Transport Phenomena.
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Lecture-02.
Fourier and Fick's Law.

So with an understanding of the transport phenomena, the basic initiation behind us, I think we are ready now to work with some fundamental concepts, something which we all of us experience every day, this is the concept of viscosity. So what is viscosity? I know that we have all noticed that if I drop of a heavy liquid on a surface, it does not spread, okay. If you like to move one layer of a thick liquid on top of the other, if you would like to stir a thick cream as compared to that of water in a glass, in the 1st case you are going to have, you are going to spin more, you will require more energy, more force to turn the spoon inside the glass.

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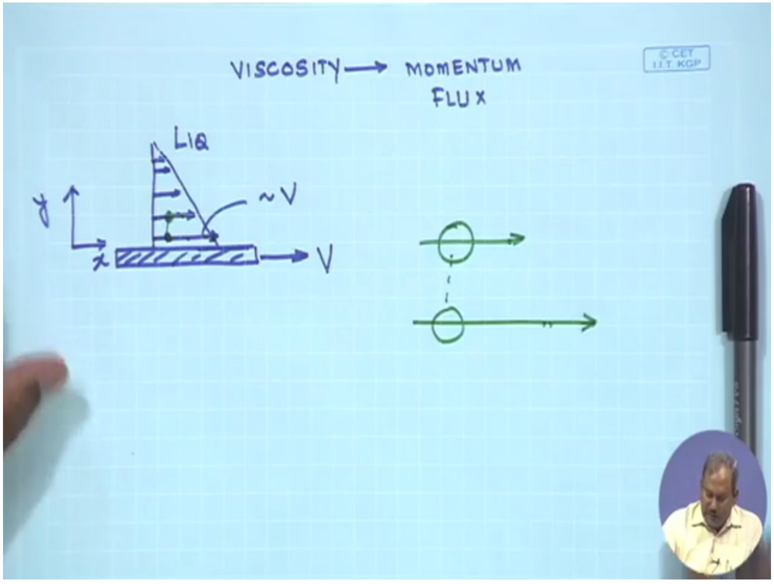

So the concept of how these liquid molecules, let us say, it is also present in gaseous molecules, how the liquid molecules credits the relative motion in between them has given rise to the concept of viscosity. So what is viscosity? We would like to start 1st with the concept of viscosity and then the concept of viscosity would lead to some idea of what is momentum, momentum flux. So for this I draw a simple system in which there is a solid plate and on top of this solid plate, so this is my x direction and perpendicular to this is the y direction.

And let us say it is a solid plate and I have a liquid in here on top of the plate and let say the solid plate is moving with a velocity V. Now as the solid plate starts to move, the liquid layer will also start to move, that is the natural tendency of the liquid in contact with the solid plate, that is near the solid plate the liquid will have the motion, will have the velocity which will be equal to the velocity of the solid plate. But as we move away from the plate, the effect of the plate will be felt lesser and lesser by the liquid layers on top of the plate.

So if I could draw the velocity profile, some sort of a very rough approximate velocity profile of the liquid velocity initiated by the movement of the plate, it would probably look something like this. So in this direction the velocity will progressively decrease and ultimately at a point far from the plate the velocity will be roughly 0. So this, this velocity which is very close to the very close to the solid plate will be approximately equal to the velocity V that of the solid plate.

And as I move away from it, the velocity will decrease. Now, that would be a liquid molecule, which lets say is associated with this layers of. And due to its Brownian motion, there is a possibility that it would jump to the upper layer and similarly a molecule from the upper layer can come to the lower layer. So the molecules when it goes from the lower layer to the upper layer carries with it the momentum associated with the velocity of the bottom layer.

So it carries with it more momentum corresponding to that of any molecule existing on this layer. So this transport of momentum with transport of momentum with transport of molecule with velocity more than that of the upper regions will carry an additional momentum, will carry an additional momentum when it goes to the top plate will carry an additional momentum which would try to force the upper layer move with a velocity close to that of the bottom layer.

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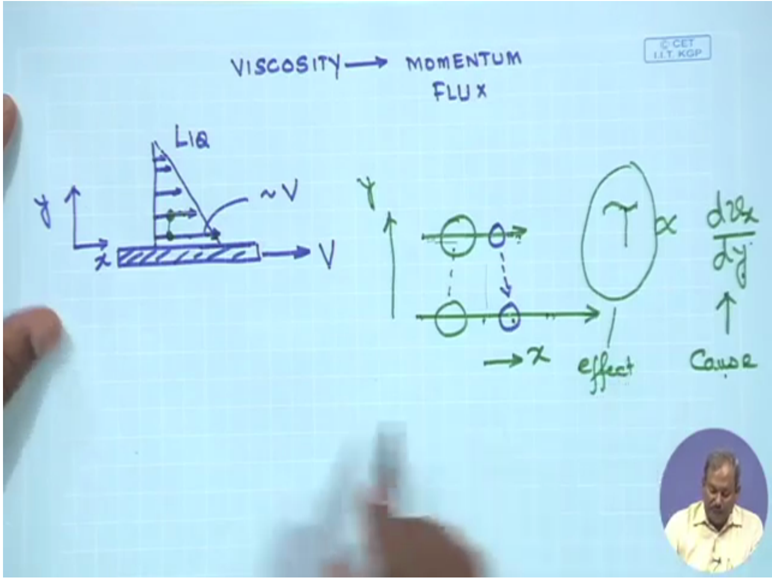

And similarly on the other hand when I have a molecule from this layer coming to this layer, the tendency of this molecule will be to slow the faster moving layer, so there will always be attraction, always be interaction between the layers as a function of y, as a function of distance from the solid plate. So this interaction, this invisible string so to say which would bind these 2 layers that are moving with different velocities is sometimes called the viscosity.

So the origin of viscosity is molecular in nature and the result of the viscosity is essentially transport of x momentum, this is x direction, transport of x momentum in the y direction. And the phenomenological equation which connects the shear stress, this results in a stress, the molecular transport of momentum is found to be proportional to the velocity gradient which is dvx times dy. Variation, the x component of velocity with y, the shear stress experienced by this layer because of the differences velocity between these 2 layers is known as shear stress that it is expressed in this form.

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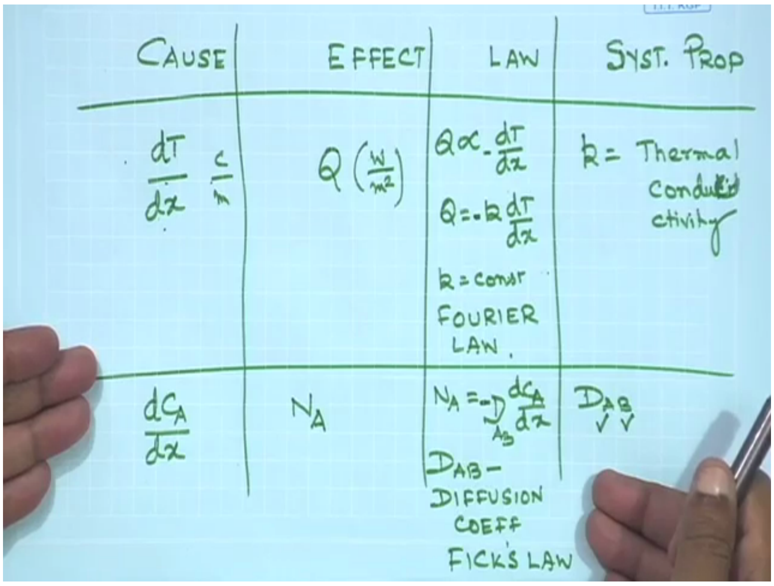

Now this you can also think of this as a cause and this is the effect. So the relation between the shear stress and the velocity gradient is nothing but the relation between cause and effect. And we are seeing these in other fields of engineering so to say and here too I would like to identify the cause, I would have the effect and then I will have the law and the system property. So, some of the fundamental laws that we see come from our understanding of what is the cause and what is the effect.

So if you think of heat transfer, so transport of heat conductive transport of heat, I am restricting this to conduction heat transfer now, conductive heat transfer, the conduction heat, the amount of heat transported by conductive heat, conductive heat transfer between 2 points essentially depends not on the temperature but on the temperature gradient. So the cause of this is dT dx, let us say I have one dimension one directional, one-dimensional case in which the temperature varies only with x, not with y or Z.

As a result of this temperature gradient, there would be some sort of heat transfer, heat flux in between in between 2 points where there exists a temperature difference and the law that relates Q as proportional to dT dx which would give, this is flux, Q equals K times dT dx where this K is a constant is known as the Fourier's law, Fourier law of heat conduction. So we also have a - sign in here denoting that heat always gets transported from higher temperature to lower temperature, so this - sign always comes in this type of equation.

So the system property that is defined by Fourier's law is thermal conductivity. So thermal conductivity is defined by Fourier law which is nothing but a relation between cause and effect. So this has the unit of watt per metre square and this is centigrade per metre. So we can find out what is the unit of K. Similarly when we talk about mass transfer, the mass flux, the diffusive mass flux of species A is given, as a result of the difference or the gradient in concentrations of species A between 2 points.

So we can only have heat transfer, we can only have mass transfer when there exists a difference in concentration between 2 points and it not only depends on difference in concentration of species A, it also depends on what is the separation between these 2 points. So it is a gradient which is important, not just the difference in concentration or species A. So the mass flux NA is proportional to dca dx which is the concentration gradient and the result of this is the law which is equal dca dx, again with a - sign since the mass transport always takes place from high concentration to low concentration.

And it is again proportional and the proportionality is constant is DAB where DAB is the decision to efficient, it is known as diffusion coefficient. So DAB as the system property, the physical property which essentially tells you that diffusion of component A in B. So if you have, let us say a component oxygen in nitrogen and the concentration of oxygen is higher here as compared to over here, then oxygen will start to move from the high concentration towards the low concentration and the amount, the mass of oxygen moving per unit area, per unit time from the high concentration to the low concentration is going to be proportional to proportional to the concentration gradient and the physical property which dictates how fast this process would take place is commonly known as the diffusivity.

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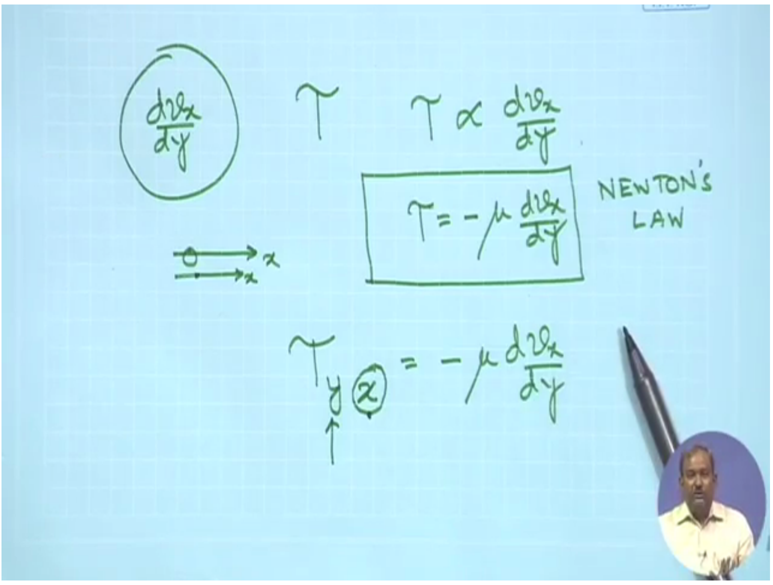

So higher the diffusivity, higher would be the amount of transfer of oxygen from point 1, higher concentration, to point 2, lower concentration. So this equation is, this relation is known as Fick's law, Fick's law of diffusion. So this equation is Fick's law so if you if you see these 2 equations, they are connecting a cause with an effect. So if in the same line I write what is the cause for momentum transfer, it is simply going to be dV x dy which is the cause this is the velocity gradient, as a result of which momentum gets transported and the relation Tao is going to be proportional to dV x dy.

And if I put the equal sign then it is going to be - since momentum gets transported from higher velocity to lower velocity times dV x dy and this is known as the Newton's law of viscosity. So all fluids which we this type of law for momentum transport because of difference in velocity are known as the Newtonian fluids. And from your fluid mechanics you are well aware that there are some specific types, some other types of fluids which do not obey the Newton's law.

So there can be pseudoplastic, there can be dilatant where the variation between the shear stress and the velocity gradient may not be, cannot be equated by simple equality by name imposing the condition of, by posing the parameter mu in there. So this brings us to the question is what, which direction this momentum transport is taking place. I understand my velocity is in x direction, so the momentum is also in the x direction. So the transfer of momentum that we are talking about is essentially the transport of x momentum.

So Tao when I write it in this way, I understand that if there is a variation in the velocity, in the in the y direction, it is the x momentum which gets transported. So that is why the subscript x on Tao denotes it which momentum, which directional momentum we are talking about. So it is the x momentum that gets transported because there is a variation in velocity in the y direction. So the 1^{st} subscript of Tao y Tao, this denotes the direction in which the momentum gets transported and the 2^{nd} subscript denotes the directional momentum that we are talking about.

So in other words what we can say is that Tao yx simply represents the x momentum getting transported in the y direction. So since there is a variation in velocity in the y direction the x momentum gets transported in the y direction because of the Thermo physical property, because of the existence of some of physical property which is known as viscosity. So more the viscosity, there would be more transport of momentum and it would be difficult to sustain a high relative velocity between 2 layers if the viscosity is more.

As a result of which glycerin flows easily but heavy oils, it is difficult to make heavy oil flow, it is the viscosity. It is the momentum that gets transported in the direction perpendicular to the flow. So this higher the viscosity, these 2 layers are going to be more bonded, connected together, strongly connected together such that they will oppose the relative motion between these 2 layers. So viscosity is that property which resists motion of layers, of relative motion between 2 layers of fluids.

We should keep in mind that whatever we are discussing, we are restricting ourselves for laminar flow from the principal direction, principal reason of momentum transfer, heat transfer or momentum transfer are molecular in nature. So due to the molecular motion of the molecular motion which is Brownian motion, the momentum gets transported. Any conductive heat transfer, due to the vibration of the molecules, vibration of the molecules or atoms while keeping the average position intact transfer the energy from one point to the other.

In species transport is the concentration gradient which makes the component move from one point to the other, there can be other type of motion which will result in the motion which is which could be the conductive motion. So I may have a let us say a slab of salt in contact with water and this water may remain stationary, in which case the dissolution of this salt will be will be due to the, due to the molecular motion. But if the top layer, top layer starts to move, then apart from dissolution and molecular motion, there can be convective motion as well.

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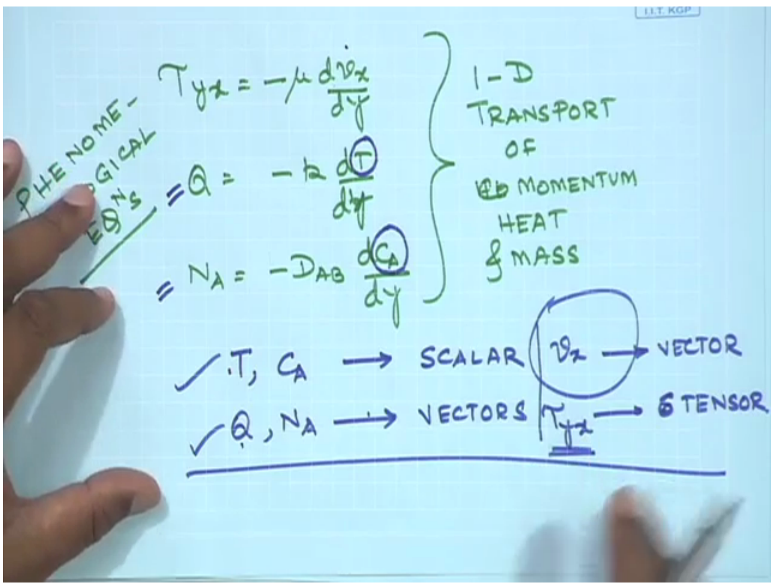

So whatever we have discussed so far, we are restricting ourselves to molecular motion only. Therefore, the 3 equations that I have written, one is Tao yx, these equations are one-dimensional, so these 3 equations are fundamental equations for one-dimensional transport of momentum, heat and mass. And since you cannot derive these equations, you only get these equations by observing a large number of data points where you calculate the amount of heat transfer is on the temperature gradient or amount of mass transfer based on the transmission gradient and observe that there is a direct relationship, direct proportionality that exists between heat and mass, heat transfer and thermal gradient, mass transfer and the transmission gradient and so on.

These equations are phenomenological in nature, so these are phenomenological equations or relations which cannot be derived, which can be observed and then you make rules, make laws out of these. So these 3 are going to be the fundamental relations of heat, mass and momentum transfer that we are going to use in this course. These 3 equations are similar, they look the same, conceptually they are the same but there exists one very basic fundamental difference. If you look at these 3 equations, this one and this one are identical in nature.

One talks about the variation, the gradient of temperature, the other talks about the gradient in concentration. If you think of T and C A, mathematically both are scalar, both are scalar quantities. So Q and the mass flux NA, they are going to be vectors, since by taking, you are taking the gradient of scalar quantities, what you end up with are vectors. On the other hand Vx in the 1^{st} equation, this is a vector quantity. So therefore Tao yx which is the gradient of the vector quantity is going to be a tensor.

So these 3 equations, these 3 equations are identical conceptually, but since one deal with the gradient of a vector, the final the left-hand side is going to be a tensor. So shear stress is a tensor, it has 9 components and will discuss about that more but shear stress is a tensor. On the other hand the heat transfer governing equation and the mass transfer one, both the temperature and the concentration are scalar in nature. So they are gradient, the heat flux and the mass flux are vector in nature.

So they are, the 3 equations conceptually are the same but mathematically there exists a difference between the 2. So we should keep these these things in mind as the complexity due to the vectorial, vector nature of the velocity and therefore the 9 possible components of Tao will come back, we will we will explore it further in our subsequent lectures lectures.

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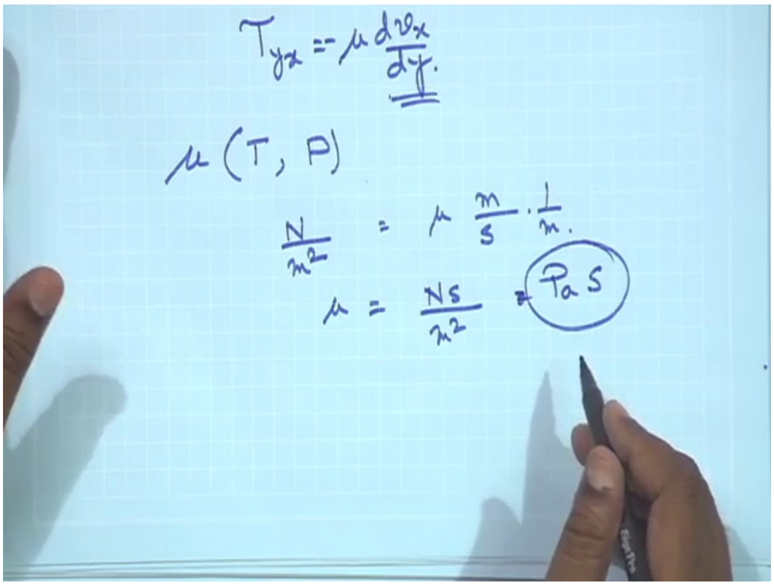

But in the 1st part I am going to this I am going to be concerned with the initial one as the Newton's law of viscosity and the use of Newton's law of viscosity under different conditions. So the property which comes out of this equation is the viscosity. And we all understand that viscosity is a very important property in the case of a fluid and this viscosity is a strong function of temperature and it is also especially the case of gases it is a function of pressure as well.

So the, there are various ways to measure the viscosity for gases, you can also predict that what would be the value of viscosity using certain theories but mostly we deal with, we deal with the variation of viscosity from a large number of experimental data. So viscosity has, you would be able to obtain the units of viscosity directly from here where the shear stress has units of Newton per metre square and then you have the viscosity, you have metre per second for velocity and then one by metre which is for the this part.

And therefore you viscosity has unit Newton second per metre square. So in other words this is Pascal second is also a unit which is the most common unit for viscosity and that is the unit of viscosity in SI units. So this viscosity has, the dependence of viscosity which temperature mostly is that for liquids viscosities increase with increase in temperature. Okay. So hot water will flow faster for the same pressure gradient as compared to cold water and so on.

So this kind of variation, the Thermo physical properties, their evaluation, their dependence on physical, their dependence on conditions such as temperature, pressure, and so on are given in detail in any of the textbooks that I mentioned, specially you can see the chapter, 1st chapter of Bird, Stewart and Lightfoot to know more about the viscosity of different liquids or more importantly the variation of viscosity of liquids and gases at different temperatures and pressure.

So those are for, I would expect you to go through it quickly and see what are the sources of these data and if you would like to know what is the viscosity of a liquid at a specific temperature and pressure, how to obtain them using your textbook. But the one that I am going to discuss more in this class is velocity distribution in laminar flow.

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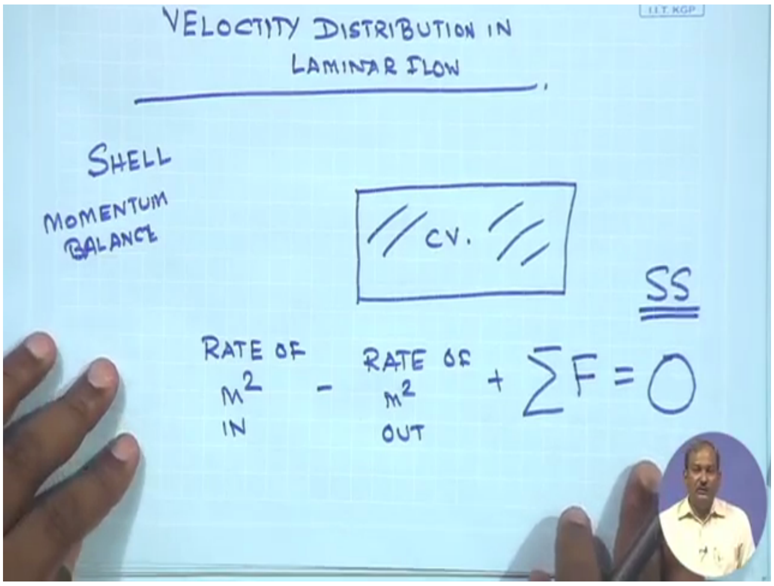

So, how to obtain the velocity distribution in a laminar flow? And in order to do that, I will introduce a concept which is known as the shell momentum balance. So if you think of a control volume like this and this is consisting of some fluids, so all the rates, all the momentum that comes in, that is rate of momentum in - rate of momentum out + sum of all forces acting on the control volume at steady-state is going to be equal to 0. So this is very important, at steady-state.

Which means to say that there is no unbalanced force acting on the control volume. But I am sure you know what you know what, what is known as control volume and what are control surfaces. But I would just go through it once again. A control surface is like this paper, it has it has no mass of its own. So if it has no mass and the control surface is only used to define what is the control volume which has a fixed mass.

So control surface has no mass, anything that comes in must go out, so the conservation equation for a control surface would be rate of something in must be equal to rate of something out, nothing gets stored in the control surface. Whereas a control volume is something like this which has a finite volume of itself. So some amount of mass may come in, let us say heat, some amount of heat may come in, some amount of heat may leave this, some amount of heat may be generated in this and the control volume itself because of its nonzero can absorb some amount of heat.

All these will result in a change in internal energy of the control volume. So for example volume the conservation equation, writing the conservation equation is slightly more in work. But for a control surface it is very easy, in is always going to be equal to out. So when we talk about a control volume consisting of a fluid, then all the forces acting on it at steady-state, the algebraic sum of all forces acting on it at steady-state must be equal to 0.

What are those forces, it could be a body force, a body force is something which depends on the mass, for example gravity. A gravity force acts on all points inside the control volume, so therefore gravity is termed as a body force. Whereas if you think of pressure, pressure acts only on surfaces, on the on the left side surface and on the right side surface. And as a result of unbalanced pressure forces on 2 sides of the control volume, the control volume will either move in this direction or move in this direction depending on which side is at lower pressure.

So pressure is a surface force, on the other hand gravity is a body force so the sum of all the forces acting on it must be equal to 0. Besides these but the pressure and the body force are static in nature, they are always there. But apart from that, some amount of momentum may come in which is like the shear stress exerted on the faces, top face, bottom face and 2 side faces of the control volume. So what is rate of momentum and when I talk about rate, it is the time rate?

So time rate of change of momentum, time rate of momentum which comes in due to the shear stress through any of surfaces and the rate at which it leaves the surfaces, they also constitute some force which are acting on the control volume. So forces can be exerted by a body force, by a surface force or by liquids which are coming in with carrying some amount of momentum with it inside the control volume. It would be more clear when I give an example taking an object.

So when you think of an object, you 1st have to identify which of the surfaces are taking part into this momentum transfer process, what is the body force acting on it, what are the surface forces which are acting on relevant surfaces. If for a control volume I can identify all the component of forces or time rate of change of momentum into the control volume. Then if it is a steady state, then the control volume has no acceleration, so at steady-state, the sum of all these must be equal to 0.

So time rate of momentum coming in - time rate of momentum going out + sum of all forces acting on the system at steady-state must be equal to 0. This fundamental relation is the is the foundation on which the shell momentum balance is developed and we would see how this fundamental relation can be used to obtain an expression for variation in velocity or expression for velocity at every point inside such a control volume.

So the shell momentum balance will will be the framework based on which we will derive our expressions for velocity, our expression for velocity gradient, shear stress, the forces needed to make a liquid block move or if the liquid is in contact with a solid, what force the liquid exerts on the solid. So I have the solid plate is in contact with a moving fluid, then in order to keep the solid plate stationary, you have to apply some forces. What is the magnitude of that force, all these answers should come from our analysis of shell momentum balance.

So in the next part of the course, next part of the class we will talk about writing the governing equations, the force balance equation for a shell of fluid in which there is variation in velocity in one direction only, simplest possible case. Velocity varies with y, velocity does not vary with x or velocity does not vary with Z, it is only one-dimensional change, variation in velocity, the entire control volume is acted by body force, only gravity and it is experiencing a difference in pressure 2 points.

So there is a pressure gradient acting on the system, there is a body force gravity which is acting on the system and the velocity is varying in one direction, it does not vary in the direction of flow, it does not very across the direction of flow, it only varies with height. What would be the governing equation and what are going to be the, what are going to be the boundary conditions for such cases and how those equations, that relation can lead to a governing equation, that is what we are going to do in the next class.