Transport Phenomena.
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Lecture-31.
Heat Transfer Basics (Continued).

So we will start with our 2nd part of the heat transfer and as we have seen what are the basic rates, basic modes by which heat can get transferred and the special considerations that are to be included for the case of convective transport process which can also be divided into free convection where there is no external agency forcing the fluid to flow over the subject, over the solid object from which heat transfer is taking place. And we have a case of force convection.

So the fundamental study of heat transfer starts with our identification of the conservation of energy principle. So for any object we 1st have to realise, 1st have to appreciate the conservation of energy principle and that conservation of energy principle when expressed in terms of mathematical relations could give rise to the complete equation describing the energy transport from or to the object. And energy conversion inside the object if that is relevant as a result of which, the total energy content of this object will change.

Energy can also be changed if you have, if you include the work done to or by the system. So work done on the system will enhance its energy and work done by the system will reduce the total energy content of the object. So all these , all these considerations must be taken into account while writing the conservation equation for any system. So we, but we are going to start with the simplest, simple conservation equation to begin with where we will at the moment not consider any work effects.

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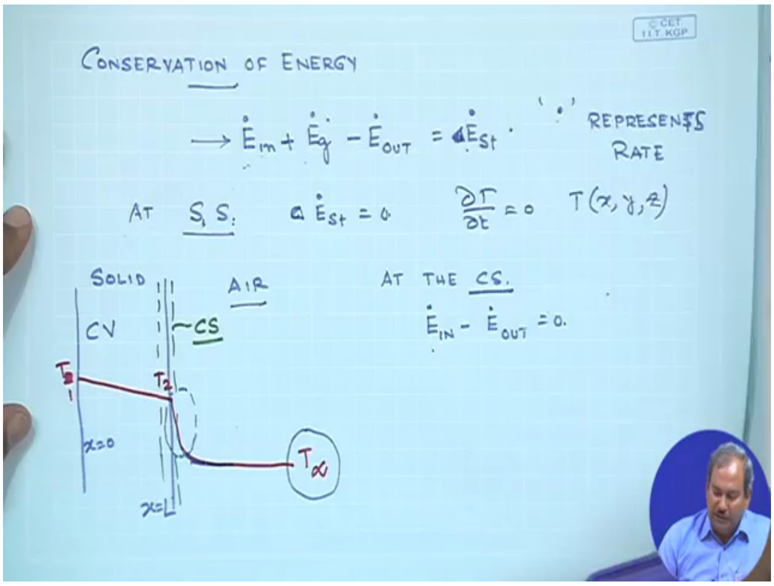

So all our consideration is the tangible form of energy which comes in or out, to or off the control volume as a result of conduction or convection. So we are, we start with the conservation of energy relation which simply says that the rate, the dot represents essentially the rate, the time rate. So rate of energy coming into the system by either convection, conduction or a combination of both + E dot G where E dot G generates, denotes the generation of energy inside the system.

This generation could be due to the ohmic heating or due to the presence of a nuclear source in the control volume - E dot out where this is the rate of energy which is going out of the control volume as, again as a result of conduction and/or convection. And the sum total of all these 3 is E dot stored. So the rate of change of stored energy in the system.

So when we talk about steady-state, SS steady-state, the E dot ST, the time, the rate of energy stored is going to be equal to 0, that means there would not be any net storage of energy as a result of these few processes, the temperatures could be a function of space coordinates, that means the temperature could be different at different points in the control volume but dell T, temperature by dell time, that is the temperature will not be a function of time, so dell T by dell time would be equal to 0.

However we understand that T could be a function of the 3 space coordinates. And if you think of a solid surface and air in contact with it, and let us assume that the air temperature is T Infinity at a point far from the, far from the solid edge is T Infinity. The inside temperature is T1 at some location, let us say this is at X equal to 0 and this is at X equal to L, the temperature is T2. So you would see later on that in absence of any heat generation in the solid, the temperature profile will be linear but then it would sharply reduce and asymptotically merge with the temperature T Infinity, that is the temperature at a point far from it.

So if you consider this region which is very close to the solid surface in the fluid where the temperature sharply changes from T2 to that of T Infinity far from the plate. And yet you see that the temperature asymptotically approaches the free stream temperature, so this by analogy with our previous discussion, you can clearly see that this essentially establishes the concept of a thermal boundary layer.

So same way as in velocity boundary layer, where the velocity increases as we move away from the flat plate and reaches the value equal to the free stream velocity, the same way the temperature the temperature changes from that of the solid and it gradually approaches to the bulk temperature or T infinity, that is the temperature at a point far outside of the effect of the solid present in contact with the fluid. So the distance over which this temperature variation takes place will in a proper way, and a slightly modified way will be expressed as the extent in which the heat transfer is taking place.

Because outside of that the temperature does not vary with distance anymore. So no heat transfer, convective or conductive is taking place in the region where the temperature reaches T infinity. So ultimately we will try to bring in the same concept as that of velocity or hydrodynamic boundary layer in the case of heat transfer as well in the form of thermal boundary layer. But coming back to the problem that we were discussing about, so if this is my control volume, this expression applies to the control volume, so E dot in, the amount of energy which comes from the left of the figure + E dot G, if this is a nuclear fuel element, then some amount of heat would be generated.

If it is not, the E dot G would be equal to 0 and E dot out is the energy which goes out to the air surrounding it as a result of which the net energy content of the control volume may change and if it does not change, then we, what we have is a steady-state situation. So in absence of energy generation and if it is a steady-state system, E dot in must be equal to E dot out. Now what is control surface? The dotted line which I have drawn can be thought of a control surface which by the definition does not have any mass of its own.

So if it does not have any mass of its own, then it cannot store any energy and no energy is generated in it. So at the control surface E dot in must always be equal to E dot out. So this is the, the these are the some of the concepts which we would use in our subsequent analysis.

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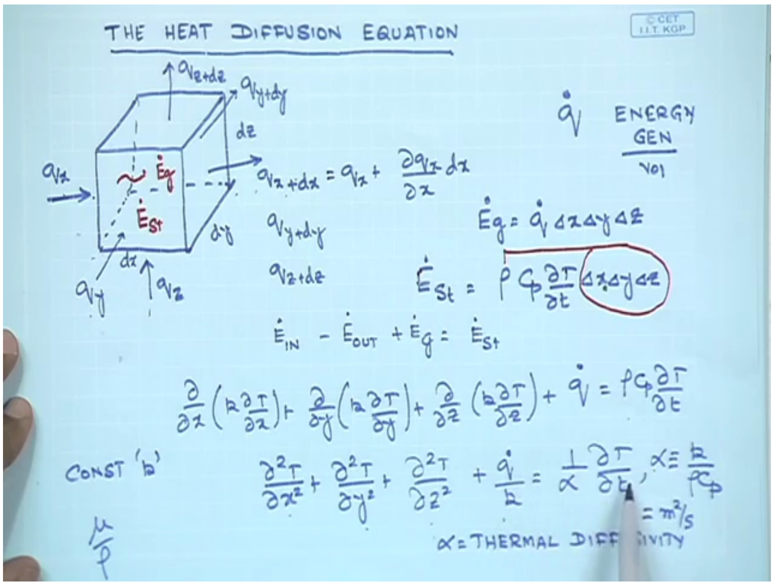

The next part that we are going to do is the heat diffusion equation, which I am sure you have done before but very quickly it tells me that if I now, if I have a control volume of size dX, dY and dZ and if we consider only conduction, let us say QX is the amount of heat coming in, QX is the flux of hits which is which is coming through the surface, through the X face and QX is going out, QZ is coming in through the Z face, QZ + dZ is going out of the Z face, QY is coming in and QY does dY is going out.

And QX + dX, that is can simply be expressed in the Taylor's series expansion, the same way we have done before. So this is for QX + dX, same way you can write for QY + dY and QZ + dZ. So that will simply be, you can simply write it and if we assume that Q dot is the energy generated by some means would be nuclear per unit volume inside this, then the total amount of heat generation in this control volume would simply be equal to dell X dell Y dell Z and E.

So this will be equals E dot G, energy generated and E dot stored would be equal to rho CP dell T by dell time dell X dell Y dell Z. If you see this row times dell X, dell Y, dell Z is simply equals to the mass of the mass of the control volume. So M CP dell T by dell time is the time rate of change of energy stored in the system. So you can see it any textbooks, when you write this in the, write the balance equation as E dot in - E dot out + E dot generated is equal to E dot stored in this.

And put in the, plug-in the values of QX, QX + dell X, QY + dell Y and so on. And you simplify, what we would get is dell dell X of K, dell T dell X + dell dell Y of K, dell T dell Y + dell dell Z of K, dell T dell Z + Q dot which is the energy generated per unit volume is equal to rho CP dell T by dell time which is the energy stored per unit volume. This is the amount of energy, net conductive heat flux into the control volume from the X direction, from the Y direction and from the Z direction.

So if we assume that it is a constant K case, the thermal conductivity of the system is constant, then K can simply be taken out of it and what you get is dell 2T by dell X square + delta 2T by dell Y square + delta 2T by dell Z square + Q dot by K is 1 by Alpha delta T by dell time, where Alpha is simply defined as K by rho CP. So there is a reason why Alpha is expressed in here instead of rho CP by K, I simply write one by Alpha because Alpha has units of metre square per second.

So this is called, the Alpha is called the thermal diffusivity of the system. When you, when we will talk about, when we spoke about the moment diffusivity which was denoted by mu by rho, that the unit of mu by rho which was the kinematic viscosity, that is also, its unit was also metre square per second. So mu by rho is sometimes called as the momentum diffusivity.

So these 2, the momentum diffusivity or the thermal diffusivity having the same units as metre square per second, it came from the concept of diffusion coefficient which we would discuss when we will talk about the Newton’s, when we will talk about the mass transfer process, the convective mass transfer process which is fixed for 1^{st} law of diffusion which connects the amount of diffusive transport of mass is equal to - dAB, where dAB is a diffusion coefficient of A in B times dell CA, dell X, if it is a one-dimensional conduction, one-dimensional mass transfers process.

And this dAB the mass diffusivity has units of metre square per second. So by simply rearranging the terms in the form of mu by rho in momentum transfer and K by rho CP in heat transfer and dAB, since all of them have the same units as metre square per second, so borrowing the term from mass transfer K mu by rho is called the momentum diffusive it in, K by rho CP is called the thermal diffusivity and dAB is simply called the diffusivity or mass diffusivity.

So that is why that the equation of energy is generally expressed not in terms of K, Rho or CP but the combination variable which is in which is expressed, which is which is called the thermal diffusivity. So this is the equation what you would get for conduct of transport of heat only for conduction, only conduct of transport of heat where the temperature can vary in X, Y and Z direction. This is the heat regenerated per unit volume and this entirely, the entire thing is the total amount of energy stored or lost in the control volume per unit volume.

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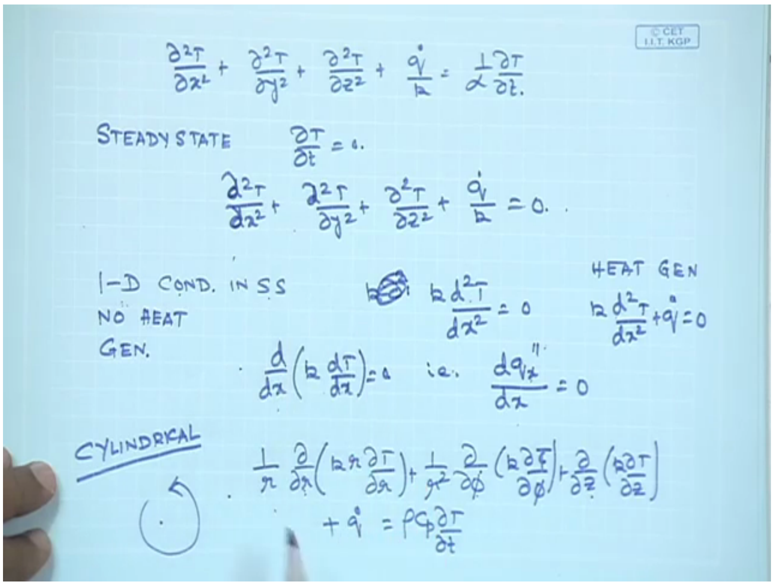

So this is the starting point. And if I write this equation one more time, what we have is dell 2T by dell X square + delta 2T by dell Y square + dell 2T by dell Z square + Q dot by K is 1 by Alpha dell time by dell T. So if it is, if it is a steady-state, if it is a steady-state, then dell temperature by dell time would simply be equal to 0 and what you will get is d2T dX square +, sorry dell, delta 2T dell Y square + dell 2T dell Z square + Q dot by K is equal to 0.

If it is 1-D conduction in steady-state and let us see the temperature is, the heat is only getting transported in the X direction, then what you would get is K times dell 2, there is no reason for writing it in partial form, K d2 T DX square is equal to 0 or you can simply write as d dX of K dT dX is 0 or in other words what you can write is, what you can see is delta QX double prime by dX is equal to 0 since K dT dX is equal to - of QX.

So the, this analysis simply tells me that for one-dimensional conduction only case at steady-state, K times d2t dX square is equal to 0 which directly follows from the previous equation and obviously there is no heat generation. So if there is no heat generation, this will be the form, if you have heat generation in the system, then you are going to have K times d2T dX square + Q dot equal to 0. And this simply can be written as dDX of K dT dX which is K dQ

X double prime, so this is the heat flux, the double prime denotes the flux, so dQ X dX equal to 0.

So these expressions are very common and would give rise to several other simplifying cases. So this is in Cartesian coordinate system, so if you write it in cylindrical coordinate systems, then you should be able to see it from the text, you would, you would see that for the cylindrical coordinate system, 1 by R dell dell R of K R dell T dell R + 1 by R square, there was no new concept involved in the cylindrical coordinate expression, it is only that when you transform from Cartesian coordinate system to cylindrical coordinate system, you express everything in terms of R, Phi and Z. Where R is the radial distance, Phi is this direction and Z is the axial direction.

So when you transfer the coordinate, the form of the equation, the form of the equation gets slightly more complicated but fundamentally it still remains the same. Similarly in the case of spherical coordinate systems, in new textbooks he would be able to see which form the fundamental equation takes when you transform it from Cartesian coordinate system to the spherical coordinate systems, that means in terms of R, Theta and Phi. So depending on which coordinate system you are using you have to choose your conduction equation.

I have derived the equation in Cartesian coordinate system but in your text you would see the same equations are given in cylindrical coordinate systems as well as in spherical coordinate system. In the Cartesian coordinate system, QX double prime, the conductive heat transport in the X direction is equal to - K times dT dX or dell T dell X. QY double prime is equal to - K times dell T dell Y and so on.

In cylindrical and in spherical coordinates, these expressions, for example QR, that is the flow of heat, the flux of heat in the radial direction, for cylindrical systems, for spherical systems would be slightly different and new terms would appear in the expression for QR in cylindrical and Q R in spherical systems. But fundamentally there is no new concept involved. It still can be derived from the basic fundamental equation that rate of energy in - rate of energy out + rate of energy generated is equal to the rate of energy stored in the control volume.

So the transformation of the coordinate system would slightly modify the expression of QX, QR or Q Theta, Q phi and so on. But there is no fundamental difference. So one should start with the right equation, cancel the term which are not relevant and then apply the boundary conditions, relevant boundary conditions to obtain what would be the temperature variation, the temperature distribution in a solid, be it a rectangular, a cylindrical or a spherical solid and to get a complete picture of the temperature profile present in such a case.

So when we talk about boundary conditions which are present in a, in the system experiencing heat transfer, let us see what are the possible boundary condition that can that can be there. The 1st boundary condition is the temperature at a point can be specified, so you precisely know what is the temperature at a specific point. So if you, if a solid let us say is in contact with a fluid and you know what is the temperature of the solid at the edge, at X equal to 0, when it is in contact with the liquid, then the temperature at this point is specified. So the 1st condition that you can think, 1st boundary condition that you can, you should look for while solving the equation is, if the temperature at any point in the control volume is given, is provided, is known to us.

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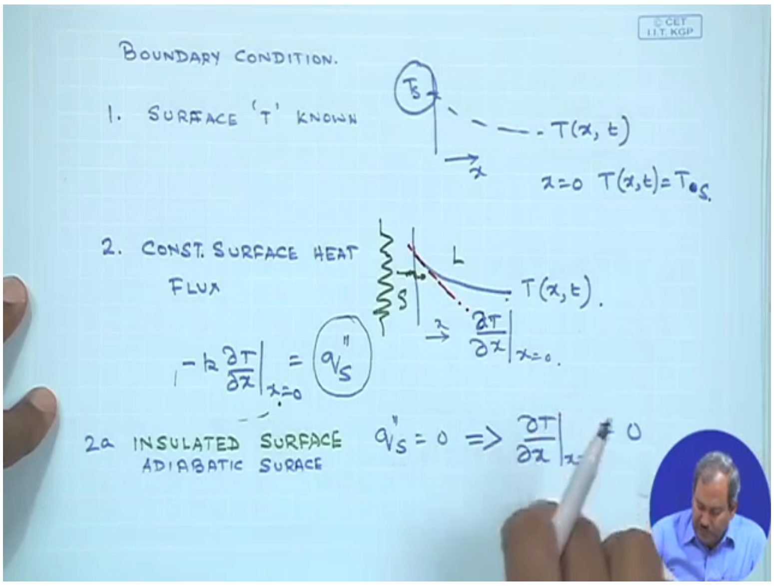

So the 1st boundary condition which is relevant in heat transfer is one is surface temperature known. So let us say this is the X direction and this T surface is known and over here the temperature may vary as a function of X and T but we know that at X equal to 0, T, X, t is simply equal to TS which is a constant. The 2nd point can be, it is a constant surface heat flux. What is a constant surface heat flux? That means you have a way, a means by creating a condition, let us say this is the temperature profile where T, X, t is known and if I draw a slope to this line, so this is essentially dell T by dell, if this is X.

Dell T by dell X at X equals 0. So if you multiply this with - K, - K dell T by dell X at X equal to 0 which is nothing but the heat flux at the surface. So it could be such that this value of Q dot double prime is provided to you. The meaning of Q dot double Prime S at the surface provided to you, it could be that you have a resistance heater in here. The resistance heater is providing a certain amount of heat into the solid which is in contact with the liquid.

So at steady-state, the amount of heat which is generated, which is supplied by the heater has to come and has to go out of the surface. So at this point which is the junction of liquid-solid of the solid liquid interface, the heat flux is a constant. And if this is the profile, then as we understand that you can never have conduction, never have convection without conduction, so conduction is the means at which heat gets transferred to the 1st layer of liquid molecules.

So - K dell T dell X evaluated at X equal to 0 which is the heat flux lost by the solid to the liquid, this must be equal to the amount of heat which is supplied to the solid. So - K dell T dell X at X equal to 0 is Q double Prime S which is another way of saying that the surface heat flux at that point is known to us. So that is the 2nd boundary condition one can, you can think of. There is the 3rd type of boundary condition is went by an artificial means you do not allow any heat to cross a specific interface.

So you have this as a solid and you are going to place a perfect installation at the side, so if you place a perfect insulation denoted by this black object, what happens is no heat can cross through this insulation and come out on the other side. So if you can apply a perfect insulation on one side of it, then going back to what we have said, what we have shown here before, this QS double prime is equal to 0. So if you have an insulated surface, then your Q double Prime S is equal to 0, so I call it as 2A, it is a special case of this.

So this would give rise to dell T by dell X at X equals to 0 is 0. So if by some means you could insulate a surface perfectly which is an idealised condition, then no heat can cross this interface QS double prime is 0, so therefore dell T dell X is equal to 0. So this is known as insulated surface or it is an adiabatic surface, the condition for this would simply be equals QF double prime to be 0, dell T dell X equal to 0.

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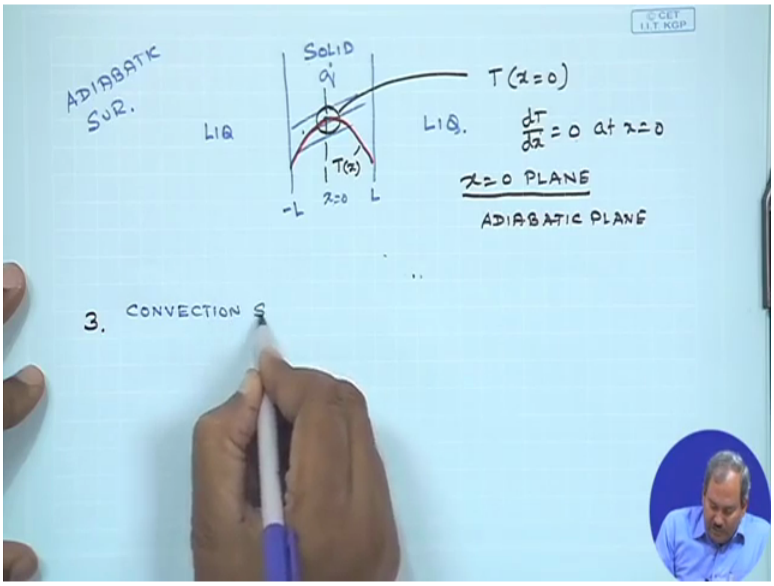

In some cases, I am describing this adiabatic surface, adiabatic surface in slightly more detail, let us say you have a solid, on 2 sides of it you have liquids and some amount of heat is being generated in here uniformly. So everywhere there is some amount of heat which is generated. I would show you later on but the profile would look something like this, with the centre line passing, let us see this at X equal to 0, this is at X equals L, this is at X equal to - L.

So when you consider this plane which is located at X equal to 0, if you see this region, this is your temperature versus X profile. At this point T at X equals to 0, if you, if you see close to this location, what you would see is dT dX is equal to 0. The nature of the curve tells you that for the special case when you have a solid and some amount of heat is generated uniformly and suppose we take which you would see later on that the profile would be like an inverted parabola, then the apex of the parabola which is located on X equal to 0 will have a slope equal to 0 at X equals to 0.

So the X equals to 0 plane, no heat crosses from the left to the right from the right to the left, so the X equal to 0 plane is known as the adiabatic plane. So for an adiabatic surface, adiabatic plane or for an insulated surface as we have seen before, so both on an insulated surface and for an adiabatic surface, the boundary condition would be the same which is dell T, dell T dell X to be equal to 0 or in the case of one-dimensional conduction case, dT d X is 0 at this.

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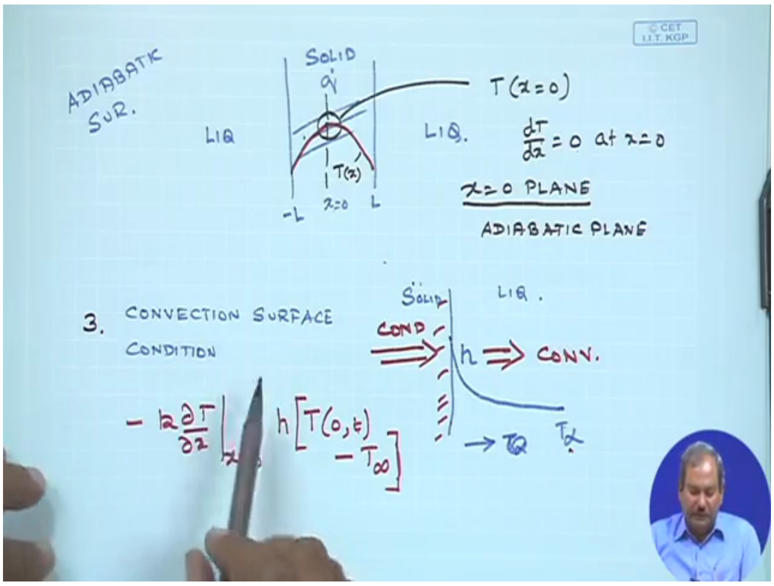

And the 3rd condition could be that I have a convection surface condition where you have surface which is solid, on this side you have a liquid let us say and the profile of temperature is this where you have T infinity, the temperature over here is T infinity and the heat transfer coefficient involved is H. So heat will come from here to here by means of conduction and from here to out by means of convection.

So what happens at steady-state is then - K dell T by dell X at X equals 0, this essentially denotes the amount of heat which comes in by conduction from the interior up to this point has to be equal to the heat which goes out of the convention and if I use Newton’s law which would simply be H times T, T at X equals to 0, at anytime - T infinity.

So the convective convective heat loss must be equal to the convective heat which is coming to the interface. So by conduction you have some amount of heat coming in and by convection the same amount of heat is going out. So this T, 0, t denotes the temperature of the interface at any given time at location 0 and T infinity is the constant temperature of the bulk fluid situated far from the solid wall. So this is the convection surface condition.

So we would, we would use the conduction equation to start with the appropriate coordinate systems with appropriate boundary conditions. And quickly solve, quickly try to solve and see if we can get the temperature profile of a solid object which is experiencing conduction. Initially we would restrict ourselves to the case of steady-state where the temperature could be a function of X, Y, Z or R, Phi, Z or R, Theta, Phi but it is not a function of time. But from our experience we know that heat, temperature can not only change with special coordinates, it can change with time as well.

When you let an object, hot object in air, it slowly cools. There are specific metallurgical processes in which you try to control the cooling rate to impart special properties in the industry in in the material that you are preparing, which is called quenching. So the quenching rates play a very important role in the final property of the solid. So in all these quenching processes, the temperature is going to be a function of time. So how temperature varies with time, which is commonly called as the transient problem or transient conduction problem will also play a very critical role in many processes of industrial importance.

So therefore we not only would try to solve a few problems, model a few problems of heat transfer at steady-state where the temperature can be a function of both X and Y and so on, we would also try to solve problems in which temperature could be function of time as well. And we would see whether, whether it is possible to make certain special assumptions such that the system can be brought into in such a way that we can club all those resistances present in such a system and the process, the method of that clubbing or lumping all the resistance is into one parameter or the lumped capacitance model, we will talk that lumped capacitance model as well.

And then we will move to convection and finally to the generalised energy equation which would be used for all subsequent problems.