Transport Phenomena
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Boundary Layers
Lecture 16

So what we have seen in the last class, is that how to treat unsteady flow behavior and while treating unsteady flow behavior, we have also seen that for the very simple case of plates sub set in motion in an infinite body of fluid. The penetration depth of the motion, the penetration depth of the effect of the motion of the plate is something which can be expressed in terms, in terms of an error function. In terms of a mathematical function, also known as error functions. Beyond that point the effect of the motion of the plate is simply nonexistent. So any velocity gradient, that you would expect is only going to be between the point and the penetration distance. Because in this, in this layer of the fluid, the velocity values from that of the, that of the, solid plate to zero because the fluid flow is stagnant. So any velocity gradient that would exist in this distance only and beyond that distance the gradient is essentially zero.

Now, that is interesting because our lesson until you have a velocity gradient, you would not have transport of momentum because the viscous transport of momentum, which is only present in laminar, which is , which is only mode by which momentum gets transported in laminar flow, it can work only when there is a velocity gradient. If there is no velocity gradient, there is no transport of momentum. So all the transport phenomenon that you can think of in terms of momentum transferred, is limiting in a layer close to the plate and nothing happens beyond that point.

So with that knowledge, with that understanding, from our previous problem, we now embark on a journey to understand what happens near a solid, liquid, solid fluid interface. This is extremely important which, with which , which applications in a, in a multitude of problems that we encounter every day in the design of a fast moving vehicle, to the design of a rocket to the sports everywhere you will see what we are going to learn, what we are going to discuss in the four or five classes from now onwards. And I will give you examples of that.

But it is time for a brief lesson in history. This theoretical hydrodynamics it existed for a long time. The concept of viscosity in the viscose equation came a bit late. So in the early 1800, the major industry at that point, the cutting technology was to design ships, how good the design of your ship is, how effortlessly you can move that ship at a high velocity in water, that is, that was the big thing at that point, at that instant of time. So the Earle's equation was available at that time.

Now the problem that the designers of ships at that time, what they found is that the existing equations do not tell you much about the force which is needed to move the ship in sea water for example. So we presume that it is the effect of viscosity, so to say the effect of drag was not incorporated into their calculations. And then came the equations, the Navier - Stokes equation and all which took into account the viscosity, aimed therefore the drag of the fluid, on the surrounding fluid on the moving ship. But the problem is not solved as it is. There is, first of all there is a huge departure between the theoretical results predicted from others equation and the practical, practical drag that is experienced by the, by the moving ship. Even when the concept of Navier- Stokes equation came, it was almost impossible to solve.

It is a very complicated equation in the effect is a two dimensional flow case then a solution of that Navier -Stokes equation in the entire flow domain surrounding the moving ship was impossible at that time. So then came someone with a bright idea, as always does. His name was Spandrel what he demonstrated, what he conceptualized and demonstrated that all these viscous transport of momentum is taking place in a layer very close to the surface of the shape. So if a ship moves in water, only in a layer very close to that of the ship, the effect of viscous transport of momentum is important. Or, in other words the Navier- Stokes of equation is applicable.

Viscosity is important, the velocity varies from that of the shape to the, to the surrounding fluid to the sea water whose velocity is, say it is zero. So the velocity varies from that of the ship to that of the sea in a region very close to the surface of the ship.

So all the velocity gradients that you can think of is confined to a region, very narrow region close to the ship. You need to solve the Navier- Stokes equation, the complete Navier- Stokes equation, equation of motion, in that thin, for that thin region and once you understand and you define the region to be very thin where viscous forces are important there would be some other approximations which can then be introduced to make our life simpler.

Any point beyond that region the fluid can be treated as inviscid and therefore Eulers equation , Donelli's equation, Eulers equation, which is the starting point for Eulers equation, which is Oilers equation is for inviscid flow , for fluids with zero viscosity, Eulers equation can safely be used for that region. So the entire flow domain, around the ship can now be defined as consisting of a very thin region and any region beyond that, viscous flow, inviscid flow. So this was the missing link between theory and experiment which when did provided with the concept called as boundary layers.

So boundary layer is that layer, is that thickness of the fluid in which there is a gradient in velocity and when we talk about the thermal boundary layer, there is going to be a gradient in temperature and when we talk about the mass boundary layer, we talk about the concentration boundary layer in which there is a concentration gradient. So, velocity gradient for hydrodynamic boundary layer, thermal gradient for temperature, and temperature gradient for thermal boundary layer and concentration gradient for the concentration or mass transfer boundary layer. These three boundary layers are different.

So this is something the concept which is something very important, which gave rise to the ideas of hydrodynamic boundary layer, in which momentum transport takes place, a thermal boundary layer in which the, the molecular, the transport of heat is taking place, and the concentration boundary layer in which the transport of a species is taking place. In essence these boundary layers combine the collective heat, mass and momentum transport process which are taking place more a solid liquid interface and which is physically the most important region that one should analyze, one should examine to, to decide or to predict what would be the total transport of heat, mass and momentum or what can be done as an engineer to ensure that you have hire heat transport, mass transport or momentum transport from that point.

And where it is going to be used? You have seen, all of you have probably seen, the racing cars. The racing cars when they are all aligned and stat to move you would rarely see they all follow a pattern. There would be one car in the front, there would be another car very close to the front car, very close to the rear end of the front car and it is going to stay as long as possible into, in this location that is very close to the front car, right over here and there would be a third car which is going to follow the second car and it going to be close to the end of the second car. So it is going to form, there would be a formation like this. Each car would like to have its nose at the end of the car just in front of it. Now why does that happen? Whenever a car moves in, at a high speed, a boundary layer is formed on the car. Now whenever a boundary layer is formed, it’s going to start, so let us say this is your car, and you see what is a design of a car. Each car is streamlined so to say, so as to reduce the drag as much as possible.

So it is moving at a very fast speed, a very fast speed and it is going to encounter, the, the air over it. So the air comes in, air flows from this boundary layer, comes to this point, and then it goes away.

The air molecules which are over here, they have a certain momentum associated with it. But at some point, they, it is going to, it is something going to happen which is called the boundary layer detachment. The boundary layer which was attached to the surface, it is going to detach itself from the, from this point and will form the wake. All of us know what wakes are.

So when anything moves at a very fast velocity, there would be a region at the back end which is a wake and which is essentially a low pressure region. So, in racing cars you would see something which is called as spoiler. So you have a spoiler which is something like this, a projected part over the end. The only purpose of that is to break the wakes which are formed on such surfaces. Whereas this is the low pressure side and this is the high pressure side. Due to the stagnation pressure, since the air is going to come, heat it and will probably at some point it will come to a zero velocity, so the high pressure at the front end and the low pressure at the back end due to the formation of the wakes, the drag the pressure drag is going to slow this car down.

So pressure drag and fiction drag these are the two major drags, but the pressure drag is going to create a low pressure region in it. So if I am in the second car and if I have studied my transport phenomenon, and I am doing intelligently , I would always like to keep the front end of my car in the wake which is formed by the first car. So the wake region over here is low pressure, so my front end does not experience a high pressure, it would, it would experience an artificially low pressure created by the wakes formed by the first car.

So the second car would always follow the first car keeping its nose in the wakes of the first car, and the third car, fourth car and so on. So if you do it this way, then the wear and tear, the wear and tear on your cars, on your engine, on your fuel consumption, everything would be less and you try to overtake and take the lead position only during the last lap where you would like to come to the front. So by the time the front car it has endured enough of high pressure, which is, which is a, which will slow it down in the final lap. So you try to overtake it towards, as late as possible and take the lead position near finish and your front end all this time has been exposed to a low pressure.

So that is an example from the design of cars, streamlining of cars, the shapes that you see in modern cars , in buses, in trains , in planes, in space shuttle, all of them the outer surface is designed in such a way to reduce drag, to improve fuel efficiency, and , and so on.

In fact the re-entry of rockets back into the earth's atmosphere is, the velocity gradient is so large near the surface that it is, it will create such a huge friction, that it is going to starts to glow, which we have seen, the pictures of which we have seen. In the field of sports, the use of boundary layers is extremely interesting. In towards, towards the end of this part of this course, I will tell you that you all know, many of you are interested, probably interested in cricket and you know that when the fast bowler bowls at you, it may start to swing. What exactly is swing? That is the ball is coming straight towards you, so you, you have taken a stance to go into the line of the ball and to play it, but suddenly, in mid air the ball changes its direction and either moves away from you which is out swing, or comes towards you which is in swing.

So, but at that point of time, you are already committed to play it in a certain way, you have already picked the line of the ball. In the last moment the ball starts to deviate from its line, then you are bound to make a mistake. So the swing bowlers always do this, and as their name suggests, the swing, they use the seam of the cricket ball to move the ball in the air either out swing or in swing. They are essentially trying to control the boundary layers on both sides of the ball. So they would purposefully try to keep one side of the ball under laminar flow conditions, the other side of the ball in turbulent flow conditions.

If you have two sides of the ball having two different roughnesses, and you are using the seam to disturb the flow, then something interesting happens. So we will see mathematically what it is later on. But always remember when now when you watch a cricket game, if you see the ball moving in air, you know that it is due to the formation of boundary layer, different types of boundary layers on two different surfaces. It is also the reason that you would see the, the fielders and the bowlers always trying to keep one end of the ball, one side of the ball shining. They will never do the same thing on the other side. So purposefully they would like to have one surface roughened and other surface smooth. If a surface is smooth, it is more likely that the laminar condition will prevail and obviously a rough surface will initiate turbulent flow. So the fielders, bowlers always try to rub the ball, keep the shine of the ball on one side and use it, let the other side get rough.

So from automobiles, to airplanes, to cricket balls, to basket ball, o golf, even to golf, you would see applications of boundary layers. You have seen the shape of the golf balls. The golf balls are never smooth. They have dimples on it. If you , you take a golf ball in your hand you will see that they have dimples in it. So these dimples on the golf ball, when you hit it with a high, hit it with a high velocity, what is going to happen that it is going to disturb the boundary layers, boundary layers on it and it would reduce the drag and it would reduce the formation of the wake , therefore reduce the drag. So if you keep two balls identical in size, shape and weight, only one is a golf ball the other is, a ball whose surface is very smooth and hit it with equal force, with the equal, in the same direction, you would, our normal understanding would be that the golf ball with dimples on it will not go further and the smooth ball will go further. But it is just the reverse.

The golf ball with dimples on it will cover a larger distance as compared to the very smooth ball which is due to the, its, the presence of the dimples how they affect the growth of the boundary layer and the formation of the wake. So the applications and the possibilities are endless. I have just talked, spoke about the hydrodynamic part of the boundary layer, so there exists a thermal boundary layer, a concentration boundary layer, and, and situations in which all these three boundary layers are present.

So think of a, think of a hot object which is moving in air, a hot object which is moving in air is going to have a thin layer, a hydrodynamic boundary layer, in which the velocity of the air varies from that of the bullet to the velocity of the air well above it. So if it is moving in still air, the velocity is zero here, but the velocity varies from that of the bullet to that of the air far from it. But, when I say far from it, its essentially a very thin region. These all boundary layers are very, very thin. Ok. All your transport phenomena are taking place in a thin layer. Now the bullet is let us say is hot, so the air is cool, so the temperature of the air close to the bullet will vary from that of the bullet, since the temperature has to be equal at the solid liquid interface, to that of the fluid. Ok.

So think, that is , that is what is called the extent to which the effect of temperature has penetrated, is called the thermal boundary layer. Now think that the bullet is made of naphthalene right now. So if a naphthalene bullet travels through air, the naphthalene is going to go through the sublimation process and the concentration of naphthalene very close to the bullet is going to be maximum and as you move away, the concentration of naphthalene is going to fall to a value equal to zero, because the air does not contain any naphthalene. As a result of which, a mass transfer boundary layer will form around the moving naphthalene bullet.

So you can see that three different types of boundary layers are possible, one which is a hydrodynamic boundary layer where we deal with velocity, the second is a thermal boundary layer where we deal with the temperature, and the third is the concentration boundary layer, where we speak about the species concentration as a function of its distance from the, from the moving object. The thickness of all these three layers can be and in most cases will be different. So we will have the governing equations will also be different. The approximations which are used to reduce to simplify those equations are similar, fundamentally similar in nature.

So the combined application of thermal, hydrodynamic and concentration boundary layer, the field is enormous. It is well researched field, but still we do not have all the answers. And if you think of a, let us say, we, we are restricting ourselves so far to laminar flow, but the boundary layer is never going to be laminar. Beyond certain distance, the disturbance of the moving object would be such that the flow inside the boundary layer, will, will turn to be, will change itself from laminar to turbulent layer.

The moment it becomes turbulent, the, the amount of momentum transfer, heat transfer will increase significantly. The thickness of the boundary layer, the layer close to the surface of the solid which is called the boundary layer, the thickness of a boundary layer in a laminar flow, moves rather slowly, but the moment it becomes turbulent, it starts to grow rapidly. So the behavior of the transport, the thickness of the boundary layer, all are going to be different when we, when we go for, that is, the transition from laminar to turbulent flow.

So that is something which we have to keep in mind. And we have to see that it is not possible always to get an analytical solution, we will have to resort to numerical techniques. Especially in the case of turbulent flow, we will have to use some approximations. Instead of the differential analysis of motion, which would give you the velocity at every point in the flow field, we at certain point will have to resort to other techniques, which are known as integral techniques, the integral methods, to deal with flow where you are more interested in finding out the averages, not the values at every point. So all those come into our discussion of boundary layers.

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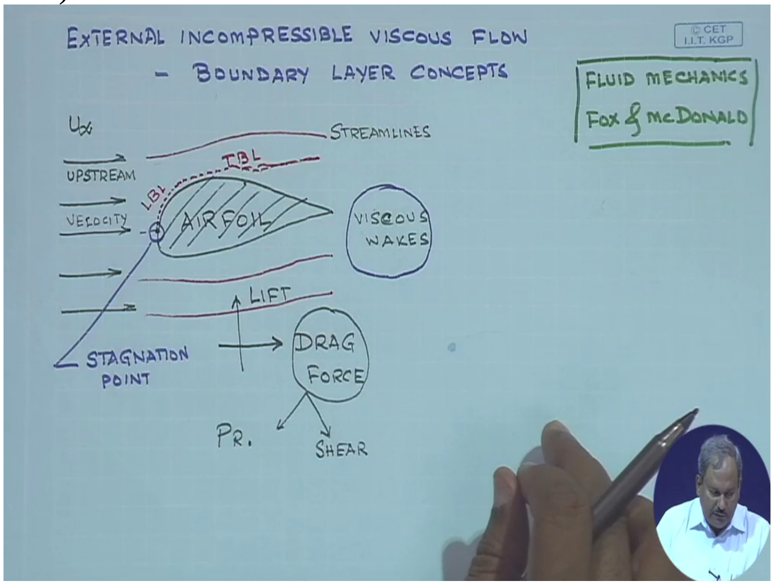

But let us look at this figure first, what we have drawn here is, this is an air foil which is moving, which is moving in air, and you have the air, this is moving so the upstream velocity, you can think of this as a relative velocity, U infinity with which the air approach the air foil. At this point, is the stagnation point, so these are streamlines of the liquid, which forms around the air foil. And the point at which the, it hits the air foil, is called the stagnation point.

And the stagnation point would give you the highest pressure and at this point the boundary layer starts to form, its first going to form the red dotted line over here, initially it will remain laminar, so its laminar boundary layer and at some point it will become turbulent and it will form, the thickness will rise rapidly and it may even detach itself from the surface of the air foil and which is going to give rise to viscous wakes that we talked about.

So the, as a result of this layers and all the air foil experiences a net force as a result of shear and pressure forces acting on its surfaces. So we all know how the airplane lifts from the ground, and the forces which are experienced by the air foil, because of its shape, there is going to be a component parallel, there is going to be a component parallel to the flow of a which is called the drag force, the component of force which is called the drag force and there will be a component perpendicular to it up to infinity which is known as the lift. So the drag force also has two components, one which is called the pressure drag and the second which is called the viscous drag or the shear drag. Ok. So these are something which we are going to discuss over here.

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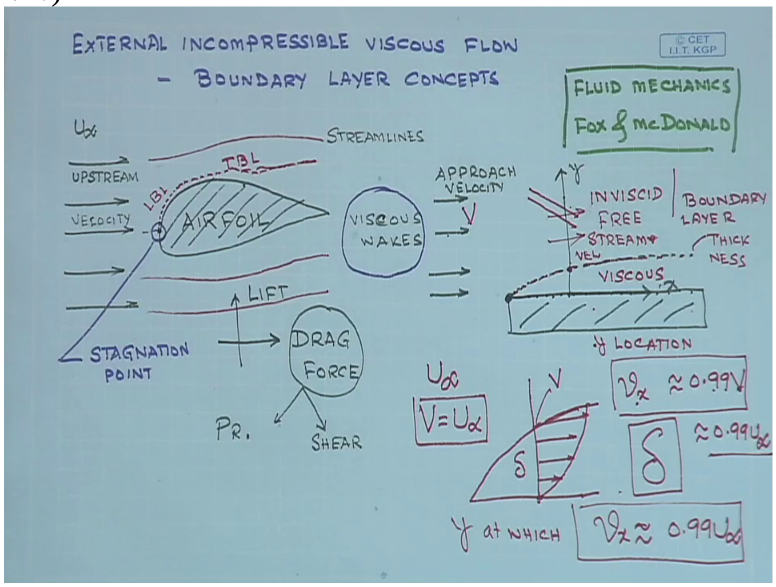

Now the situation here is complicated as you can see. There are so many things we have to keep in mind. The problem that we have in here is , let us see the simplest possible case : I have a solid plate over which some field is approaching this. This is my Y direction and this is the X direction. So at this point and on the, on the solid plate the velocity, a velocity is going to be zero. But if I move slightly up, there is going to be a velocity in here which is not going to be equal to the velocity of the approach velocity.

This is called the approach velocity of the string, it is going to be slightly lesser than that. So if I draw the contour of the point at which I can see, I can sense the velocity, then this is known as the boundary layer thickness. So the velocity is going to be zero here and the velocity is going to be equal to the approach velocity in this case. So the imaginary layer is known as the boundary layer. So if I draw it, if I simply magnify it this region and the velocity here is going to be zero, the velocity is going to be slightly more and more, even more and at this point over here its more or less going to, if this is V, the approach velocity V, the velocity is going to be equal to V. So the profile looks something like this.

So the velocity changes from value equal to zero to that of the approach velocity for flow over a flat plate over certain distance and this distance where the velocity changes from zero to V is called the boundary layer thickness or the disturbance thickness. This boundary layer thickness is once again a, I would say that to repeat, that the velocity changes from zero to that of the velocity above it over a certain distance and then this point, the velocity does not vary at all. So from here to here the velocity varies from zero to V, but once it reaches V at this point, from this point onwards the velocity does not vary.

So the location at which the velocity reaches the first thing velocity is called the boundary layer thickness. The point beyond this, there is no change in velocity and the flow here is inviscid. That is there is no viscosity; in here the flow is viscous. Generally it is not the attendant of the velocity, equal to the approach velocity which is used to demarcate the boundary layer. It is when we say that Vx is about 99 percent of V, 99 percent of the approach velocity for a flat plate. This location, the Y location where Vx is equal to point 99 V (0.99 V) is called the boundary layer thickness delta. So once again the boundary layer thickness delta is defined as the Y location, where the velocity reaches 99 percent of the free string velocity. The velocity is called the free string velocity. Why is it called free string velocity, because it is free from the effects of viscosity and for the case of a flat plate, this approach velocity and free string velocities are equal? In general free string velocity is defined as U infinity for a flat plate the approach and free string velocities are equal.

So whether you write point 99 V or point 99 U infinity it really does not matter for a flat plate, because V and U infinity are same. But on a curved surface, U infinity can be different from V. So I think the correct definition would be zero point 99 U infinity. For a special case, U infinity is equal to V for flow over a flat plate but the definition of boundary layer should be the value of Y at which Vx is 99 of U infinity. We will discuss it once again in the next class but what I have done here is introduced the concept of boundary layers from a historical perspective, discussed some of the interesting applications of it and started to give you the definition of what is a boundary layer thickness. There are so many things to cover in this part, we will start with the thickness of the boundary layer, whether its approach velocity, free string velocity and we would see that it is very difficult to experimentally decide, measure what is a boundary layer thickness because it varies slowly and merges asymptotically with the free string.

So if it is varying slowly and merging asymptotically with the free string, it is difficult to pin point the exact location , where the velocity inside the boundary layer becomes equal to 99 percent of the free string velocity . So there are various different methods have been suggested to alleviate this problem, to address this problem where we can say with some confidence that whatever we call as the thickness of the boundary layer, is correct. There could be large experimental errors to decide the location where V is equal to, V the velocity inside the boundary layer is 99 percent of the free string velocity. So those things we are going to, those more detailed description of boundary layers, both descriptive as well as mathematical one, we will take up in the next class.