Transport Phenomena.
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Lecture-25.
Turbulent Boundary Layers.

So let us start with turbulent flow in this class. Before we start the turbulent flow, the treatment of turbulent flow, which as I mentioned before is quite complicated, I would start with something very simple. Let us think of fully developed flow in a horizontal pipe. So whenever you have flow in a horizontal pipe and if we consider steady-state and since it is horizontal, there is no effect of body forces. So what it, the flow only takes place as a result of applied pressure gradient, so if you apply pressure the fluid starts to flow.

But when it attends a steady-state condition, that means the velocity at any specific location is not a function of time, then this pressure gradient must be balanced by an opposing force. And the opposing force is provided by the fraction of the fluid. So if one writes the Naviar Stokes equation, the Z component, the axial component of the Naviar Stokes equation in a cylindrical coordinate system for steady flow, steady fully developed flow in a horizontal pipe, the form of that equation would look something like this.

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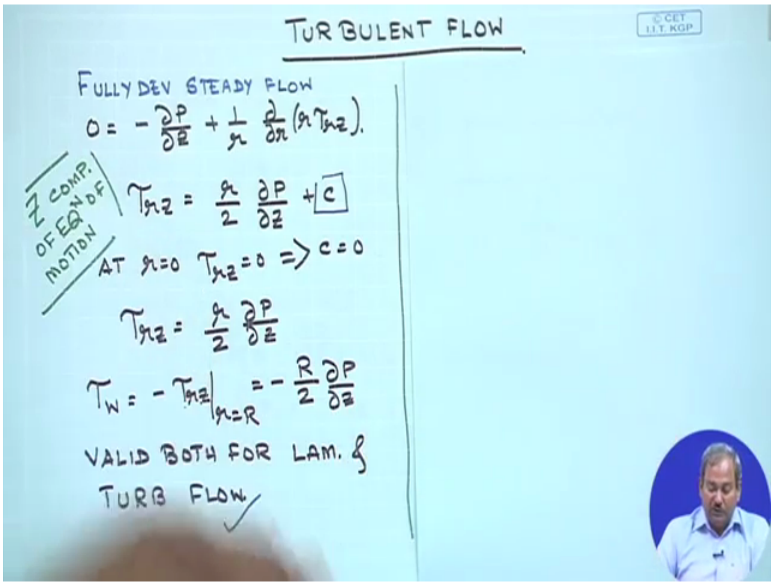

Where the all the convective terms are going to be 0, this is Dell P Delta Z with a - sign. And since it is cylindrical system, this denotes the viscous forces acting on it. So this is the gradient of pressure, the pressure gradient which is forcing the fluid to flow and this is the shear stress which is opposing the motion of the fluid. So at steady-state fully developed condition, fully developed condition, this would be the equation for the Z component of equation of motion.

So one would be able to integrate this equation and to obtain concert of integration as C. And we all know that at R equal to 0, that at the Central line, the shear stress must be equal to 0. So if the shear stress is 0 at the Central line, that is at r equal to 0, Central line of the pipe, at R equal to 0, which should give us the integration constant to be equal to 0. So the expression for shear stress in a flowing fluid would simply be equal to r by2 where r is the radial distance coordinate and Dell P Delta Z is the pressure gradient in the, which is forcing the liquid to move from lower values of Z towards higher values of Z.

This is the shear stress on the fluid, so when I express it in terms of the wall shear stress, that is the force experienced by the solid wall in contact with the fluid, then tao W would be simply equal to - Tao RZ but the location here is that the wall at which point is the r is going to be equal to capital R Where capital R is the radius of the pipe. So substituting the value of R in here, one can obtain the wall shear stress to be equal to - R by2 Dell P Delta Z. So this is valid both for laminar and turbulent flow.

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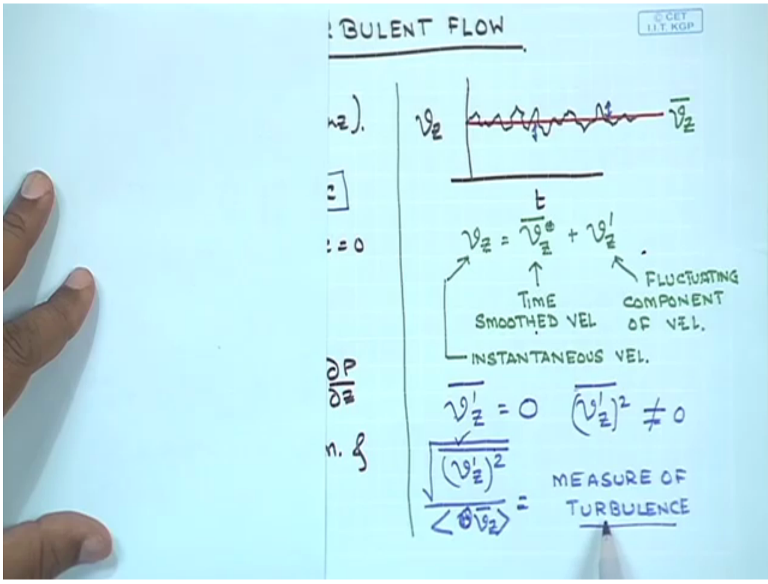

We will use this later on in our discussion. But let us come to this part of the turbulent flow. What I have drawn over here is the instantaneous velocity which is denoted by VZ. So this is the axial value of velocity at any given location as a function of time. So far in laminar flow we used to get only one value of velocity, that is the velocity is a constant and therefore it will not fluctuate. But in turbulent flow whether is going to be a lot of intermixing, the exchange of Eddies, the formation and exchange of Eddies and so on and if you are measuring the velocity of the fluid at a given point, the velocity is never going to be constant.

It will be an arbitrary oscillating function of time. So at one point instant of time the velocity could be high and the next instant it could be low, set would arbitrarily oscillate as a function of time. Now if you integrate this over entire time, if you find the time average of this velocity where the averaging is done over a timescale which is large enough in comprising do they timescale of the selection, so if it is oscillating like this, you are averaging over a large time. And therefore the value you does obtain the would be a constant, it would be independent of oscillation and this is called the time smoothed velocity.

So if you can integrate the fluctuating velocity over a large time domain, then the constant value that we are going to get out of it is known as the time smoothed velocity. So that is the 1st difference between laminar flow and turbulent flow. In laminar flow the velocity is constant but in turbulent flow the velocity can fluctuate. So the instantaneous value measured at different different points of time could have different value. So it is better always to express a time smoothed velocity rather than the instantaneous velocity.

So what I have drawn over here, this V that denoted by the black line is the oscillating velocity whereas if you take a times smooth average, then the red line that I have drawn and denoted by V bar Z is the time smoothed velocity. So one can express the instantaneous velocity as the sum of time smoothed velocity and then a fluctuating component, whatever be this fluctuating component. The fluctuating component can be a positive, the fluctuating component can be negative.

So the instantaneous velocity is a, is a function of time smoothed velocity and the fluctuating component of the velocity. Now if you look at the fluctuating component of the velocity and if you decide to take a time average of the fluctuating component, then you can clearly see that this would be equal to 0. That means that fluctuating component when you take the average over a large time domain, that the fluctuating component will be 0, however V prime Z square if you take the time average of this, this is not going to be equal to 0. Okay.

In fact the fraction VZ square times smooth, the square root of this divided by the, divided by VZ is known, it is it is essentially a measure of turbulence. So the points which I am making over here is that they instantaneous velocity is the sum of the time smoothed velocity and the fluctuating component of velocity. The fluctuating component of velocity, since it can be and negative, it can vary both it can become both positive or negative, so if you allow, if you if you find out the time smooth over a large, over a large time domain, then V prime Z would be equal to 0.

But V prime Z square, if you take the time Smoothing of that will not be equal to 0. So sometimes the measure of turbulence is expressed as the square root of time smoothed velocity square divided by the velocity, divided by the time smoothed velocity and this is known as the measure of turbulence. But again the point here is that the velocity fluctuates, the fluctuating part when you take the time smooth is equal to 0 but the square of that is not going to be equal to 0.

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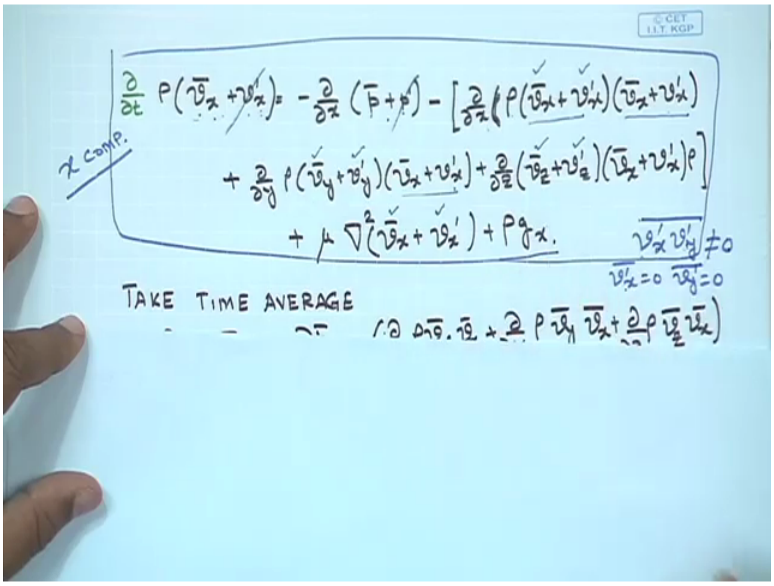

So now we move on to the Naviar Stokes equation, the equation of motion where all the velocities are expressed in terms of a time smoothed velocity and a fluctuating velocity. This is written for the X component of Naviar Stokes equation. Similar to velocity when we come to the pressure term, the pressure will also be in turbulent flow, the pressure will also have a time smooth component and a fluctuating component. So the 2nd the other terms are going to be Dell Dell X of rho VX times VX, like before but the velocities are now expressed as the sum of time smooth and fluctuating.

Similarly for VY time smooth fluctuating, this is the same for Z also I can have the sum of time smooth and fluctuating and the shear stress term will also have a time smooth and a fluctuating and I have the normal body force. So there is nothing difference, nothing unusual about this equation except all velocity terms and the pressure terms are expressed as a sum of the time smooth component and a fluctuating component. The same equation can, the similar type of equation can also be written for the equation of continuity.

What we do this at this point is we take a time Smoothing of the entire Naviar Stokes equation. So if you take the time Smoothing of this equation, then obviously this term would disappear, this term would disappear, wherever I have V prime X alone, it will disappear at here you would see you have V prime X square, it will not, if you are, the time Smoothing of that would not be 0. And here for example you are going to have V prime X, V prime Y and if you take the time Smoothing of that, it is not going to be equal to 0.

Whereas individually V prime X would be 0, time Smoothing of that, V prime Y, if you take the time, time average of this, it would be equal to 0. But the product of these 2, when you take the time average of this, they may not be equal to 0. So when you take the time average of the entire equation considering the facts that the products are not be equal to 0 but individually they are 0, then some of these terms will drop out but some of these terms will remain as additional terms into the Naviar Stokes equation.

And most of the additional transport, additional transport of momentum that one would encounter in turbulent flow appears as a result of these fluctuating components of velocities. These fluctuations are characterised by the formation of eddies and therefore these fluctuating components would give rise to additional stresses which are not quick are not visible, which are not important for the case of laminar flow whereas we have a velocity which remains constant overtime but in turbulent flow it keeps on changing.

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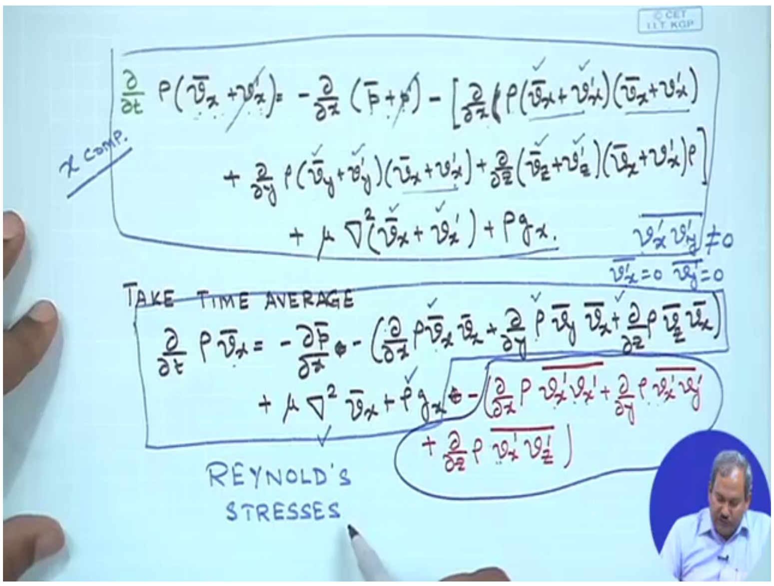

So the products of these fluctuating components denote additional transport of momentum unlike in laminar flow. So these are, these explain the presence of these terms explains why the transport of momentum is more in the case of turbulent flow as compared to laminar flow. So when you take the time smooth of this equation, what you get is as I mentioned, the left-hand side would simply be this term, the pressure term would only have the time Smoothing term and these 3 terms, they are identical with the question that you would get in turbulent, in laminar flow.

This term is identical that you would get in laminar flow, the GX term, GX, the body force term would also be identical as in laminar flow, however these 3 additional terms appear automatically in Naviar Stokes equation which are products of the fluctuating components. Remember I am writing for the X component, so therefore V prime X appears in all 3, the time smoothing of the product of these fluctuating components will not be 0 and collectively they are referred to as Reynolds stresses.

And the Reynolds stresses, they are responsible for the transport, additional transport, turbulent transport of momentum over and above that one would expect in the case of laminar flow. So up to this part, there is, this is the laminar flow equation and the 3 terms known as the Reynolds stresses, they essentially refer to the Reynolds, the Reynolds stresses or the additional transport terms. So the next question is, we understand that in the case of laminar flow of, flow of liquid through a pipe, we get a velocity profile.

We know that the velocity profile is parabolic in nature with a maximum in the velocity at the centre line. We also have an idea of how would the velocity vary inside the boundary layer in laminar flow. But can there be an Universal velocity profile which would explain which would tell us about how the velocity varies in the case of turbulent flow. Unfortunately there is no, till today there is no such universal velocity profile which are going, which can be used from the very from a point very close to the solid surface all the way up to the centreline of flow when the flow is taking place in a pipe.

Rather it is customary to express velocity profiles in 3 distinct layers depending on which forces and which mechanism is important in those 3 layers. So if you think of the layer which is very close to the solid surface, very close to the solid surface, the effects of viscosity are important. So this region where the effect of viscous forces are important, this region is termed as the viscous sublayer.

So in the viscous sublayer, the viscous forces are important, whereas if you go towards the core of the pipe, in that zone which is far from the far from the pipe wall, the effect of viscosity would be negligible and most of the momentum transport is going to be governed by the formation exchange and transport of eddies. So the momentum transport in the core region is controlled by the formation of eddies. So that is the turbulent core region.

So we have a turbulent core region and a viscous sublayer very close to the wall. And in between the 2, in between the viscous sublayer and the turbulent core, there exists another hypothetical layer which is known as which is known the transition region and in this region both the viscous forces and the eddies, the turbulent transport of momentum, both are going to be important. So in a turbulent flow field, the velocity the region the flow region, the region, 3 regions, distinct regions are introduced the viscous sublayer, the transition region and the turbulent core.

And we have expression for velocity in turbulent, in in the viscous sublayer, in the transition region and in the turbulent core, 3 distinct expressions of velocity, unlike in laminar flow where we just have one expression for velocity for the entire flow field. So turbulent, turbulent velocity profile is much more complicated than that of the laminar velocity profile. Additionally the expressions for velocity in turbulent flow, be it in viscous sublayer or in turbulent core, they are, they are obtained using a semi-empirical approach.

So the constant, the expressions are provided in the text, in the literature, these expressions are essentially averages of a huge amount of experimental data points. Okay. So these are semi-empirical in nature and they essentially denote, they essentially show, they essentially provide, denote averages over a large amount of experimental data and their Genesis is however semiempirical in nature. The 3 the points to remember are for 3 distinct regions we get 3 velocity profiles.

The velocity profiles are obtained is in a semi-empirical approach. And the demarcation of these 3 regions are also a result of experimental observations. So I would simply provide you with the 3 velocity profiles, just for the sake of completeness and to show how complicated turbulent velocity turbulent flow is even for the case of flow in a pipe and I will give you the expressions for the velocity profile in those 3 regions.

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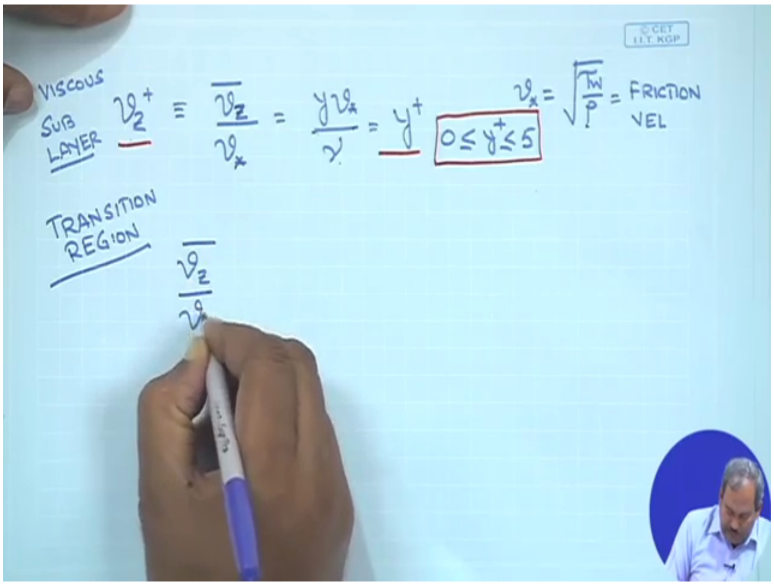

The 1st is the turbulent velocity, 1st is the viscous sublayer where VZ + the velocity, axial component of velocity in dimensionless form, where this is VZ by V Star, this is the time smoothed velocity is equal to Y V Star by mu, the kinematic viscosity. And this V Star is essentially root over tao W by rho, root over tao W by rho has the same unit as that of velocity, so this is used to nondimensionalize velocity and it is called the frictional velocity. And Y V Star by mu, again it is the non-dimensionless distance from the wall so it is denoted by Y +.

So for viscous sublayer where the viscous forces are important, the velocity profile would simply be equal to VZ + which is by definition the time smooth axial velocity divided by frictional velocity, this is the definition of racial velocity is equal to a nondimensional, Uhh is equal to a dimensionless distance from a solid wall defined as Y distance from the wall, V Star, the frictional velocity and the kinematic viscosity and this is, this is expressed as Y +.

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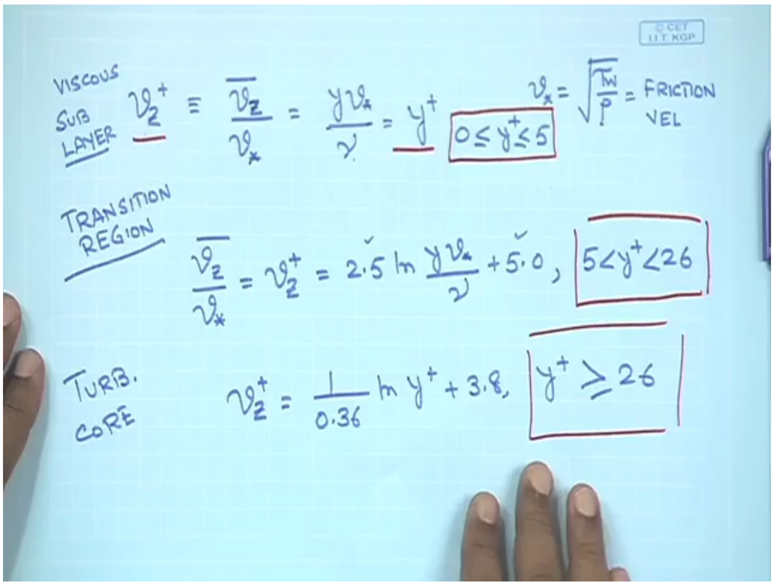

So VZ + equal to Y + and this gives valid, the extent of the viscous sublayer is denoted by this. So this is the expression for velocity in viscous sublayer. In transition region, the proposed profile is VZ by V Star, frictional velocity again, that means which is equal to VZ + is 2.5 ln YV Star by mu + 5.0 where the reason of applicability is 5, this. So this expression which is obtained semiempirically and the constants essentially denote the are derived from a large experimental dataset.

So their average is over many experiments and for the turbulent core, the expression is VZ + is equal to 1 by 0.36 ln Y + + 3.8, where Y + is greater than equal to 26. So in other words, there is nothing called a universal velocity profile in turbulent flow. Okay, people have tried different means, different statistical techniques to obtain an expression for velocity profile in turbulent flow.

However the most common accepted way of treating turbulent velocity profile is to use 3 different zones and 3 different expressions for velocity that are obtained semiempirically and the constants are essentially averages over a huge quantity of experimental data which are obtained, so viscous sublayer, transition and the turbulent core. And you are obviously can sense how difficult it is to express everything in terms of a universal velocity profile in turbulent flow.

So there is no such, there is, it is so difficult to use a universal velocity profile. However, there is a velocity profile entirely empirical in nature which is known as the one 7th power law that has been very successful in expressing the experimental results over a large domain of distance from the solid wall. So here we have an expression of velocity as a function of distance from the solid wall which is expressed, it is totally empirical in nature. But for some reason this empirical velocity profile has been proven to be very successful in expressing the velocities, in velocities over a large value of Reynolds number and this is known as the one 7th power law profile which are going to show now.

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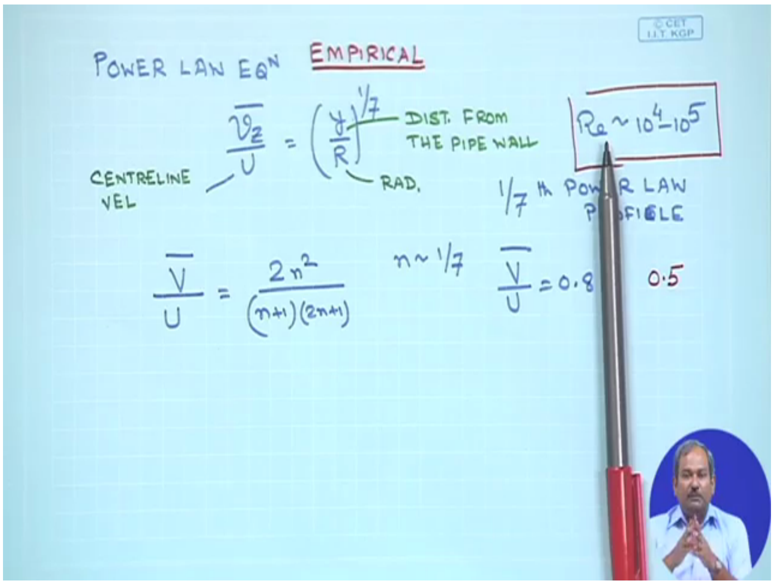

The power law profile, the power law equation simply tells that VZ by U is equal to Y by R to the power one 7th where Y, U is the centreline velocity, the R is the radius, the Y is the distance from the pipe wall. The region of applicability of this equation is, if Reynolds, Reynolds number is within the range of 10 to the power 4 to 10 to the power 5. So if Reynolds number is in this zone, it is entirely empirical in nature, so this is empirical in nature, so this would give us, this will give us a profile V Z by U to be Y by R to the power one 7th, this is also known as the one 7th power law profile.

And if you find out the average velocity, if you integrate it and you find the average velocity, this is 2N square by N +1 by 2N +1 where N in this case we have taken it to be equal to one 7th, so for N equal to 1 by 7, then V bar by U is equal to 0.8. Compare this with the case of laminar flow where this constant is equal to 0.5. So compared to Uhh compared to a linear velocity, laminar velocity profile, the profile in turbulent flow is going to be flatter near the centre.

So for the case of laminar flow the velocity profile is parabolic in nature and for the case of turbulent flow it is going to be flat at the Centre. Higher the value of Reynolds number, the more turbulent it is, the length of this flat region, relatively flat region where the velocity profile is not a function of R will keep on increasing. So in the extreme limit when you have a very highly turbulent flow at the entire pipe can also can approach a velocity profile which is flat.

Okay so you are you are going to go from a plugged flow in the case of a laminar, sorry parabolic velocity distribution in the case of laminar flow to almost like a plugged flow in the case of highly turbulent flow. So this expression, the one 7th power law is extremely successful in fitting experimental data with relatively high accuracy, however there is one catch. The catch in this is that the one 7th power law profile can fit the data points of the velocity very well when the distance from the solid wall or it the solid wall is slightly higher.

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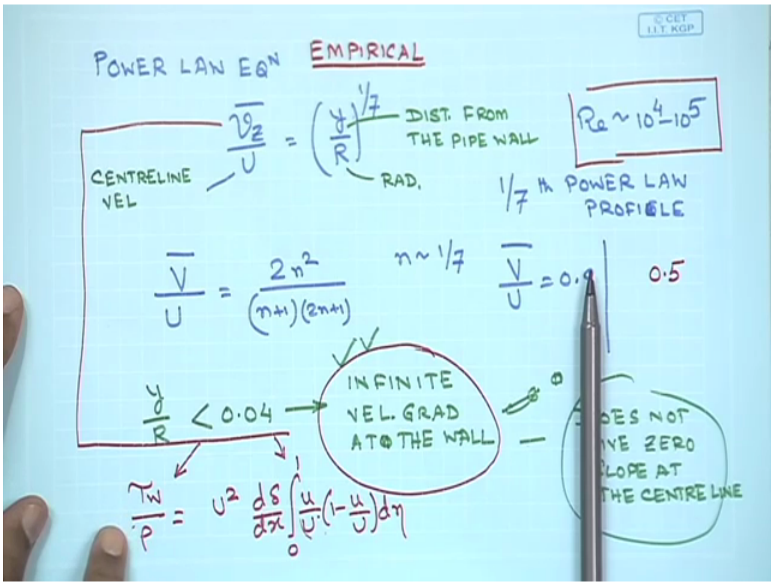

In other words, this profile fails when you approach solid wall. So the one 7th power law though successful in expressing in expressing the velocity Uhh within, with sufficient accuracy, it cannot track the profile when you approach the solid wall. In fact when the value of Y by R, that is the distance from the solid wall nondimensionalized by the radius is less than 0.04, it gives and you can work it out infinite velocity gradient at the wall, which is not possible. You cannot have an infinite velocity profile at the wall, so the, at region of applicability of this one 7th power law equation is definitely not in, not near the wall.

So therefore one must reemphasise that although the profile fits the data close to the centreline, it fails to give a 0 slope at the centreline, that is the 1st, 1st problem and the 2nd problem is that it gives Uhh an infinite velocity gradient at the wall. So infinite gradient at the wall and does not give 0 slope at the centreline. Of these 2, this is going to create the maximum problem if we try to use this expression for the wall, for the use of the, for the use of the momentum integral equation.

Because the momentum integral equation if you recollect is simply Tao W by rho is equal to U square D Del DX and then over here, integration of u by U1 - u by U times T Eta, Eta varies from 0 to 1. So if you take the simplest possible form of the work, the moment the work, momentum integral equation, on the left-hand side I have Tao W, on the right-hand side I have a velocity profile and yet I have a very handy power law equation which can be used here in order to express it as u by U equal to Eta to the power one by 1 by 7 or I can express it, I can plug it in here in order to obtain the expression for tao W.

If you look at these 2 sides, it becomes apparent that you may be able to use one 7^{th} power law on the right-hand side since you are integrating the expression over the entire flow region, that is you are integrating from 0 to, 0 to 1 over d Eta. So whenever we integrate an expression that contains some experimental error, the error gets minimised, the error gets reduced. But what happens in Tao W, in Tao W you are to express Tao W in terms of velocity gradient.

That means you are differentiating the velocity profile, the moment you differentiate the, differentiate a data containing error, you magnify the error. So power law equation since it fails near, it fails and it gives infinite velocity gradient at the wall and the velocity gradient at the wall is connected with the shear stress at the wall, so, power law equation in this form cannot be used with now, the left-hand side of the comment integral equation where were we have Tao W.

On the other hand the right-hand side essentially denotes an integration over the velocity profile, so power law, one 7^{th} power law is an ideal candidate to be used on the right-hand side of the momentum integral equation. So when we try to plug in a universal velocity profile on the left-hand side of the momentum integral equation, the problem we see is that there is no such thing as an universal velocity profile. At the best we have 3 velocity profiles in 3 regions, which makes it difficult, cumbersome to be used with the momentum integral equation.

And the whole purpose of momentum integral equation is to simplify the entire process, so it cannot use those 3 profiles. Now we have a one 7^{th} power law profile which fits the data rather well near the centreline but it does not do it so good a job near the pipe wall. So we can use it on the right-hand side of the momentum integral equation when we integrate the profile. The left-hand side of the momentum integral equation contains wall shear stress and wall shear stress is generally expressed in terms of velocity gradient at Y equals 0.

And we know that one 7^{th} power law fails miserably when Y approaches 0. It predicts an infinite gradient which is impossible, which is physically indefensible and therefore one 7^{th} power law cannot be directly used to evaluate the wall shear stress and therefore the left-hand side of the momentum integral equation. So in order to use momentum integral equation for turbulent flow, we will use one 7^{th} power law for the right-hand side but we cannot use one 7^{th} power law for the left-hand side and we have to devise something else. So that is what we would discuss in the next class.