Transport Phenomena.
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Lecture-32.
1-D Heat Conduction – Temperature Distributions.

So previously we have discussed about the heat diffusion equation, the different modes of heat transfer, realising that heat transfer is nothing but an energy in transit where thermodynamic tells us the states, the conditions of the end states, the energy content etc. of the end states but how energy gets transferred from one point to the other, from one control to the other control volume, the process is essentially the heat transfer process.

We identify the different modes of heat transfer, the requirement of having a medium in conduction and convection, that the convection can be divided into natural or free convection in which there is no imposed velocity and forced convection wherein an external energy forces the fluid surrounding the solid object over it, therefore enhancing the heat transfer from the solid object.

We have also seen that based on a simple energy balance, where the rate of energy in - the rate of energy out + any amount of heat which may be generated inside the control volume, the algebraic sum of these 3 terms would give rise to a time rate of change of the internal energy, total internal energy content of the control volume. So we wrote that E dot in - E dot out + E dot G, where E dot G is the energy generated per unit, energy generated within the control volume should be equal to the time rate of change of energy of the control volume.

We do not, we did not consider the effects of work done by the system or on the system which would change the total energy content of the control volume. So we would have to have a generalised equation that would take care of all possible sources of energy, all possible processes in which the total energy content of the control volume would change. But that we would pick up at a later point of time, so right now we are restricting ourselves to conduction only, to conduction only process.

As a result of conduction in and out of the control volume, the total energy content of the system will change and there may or may not be energy generation and this energy generation could be simply an ohmic heating or it could even be a nuclear heat source which is distributed inside the control volume. So we would like to see using simple methodology and towards the end of this class using a shell heat balance, same way as we have done shell momentum balance, a shell heat balance to obtain the temperature profile in a system where we have heat generation.

And we would also see that this kind of shell he would balance would work for systems with simple geometry. As the geometry gets more complicated, it would not be possible to use a shell momentum balance, thereby underlining the need for a more general scheme to solve such kind of problems or in other words the need for the development of energy equation would be felt as we move into move to problems with more and more complicated geometries for systems in which let us say distribution of heat throughout the or the generation of heat inside the control volume may not be uniform, it could be nonuniform and so on.

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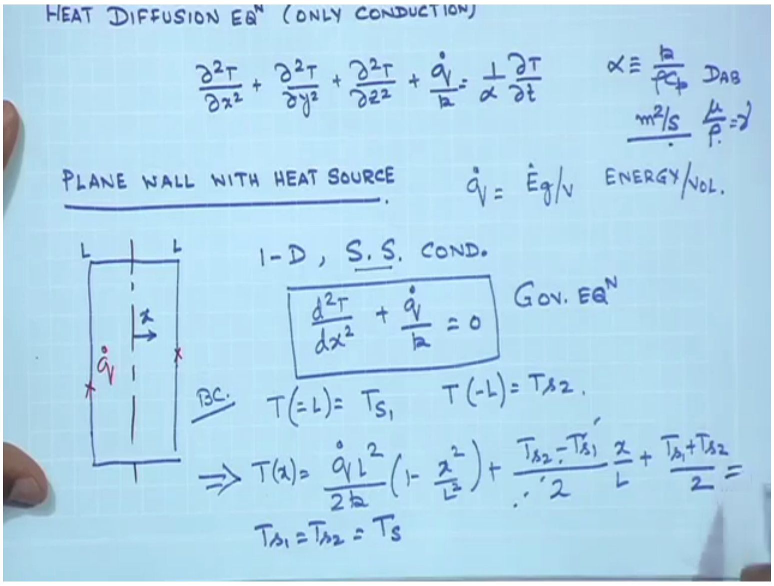

And how to handle force convection in which you not only have conductive heat transfer but you also have significant convective heat transfer process. So we will talk about that subsequently but let us concentrate on the heat diffusion equation which we have derived in the last class. So these are temperature variations with X, Y and Z and this Q dot is the energy generated per unit volume, this E dot G is the rate of energy generation by V where V is the volume of the, V is the total value, so Q dot is simply energy generated per unit volume.

This K is the thermal conductivity and Alpha is the thermal diffusivity which is defined and I have described, discussed what is the significance of Alpha which is K, the thermal conductivity of the solid, rho the density and CP the heat capacity and the unit of Alpha would be metre square per second. In this way it is going to be similar to, similar in concept to mu by rho which is kinematic viscosity and DAB which is the diffusion coefficient of A in B, all 3, Alpha, mu by rho which is denoted by this symbol, commonly denoted by this symbol, so the thermal diffusivity, the momentum diffusivity and the mass diffusivity will have the same unit as metre square per second.

But let us come back to this problem in which we have a plain wall with a heat source that is distributed uniformly inside the control volume, so some heat is going to be generated in this and on these 2 sides it is open to, let us say atmosphere and we would like to solve this equation is at steady-state for this system in which you have a Q dot, some amount of heat which is generated inside the control volume by whatever, for whatever reasons, it could be ohmic heating or it could be a heat source system which is distributed inside the control volume.

And if we assume it is a one-dimensional steady-state condition, if it is one-dimensional steady-state condition and this is a solid and if this is my X direction, then T is not a function of Y, it is not a function of Z and it is definitely not a function of time, since it is assumed it is steady-state. So what you have then is d 2T dX square, no need to use the partial, + Q dot by K would be equal to 0. So this is the governing equation for conductive heat transfer in a plain wall system where the temperature, where the heat conduction is one-dimensional, it is only in the X direction and it is at steady-state, so therefore there is no time term present in here.

And let us assume that T, the boundary conditions which are available to us is the T at L is equal to TS1 and T at - L is equal to TS2. So the 2 temperatures at these 2 points, they could be different, these are TS1 and TS2. So when you solve these equations and these boundary conditions, the temperature profile that you are going to get is Q dot, the amount of heat generation for unit volume by twice K, 1 - X square by L square + TS 2 - TS 1, the 2 known temperatures at the 2 extremities, by 2 XY L + TS 1 + TS 2 by 2.

So this is the complete profile of the temperature inside the, inside this this solid. So if you if you see that, if you so this is for the case where TS 1 and TS 2 are different. So if you have a symmetric situation in which the 2 end temperatures are identical or another word TS1 is equal to TS 2, so if you look at the discussion, you clearly see that TS would simply be equal to would be 0 and this is going to be simply and if we assume that TS 1 is equal to TS 2 is equal to some constant TS, then this expression would result in the following simplified expression.

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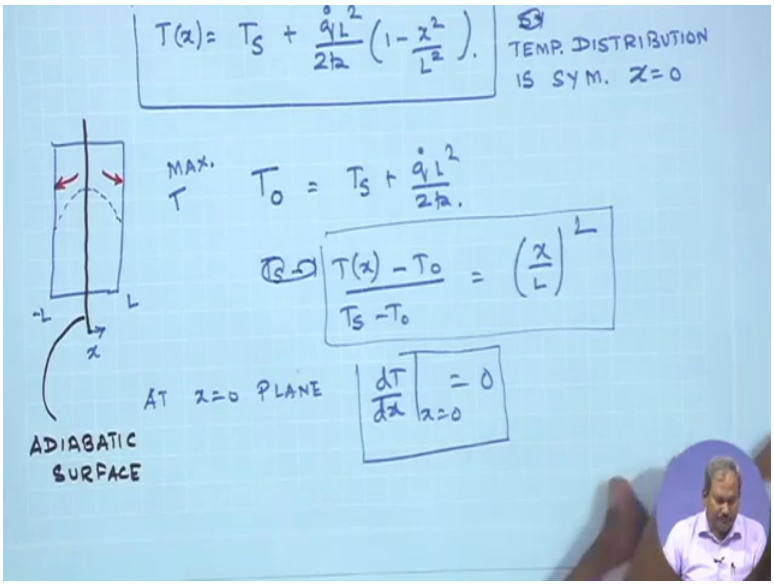

As TX is TS + Q dot L square by 2K, 1 - X square by L square. And so therefore you have this expression where we have symmetric heat generation. The temperature distribution is symmetric around X equals 0. So if you, this was the plain wall and this is the X equals to 0 plane, this is L and this is - L, so looking at this profile, it is clear that the temperature profile is going to be symmetric around the X equal to 0 plane. So it would probably look like this, it is an invert, it is going to be a parabolic, it is going to have a parabolic distribution and any heat generated inside the wall is going to travel in this direction and then out of it.

So your temperature, the maximum temperature in this case and if I consider, if I call it as T0 and it is clear from here that the maximum temperature exists where X equal to 0, that means at the midplane and T at that point, if I call it as T0 would simply be TS + Q dot L square by 2K. Okay. And if you just rearranging the symbol 2 equations, what you get is T of X - T of 0 by TS - T0 is equal to X by L whole square.

So the dimensionless temperature distribution inside a wall where there is uniform heat generation and where the 2 ends of the wall are maintained at constant equal, constant equal temperatures is going to resemble a parabola and this would be the dimensionless form of the equation. And at X equals to 0 plane, what you see here is that dT dX at X equals to 0 is 0. So the plane you have over here at which dT dX is 0 can be called as an adiabatic surface.

That is at X equals 0, the plane, at this plane X equal to 0 would simply V and adiabatic wall since you have dT dX equal to 0, so no heat travels or no heat losses, no heat flow process this plane in either direction. So this is truly an adiabatic surface. Let us say in some cases this plane wall which is generating heat, the 2 end temperatures are difficult to evaluate.

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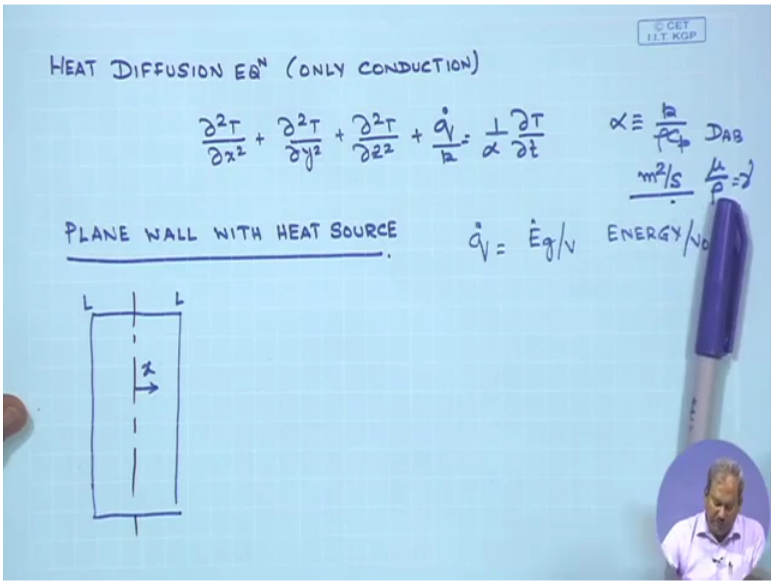

So if we have a situation in which the end temperatures of this wall are not known, however they are placed in a liquid, whose temperature, the surrounding temperature TS, this is known, the temperatures at the 2 walls are not known. So in that case the boundary condition to be used, you remember, you remember that previously the boundary condition that we have used is at X equals L, TX is equal to TS which we cannot use now, since the value of the temperature at X equals L at this point is not known to me, however the surrounding fluid temperature at a point far from the wall is known to me.

So we need to express our boundary condition, the new boundary condition that we we are going to use must concentrate on this surface. So we are going to take this surface as our control surface, so if I take this as a once control surface, what we have done is whatever heat comes to the control surface must be equal to the heat that gets convected out of the solid into the liquid, so this is the liquid and this is a solid wall. So if I consider this control surface from the, from the solid side, I am going to have conduction heat transfer towards this control surface and from this point I am going to have convective heat transfer out of the control surface.

And at steady-state, these 2 must be equal. So if these 2 must be equal, what can, what I can write for the control, what I can write for the conduction of heat towards the control surface would simply be equals K dT dX at X equals L, so this is a conduction heat transfer and this must be equal to H times TS - T infinity, this should be T infinity. So this is the temperature of the fluid at a distance far from the solid and TS is this temperature that would not know, however this temperature is experimentally known to me.

So I am expressing TS in terms of T infinity by invoking the equality of conduction and convection at the control surface located at the junction of the solid and the liquid. So we already know that is TX is equals Q dot L square by Twice K1 - X square by L square + TS, so when I put that in here and what I would get is simply TS is equal to T infinity + Q dot L by H.

So putting this expression of T in here and simplifying what you would get is the temperature of the surface is going to be the temperature of the surrounding fluid + a term which has in it the amount of heat generated per unit volume, the half width of the solid plate and H is the parameter which is related to convective heat transfer. So this is a nice example, a simple example of how to treat heat is generated inside a control volume for a planar system where we have heat generation in a plane system.

The system, the problem should be slightly different and we have, I can give you some examples, example problems for you to work on, so I will simply write the problems and give you with the numbers, what you have to do is using the concept already developed, find out the answers to the problem.

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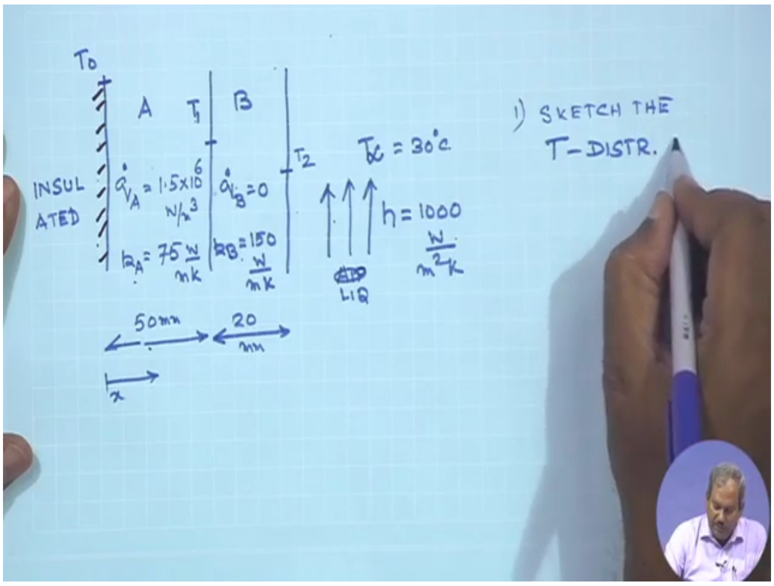

So the problem that we have, I am going to give you as a test problem is, I have 2 surfaces A and B and this side of A is insulated. So if this side of A is insulated some which simply means no heat crosses from A to the outside and let us assume that the temperature over here is T0, the temperature over here is T1 and the temperature over here is T2. I am placing the values T0, T1 and T2 just, T1 is just below T0 and T2 is just below T1 but I am not exactly sure what would be the value of T1, relative value of T1 with T0.

It can very well be that T1 would be the same as T0, T1 could be more than T0 or T1 could be less than T0 and so is for T2. So we have to use your simple logic to find out, to see whether or not T1 is going to be more or less than T2, think about the way the heat always travels from high-temperature to low temperature and that would give you some indication of whether T1 is going to be less than T0 or more than T0. So I will leave that for you to figure out.

This in A, I have some amount of heat which is generated that I denote as Q dot by K as 1.5 into 10 to the power 6 watts per metre cube, the K of A is 75, the thermal conductivity is 75 watts per metre Kelvin, the thickness of the plane wall A is 50 millimetres, my X starts from this point and the thickness of B is 20 millimetre, Q dot B is 0, that means no heat is generated in B. The thermal conductivity of the B material is 150 watts per metre Kelvin.

And on the outside, outside surface of B, I have flow of air or flow of any liquid which is moving past this outer surface, the T infinity, that means the temperature of the liquid, the fluid which is flowing along this is equal to 30 degrees centigrade. And convection condition outside of B maintains a convective heat transfer coefficient of 1000 watts per metre square per Kelvin.

So the system once again is, you have 2 walls A and B, A has heat generation, B does not have any heat generation, the thermal conductivity of A, the dimension, the thickness of A, thermal conductivity of B and the thickness of B is provided, the other side, the left-hand side of A is perfectly insulated, the right-hand side of B, the outer side of B is exposed to a convection environment where the fluid temperature is 30 degrees centigrade and the heat transfer coefficient, the convective heat transfer coefficient is 1000 watts per metre square per Kelvin.

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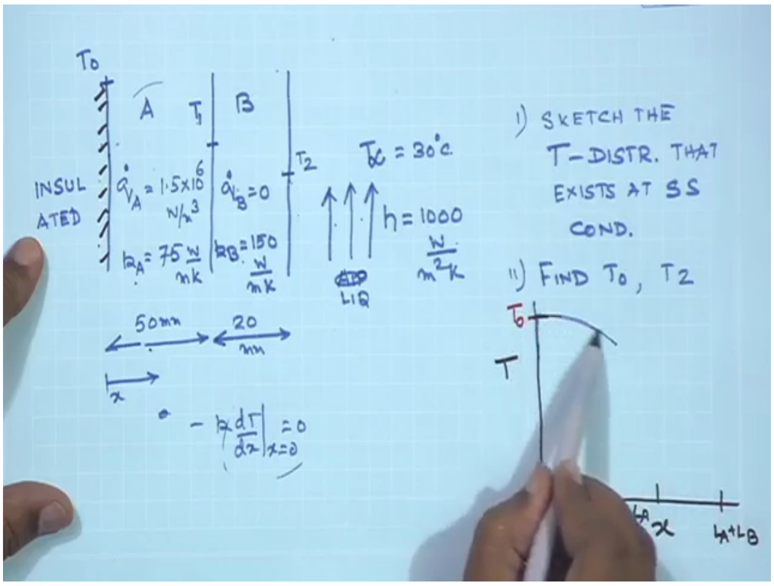

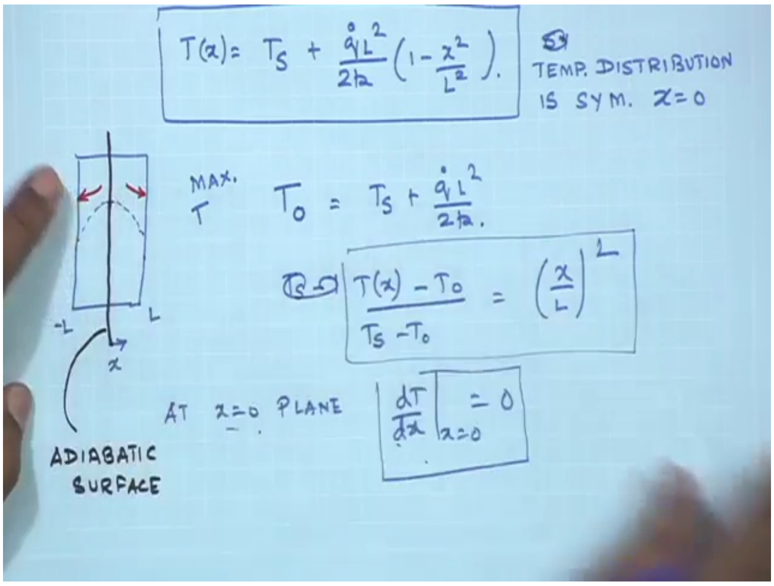

What you have to do is the 1st part is the 1st word is sketch the temperature distribution that exists at steady-state condition in the system and the part 2 is find out, find T0 and T2. Find out the temperature of the insulated surface and temperature of the surface that is cooled by through the use of convection. So find out what is T0 and what is T2. So I will not solve this problem but I will simply give with the pointer about how to proceed about it.

If you look at this one, you have a uniform generation of heat and no heat crosses this side. So if no heat can cross to this side, so this mark, at this point the convective heat must be 0, so - K times dT dX at X equal to 0 must be equal to 0. So if I am if I am plotting the temperature profile as a function of X at X equals 0 and that say this is the thickness of the 1st part and this is the thickness of, this is L, this is L A + LB, soft to this point is the thickness of this and L A + LB is the total thickness.

And let us see we start with, this is the point the temperature of the insulated plate. Now if dT dX is 0, then whatever be the profile, it must approach this with a 0 slope. And I would bring to the profile which we have obtained in this case. Here also see if you only consider this half of the of the plane wall in which he had is being generated, we saw that at this point, at X equal to 0, dT dX is equal to 0. So at this point dT dX is 0.

So if you place these 2 one after the other, this one below this, you would see that there is not, no difference between this half of the plane wall and A where some heat is generated, heat is generated, the boundary condition at X equals 0, is dT dX is 0, the boundary condition at this point is dT dX is 0 since you have an insulated wall. So whatever be the nature of the profile over here, the same profile should also exist in this.

That is if I draw the temperature profile from T0 to T1, it must look like a half of a parabola where the slope at this, where it would approach the plane wall with 0 slope, so that, such that the temperature over here is going to be 0 slope and it is going to decrease all the way up to this point where the temperature is going to be equal to, this temperature is going to be equal to T1. So the reason is clear right now and then we get in get into this part. In this part the temperature over here is T1 and the temperature at this point is T2.

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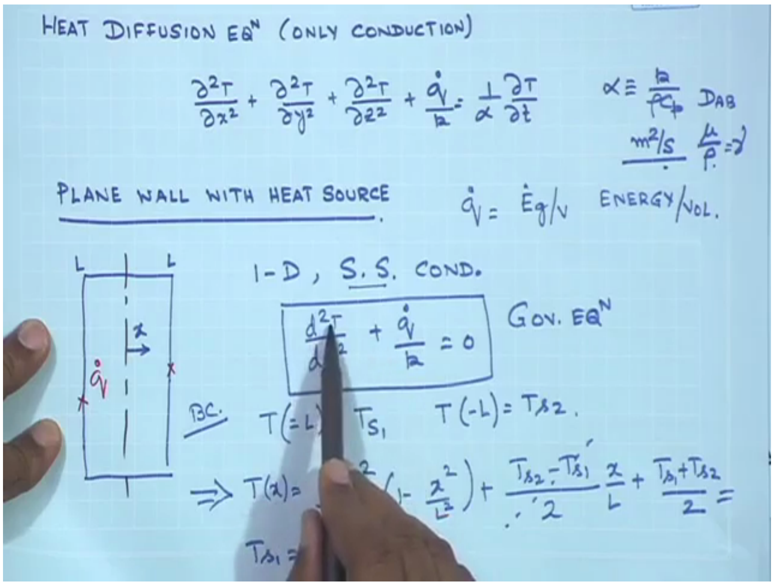

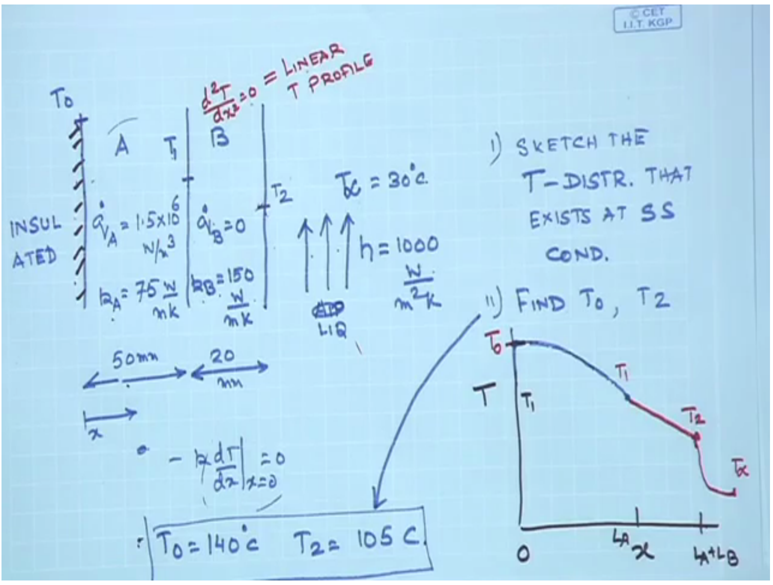

For the heat to transfer from left to right through A and through P, T2 must be less than T1. So let us assume this is my value, this is my value of T2. Now in this we have a plane wall where there is no heat generation. So if it is a plane wall with no heat generation and if we look at the governing equations in this case, then Q dot being 0, d2T dX square would be equal to 0. So the governing equation for the, for this is simply going to be d2 T dX square to be equal to 0 and which would give rise to a linear temperature profile.

So the temperature profile in between T1 and T2 would simply be linear. The slope of this profile would depend on the value of the heat transfer, value of the conductive, the thermal conductivity of B. More higher the value of thermal connectivity, lesser is going to be the slope of the light, straight-line connecting T1 and T2. And if you have a very low conductive wall, if the thermal conductivity of B is extremely low, then this slope will be even more. Beyond B, it is going to be only convection.

And we know from our discussion in previous classes, that there would be a thin layer of liquid very close to the hot wall in which most of the transport processes are going to take place. Beyond that thin layer, nothing will happen, there would be no transport, no effective transport of energy from that point to the bulk. So the temperature is T2 will asymptotically reach T infinity over a very short distance near the wall which is essentially the concept of thermal boundary layer.

So temperature of the solid wall in contact with the liquid will approach the liquid temperature over a very thin layer, over a very small distance from the solid wall itself. So there is going to be sharper drop of temperature in a region close to the outside of the wall and then the temperature will asymptotically reach the value of free stream, value of the fluid which is moving at some velocity.

So it has been given that T2 is equal to, sorry the T infinity here, if it is T infinity over here, then what you would get is, there would be a sharp change in T2 and then it will asymptotically reach the value of T infinity. So this sharp change takes place over a very small value of X and this can roughly be called as the extent of the thermal boundary layer over which the temperature changes from T to T infinity. So this, I would provide you with the values which you can check on your own.

The value of T0 you should find to be equals 140 degrees centigrade and the value, what was my final T0 and P2 I think you are, you do not have T2 would be equals 105 centigrade. So these are the answers for part 2 of the problem which you can do on your own. In the next part I will quickly draw the figure but continue this in the next class.

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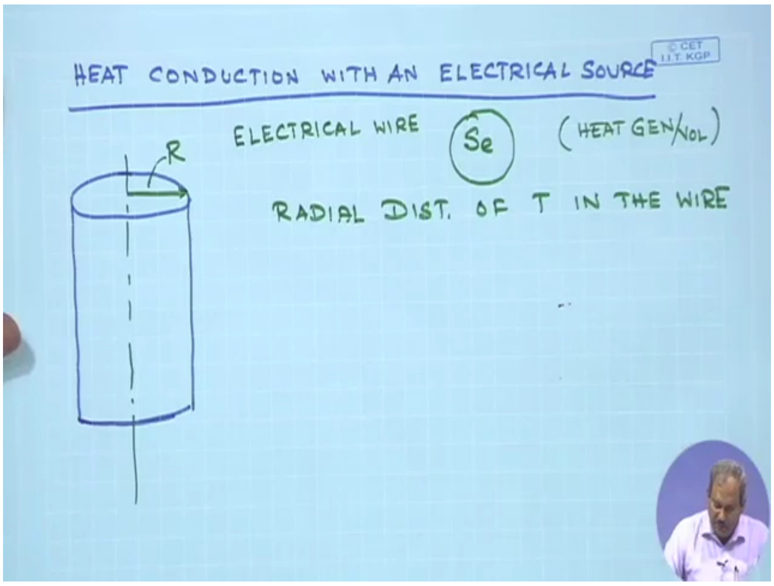

What we need to do is heat conduction with an electrical heat source. This is the process scheme that we are going to model in the next class, heat conduction with an electrical heat source. This is a very common situation in which let us say I have a electrical wire through which a current is being past, it is cylindrical in nature. So what I have then is, this is the centreline and the radius of this wireless is equal to R. So this is an electrical wire, the example could be an electrical wire which has obviously a resistance and when current passes through it, there is going to be some amount of heat generation.

And we will call this heat generation per unit volume, so this is heat generation per unit volumes due to the flow of current through the electrical, through the electric wire. And I am going to have ohmic current which is simply going to be related to I square R square I is the current and this is the amount of heat which is being generated in here. So our job is to find out the radial distribution of temperature in the wire. So this is the problem that we are going to do in, we are going to continue in our next class.