Transport Phenomena.
Professor Sunando Dasgupta.
Department of Chemical Engineering.
Indian Institute of Technology, Kharagpur.
Lecture-28.
Drag.

So we are going to start with what is drag, how the drag can be expressed in terms of the quantities that we have derived so far, mostly in terms of the expression for shear stress and so on. 1st of all any moving object in a stream of fluid will experience is retarding force, this retarding force is commonly called as drag. And we understand that it is easy to move, let us say a ruler in air compared to a box, okay, the reason for that is even though the surface area more or less remains constant, the front end of the box will create an additional resistance for move, for its movement through air.

So the frontal end gives rise to something which is known as the pressure drag. Whereas the interaction of the surfaces which are parallel to the flow through viscosity gives rise to frictional drag. So drag can be, can consist of 2 contributions, can have 2 contributions, one from the pressure drag and one from the frictional drag. So in this part of the course, we will restrict ourselves to frictional drag only.

And we will see what would be the expressions for the drag coefficients, 1st of all what is the definition of drag coefficient and how the drag coefficient is related for flow over a flat plate, that means we will not consider the pressure drag this specific case and this drag coefficient would be function of the regime, flow regime that we have, whether it is laminar flow or whether it is going to be turbulent flow. What would be the expressions for the drag coefficient for these 2 cases?

And we also know that in reality, we do not have turbulent flow from the very beginning at the edge of the plate itself, at X equal to 0. You may not have turbulent flow at X equal to 0 and therefore any value of CD in turbulent flow needs to be corrected to take into account the portion of the place which is under laminar flow and transition takes place at a certain point and then you have turbulent flow for the rest of the plate. So starting with the expression for the friction, the drag coefficient, for completely turbulent flow from the very beginning, we need to have a correction factor included in the expression for CD to for cases where you have mixed flow.

And the mixed flow is the one in which initially we have laminar flow followed by a turbulent flow and so on. So therefore it is, it is it is important to note what are the, what are the relations, how we can derive them based on whatever we have done so far, what is drag and implication of drag in citing some of the interesting example is that we all are familiar with.

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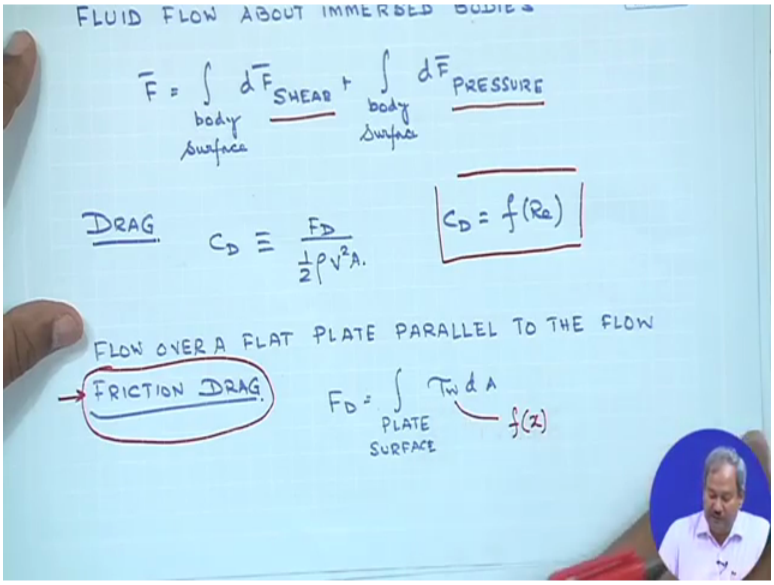

So we will 1st see the, if you want to analyse fluid flow about immersed bodies, we understand that the force will have 2 components, the the integration over the body surface which is a shear stress component and which is a component from shear contributions structure and the contribution from pressure. The drag is generally expressed in terms of drag coefficient which is denoted by CD, where CD is simply FD, where FD is the drag force. So this little FD is simply the force exerted, of course experience by the moving object in the mind for example let us say air.

And so CD is defined as the force divided by half rho V square A. If you recall the definition of CF, the friction coefficient, this FD over there was replaced by tau W and in the denominator we only had half rho V square. Whereas in the definition of CD, it is defined as FD, the drag force divided by half rho V square A, where V is the velocity, free stream velocity. And we also, we also realise that drag coefficient is going to be a function of Reynolds number.

Whether it is a function, whether it is in laminar flow or in turbulent flow, the value of drag coefficient will be different for different situations. So we are going to analyse the flow over a flat plate which is located parallel to the flow and therefore we are all the talking about, only talking about frictional drag. So any discussion that we have from this point, we only refer to the frictional drag because you are dealing with flow over a flat plate.

So using the, using the definition, using the concept that we have derived so far, FD, the drag force, if we integrate over the plate surface, this is tau W times d A. The same approach we have we have used for the solution of the previous problem. So you frictional drag is simply the integration of tau W D A and we understand this tau W among other things is a function of the axial location. So this is this is something which we need to evaluate.

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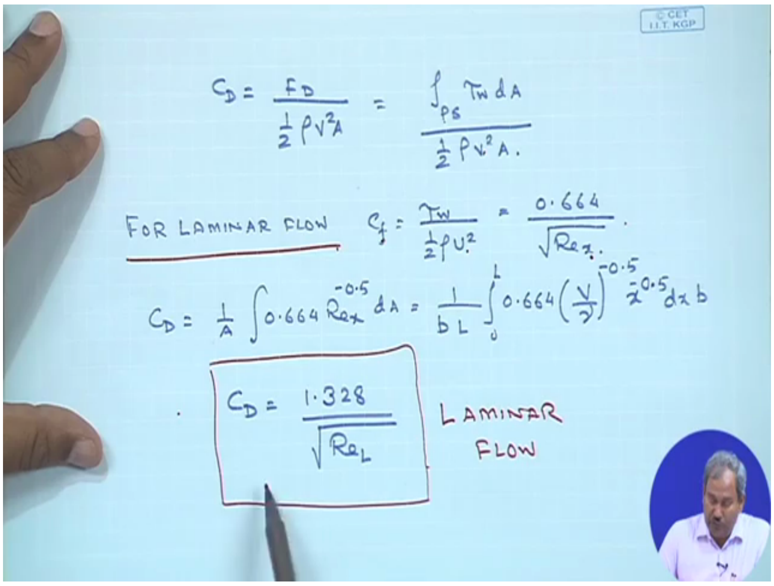

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{\int_{ps} \tau_w \, dA}{\frac{1}{2}\rho V^2 A.}$$

FOR LAMINAR FLOW $C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}.$

$$C_D = \frac{1}{A}\int 0.664 \, Re_x^{-0.5} \, dA = \frac{1}{bL}\int_0^L 0.664 \left(\frac{V}{\nu}\right)^{-0.5} x^{-0.5} \, dx \, b$$

$$\boxed{C_D = \frac{1.328}{\sqrt{Re_L}}}$$

LAMINAR FLOW

So what I do then is the CD, the definition and therefore the force is replaced by over the plate surface integration over the plate surface is equal to tau W times DK and half rho V square A will remain in the denominator. We are already aware that for laminar flow, the expression for CF, which is tau W by half rho U square is the 0.664 by root over REX. This we have defined, this we have derived before. So therefore if you plug-in the expression for tau W in here, what you get is CD to be equals 1 by A, the rho V square part, the rho V square part will cancel out, since it is a flow over a flat plate the approach velocity and the free stream velocity is equal, so they will cancel out and what you have simply is 1 by A and instead of tao W I put in this which is 0.664 REX to the power - 0.5 D A and D A is simply B which is the width of the plate times L where L is the length of the plate.

So the integration of changes from X equal to 0 to X equals L and when you bring in the X outside, you simply have to perform this integration and what you get is an expression for CD to be equals 1.328 by root over REL which is laminar flow. Note the difference between the expression of CF and CD. CF is the friction coefficient at a specific X, the subscript of Reynolds number is RX, so it is that REX, so depending on the location of the, location of the point, the value of CF will be different.

But if you look over here, the CD expression contains REL, so this is the Reynolds number based on the entire length of the plate. So thus CD is a constant, the moment you specify the geometry of the plate and the flow condition and the property, you have one value of CD which is for the entire plate surface. On the other hand, the expression force CF Contains X, the Reynolds number there has the subscript X, so depending on whatever be your axial location, you will get a different value of CD, different value of CF based on your location.

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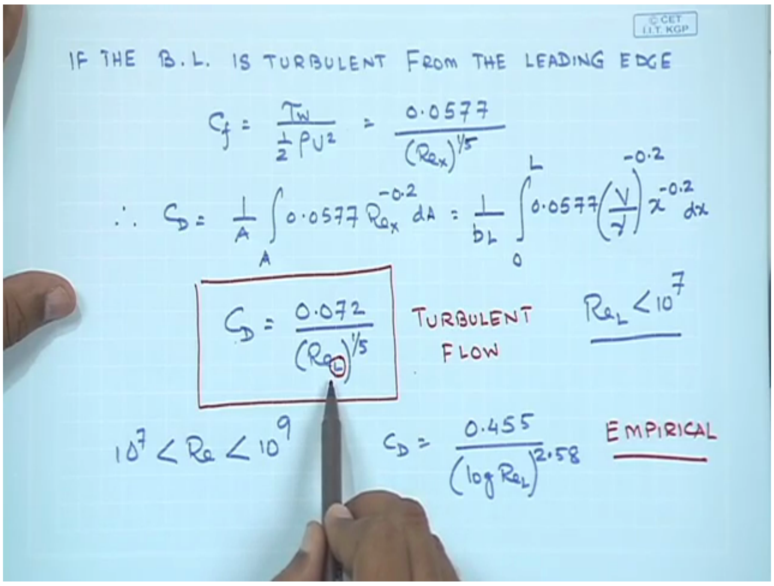

IF THE B.L. IS TURBULENT FROM THE LEADING EDGE

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0577}{(Re_x)^{1/5}}$$

$$\therefore\ C_D = \frac{1}{A}\int_A 0.0577\, Re_x^{-0.2}\, dA = \frac{1}{bL}\int_0^L 0.0577\left(\frac{V}{\nu}\right)^{-0.2} x^{-0.2}\, dx$$

$$\boxed{C_D = \frac{0.072}{(Re_L)^{1/5}}}$$ TURBULENT FLOW $Re_L < 10^7$

$10^7 < Re < 10^9$ $C_D = \frac{0.455}{(\log Re_L)^{2.58}}$ EMPIRICAL

However CD is just one value. So this is for laminar flow that one can see. If the boundary layer is turbulent from the leading edge itself, that is from the very beginning, then we know that our expression for CF as we have seen before 1 by, this is 1 by 5, 0.0577 by REX to the power 1 by 5. So using the same methodology as before and this would simply be equal 1 by B times L and this will be 0 to L, 0.0577 velocity, free stream velocity by nu to the power -0.2, X to the power 0.2 times DX.

So what you get then out of this is CD equals 0.072 by REL to the power 1 by 5. So this expression for CD is for turbulent flow and where the turbulent flow starts from the very beginning itself. Okay. This expression is valid for a Reynolds number which is less than 10 to the power 7, the same constraint that we have done, we have used for the case of CF over here. So if Reynolds number is greater than 10 to the power 7 and up to a Reynolds number of to the power 9, the empirical equation, it is empirical nature, the CD would simply be equal to 0.455 log of REL to the power 2.58, so this is entirely empirical.

So turbulent flow CD, again you note that the subscript here is L, so this is the drag coefficient for the entire length of the plate, exactly like what we have done for the laminar flow and this is valid for a Reynolds number less than 10 to the power 7. And if you have Reynolds number is beyond 10 to the power 7, then we have to use, we have to take recourse of an empirical formulation, empirical relation which is, which is given in terms of log and so on. And the reason, region of validity is between 10 to the power 7 and 10 to the power 9 and we have an expression for CD here as well.

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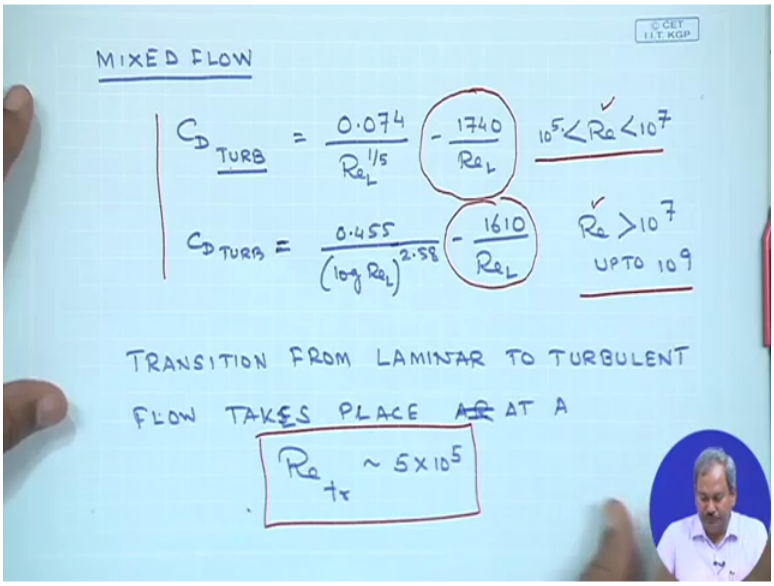

MIXED FLOW

$$C_{D_{TURB}} = \frac{0.074}{Re_L^{1/5}} - \frac{1740}{Re_L} \qquad 10^5 < Re < 10^7$$

$$C_{D_{TURB}} = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1610}{Re_L} \qquad Re > 10^7 \text{ UPTO } 10^9$$

TRANSITION FROM LAMINAR TO TURBULENT FLOW TAKES PLACE ~~AR~~ AT A

$$\boxed{Re_{tr} \sim 5 \times 10^5}$$

What happens in mixed flow? In mixed flow the boundary layer is initially going to be laminar and it would undergo transition at some location on the plate. And therefore CD, the turbulent one that we have used must be corrected and it is corrected in this form REL to the power 1 by 5 - 1740 by REL where Reynolds number is greater than 10 to the power 5 but it is less than 10 to the power 7 and CD turbulent 0.455, the same expression that we have used before where Reynolds number is represented to the power 7, all the way up to 10 to the power 9.

So these 2 equivalent relations are there for 2 different values of Reynolds number and I am not sure whether I have mentioned it but I should say it again, the transition from laminar to turbulent flow takes place at Reynolds number transition to be equals 5 into 10 to the power

5. This is to be kept in mind, the same way in the case of flow through a pipe, we know that there exists a Reynolds number at which beyond which the flow can be treated as turbulent, similarly for the growth boundary layer over a surface, it is assumed that REX upto 5 into 10 to the power 5 or the X corresponding to a Reynolds number of 5 into 10 to the power 5, the flow remains laminar.

And that is a transition in between laminar and turbulence and from that point onwards, the flow becomes entirely turbulent. So this is convenience, this is based on a number of experimental observations, so one value was chosen to be the transition value between laminarity and turbulence. However it is important to realise that the onset of turbulence starts well before 5 into 10 to the power 5 and beyond 5 into 10 to the power 5 depending on where you are what is the Reynolds number you are going to have more and more turbulence.

So the transition from laminar to turbulent does not take place at a specific point as we are taking over here to be equal to 5, Reynolds number corresponding to 5 into 10 to the power 5 but it takes place over a region. But for convenience sake, the transition Reynolds number is always taken to be for flow over a flat plate, the transition from laminar to turbulent will take place at a value equivalent to Reynolds number 5 into 10 to the power 5. But we realise that it is only an approximation the mighty the range over which the transition takes place.

So coming back to the to the formula once again, the CD turbulence when you incorporate the correction due to the mixed flow, due to the presence of a laminar layer before laminar boundary layer before the turbulence sets in for 2 different ranges of Reynolds number, these are the expressions for CD in turbulent flow when the Reynolds number is less than 10 to the power 7 and when it is greater than 10 to the power 7.

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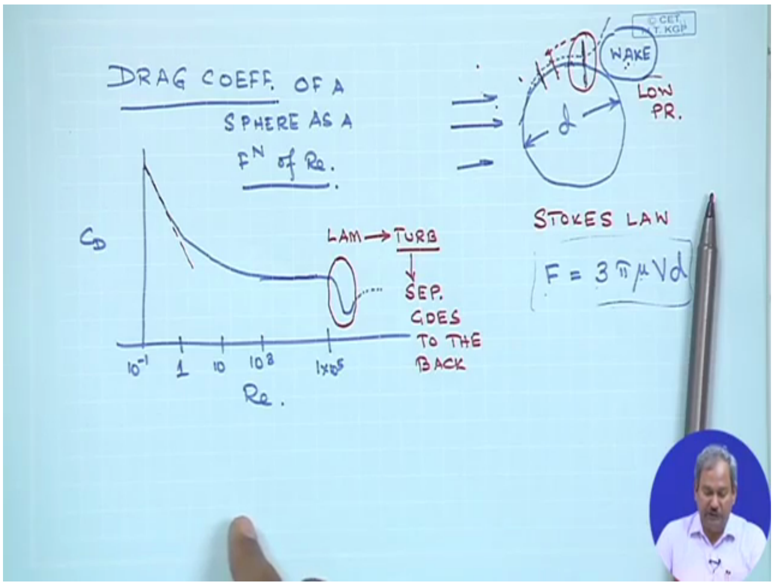

So transition and it is assumed that the transition from laminar turbulent flow takes place takes place at a Reynolds number equal to 10 to the power 5 into 10 to the power 5. So with that I think we have, I have covered most of the things that I wanted to cover in this part but there would be one curve that I would like to show you of drag coefficient of a sphere.

So if we have a sphere and if we plot the experimentally what is, how does CD vary with Reynolds number, initially the going to see that it is going to be almost like a straight line, it becomes more or less a constant, though slowly decreasing and at certain value of Reynolds number, it is going to dip suddenly and then itself slowly increase. So this value, this is about 1 into 10 to the power 5, this is a log log scale it is I am not drawing it true scale, but this is drag coefficient of a sphere as a function of Reynolds number.

So we understand that the Stokes law gives the force experienced by a spherical particle when a fluid starts to move over it and the well-known Stokes law is given as X equals 3 pie mu, the viscosity, V the velocity and where d is the diameter of the sphere. So this is the Stokes law which is truly valid for very slow flow. So when Reynolds, the strokes regime will be valid for a Reynolds number roughly about 1, up to a Reynolds number of equal to 1.

So when Reynolds number is 1 and beyond, the linear relation, linear relation between the force, this relation is valid only up to a Reynolds number equal to 1. So if you plot CD, you would see that the linear relation of CD with Reynolds number will be valid up to, up to a value of Reynolds number to be equal to 1. As Reynolds number is increased beyond, let us say up to 10 to the power 3 and all, CD drops, CD starts to drop continuously, decreases continuously and as a result of flow separation the drag is going to be combination of frictional drag and pressure drag.

As I mentioned the formation of the wake would create a pressure drag in the system and therefore the presence, with the increase in Reynolds number, the wake, the drag becomes more, the drag becomes more prominent and pressure starts, the pressure drag starts to become more important and the, they will contribute to the overall drag force experienced by a sphere. So up to a Reynolds number equal to 1, the CD more or less remains linear with Reynolds number, however beyond 1 and all the way up to 10 to the power 3, the value of CD decreases slowly with Reynolds number.

A turbulent wake, whenever you have flow like this and you have the formation of a boundary layer and at some point in the boundary there starts to separate and you have the formation of wake in here. A turbulent wake is developed when you have such a flow and it grows at the rear of the sphere as a separation point, the suppression point initially it was here, the separation point moves from the rear to the front.

So this is the location of the separation point in this specific case where the boundary layer detaches from the surface, but as the Reynolds number is increased, this separation point will start to move towards the front and you have new values of separation, new values of suppression points at higher values of Reynolds number. So the wake is low-pressure region and since you have high pressure on this side and low-pressure, so this is a wake is a low-pressure region, so the presence of a low-pressure region and a high-pressure region at the front of it would lead to a large pressure drag.

For Reynolds number greater than, roughly about as you can see in the figure, roughly about 2 into 10 to the power 5, transition occurs in the boundary layer on the forward portion of the sphere. The point of separation, the moment it becomes turbulent, it has a high velocity and if it has a high velocity, the molecules inside the boundary layer will carry more momentum. And this more additional momentum due to the high-speed motion in turbulent flow will push the separation point downstream from the centre of the sphere and the size of the wake is decreased.

So if you are in laminar flow, when I say that the transition point starts to move from the back to the front as you increase the Reynolds number, we are referring to laminar flow only. So with increase in Reynolds number, the point of separation would start to move forward in laminar flow but the moment it becomes turbulent, than with increase in Reynolds number, the point of separation would start to go back and the size of the wake is reduced and therefore the pressure drag, the net pressure force on the sphere is reduced and the drag coefficient decreases abruptly.

So this abrupt change of the value of CD of Reynolds number is a direct result of transition from laminar to turbulent and in turbulence, the separation, the point of boundary layer separation goes back, go to the back of the object and therefore the wakes are going to be smaller and so on. So the turbulent boundary layer, since it has more momentum than the laminar boundary layer can better resist an adverse pressure gradient. Consequently turbulent boundary layer flow is desirable on a blunt body.

So if you think about these different flow situations, what you have is there in laminar flow with increase in the velocity of flow or the case of laminar flow, the point of suppression would start to move forward. Okay. And when the point of separation starts to move forward, the size of the wake at the back of the moving sphere would increase. So the low-pressure region at the back would increase and that is balance of pressure between the front end and the backend would give rise to significant pressure drag, in fact most of the drag that is experienced by the moving object, moving spherical object in air is due to pressure drag.

It is the frictional drag constitutes only about 5 to 10 percent. But the situation gets reversed when you go into turbulent boundary layer. The turbulent boundary layer, since the molecules, the fluid molecules carry more momentum with it, the point of separation on the sphere would start to move backwards resulting in smaller wakes and smaller adverse pressure gradient. This is the reason that turbulent boundary layer is often preferred over a blunt body. Okay. So for a blunt object, it is rather, we would rather have turbulent flow rather than laminar flow on a boundary layer.

So this figure of CD drag coefficient versus Reynolds number, it is a very well-known figure and of course for different objects, the values of this CD would be different at its variation with Reynolds number would also be different. But this roughly gives you an idea, starting at the Stokes regime, the laminar flow and the turbulent flow, how CD changes, how wakes are formed, how wakes are reduced and so on.

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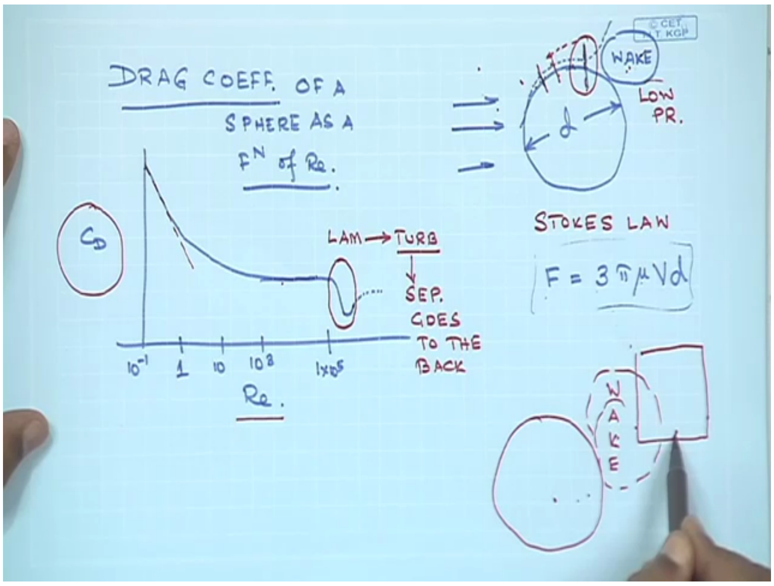

So if this is the object and you have a low-pressure region at the back of it, if this is the wake, then I was telling you, that if this is a car, then the next car would like to be in the wake formed by the 1st car and so on, such that it would have a special track. So sometimes intelligent use of the wake formed by the previous object would allow this object, the object next to it in a more intelligent fashion with less effort. So that is what is, what we are going to, we are going to explore further with our example from cricket.

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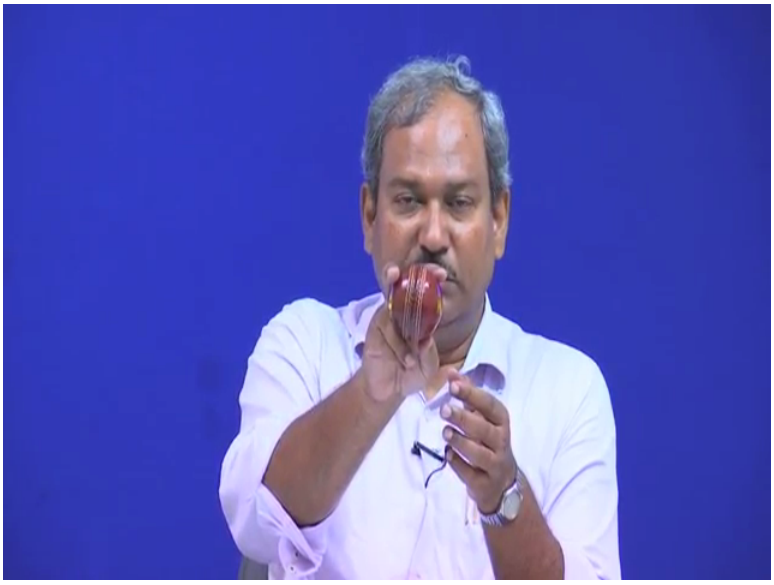

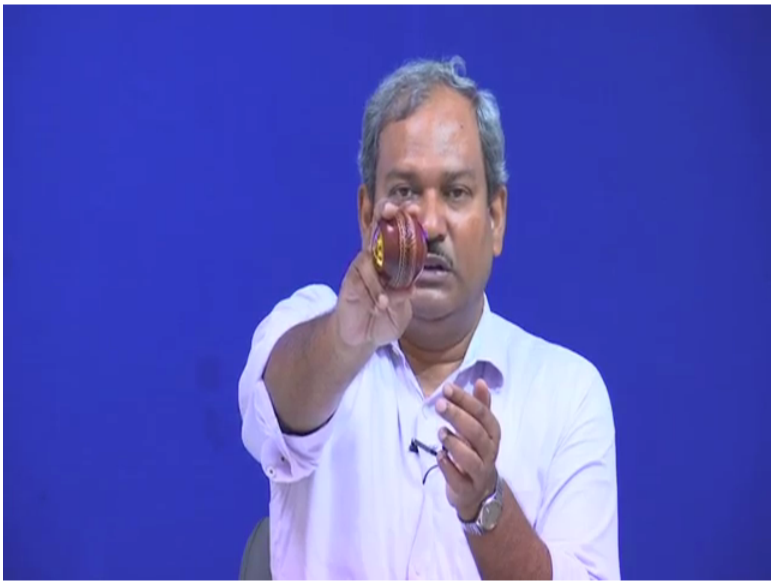

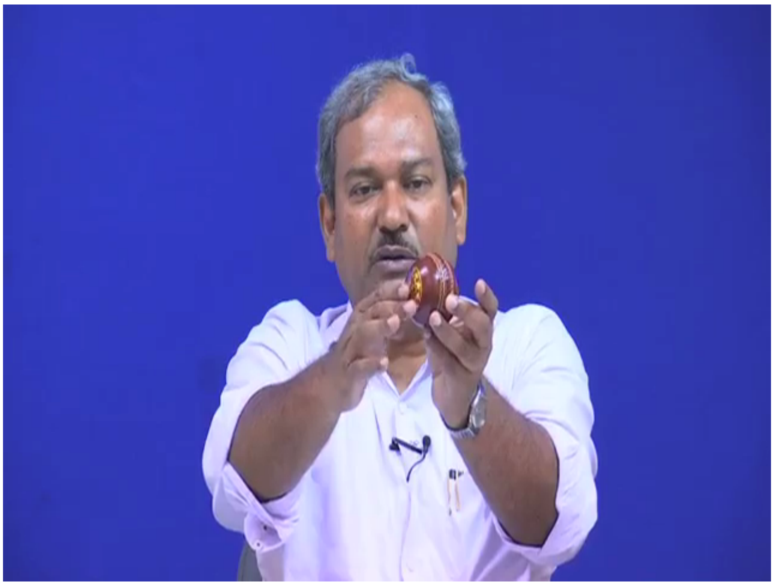

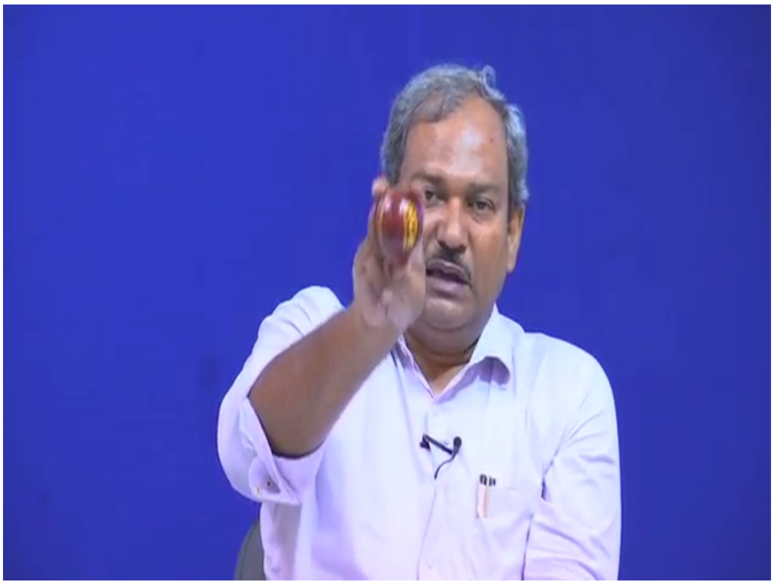

So how do we have an object, spherical object, the cricket ball, when it moves in air, how does it change its direction? So I have brought a cricket ball into the class today. So you are, all of you are familiar with this cricket ball. So it is roughly spherical in shape so might have seam and you would see that the fast bowlers, all bowlers use the seam, its position and its direction intelligently in order to make the ball move while it is coming towards the batsman.

So the seam, if it is straight pointing to the batsman, then the ball would simply go straight towards it because both the sides are exposed to similar conditions. But if you see, if you have the seam in this direction, the seam is pointing towards the 1st slip, therefore the, when the air comes towards it, it encounters seam when it travels this site but it does not encounter the seam when it travels to this side. So therefore using and pointing the seam towards the slip, you create turbulent boundary condition on this side of the ball and laminar boundary condition on the other side of the ball.

The situation would be different if it is pointed like this, then you have laminar over here and the presence of the seam disturb the flow and creates a turbulent condition on the other side. And you know what, when you, when your seam points towards the bound, towards the slip petition like this, if you, if you hold the ball like this and if you bowl to a batsman, you would see swing. That means the ball while moving, it would start to change its direction and it would move away from the batsman, which is commonly called as the outswing.

On the other hand if it is like this and if you can bowl it perfectly in the right way, then the ball will start to move, it starts outside of the off stump and would come towards you which is known as the in swing. So through the use of a problem in the next segment we would try to show you the physics of swing ball, that is what we will do in the next segment.