the viscous dissipation term.

So viscous dissipation term appears only in specialised cases where the velocity gradient is very large, in most of the normal ordinary energy equations we do not need to include that term. But if we do include, then the velocity expression must be obtained up priorly before we attempt to solve the energy equation. So as long as the Thermo physical properties of the system remain constant, there will always be a one-way coupling between momentum transfer and heat transfer.

But if the properties start to change, then that, then those equations, the momentum equation and the energy equation will have to be solved simultaneously. But as long as the equations are coupled in only one direction, you need to solve independently the expression for velocity and then plug that in 2 or the energy equation that we have just derived which is the case here in.

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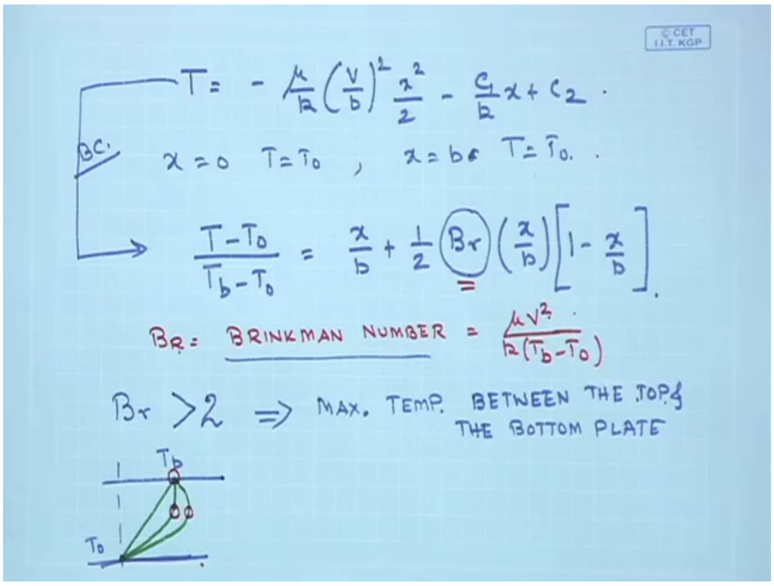

So with this, with this I would be able to I would be able to express this in terms of Fourier's law and with Fourier's Law, the temperature expression can be obtained, this you can find out on your own, it is going to be mu by K V by B whole square times X square by 2 - C1 by K X by C2. And the 2 conditions, the boundary conditions that we have at X equals 0, T equals T0 and at X equals B, T of the surface is going to be equal to T0. With these boundary conditions, this temperature profile cannot be written as T - T0 in dimensionless form by T B - T0 equals X by T + 1 by 2 BR X by B 1 - X by B.

This BR, this is, this BR is known as the Brinkman number, which is defined as mu V square by K times Tb - T0. Which would come directly if you solve this equation with these boundary conditions and express the dimensionless form, then the Brinkman number you would see would be equal to mu times V square by K TB - T0. So the Brinkman number is important, bring number essentially tells you whether or not because you can divide both sides by L, the numerator and the denominator by L.

So Brinkman number essentially tells you how much, how far viscous heating is important relative to the heat flow from the imposed temperature difference. Once again I will come to that later. If you do not have any discuss heat generation in the system, what is going to happen, what is going to happen to the expression that we have just derived? If you do not have any heat generation present in the system, the entire term, the 2nd term on the right-hand side would be 0.

And what he would get is a linear distribution of temperature. Since you have viscous heat generation where the Brinkman number essentially tells you the importance of the generation of viscous heat with respect to, with respect to the heat that would flow because of an imposed temperature difference, I am going to have a non-linear term present in the expression for temperature. So Brinkman number tells me how important viscous heating is.

And if viscous heating is important, that means for higher values of Brinkman number, if viscous heating is important, the value, the value of Brinkman number would increase and an interesting thing would, can be seen for a value of Brinkman number greater than 2. If value of Brinkman number is greater than the numerical, greater than 2, then you are going to have a temperature, maximum temperature, you would see the existence of a maximum temperature between the top and the bottom plate.

So here I have 2 plates, one is at TB and the other is that T0. Normally I would get a profile like this. So the maximum temperature would be at the temperature of the top plate. But as this effect of viscous heating starts to become important, that means the value of Brinkman number starts to increase, there would be a value of Brinkman number greater than 2 for all values of Brinkman number greater than 2, the profile would probably look something like this.

That means the maximum temperature which was here when Brinkman number is 0 or less than 2 and for these cases the Brinkman, for bring my number greater than 2, as it progressively becomes more and more, the maximum is going to be somewhere in between the top plate and the bottom plate. I leave the derivation of this that whether, when Brinkman number is greater than 2, the maximum temperature is going to be located in between, in the lubricant in between the top and the bottom plate.

I will leave that to you to solve. But the interesting phenomena what you see here is that for Brinkman number greater than 2, the maximum is going to be in between the 2 top plates. So now comes the question of the selection of the lubricant. Each lubricant has a specific value of temperature up to which it will retain its lubrication properties. So if it is within if it is within the lubrication zone, if it is within that temperature, the lubricant will work perfectly.

But if for some reason the temperature in between the 2 moving surfaces, if the temperature of the lubricant exceeds that of the that of the higher temperature surface and it will keep on increasing as the velocity, relative velocity between the 2 plates increase, then you may get a temperature which is more than the safe operating temperature of the lubricant. So before you choose the lubricant, you 1st find out what is the temperature at which the lubricant can work safely.

And then try to solve the problem with the known velocity differences between the top and the bottom plates and see what is the maximum temperature that can be attained by the lubricant due to the motion of one of the plate. So the frictional heat generation plays a critical role in the choice of the lubricant. In the, for the, in the performance of the lubricant, what kind of a velocity difference in a lubricant can sustain that is something which one has to consider before choosing the lubricant.

And Brinkman number would tell you, would tell you an idea of whether or not you are going to get a higher temperature in the lubricant as compared to any of the temperature, any of the 2 temperatures of the 2 solid plate which are in motion.

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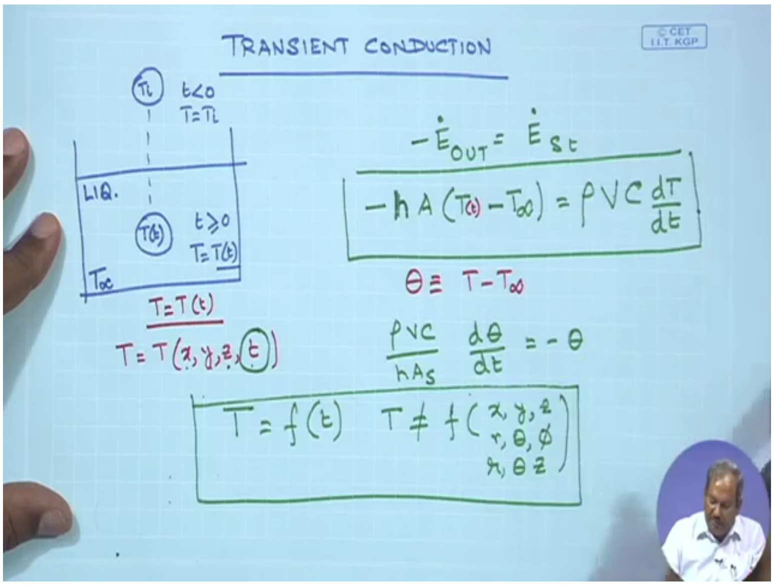

So viscous heat generation, viscous heat dissipation is can be important in some applications. Now let us move onto a different type of phenomena which we have not, which we have not talked about before and it is transient condition. Transient conduction, a transient conduction is something which you see, which you can easily visualize, let say this is a coolant liquid which is there, whose temperature is T infinity. And I have an object, it is a spherical ball whose temperature is Ti, so at T + than 0, the temperature of the solid object is equal to TY.

Then you drop it into the coolant liquid, then what you see is that the temperature of the solid object is now a function of time. So any T greater than 0, the temperature would simply be a function of time. So as long as the temperature is going to be a function of time, this is a transient, transient conduction problem, where the temperature is a function of time as well as the temperature could be a function of X, Y, Z or R, theta, Z or R, theatre, Phi depending on whether it is spherical object, a rectangular object or a cylindrical object, it is going to be a function of time.

So the presence of this time part, Time term makes the situation much more complicated in the sense that my temperature is not a function of spatial coordinates, it is also a function of time. So any problem that deals with the temperature, when the temperature is can vary with respect to time is commonly termed as the transient conduction. Spatial as well as time-dependent, it is called the transient conduction.

Now if we if we write, if we think of a spherical ball which is dropped and the liquid coolant at its time whose temperature is lower than the temperature of the hot spherical ball, then if I write the conservation equation. There is no E in, E dot in - E dot out + generation is equal to accumulation. So what you have I write this equation then, there is no E dot in, that means no heat that comes into the spherical ball, there is a heat out, so E dot out would be there, there is no heat generation in the system, however there is a change, on the right-hand side there is a change in the energy stored in the spherical object because it is now in contact with take cold liquid, cooler liquid.

So the governing equation, the physics, physical equation which would describe transient conduction is - E dot out is equal to energy stored, change in energy stored is in the system. So it would simply look like as E dot out is equal to E dot stored. And what is E dot out, the energy that goes out of the spherical balls is mostly by conduction, convection. So if this H is the convective heat transfer coefficient times A - temperature of the solid objects - temperature of the liquid which is T infinity, must be equal to rho V, so that is the mass, C, that is, that is the, that this is the heat capacity times the rate of change of temperature with time. So this would be the governing equation for transient conduction problems.

And we realise that this, this is a function, the temperature of the solid object is a function of time as well. So if we define theta, the temperature to be as T - T infinity, then this equation takes the form as rho VC divided by HAS d theta by dT is equal to - theta. Now one thing has to be mentioned here is that the assumptions we can make is that T is a function only of time, T is not a function of space coordinates like X, Y, Z, or R, Theta, Phi or R, theatre, Z.

If we can make an assumption like this, if this assumption is valid, then the governing equation can simply be transformed to this and can be integrated. But the fact the assumption that the temperature of the solid object is a function of time but it is not a function of positions, this is called the lumped capacitance model. So a lumped capacitance model allows me to simply integrate the equation with respect to time while assuming that at any point the object, the solid object is space wise isothermal.

That is the temperature of the solid ball is going to be, going to vary depending on time but I take a specific time, there is no variation of temperature inside the solid ball. It Centre at a point halfway between the Centre and the periphery, as well as the periphery, all these temperatures are the same. If that condition, if that assumption is made, if that assumption is valid, then it would be easy to solve the problem of transient conduction and the assumption that the temperature of the solid object is space wise isothermal, it depends only on time, this assumption is known as the lumped capacitance model.

How and when we can make this assumption and how does that help us in solving problems of transient conduction, those 2 things we are going to look into the next class with the help of examples and numbers and you would see that there would be many conditions, many cases in which these assumptions can be made. So from next class onwards we are going to start with our treatment, our analysis, our study of the transient conduction process with lumped capacitance as the 1st step.