Transport Phenomena.
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Lecture-33.
1-D Heat Conduction – Shell Heat Balance.

So as I mentioned I am going to continue with a system in which let us say we have an electrical wire and some amount of heat is generated when current passes through it, it is a simple, the due to the resistance, there will be some heat regeneration and let us assume that this heat generation can be expressed, can be termed as S subscript e which is the heat generated per unit volume. And we would like to find out from 1st principles what would be the dissolution of temperature, the form of the temperature distribution inside such a wire.

What we have seen for the case of plane wall, now we are going to do it for a cylindrical system in which we consider the radial distribution of temperature. So what we assume is the temperature varies only with r, it is not vary with Z or with Theta and that we are dealing with a steady-state system, so on the, so heat in - heat out + heat generation or in terms of rates must be equal to 0. Since it is there it is a steady-state, so no accumulation of heat, accumulation or depletion of heat inside the control volume which is this case if the entire wire.

So the same way we have done for the case of shell momentum balance, here also I will have to assume a shell of some thickness of some small dimension and write the corresponding terms of in and out and generation if any and then equate them to at steady-state equal to 0. So the algebraic sum of heat in, heat out and the heat generation must be equal to 0 at steady-state. And the same way for the case that we have done for the case of momentum transfer, the trick to choose the smaller dimension of the control volume is to realise which is the, in which direction the temperature is changing.

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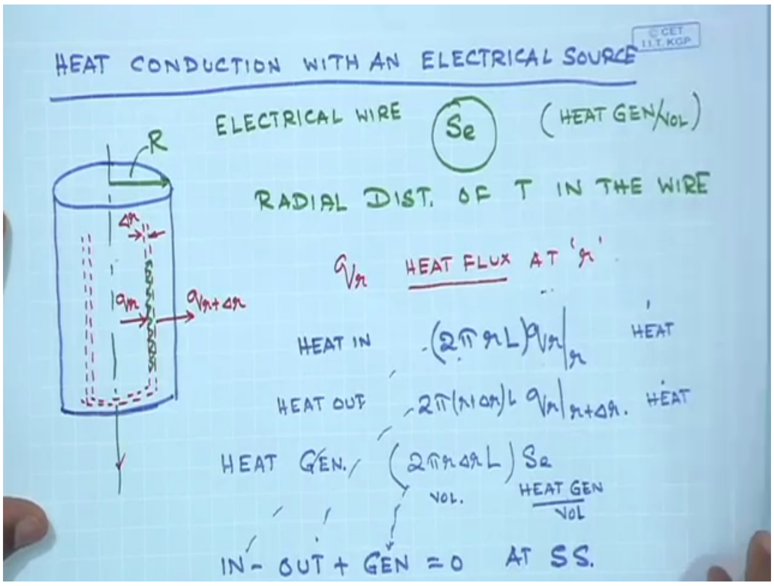

So if you consider the problem at hand over here, temperature is obviously will change with r, temperature will not be a function of Z and temperature is definitely not going to be a function of Theta. So since the temperature is changing with r, my choice of the control volume should be, should have a thickness which is, which we can call it as delta r. So on this side also, so this is the shell, cylindrical shell which is coaxial with the axis of the electric wire and we are talking about this shell.

So if you think of this shell, the cylindrical shell, then I am going to have some amount of heat which is coming in by conduction, the heat which is going out of the shell by conduction and some amount of heat will be generated in this control volume because of the distributed heat generation term which is heated generation per unit volume. So if we think QR is the heat flux at r, some value of r, so this is Q r which is entering and QR + delta r, says the thickness is delta r that is leaving this interface.

So if this is heat flux at r, then the total amount of heat in would simply be equals twice pie r times L, this is essentially the area of the inner surface of the assumed shell multiplied by Q r evaluated at r, heat out would be twice pie r + delta r times L times Q r evaluated at r + delta r. So look at the similarity of the shell momentum balance that we have done previously and the heat generated would simply be equals the volume which is twice pie r delta times L.

So this is essentially the volume of the control volume that we have chosen. So this has units of cube, multiplied by Sc which is heat is generated per unit volume. So this is volume and this is heat generated per unit volume and these are all rates heat, heat, the rates, these are rates of heat coming in, rates of heat going out and rates of heat which are generated in here. So what you do is you write this equation and in - out + generation is equal to 0 at steady-state.

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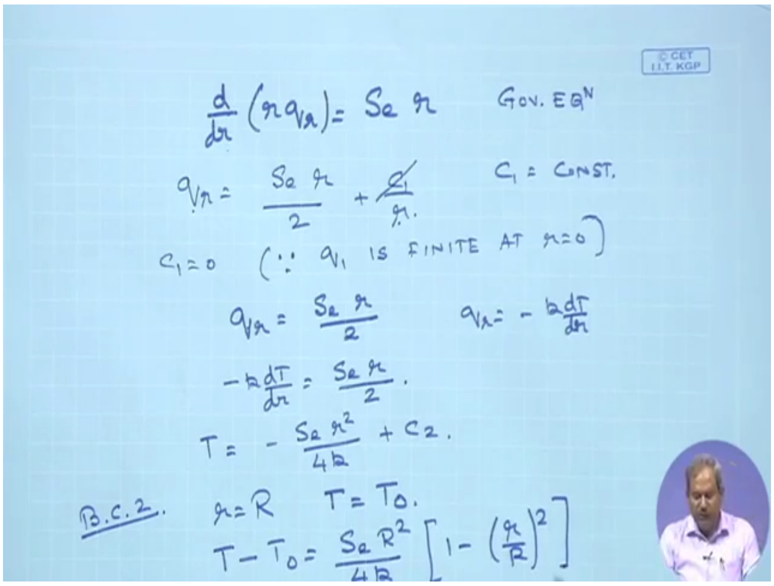

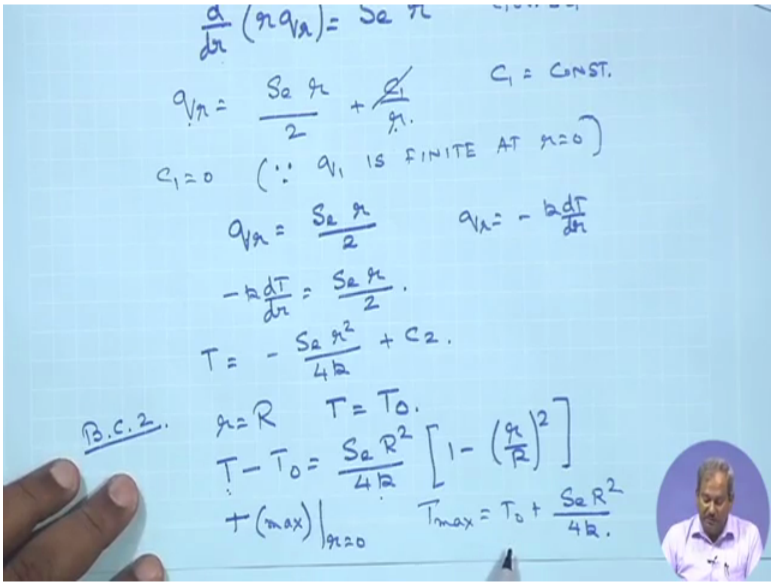

So we put these terms over here in - out + generation is equal to 0 and divide both sides by, divide all sides by delta, cancel out pie, L, etc. and divide both sides by delta r. And when you do that this difference in question would slowly would definitely be converted into the differential equation rQR is equal to Sc Times r. So this becomes the governing equation for heat transfer when you have generation of heat inside a cylindrical system.

So in - out + generation equals 0 divide cancel the terms, cancel 2 pie L, etc. and divide both sides by delta r would simply give you the expression, the final expression to be as d dr of r

QR Times Sc r is equal to 0. This can be integrated to obtain S ER by 2 + C1 by r, where C1 is the constant of integration. Now we have to evaluate C1 and C2. As you can see the form suggests that QR would be would, the QR cannot be finite when r is equal to 0.

So at r equals 0, at this location QR has to be finite. So if qR has to be finite at our equal to 0, then C1 has to be equal to 0. Since Q1 is finite at r equals 0, so C1 has to be equal to 0. So if C1 is equal to 0, then what you would get his QR is equal to Se, the heat generated per unit volume by 2 and substitute Fourier's law which is QR equals - K dT d r in here and what you would get is - K times dt dR is Sc r by 2 and you integrate it once to get - Sc r square by 4K + C2.

And the boundary condition 2 could be that at r equals Capital R that is the at the outer edge of the electrical wire, the temperature is some known temperatures T0 and if you use this boundary condition, then the temperature distribution would simply be equals Sc R square by 4K 1 - small r by Capital R the whole square. So what you see here then, what you see here, I will point to this equation once again.

The temperature distribution which is a function of r, T0 is the temperature at the, of the outside of the wire is a parabolic function of r continuing terms such as the thermal conductivity of the solid wire and the heat generation per unit volume due to the passage of current through it. So it is a parabolic distribution and with the maximum temperature, T max would obviously be at a point where r equals 0 and T max would be equals T0 + SER square by 4K.

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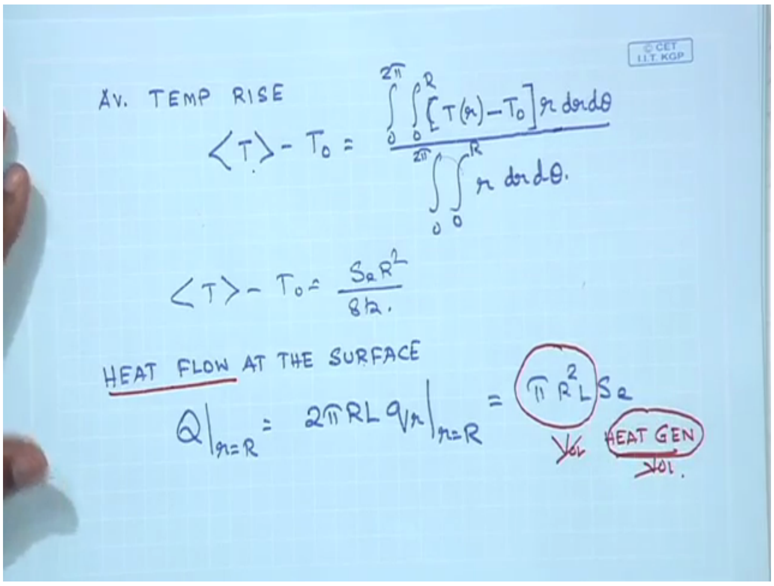

So this would be the maximum temperature of the rise, temperature rise of the wire. And additionally if you would, if you are interested not only in the temperature at every point at some sort of an average temperature rise which is some sort of an average and all these averages are mostly area averages. So what I have to do is I have to find out the area average of T which is a function of r - T0 Times r dR d Theta and this must be equal to the area, the cross-sectional areas which is r dr d theta.

So this is a standard method of expressing with the temperature rise or temperature, average temperature, the same way we have expressed the, the average velocity when I have a parabolic distribution of velocity inside a pipe due to a result in less safe pressure difference, apply pressure gradient at the liquid flows follows a parabolic velocity profile. Same way when you have heat regenerated inside a wire, the temperature distribution will turn out to be parabolic in nature.

And we expressed the average velocity when we did the averaging of velocity, it was always averaged over the flow cross-sectional area. So simply replace fluid flow by heat flow and the temperature is therefore averaged over the average, over the average flow area that is perpendicular to the direction of current passage. So you are, you are not to obtain the thermal, the temperature average, you are everything over the entire cross-sectional area.

And for an electrical system, the entire cross-sectional area is going to be double integration, one is from 0 to pie, the value of theta can vary from 0 to pie and the value of r, small r can vary from 0 to Capital R and the area would be simply r dr d Theta. So when you do this r dr d theta, in this specific form, you would see the result for this - T0 would simply be equal to Sc r square by 8K. And the last part that remains is what is the heat flow at the surface, at the outer surface?

The heat flow at the outer surface, if I denote it as Q and r equals r, Capital R should be equals the area which is twice Pi rL multiplied the heat flux evaluated at r equals Capital R. Which when you do it, it would turn out to be pie r square L by Se. If you look carefully, pie r square L is simply the volume of the control volume and Sc is the heat generated per unit volume. So the heat flow at the outer surface, these 2 volumes will cancel, so the heat flow at the surface must be equal to the heat regenerated.

That is a condition which must be maintained in order to obtain, in order to reach the steady-state. So at steady-state, all the heat which is generated inside the control volume must be conducted out of the control volume and that is the result which, that is our physical understanding and you would see by plugging in our expression for T, expression for Q, multiplying it with the volume and so on you get the exact same result. So our results are consistent with our understanding, our physical understanding of the basic fundamentals of the process.

So this example essentially shows you how to start with the model, to start with, how to start converting your understanding of the problem in terms of difference equation by the choice of a shell across which you are going to make heat balance. And from that choice of your shell, you would be able to derive what is the governing equation. Once you have the governing equation, use appropriate boundary conditions to get the temperature distribution. The same way we have done for the velocity distribution.

Once you have the temperature distribution you can find out what is the average temperature by averaging it across the cross-sectional area of your control volume. And if you have the average temperature, then the average temperature can be related to other parameters which you would see later but this example establishes or re-establishes the ease with which one can use a shell heat balance to obtain that temperature distribution inside a control volume.

However the moment we have this is true for the case of conductive heat transfer, the moment we have convective heat transfer the system starts to get complicated and a simple shell balance will probably turn out to be inadequate in addressing slightly more complicated problems, so we will see them next. What I will like very quickly do is give you a problem to practice on.

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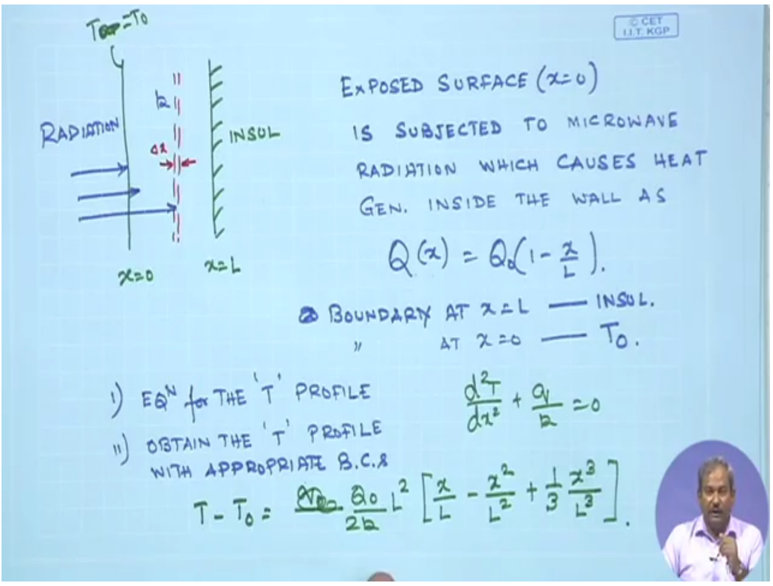

And the problem you are going to practice is, let us assume that you have again a plane wall where one surface is maintained at T0, this surface is insulated, this is located at X equals to 0 and this is at X equals L.

And it says that at exposed surface and by exposed surface I obviously mean at X equal to 0, so this is the exposed surface. Its thermal conductivity is K is subjected to microwave radiation which causes heat generation inside the wall according to the function as Q, the heat generation which is a function of X is equal Q0 which is a constant times 1 - X by L. And X boundary at X equals L is insulated and boundary at X equals 0 is maintained at a temperature equal to T0.

The things that you have to find is derive equation for the temperature profile and 2nd is obtain the temperature profile with appropriate boundary conditions. So once again we have a wall, one side of it is insulated, the other side is exposed to microwave radiation. So you have radiation falling on it and getting absorbed inside the whole material and the radiation causes the heat generation inside the wall as QX equals Q0 which is a constant 1 - XYL.

So as X increases, the amount of heat which is generated gets smaller and smaller. The boundary at X equals L is kept at insulated condition and the boundary at X is at 0 is maintained at a temperature equal to 0. So as before you can either think of a thin shell of thickness delta X and do the balance or you can directly write, since it is a conduction only case, you can directly write the governing equation Q by K to be equals 0.

Put in the expressions, integrate, use the appropriate boundary conditions which I would not discuss and what you would get is, the answer you should get is T - T0, that is temperature which is a function of X, T0 which is a constant temperature, Q0, sorry this is Q0 by Twice K times L square X by L - X square by L square + 1/3 X cube by L cube. So this is the temperature profile which you should get. That is for this example problem, you can you can use the appropriate boundary conditions, write the equations, solve it and this is going to be one of your practice problems.

Now we move onto something which you probably have done in your heat transfer, so I need not spend too much time on it. The system of the heat conduction in a cylindrical systems is slightly different than the heat conduction in the planar system. And it would give rise to interesting phenomena where you would see that in most of the cases when you add insulation to pie, the heat transfer from the pipe would reduce. So if you have a pipe which is carrying steam as a few put an insulation around it, the total amount of heat transfer from the pipe to the surrounding would increase, would decrease.

If you increase the thickness of this insulation, that means you are adding more resistance to the path of heat flow from the tube to the surrounding, therefore increase in the thickness of the insulation increases the resistance and hence decreases the total amount of heat transfer from the pipe. That is normally what would expect and that is what we generally see when we add insulation to a cylindrical system, or cylindrical or to a spherical system. But there are situations in which adding insulation may result in the decrease of resistance for heat transfer.

So if you if you are in that condition, that means adding an insulation reduces the amount of, reduces the thermal resistance and therefore enhances the heat transfer, that thickness of insulation is known as the critical insulation thickness. So there is a concept called critical insulation thickness, if you are beyond that critical insulation thickness, adding more insulation on that system could follow the logically what we think should happen, that means increasing the insulation decreases the heat transfer.

But if your system size is below that critical thickness of insulation then and if you add more insulation to it, then what you would expect is contrary to your expectation, that is you are putting insulation but you are enhancing heat transfer. So I am sure this concept is probably known to you but since it is such an interesting concept, so I will quickly show you how, how this is possible and why it is possible only for cylindrical and spherical systems in which the flow area changes with respect to the distance from the centre.

For a plane system, the flow area, the heat flow area does not change as you move away from the centre plane. But when you think of, think of a cylinder, if you increase the radius of the cylinder, the flow area available for heat transfer increases. The same happens for the case of sphere. So a sphere when you increase the size of the sphere, the area available for conductive flow of heat out of the sphere will increase. So the concept of critical thickness of insulation can only be found in the in those situations, in those geometries where the area increases with increase in its distance from the centre point or centreline.

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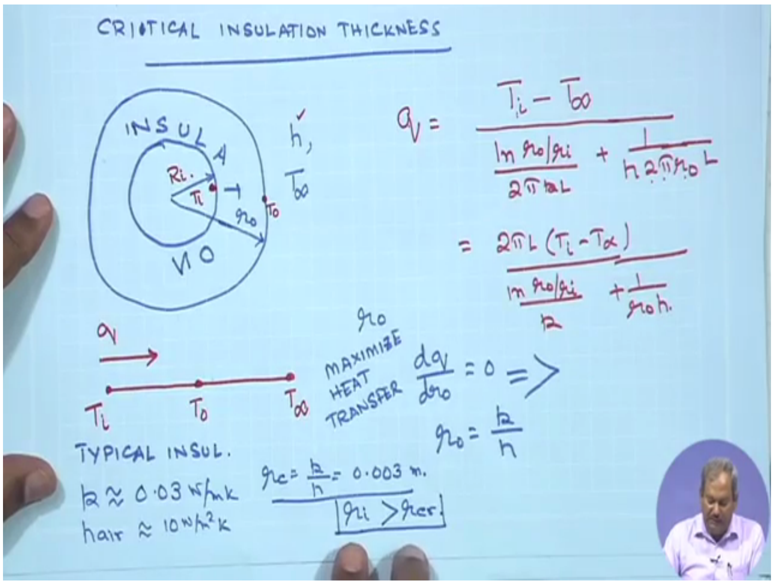

So I will quickly show you how, what is critical thickness of insulation, where you have a system, you have a pipe and an insulation around it. So this is, so this is an insulation around it, let us assume that the radius, inner radius of this is r I and the temperature at this point, that is over here is equals T I and let us see the radius of the installations is r0. So the total amount of heat flow, total amount of heat flow from the system is simply the potential difference which is T I - T infinity and divided by the resistance, the conductive resistance of heat transfer and it is in contact with a fluid with a value of H, with the value of heat transfer coefficient, convective heat transfer coefficient H and the temperature as T infinity.

So we are writing, we are drawing the circuit between TI, which is the inner temperature of the insulation and this could be some temperature TO which is the outer temperature, outside temperature of the insulation and this is T infinity. And some heat Q is flowing from the inside to the outside. So Q is going to be equal to the cause, the cause, this is the effect, so this is the cause, the temperature difference and the sum of resistance. And we know that for cylindrical systems, it is simply going to be r0 by r I by twice pie KL and for convection it is going to be 1 by HA, where H is the convective heat transfer coefficient and K is the area available for convection.

So area available for convection is simply going to be twice pie r0 times L, this is a cylindrical system. So one can write it as twice pie L, T I - T infinity by LN r0 by r I by K, +1 by r0 H. In order to evaluate the radius of the insulation, which is r0, that would maximise heat transfer, dQ by d r0 must be equal to 0, so this is maximise heat transfer with respect to the thickness of insulation. So if you put dQ by d r 0 to be equal to 0, this, I will not do this math, you can do it on your own, what you would see is r0 to be equal K by H.

So r0 equals K by H is an interesting formula. It simply tells you that if your radius of insulation is less than this value of r0 given by K by H, then by putting more insulation you increase your heat transfer. And this will keep on increasing till you reach r0. The moment you reach r0, the heat transfer at that point from the cylindrical system is maximum. Beyond r0, if you keep on adding insulation, the amount of heat transfer from the cylindrical system will keep on decreasing, that is what we would normally expect.

So your heat transfer starts to increase with r, so if I plot heat transfer on the Y axis and radius of insulation on the X axis, you would see that it, the heat transfer would increase, reach a point where it is maximum corresponding to r 0 which is equal to K by H and if you, since it is maximum, if you go beyond that, the heat transfer would start to fall. So this r0 is known as the critical thickness of insulation.

Now if you see the values of K and the values of H, you would normally, in normal applications you do not come across the critical insulation thickness because the thermal conductivity of most of the common metals or common objects and the value of heat transfer coefficient that is available to us are such that the value of r0, when a you have even a thin layer of insulation on something, you are beyond that critical thickness of insulation.

So add, in a steam pipe, you add insulation you reduce heat transfer. But there is only a few example, only only few examples, one example being that of an electrical wire. In an electrical wire, the combination of conductivity and the value of the heat transfer coefficient is such that you will probably be within the limit of critical thickness and therefore by adding an insulation and answers heat transfer from the electric wire. I will give you, put you give you some of the numbers which would which would be easier for you to understand.

So for the case of an electrical wire, for a typical insulation, for a typical insulation, thermal conductivity is about 0.03 watts per metre Kelvin and the value of H for the case of air, heat transfer of the air is about 10 watts per metre square Kelvin. So your rC would be equals K by H to be equal to 0.003 metres. So mostly what you would get is rI is greater than r critical. So r I is greater than our critical and therefore you do not encounter the fact, you do not encounter the effect that by increasing the insulation thickness you are going to get you are going to get enhanced heat transfer.

But for some situations this is there and even though you would need the basic purpose of insulation, providing insulation is to reduce heat transfer. So if you cannot do that, if you enhance the heat transfer by adding insulation, then you are defeating the purpose of putting the insulation in the 1st place. Fortunately as I said in almost all of the cases you do not encounter this. But if you are really working with a very thin electrical wire and you put a thin insulation, the combination would be such that by putting insulation you enhance heat transfer, which is good for the case of electrical wires because electrical wires, when current passing through it generates heat.

Unless you dissipate that heat, if you are putting an insulation on top of it, to make it electrical insulation on top of it to, for safe handling of it, then what you would get, what you would expect is that the heat transfer, the wire would get more heated, wire will get more hot which you do not want. What we have then is it a thin insulation, enhance heat transfer and let the wire operate at a safe temperature. So critical thickness for insulation may be desirable at certain certain cases, certain very special cases.

But mostly the fact we do not encounter because of the relative values of K and H in most of the heat transfer systems and we will always operate beyond the critical insulation thickness values. So what I would do in the next class is show you some more examples, some more interesting examples of shell heat balance in a reactor. So if we have a catalyst field reactor with a zone which is not where there are no catalysts, a zone where there is a catalyst and a zone where again there is no catalyst, so if the reactant mixture succumbing preheated at some point and then it encounters a catalyst and a reaction is going to take place.

As we move along and reach the end of the reactor, when it goes out of it, then obviously the reaction stops. It is an exothermic reaction and we would like to see whether having an exothermic or endo or having an exothermic or endothermic reaction in a reactor, in a catalytic reactor, how does that affect the temperature profile of the reacting gases as they approach the reacting zone, inside the reacting zone and as they leave the reacting zone. So that process from the fundamentals will try to model using the shell heat balance method that we have adopted so far.