Transport Phenomena.
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Lecture-03.
Shell Momentum Balance.

So we are going to talk about the fundamental shell momentum balance, the shell momentum balance approach and how it can be used to solve a very simple problem. So we have identified that in a shell of a fluid, some fluid is coming in, some fluid is going out and there can be also variations in velocity which exists in a fluid. So when something is flowing into a control volume, it comes in with a velocity, some mass of fluid coming in with a fixed velocity carries with it some amount of momentum.

And the rate, time rate of this momentum which comes in to the control volume and the rate at which it leaves the control volume, there would be a net addition of momentum to the control volume per unit time. Also due to the variation in velocity, there can also be momentum fluxes as per the Newton's law of viscosity. So there can be molecular transport of momentum which is given by Newton's law and there can be the overall convective transport of momentum which is due to the flow.

Apart from that the other forces, since it is time rate of change of momentum, so all these are essentially have the same units as that of force. The other forces which are acting on it can be divided into 2 parts, one is the body force and the 2nd is the surface force. The most common example of body and surface forces are the gravity and the pressure difference respectively. So if I can write in mathematical terms what I just said, it is going to give rise to an equation which is a difference equation which is written in terms of Dell X, Dell Y, Dell Z.

Where Dell X, Dell Y, Delta Z are the components that specify the space and it is also going to have DP the pressure gradient and it will have the effect of G which is gravity. So if we can write these difference equations and then if we can convert the difference equation to a differential equation, what I then would have is the governing equation for the flow, for the for the flow, let us say over a flat plate. And if I can solve these fundamental equations with appropriate boundary conditions, then I would get the velocity profile which is my ultimate aim.

In order to do that, I 1st need to identify what are the relevant boundary conditions that we can think of when a solid and liquid are in contact. So let us assume that I have a solid plate, I have liquid, so this is the liquid layer, the solid plate is slightly inclined, so the liquid starts to flow along the solid plate. So this is my Y equals 0 and there is, there is a film of the liquid, so let us assume that the thickness of the liquid is delta.

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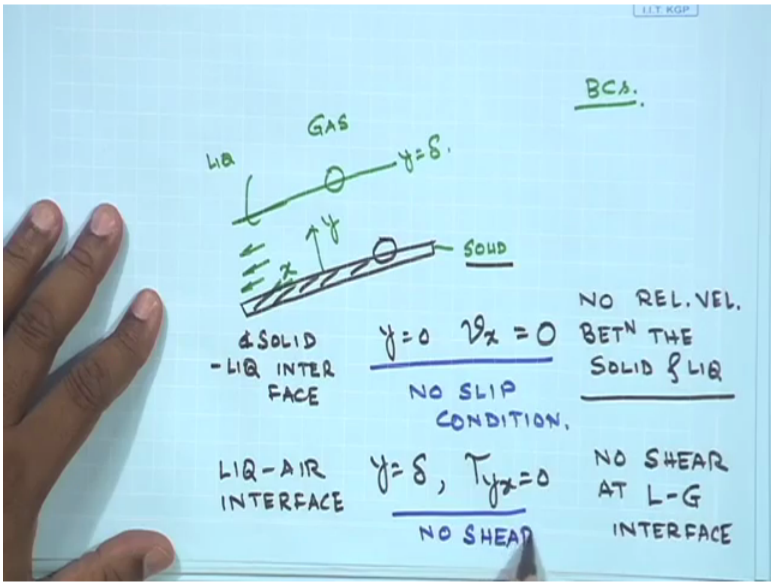

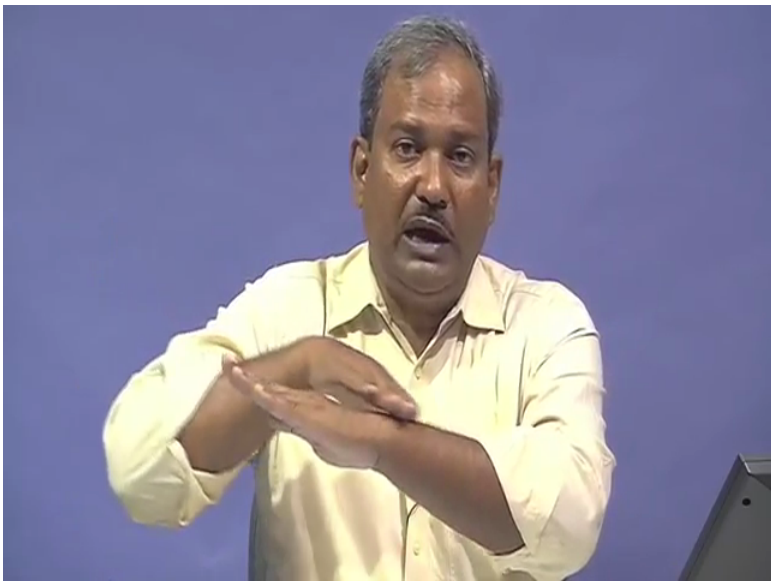

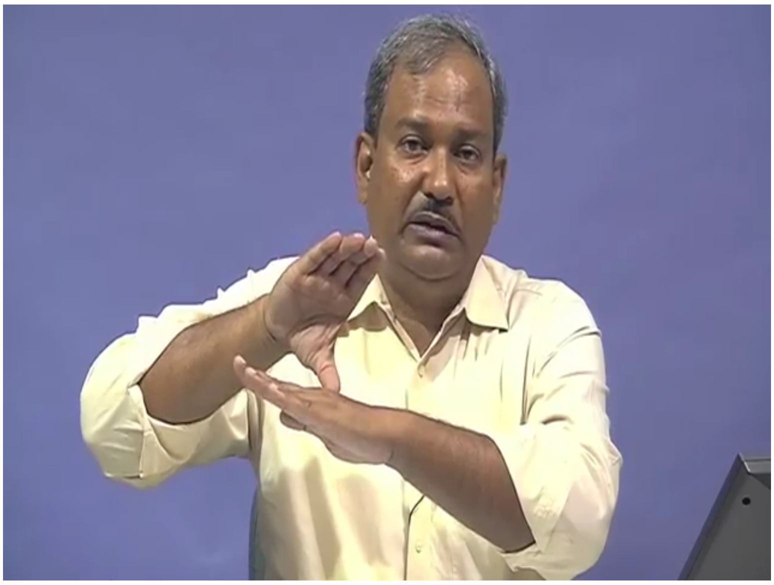
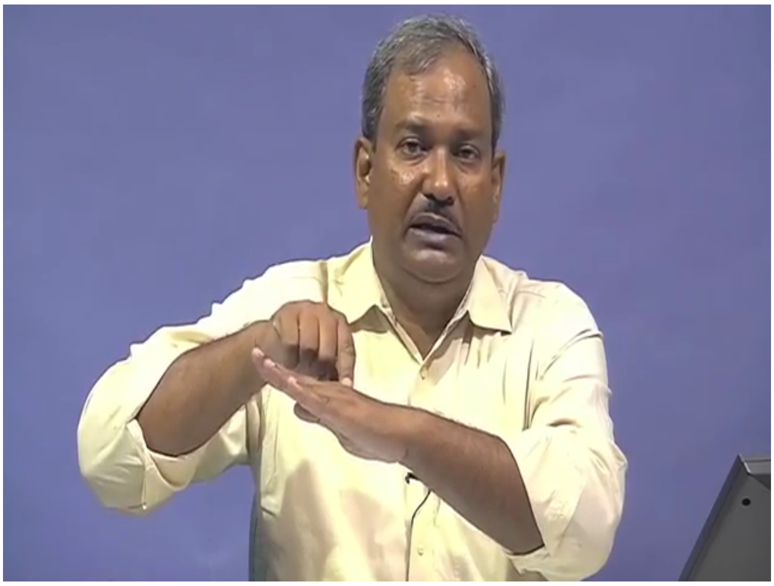
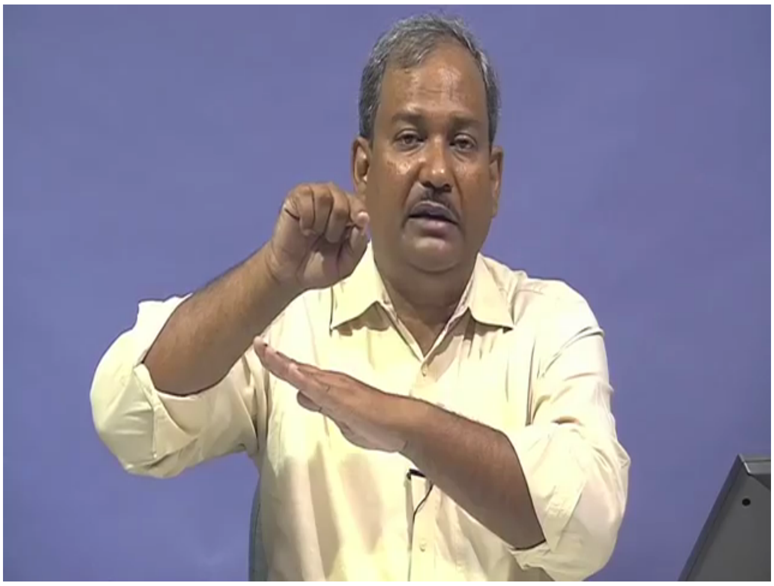

So Delta is 0 here, Y is 0 here and Y is equal to Delta at the end of the film, so what are the boundary condition that one can think of? Now I will draw this picture once again, so let us say I have this as the solid plate inclined with some angle and I have a liquid film which is falling, so this is my liquid and over here I have gas, most probably air and this is a solid. And I would like to know what are the boundary conditions which are present in here. Over this is my Y direction and this is the X direction.

And this is Y equals delta. So the boundary condition that one can expect a is is what is the velocity at Y equal to 0. So at Y equal to 0, since the solid plate is static, the VX, the velocity in the X direction would also be equal to 0. So what it means is that no relative velocity between the solid and liquid, this situation, this boundary condition commonly known as no slip condition. So at Y equal to 0, VX equal to 0, that means that the liquid solid interface, there is no, no relative velocity that exists between the liquid and the solid are known, commonly known as no slip condition.

The 2nd boundary condition is at Y equals delta, which is the liquid air interface, this is solid liquid interface. So at Y equals delta Tao YX equal to 0. So this is known as no shear condition. These 2 are the fundamental relations, fundamental boundary conditions which are grounded in our concepts because if the liquid molecules are clinging to the surface of the solid, this simply cannot move. But the layer of molecules just above these stationary molecules, they are free to move.

So the velocity will always started the solid is stationary, velocities always start at a value equal to 0 and it would progressively increase as I move away from the solid plate. Okay. So at every solid liquid interface, in most of the cases, relative velocity would be 0. So if the solid is static, the liquid molecules on top of these, on top of this solid plate will also be static. This is known as no slip condition.

There are some special cases where this no slip condition would not be valid, there is a special branch of fluid mechanics which deals with fluid mechanics at very small-scale where the flow through which, through a nano tube or Micro micro tube is being considered where the flow of gas is being considered at a very low pressure. In some special cases like that, the mostly condition would not be valid, okay.

And you are probably going to get a case in which there would be a slip flow, there would be a velocity, nonzero velocity component that exists, that may exist on at the solid liquid interface. But those special cases we will not consider in this course. For the majority of the situations, for the vast majority of the situations, the flow at the liquid solid interface will always be 0 relative velocity, so it is in no slip condition. The other condition is normally again, there exists a significant difference between the viscosities of the liquid and the gas, okay.

So because of this difference in viscosities, the momentum transfer Tao YX, X component being transported in the Y direction would be insignificant at the liquid vapour interface. No matter the liquid velocity, the moving liquid film will not be able to impart momentum onto the gas and vice versa. Therefore even if the gas is flowing, the liquid can still remain stationary and no momentum from the gas gets transported to the liquid and so on. The exceptions well, when the relative velocity is high.

When the relative velocity between the liquid and the vapour and the gas is large, then this relation would not be valid. So you would see the formation of the waves, the Shearing, Shearing or entrainment of the liquid when there is a fast moving vapour on top of the static liquid film, those are exceptions, those are situations in which the no shear at the liquid, at the liquid vapour is interface, that relation would be valid. But again for the time being we will, we are assuming where the relative velocity between the liquid and the gas is not too high, so therefore the no shear condition would prevail.

So we will deal with, we will use 2 major conditions, boundary conditions which are physical, observations, principles, written mathematical form, one is the velocity equal to 0 at the solid liquid interface and shear is 0 at the liquid vapour interface, these 2 will be our governing equations. That background clarified, now we will try to solve a problem which is the simplest problem where there is an inclined plate and the liquid is flowing because of gravity there is no imposed pressure gradient.

So the flow is taking place only because of gravity and the body force but there is no pressure force, no surface force. So when it starts to flow along the inclined plate with some thickness, what we can clearly see from commonsense is that the no slip condition is valid at the liquid solid interface and at that the velocity is 0. And as we move away from the solid plate, the velocity will progressively increase and the maximum velocity will be at the top layer of the fluid, top layer of the liquid which is which is far from this solid plate.

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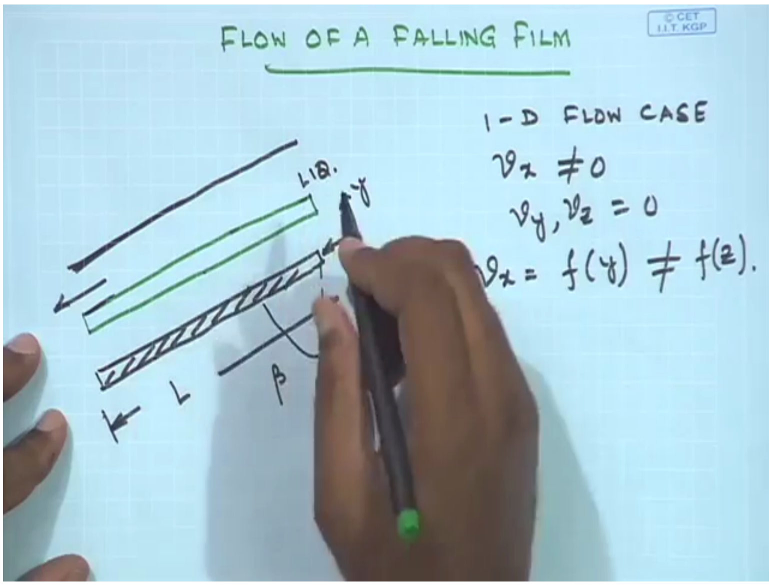

So we will try to see based on Shell momentum balance how that can be done. So the problem that we are looking at is flow of a falling film. And the flow of a falling film, let us draw this physical situation in which I have a solid plate of some length L and it is making an angle with the vertical and this is the liquid film where this is X direction, this is Y direction and it is a 1-D flow case. What I mean here is that VX is not equal to 0, may not be equal to 0 but VY and VZ, both are equal to 0.

And if you also can see carefully, you would see that VX is going to be a function of Y but it is not going to be a function of Z. So the gravity is acting in this direction and due to the variation in velocity, there would be shear force which is acting in the Y direction. So this system is to be modelled thinking about what is going to be the different forces acting on a small volume, a control volume, a differential control volume that we have, that one can one can assume.

When I say differential control volume for this 1-D case, there would be a finite length, finite width and a finite depth, finite thickness of this. We understand that the variation in velocity is only in the Y direction, it does not vary with Z and it is not going to vary with X direction. So the film is falling freely along the side of the solid. Whenever you encounter such a situation, the smaller dimension of the imaginary control volume will always be the dimension across which the variation in velocity is taking place.

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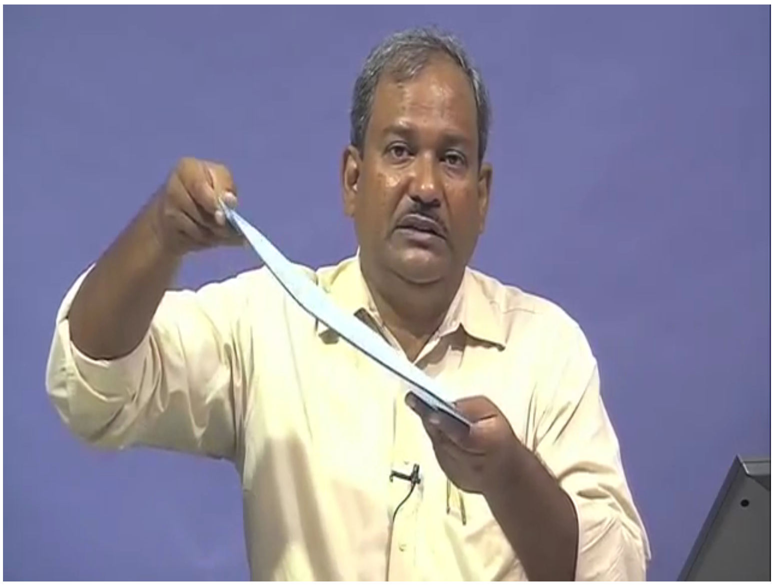

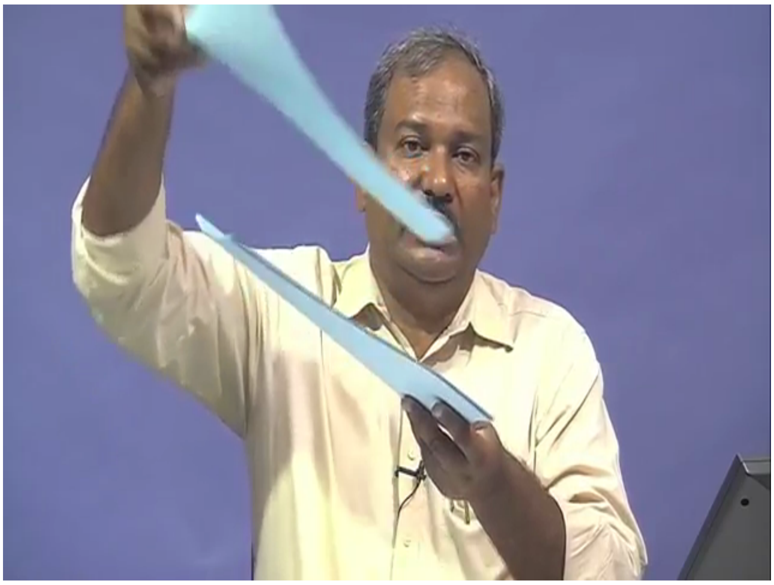

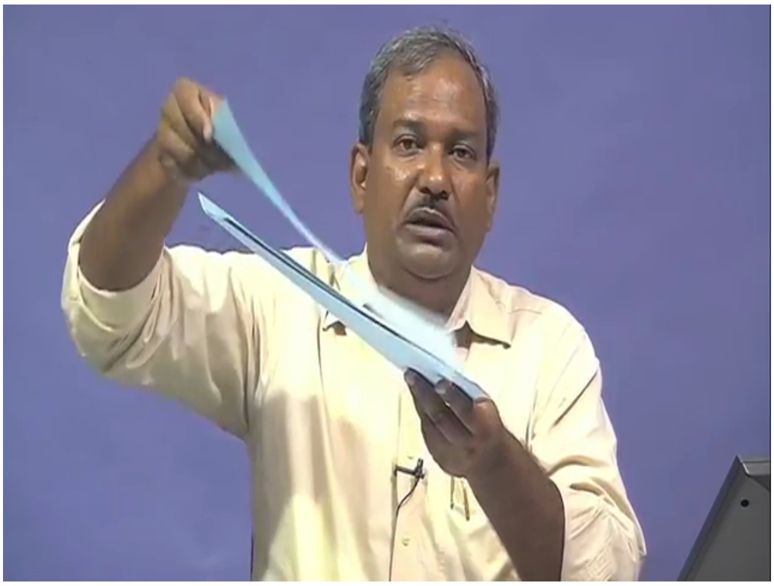

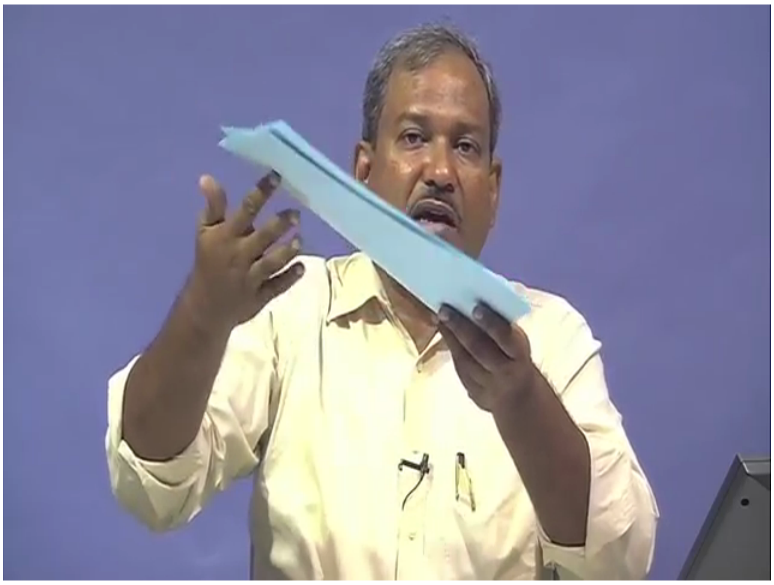
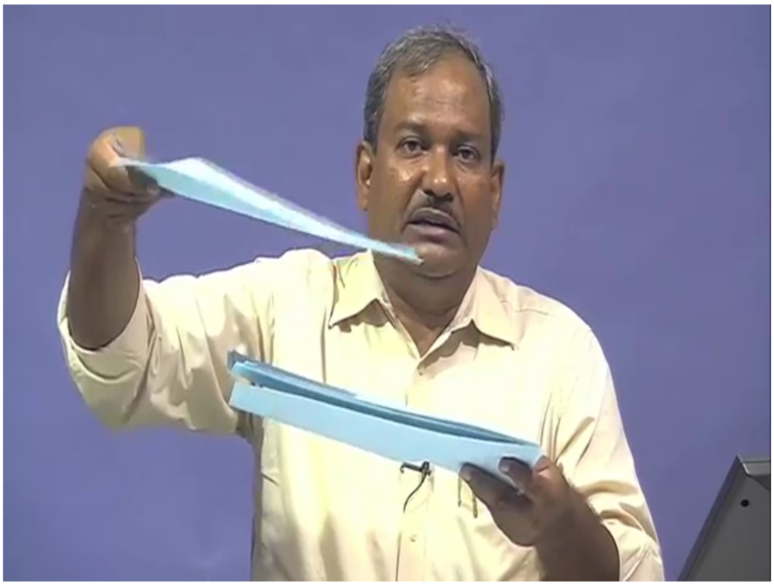
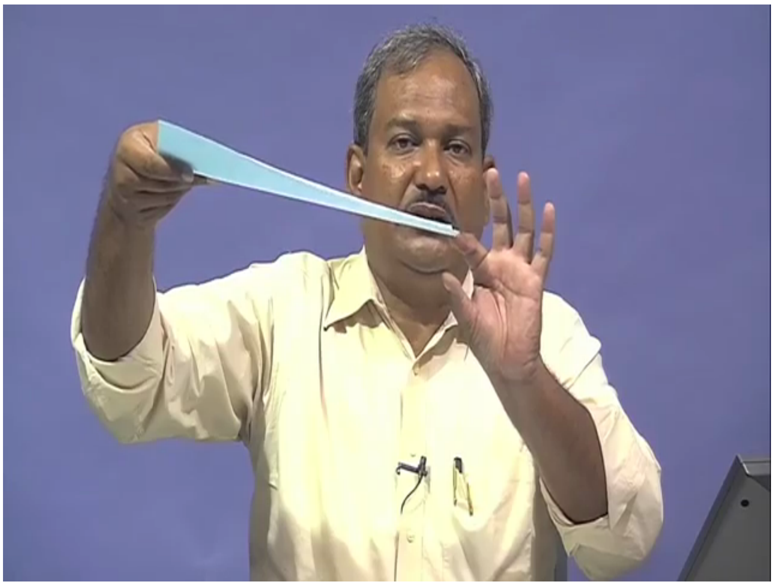

So in this specific case it does not matter what is the length of the control volume that you take, what is the width of the control volume that you take but what you are going to take as the thickness of the film, that is going to be, that is the control volume will have any dimension in X, any dimension in Z but Delta Y in the Y direction. So is the the this these are the different layers that constitute my film, this is the top layer which is moving with the highest velocity and the bottom layer which is in contact with the solid is static, it does not move.

So there is a variation in this direction in the Y direction in velocity. The velocity does not vary with X, the velocity does not vary with Z, it only varies with Y. So in that case in order to model such a process, I will take this this as a layer, this plate as a layer, this is going to be my differential control volume which is situated at an arbitrary distance X from here. So the bottom face of this imaginary control volume is situated by a distance X from the solid plate. It is of it is of thickness Delta Y and we are going to we are going to find out what are the different ways by which the momentum can come into this control volume.

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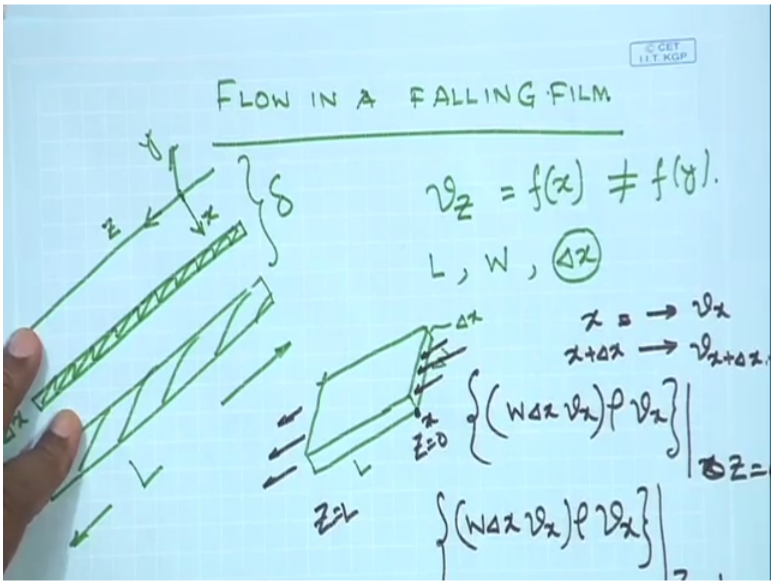

So the control volume that I have, that I am going to that I am going to use is this. This is my control volume, I will I will draw it once again. This is a solid plate, this is a liquid, this thickness of the liquid is Delta and you can find you can use any coordinate system that you like, but let us say this is my X direction and this is my Z direction and in the Y it does not matter because the nonzero component of velocity is VZ which is a function of X, it is not a function of Y.

The size, the length of this is L and I am going to, since it varies with the X direction, the control volume that is chosen is of size Delta L. So this control volume has a length L, let us assume the width in the Y direction is W and in the X direction it has a thickness of Delta X. So it looks something like this, this is the control volume, this is L, this is W and this is Delta X. We are going to make the momentum balance in this case. The velocity at, this is at X, so velocity at X is equal to VX and let us say velocity at the upper layer of this control volume, that is X plus dell X is denoted by X plus Delta X.

The there would be some fluid which will be coming in through these faces and they are going to go out of these faces. So if it is like a box, if it is a box like this, what you are going to get is the liquid is coming down, the velocity is only in the X direction, so some amount of fluid is going to and this is my control volume, some amount of fluid is going to enter through this face. What is the area of this face, it has width W and thickness Delta X. So the area, cross-sectional area of this face is simply going to be W Times Delta X.

What is the mass of fluid is coming in through this face per-unit time, any fluid element, let us say the fluid velocity is VX. So if this is the VX length, then any molecule which is situated over here in in per-unit time, it is going to cross this surface. So the volume of fluid which will enter through this face having area W Times Delta X must be equal to W Times Delta X times VX, that is the amount of fluid which enters through the surface. I will go through it once again.

The area that you see over here is W Times Delta X, the amount of fluid which crosses this per unit time must be equal to W Times Delta X times VX because only if it is VX, it enters, if it is outside of VX, then it cannot enter at that in 1 seconds. In the next instant it is going to come through this surface. So the volume which crosses this per-unit time is W Times Delta X times VX. This volume has a mass of its own. So the amount of mass which comes in to the control volume through this face is W Times Delta X times VX times rho.

So that is the mass that comes in in kg per second through this surface. What is the momentum associated with it, whatever will be the mass multiplied by the velocity. So the amount of X momentum, this is X direction, amount of X momentum which is coming through this face is W, Dell X, VX, volumetric flow rate times rho, mass flow rate times VX which makes it momentum. So rate of momentum coming in through the face would then be equal to W Dell X times VX times rho times VX and everything here is to be evaluated at the X location which is this point which is X, which is I mean at Z equal to 0.

This is Z, so this is Z equal to 0, this is Z equal to L. So this is the mass, this is the momentum of the fluid which enters through the face at this point. What is the one that goes out of it? It is the same Times VX but it is evaluated at Z equal to L. So you see that there is no difference between the momentum that comes in and the momentum that goes out, except these quantities are evaluated at X equal to 0, Z equal to 0, these quantities are evaluated at Z equal to L.

So W and all these are constants and if we assume that it is an incompressible fluid, then the rho at these 2 points are also constants. Now this face being perpendicular to the direction of flow, no liquid is entered through this, similarly no liquid enters through the side and if it is one-dimensional flow, if it is one-dimensional flow, then liquid can enter only through this face, it cannot enter through the surface, this face, this or this.

So the momentum that comes in to the control volume, due to convection can only be through these 2 faces, not the other for faces, remaining faces. And we also understand that it is an incompressible fluid, if it is an incompressible fluid, rho does not change. Whatever the value of density at this point, the density at this point will also be the same. So therefore the amount of mass that comes in must be equal to the amount of mass that goes out, otherwise if it is not so, there will be accumulates on of mass inside the control volume which would violate our steady-state assumption.

Since we are using at steady-state, the amount of mass that comes in and the amount of mass that goes out are same. The velocities, the momentum that is the velocity that is coming in and going out, velocity is a function only of this, it is not a function of the Z position in this case. So VX at Z equal to 0 is equal to VX at Z equal to L, so if the masses are same, the velocities are same, what you can see is the amount of convective mass in must be equal to the convective mass out.

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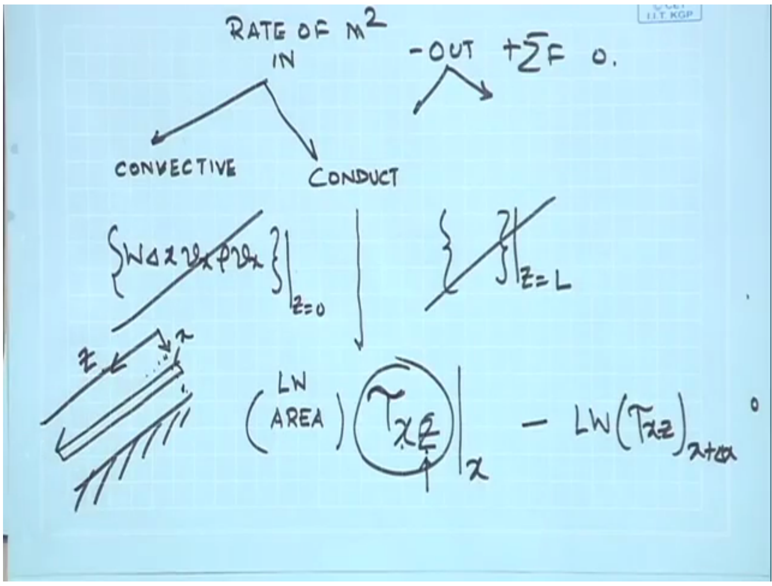

So your governing equation in that case will be rate of momentum in minus out plus sum of all forces equal to 0 and this in, there can be one is the convective momentum, the other one is going to be the conductive momentum which we have not done yet. And the same applies for this. So the convective case is, which we have identified is W Times Dell X VX rho VX, everything evaluated at Z equal to 0. The out term is the same evaluated at Z equal to L. And these 2 will simply cancel each other.

So what are remaining are the conductive terms, conductive terms in and conductive terms out. Now how to evaluate that? Now in order to do that, we need to realise that this is the X direction, this is my Z direction and this is the solid plate. And my control volume is situated over here. So if my control volume is situated at this point, at this location, then I have the top plate like this. I have the control volume like this, it is in this direction in which the convective moment, the conductive momentum, the shear stress is acting and the shear stress goes out of this.

The convective flow is that this location, convective flow out is at this location. So when I talk about the conductive or the molecular transport of momentum, I understand that the shear stress that I am going to do, going to associate is Tao XZ. What does this Tao XZ mean? This shows that it is a principally the Z component of momentum due to viscosity gets transported in the X direction which is obvious. You can see that it is the Z momentum that I have an the velocity varies at different, based on the location over here.

So because of the variation in the velocity, the momentum gets transported in the X direction due to viscosity. So it is the the shear stress or the molecular transport of momentum, molecular transport of Z momentum in the X direction. So this has units of force per unit area. So in order to obtain the rate of momentum in, I must multiplied it with some area, the area on which the shear stress is acting on. So what is the area on which the shear stress is acting on?

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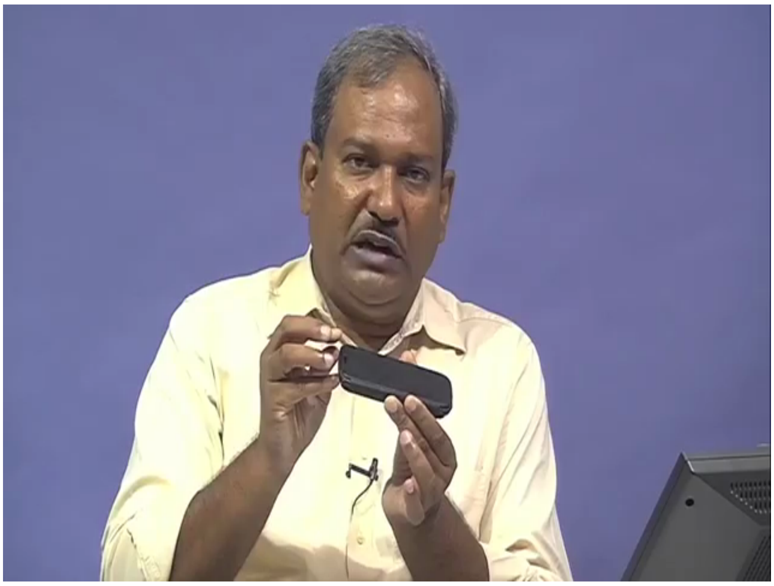

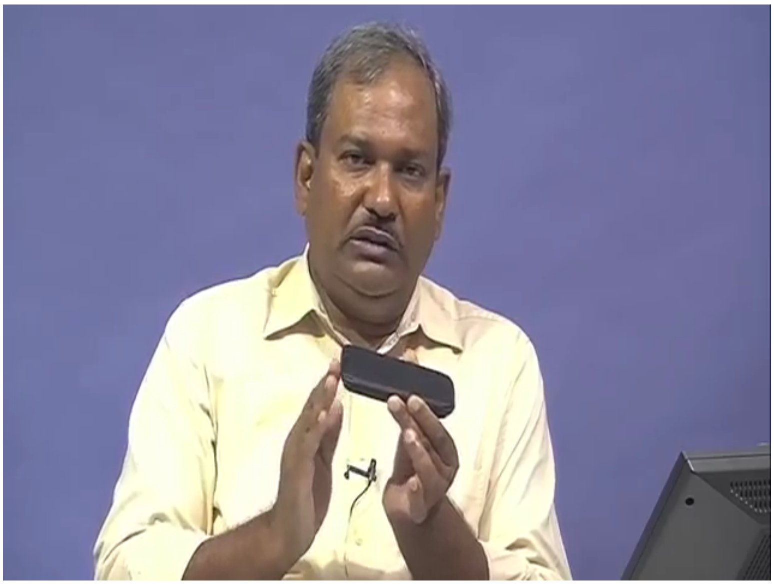

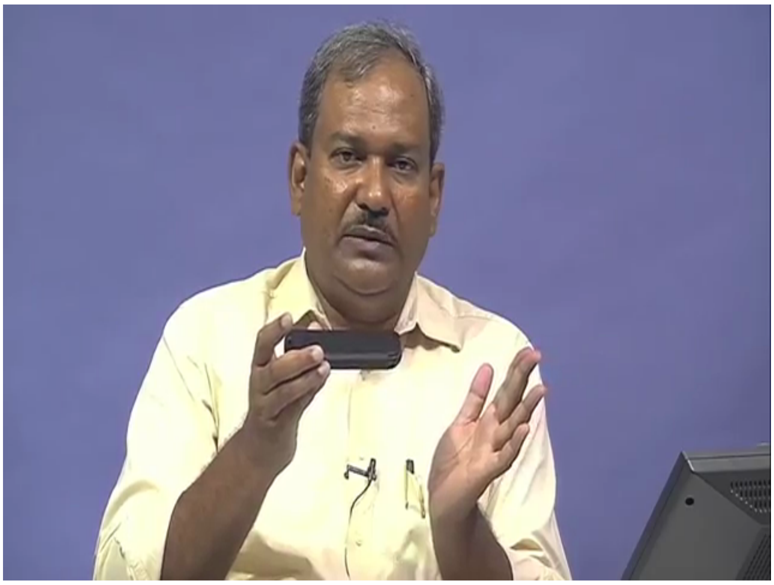

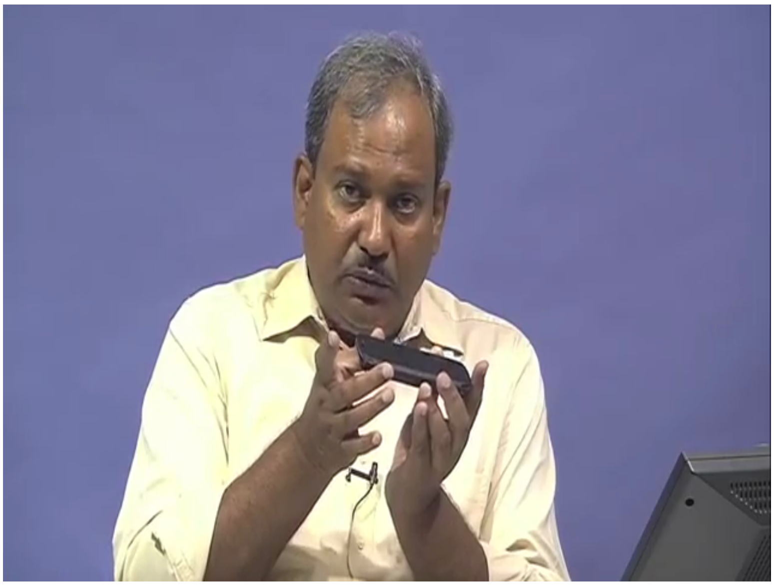

Let us think of this this once again. It is this area which is on which the shear stress is acting and this is the area on which shear stress is acting. The velocity is the same at between these 2 points, so it does not matter, it is only this area which is length times W. So L by W is the area on which the shear stress is acting. So my this is going to be L times W which makes it the force which is acting on it and it is at X and the one that goes out would be LW Tao XZ at X plus Delta X.

This if we understand these 2 things, then the rest would be very simple. Shear stress, flow is in this direction, shear stress acts on this area and on this area, the convective flow, the amount of fluid which comes into the control volume is through this face which has an area of W times Delta X and it goes out through this area. Since the velocity does not vary with X, velocity does not vary with, velocity does not vary with Z, velocity does not vary with Y, we do not have any shear component acting on this face or acting on this face.

So the net momentum, rate of momentum coming into the control volume, convection in, convection out, shear stress in, shear stress out, so if I make the algebraic sum of all this, this one, the convective in and convective out cancels out. But since the velocity is varying, their shear stress, momentum transport by shear stress from the top to the from the top surface and to the bottom surface, they do not cancel out, they are the remaining terms in the governing equation. So what I am going to the next class is see what are the other forces, body and surface forces, which are acting which are relevant and then write the complete governing equation and try to solve. So that, I will take up in the next part.