Transport Phenomena

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Lecture Number 30

Heat Transfer Basics

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So far we have studied fluid mechanics that is momentum transfer and details about the boundary layers. What we saw is that the flow of momentum because of a difference in velocity in between adjacent layers of fluid can be expressed in terms of Newton's law. So it is the shear stress which is the molecular, which also signifies the molecular transport of momentum; that was the basis of our analysis of fluid motion. And there we have chosen initially a control volume and saw what are the different methods, ways by which momentum is entering into the control volume.

So the rate of momentum coming into the control volume through convection as well as molecular means which is conductive momentum transfer, so rate of in minus rate of out plus sum of all forces and when I say all forces they can be body force, for example gravity or surface force acting on the control volume at steady state would be equal to zero. If it is not at steady state that means if all these contributions are not balanced then the control volume is going to have an acceleration of its own, acceleration or deceleration of its own. So that was the basis; that was the starting point for the derivation of Navier–Stokes equation of motion of which Navier–Stokes equation is a special case where the viscosity and other properties are constant. So when we got to Navier–Stokes equation, we saw that the left hand side of Navier–Stokes equation was rho, the density times substantial derivative of velocity so which contains not only the transient effects, that means del v del t, at the same time it also has all the convective transport of momentum terms associated with it.

When we move to the right hand side of Navier–Stokes equation, we got a gradient in pressure which is one of the surface forces, another term was due to viscous forces and the third term that we got in Navier–Stokes equation was the effect of body forces denoted by rho times g where g is the acceleration due to gravity. So that was the equation of motion which we have derived out of the simple consideration of Newton's second law for an open system in which mass is allowed to come in with some velocity and go out as well. We have seen examples of the use of Navier–Stokes equation in different coordinate systems and we saw how the use of Navier–Stokes equation has simplified the overall treatment of fluid mechanics leading to the accurate evaluation of the velocity in a flowing fluid.

The next part of our, our, our study in transport phenomena so far start, dealt with the concept of boundary layers. What we know now is that all the transport processes are located, are taking place in a region which is very close to the solid liquid interface. And outside of this solid liquid interface there is no effect of the interaction of the liquid, of the fluid with that of the solid. So Navier–Stokes equation in its full form has to be solved in a very thin layer close to a solid surface outside of which a simple Euler's equation which is valid for inviscid flow can be used. And because of the length scale and because of the velocities in the x and in the y direction and the relative values of their magnitudes, that is del v x del x or del v x del y we could simplify Navier–Stokes equation and we have shown that for the simplest possible case of flow over a flat plate, the Navier–Stokes equation in absence of a body force can simply be reduced to v x times del v x del x plus v y times del v x del y is equal to nu, the kinematic viscosity times del square v x by del y square.

We then solved that equation using Blasius method, Blasius solution method by combining the two independent variables x and y into one independent variable which is eta and v x and v y are both expressed in terms of the stream function. So when all these are combined we obtain, and from a partial differential equation we obtain an ordinary differential equation in terms of the dimensionless, dimensionless stream function and in terms of the dimensionless distance which we refer to as eta. We had a numerical solution even after all these simplifications and analytical solution was not possible, a numerical solution was used. We then use an approximate method known as the momentum integral method which is not limited to the case of laminar flow or flow over a flat plate which also allows the possibility of variation of the free stream velocity with the actual distance. So the momentum integral distance that we have obtained out of this exercise is more general and it takes care of all the assumptions, all the simplifying assumptions that one had to make in for the case for Blasius solution.

So momentum integral equation is easy to use. It gives an o d e instead of a p d e for the growth of the boundary layer and we have solved the, the momentum integral equation for simple cases of flow over a flat plate and we establish that our method is correct. It more or less gives within about 5 to 10% errors. It gives the same form of the equation for the growth of boundary layer, the same form of the equation for the shear stress coefficient and so on. So having established the utility and accuracy of momentum integral equation, we then proceeded to obtain the growth of the boundary layer thickness and the value of the friction coefficient for the cases of turbulent flow. There also we had to use some simplifications which I have discussed in detail and this has given us the variation of boundary layer thickness and we saw the variation of boundary layer thickness unlike in the case of laminar flow varies with Reynold's number to the power minus 1 by 5.

So it shows that the boundary layer grows more rapidly for the case of turbulent flow. We also obtain an expression for the friction coefficient c f and we could see the difference in the expression and the magnitude of delta for laminar and boundary, laminar and turbulent cases as well as the values of c f between laminar and turbulent cases. With all these approximations it is surprisingly close. The predictions from momentum integral equation are surprisingly close to the experimental data and which was a direct result of the very small thickness of the boundary layer and our correct identification of the boundary conditions at the solid liquid interface and at the, at the edge of the boundary layer.

The next important thing which we have done is the concept of drag. Whenever an object, a solid object is in a flowing fluid stream, it experiences certain forces. One force, one is the drag force which the solid experiences in the direction of the flow and the other, is due to the pressure difference which is called as pressure drag. So we have pressure drag and we have friction drag. In the last part of the, previous, last part of that, we have seen what is drag coefficient, its definition, its value or its expression for the case of laminar flow, for the case of turbulent flow when the flow is turbulent from the very beginning that is at x equal to zero and I have also shown, given you the expressions for c d, the drag coefficient for the case of mixed flow.

When the flow, as it should be is initially laminar and beyond a certain point mostly characterized by a Reynold's number value of 5 into 10 to the power 5, it changes from laminar to turbulent flow. So the kind of correction that one has to incorporate in the expression of c d when we have mixed flow on such a case. And finally we have seen, we have, we have solved some of the very interesting practical examples of how the different types of boundary layer, on different surfaces can lead to some interesting phenomena what we we see in, in real life.

I am going to change the topic in the remaining part of the course. So far we were discussing about fluid flow and momentum transfer. I think we have now learned enough to go into the same type of modeling exercise for the case of heat transfer and for the case of mass transfer. And then not only we will go through what the basics that you have probably already read in your heat transfer and your introductory mass transfer but we will quickly shift to the point where you would be able to model a specific heat transfer process based on the modes of heat transfer as well as on fundamental modeling the same that you have used for momentum transfer.

So in the case of momentum transfer the fundamental equation which you have used is rate of mass coming in, rate of momentum coming in to your control volume minus the rate of momentum which goes out of it plus the sum of all forces which are acting, which are acting or in the, on the control volume, the sum total of that is equal to the mass times acceleration of the control volume. So we would do the same thing for heat transfer and for mass transfer. But before we go into heat and the final part of this course I would try to show you that all these three different transport processes, heat, mass and momentum transfer are essentially similar.

That means if you know how to solve one problem you should be able to extend the same type of methodology and in many cases the same expression to solve, to find out the heat transfer coefficient, the mass transfer coefficient or the friction coefficient. These three are the most important engineering parameters that we come across in heat transfer, momentum transfer and mass transfer namely in all these transport processes. So what is the relation between all these three parameters, the coefficients, the friction coefficient, the heat transfer coefficient and the mass transfer coefficient in terms of dimensionless parameters?

So you would express everything in terms of dimensionless parameters and you would see how and when these different transport processes can be expressed by same or similar governing equations and identical boundary conditions. So that kind of similarity between systems having or experiencing heat transfer in one case and mass transfer in the other case, how do we relate these two? That was the objective of this specific course.

And after a brief introduction and after a brief introduction on heat and mass transfer some of which you have already read in your previous courses, we would quickly move in to the modeling of these processes for slightly complicated systems and ultimately we will go to this similarity between, establish the similarity between these processes and see how they can be, how one relation, for example in heat transfer can be interchangeably used for mass transport process. So that was the broad objective and that is what I am going to do in this part of the course. But let's first start with heat transfer. So what is heat transfer?

Whenever we talk about heat transfer, the other thing that comes to our mind is thermodynamics; so if you have a system at, at a state a and a system at a state b and if you know the conditions of the state, the thermodynamics would tell you what has changed between a and b. So thermodynamics essentially deals with the end states of the process. But how do you achieve, how you are going to get heat transfer from one point to the other, through the transport of heat from system a to system b is governed by heat transfer, is denoted, is defined as heat transfer. So the transport of energy from a system to another system is dictated, is described by the heat transfer process.

So therefore heat transfer is energy in transit. Thermodynamics deals with the end states but heat transfer deals with what happens when energy, what and how and when the energy can get transported from location from system 1 to system 2. So that is why in, we call heat transfer as its energy in transit. Now when we talk about heat transfer, we also know there are three types of heat transfer which are, which are possible. One is the conductive heat transfer in which there is, you require a medium but there is no net movement of the medium. So the conduction is, the heat conduction like most of the other conduction processes in other areas, other fields where there is no net motion of the molecules and heat gets transported from one point to the other point through a solid, through a medium, it could be solid, liquid or even gas.

It passes through this medium and heat always travels from high temperature to low temperature. And whether or not heat can pass easily through a system is denoted by a property of the system which we all of us know to be the thermal conductivity of the system. And the fundamental law, fundamental rule which dictates the amount of heat transfer, amount of conductive heat transfer from point 1 to point 2 is Newton's, is Fourier's law of conduction. So what is Fourier's law of conduction? The Fourier's law of conduction, it's a phenomenological equation; you cannot derive Fourier's Law from first principles.

So it's a result of seeing and analyzing a large amount of experimental data and to establish a relation between the heat flux which is the amount of heat that is flowing per unit area per unit time. Heat flux and temperature gradient, say it is, if it is, if you have heat transport only in the x direction, so your Q x that is the amount of heat that gets transported in the x direction is proportional to d T d x. So it is, it's not proportional to the temperature difference.

It is the temperature gradient. So s Q is proportional to d T d x and when you plug in the proportionality constant which is going to be the property of the medium which would dictate the ease or difficulty with which heat gets transported is known as thermal conductivity and complete form of equation is Q x with double prime in it. The double prime shows, the double prime denotes its flux and not total quantity so Q x double prime is equal to minus k d T d x. The minus sign denotes, the minus sign denotes the physical observation that heat always flows from high temperature to low temperature. So if we see the equation over here now and what you would know that

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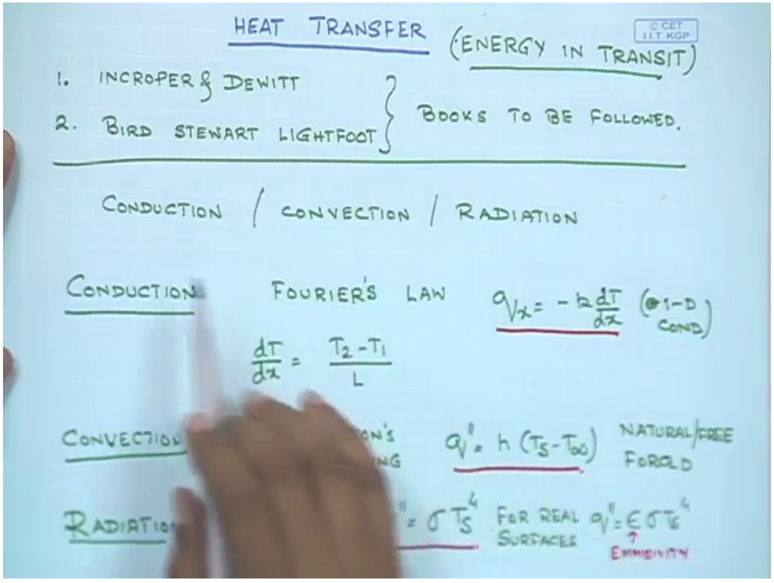

HEAT TRANSFER (ENERGY IN TRANSIT)

1. INCROPER & DEWITT
2. BIRD STEWART LIGHTFOOT

} BOOKS TO BE FOLLOWED.

CONDUCTION / CONVECTION / RADIATION

CONDUCTION FOURIER'S LAW $q_x'' = -k\frac{dT}{dx}$ (1-D COND)

$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$

CONVECTION ON'S NG $q'' = h(T_S - T_\infty)$ NATURAL/FREE FORCED

RADIATION $'' = \sigma T_S^4$ FOR REAL SURFACES $q'' = \underset{\uparrow \text{EMMISIVITY}}{\epsilon} \sigma T_S^4$

there are three modes of heat transfer, conduction, convection and radiation By the way the two books which I am going to follow for my treatment of heat transfer, one is Incropera and DeWitt and the second is, the other textbook Bird, Steward and Lightfoot. So this book is on heat and mass transfer. And this is the book of transport phenomena. So these two books I am going to follow in the rest of the course that, we will deal with heat transfer as well as mass transfer and the similarity between heat, mass and momentum transfer. So when you come to conduction, conduction it is the Fourier's Law which is going to be most important, which as I said for one dimension conduction case, Q x is minus k d T d x and d T d x is temperature gradient. So in difference form, it can be expressed as T 2 minus T 1 by L where L is distance between point 1 and point 2. There is, but, so, it's, there is no net motion in this case. However you still require the presence of

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a medium So this, that is

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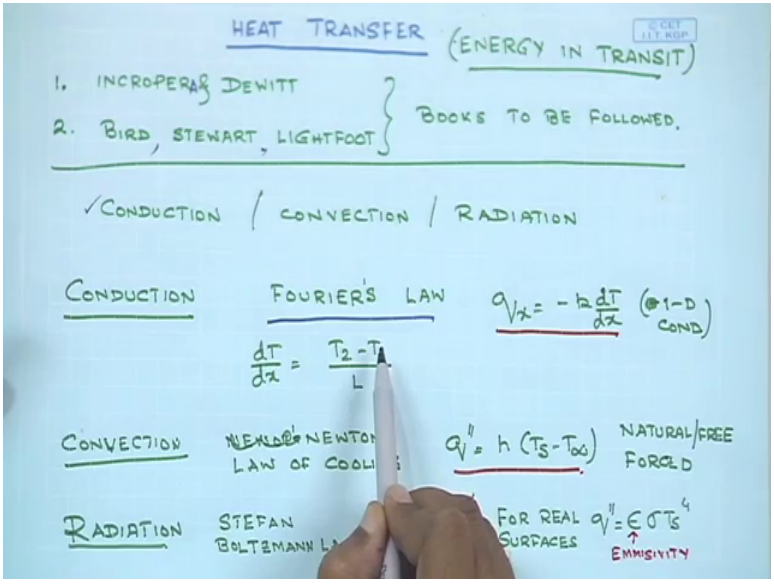

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EMMISIVITY

the fundamental of conduction Next we come to convection in which there is, we still require a medium but the medium is moving and therefore many of the common

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examples of what we see around us is a combination of conduction and convection process. In some cases convection is more common than conduction. So if you are sitting in a room listening to my, my lecture and if you have a fan or an air conditioner, the flow of air above you which helps to cool or reduce your body temperature is an ideal example of convection. So you have movement of the medium past you and that is a basic requirement of convective heat transfer process. But you still require a medium. And one more thing I would like to point out here is that you can never have convection without conduction. You can have a system in which you have only conduction. For example, if you have

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a solid object and if you maintain one side of it at a higher temperature as compared to this side, there would be transport of heat even when there is no convection. But let's say this object is at 100 degrees centigrade and you keep it in a room, in which a fan, a blower is making the air moving over it, moving over the hot object with a certain velocity, then you are going to have convection. But even at that point the molecules which are very close, the gas molecules, the air molecules which are very close to the solid surface, they will cling to it due to no-slip condition and they are going to gain, those molecules which are stuck on the solid, they are going to gain energy from the hot object by means of conduction and then it will transfer that energy to the mobile molecules just above it by means of convection.

So the convection process in between the solid and the convective flow of air above it, you have a layer, layer of molecules which due to no-slip condition, that layer is not moving. So through that layer you have conduction. So therefore in order to achieve convection you will still have conduction. So conduction is there in convection process but you can have a purely conductive heat transfer. But you do not have something called purely convective heat transfer. There would be one stagnant layer which is going to get energy or lose energy with the adjoining surface, through the adjoining surface through the conduction and the law which is, which describes the convective heat transfer process, you already know, it is known as the Newton's law of cooling which simply tells us the amount of heat lost from the surface per unit time is Q equals h times a times delta T and this h, the convective heat transfer coefficient is one of the most important engineering parameters in heat transfer.

Many of our studies in convective heat transfer, if you recall from your heat transfer course is essentially to find what is the expression of h, the convective heat transfer coefficient at different conditions. And this amount of heat transfer by convection from a surface would be different based on whether you have laminar flow or you have turbulent flow around the solid object. And of course if your velocity is more, if your flow is in the turbulent region, you will lose or gain more energy by convection. So a natural convection or a free convection is going to dissipate lesser amount of heat by convection as compared to the forced convection method in which you are foreseeing by an external agency the fluid to move over the solid at a higher velocity and thereby creating the right conditions for additional heat transfer where as in free or natural convection you are not forcing, there is no external object, external agency which forces the fluid to move.

The fluid adjourning to a hot plate simply gets heated and it will rise to be replaced by cooler air from the surrounding. So hot object placed in a room full of static air will create a current of, current in that static room due to the change in buoyancy of the gas or the air, let's say of the air which is caused by its interaction with another object, with solid object of higher temperature. So this convective process will start without the aid of an external agency and it is known as the natural or free convection. So this is what I have written over here. The Q double prime

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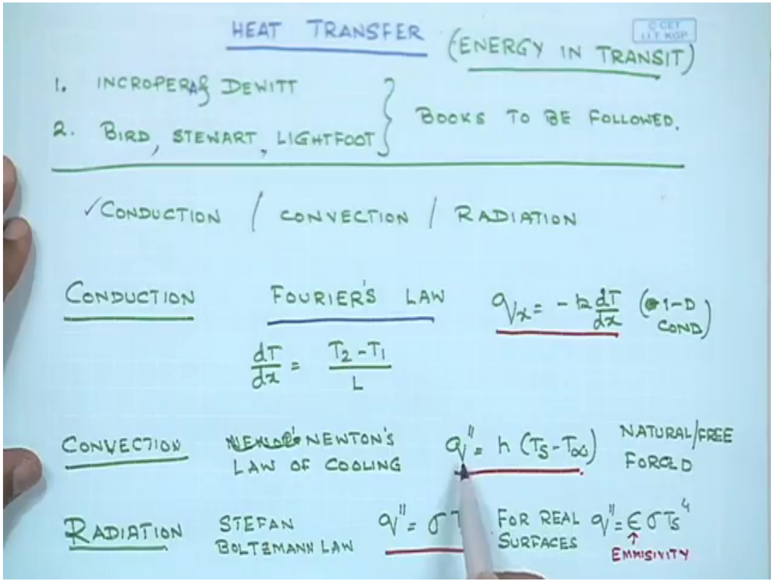

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EMMISIVITY

which is flux, whenever we use double prime, it refers to flux, is h times T s minus T infinity where T s is the temperature of the solid substrate and T infinity is the temperature of the, of the fluid at a point far from that of the solid and of course convective, convection can be of two types, natural and free or forced. The third one and it still requires the presence of a medium, the third one is radiation which does not require the presence of a medium and therefore the radiation law, the common law of radiation which expresses the amount of heat, which gets transported as a result of the temperature of the substrate is given as sigma which is Stefan Boltzmann's constant times T s to the power 4 where T s is the temperature of the solid substrate in Kelvin. However for real surface, the, its, there is a factor emissivity which is brought into this formula to ensure, to emphasize that real surfaces do not emit, radiative heat radiation as efficiently as that of an ideal substrate which is where the value of emissivity is equal to 1. So you have emissivity is at different, the different values of emissivity is for different surfaces, there are

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tables and so on and the radiative heat transfer is in itself a separate, a separate subject of heat transfer in the, you would have the concept of the transmittivity, the reflectivity and so on so that's another thing that there is a concept of black body, there is a concept of gray body and you are, you have probably done the network method of radiation exchange between surfaces which are forming let's say an enclosure, the concept of view factor and so on. So I, I am sure you have studied that, those in heat transfer so I will not discuss about them in this transport phenomena course. In transport phenomena I will restrict myself to conduction and convection, and try to model, try to write, generalized equations and develop models which would describe the convection process and so on.

And same as in the case of momentum transfer we would see that defining or assuming a shell and making a balance of all the heat inflow and outflow terms and the amount of heat generation in the system etc, after a while it becomes very difficult to visualize and solve a system assuming a shell only. So the same way, a generalized method in the form of Navier–Stokes equation was used. Similarly for heat transfer also, we will also develop an equation which not only takes care of all the heat flow in and out and heat generation but it would also take into account the work done by the system or work done on the system because that would also, work done by the system or on the system would also affect the total energy content of the control volume.

So generalized equation which would take the heat as well as the work form of energy into considerations in order to obtain a generalized energy equation, that can be used for any system undergoing conduction or convection free or forced, in presence or absence of body forces and so on, that equation we are going to derive in this course and we would see then, as in the case of Navier–Stokes equation, a simplification of the energy equation for the problem at hand would make our life a lot simpler. So you would simply write the energy equation in the correct coordinate system, cancel the terms which are not relevant, which are physically not present or relevant in for the problem that we are discussing and then what we will have in the end is the governing equation. And once we have the governing equation, we will also try to see what would be the, what would be the pertinent boundary conditions to, for that specific problem.

And then it is a question of simply solving it to obtain the temperature profile. And once you have the temperature profile, you can find out the gradient of temperature at a specific location to obtain how much heat that surface is receiving or losing and how do we connect the amount of heat loss to the heat transfer coefficient and thereby obtain a relation that contains h, the convective heat transfer coefficient and the length scale and property of the system so as to combine the edge and other properties including the geometry of the system, we will bring the concept of Nusselt number. So the expression for Nusselt number is the most sought after while describing convective heat transfer process. So our whole emphasis will be to start the energy equation and obtain if possible an expression for Nusselt number for the heat transfer taking place in that specific geometry under that specific conditions. So that is what we are going to do in our treatment of heat transfer from now on.