Course on Phase Equilibrium Thermodynamics
By Professor Gargi Das
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Lecture 15
PvT Behaviour (Contd.)

Well to continue with the discussions of the last class, what did we do? We first found out the graphical representations of the PvT behavior and after the graphical representations of the PvT behavior then we came to the mathematical representations of PvT behavior and then we discussed we found out that there were different equations of states, the cubic equation the Virial equations of state and we started discussing the cubic equations of state and there we found out that the cubic equations of state they have got some very strong advantages.

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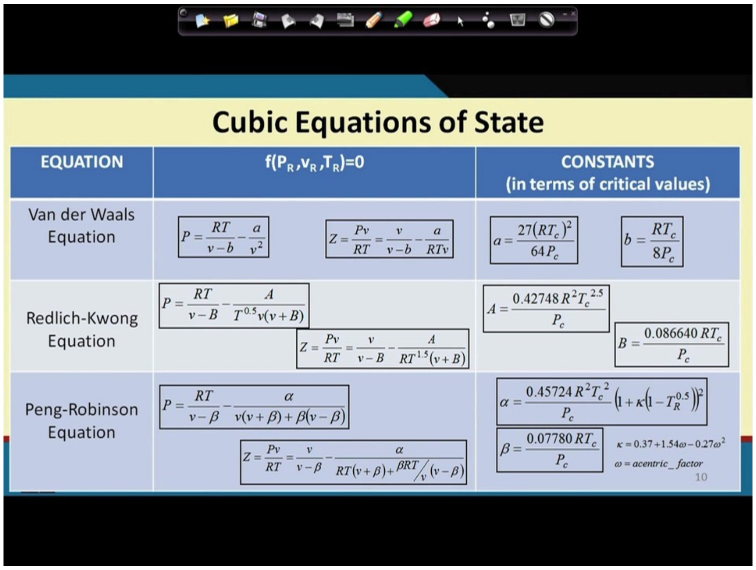

Cubic Equations of State

EQUATION	$f(P_R, v_R, T_R)=0$	CONSTANTS (in terms of critical values)
Van der Waals Equation	$P=\frac{RT}{v-b}-\frac{a}{v^2}$ $Z=\frac{Pv}{RT}=\frac{v}{v-b}-\frac{a}{RTv}$	$a=\frac{27(RT_c)^2}{64P_c}$ $b=\frac{RT_c}{8P_c}$
Redlich-Kwong Equation	$P=\frac{RT}{v-B}-\frac{A}{T^{0.5}v(v+B)}$ $Z=\frac{Pv}{RT}=\frac{v}{v-B}-\frac{A}{RT^{1.5}(v+B)}$	$A=\frac{0.42748R^2T_c^{2.5}}{P_c}$ $B=\frac{0.086640RT_c}{P_c}$
Peng-Robinson Equation	$P=\frac{RT}{v-\beta}-\frac{\alpha}{v(v+\beta)+\beta(v-\beta)}$ $Z=\frac{Pv}{RT}=\frac{v}{v-\beta}-\frac{\alpha}{RT(v+\beta)+\frac{\beta RT}{v}(v-\beta)}$	$\alpha=\frac{0.45724R^2T_c^2}{P_c}\left(1+\kappa\left(1-T_R^{0.5}\right)\right)^2$ $\beta=\frac{0.07780RT_c}{P_c}$ $\kappa=0.37+1.54\omega-0.27\omega^2$ $\omega=acentric\_factor$

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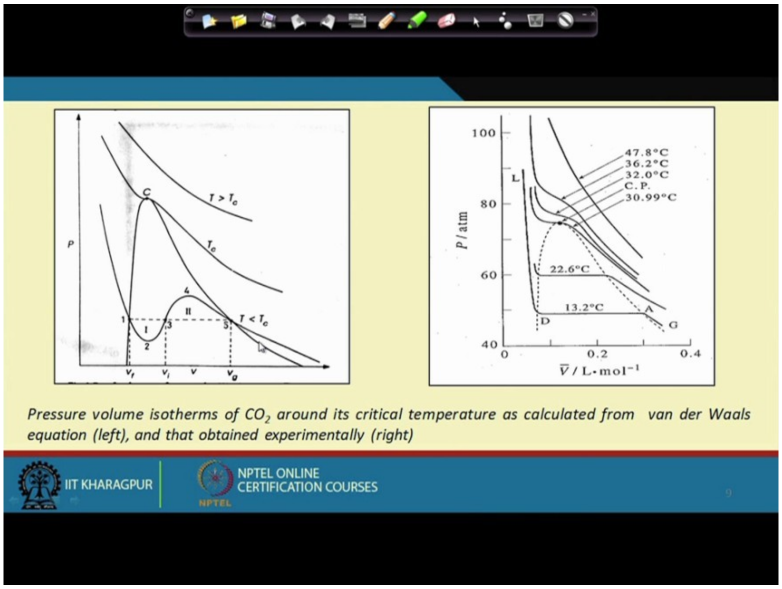

Pressure volume isotherms of CO_2 around its critical temperature as calculated from van der Waals equation (left), and that obtained experimentally (right)

The first advantage is that we can, as we have found out in the last class the first advantage was it, since it is cubic in volume it is capable of predicting not only the behavior in the gaseous phase but also in the 2 phase region as well as in the liquid region that was number 1 and then we also found out the drawback in this particular case that in the 2 phase region we found out that it was not capable of predicting the isotherm the nature of the isotherm it showed a continuous curve we had discussed all these things.

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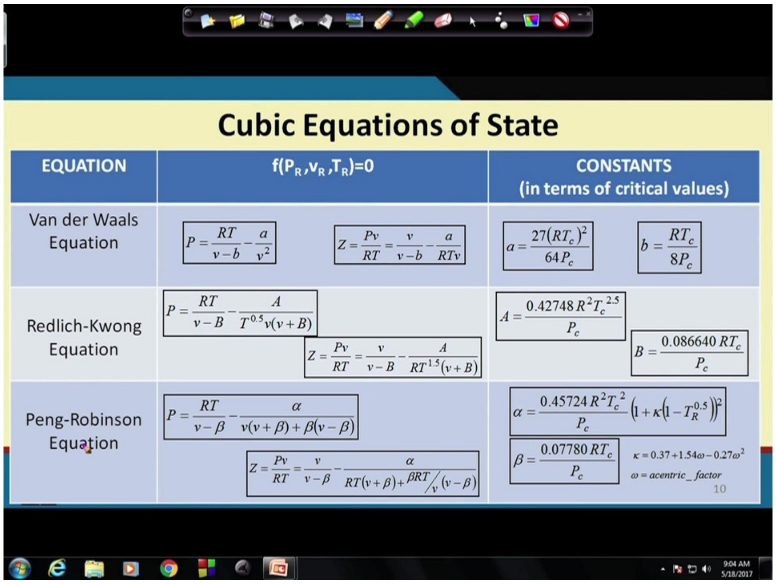

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Well, the next thing and I had also shown you the list of the different cubic equations which have come up and for the present class I had already mentioned that these 3 equations they are the most important equations which we will be dealing with as we have seen in all the 3 equations there are 2 empirical constants which are characteristics of the particular depth and nature of the gas and when we are dealing with mixture, naturally it is a function or it is a mixture property which can be expressed in terms of the pure component properties as we shall see later.

Now for each and every gas if you are taking up say for example we Van der waals equation, we need to find out a, b for everything, so we are just increasing the number of known parameters that we need for when we are going to deal with PVT behavior of gases that is not a very desirable situation.

So therefore in order to reduce the number of unknowns usually what is attempted is to express the empirical constants in any cubic equation of state in terms of the critical constants because as we have already seen the critical constants they are true constant characterizing any particular pure substance.

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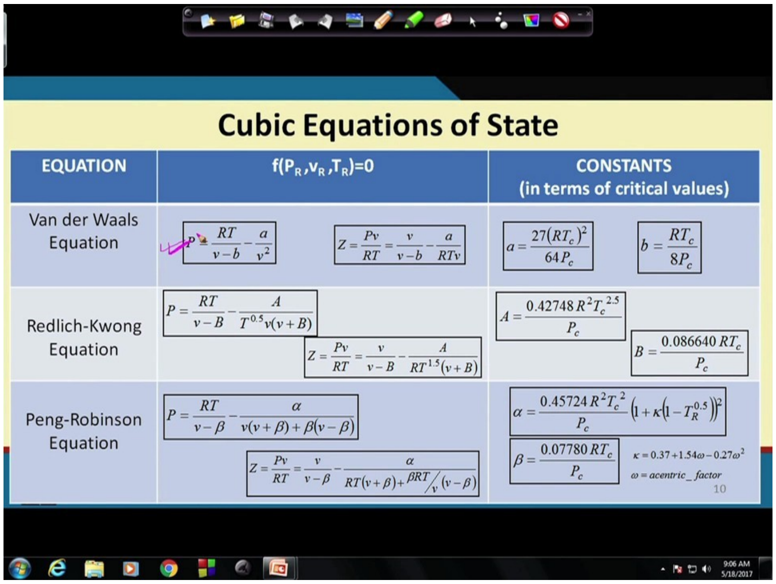

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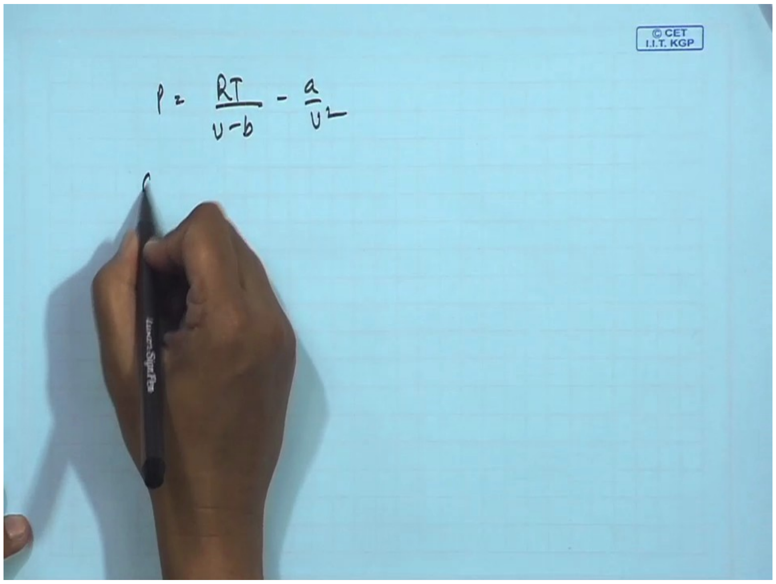

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

So therefore there has always been an attempt to predict the empirical constants in any cubic equation of state in terms of the critical constants and it is very easy to do it, say for instance if you take up the Van der Waals equation of state, this is the equation, right? So therefore we have one equation as P equals to RT by v minus b minus a by v square we can have another equation if you differentiate it this can be one equation this can be the other equation, so from these 3 equations we should be in a position to find out a and b, right?

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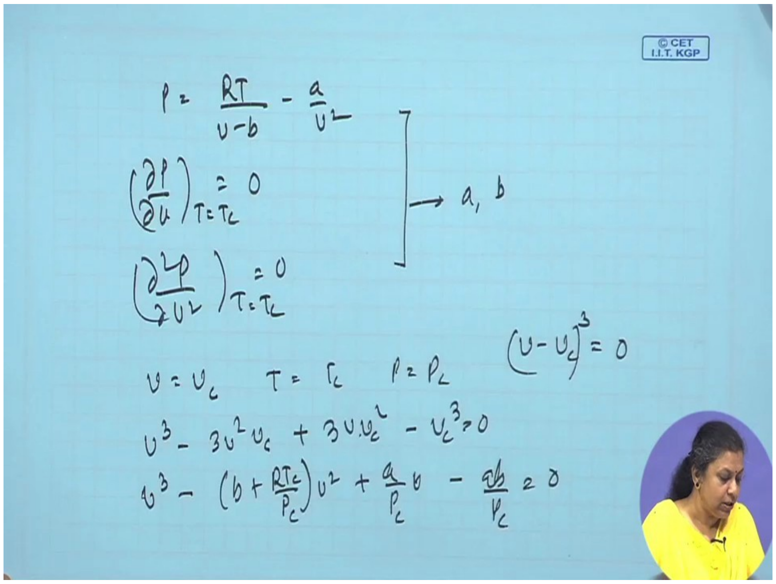

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\left(\frac{\partial P}{\partial v}\right)_{T=T_c} = 0$$

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_c} = 0$$

$$\rightarrow a, b$$

$$v = v_c \quad T = T_c \quad P = P_c \qquad (v - v_c)^3 = 0$$

$$v^3 - 3v^2 v_c + 3 v v_c^2 - v_c^3 = 0$$

$$v^3 - \left(b + \frac{RT_c}{P_c}\right) v^2 + \frac{a}{P_c} v - \frac{ab}{P_c} = 0$$

And these are at T equals to Tc, sorry. So from this we can find out a, b but a more straightforward technique of doing this is that if we remember that at v equals to vc T equals to Tc and P equals to Pc, so what we can do? We can just write down the equation in the cubic state its v cube minus 3 v square vc plus 3v vc square minus vc cube equals to 0, from where did it come? We know v minus, v equals to vc v minus vc equals to 0. So v minus vc whole cube will be equal to 0 I have just expanded this, right?

And then I have compared this with the original the van der waals equation written in the cubic volume, what is it? This is going to be v cube minus b plus RT by P v square plus a by P v minus ab by p equals to 0, right? And here we know that at the critical point, what do we have? We have Tc, we have Pc, we have Pc, we have Vc here, right? So therefore this equation has been written down as a cubic in volume at the condition of v equals to vc, this has been written down in the cubic of volume at the condition of v equals to vc.

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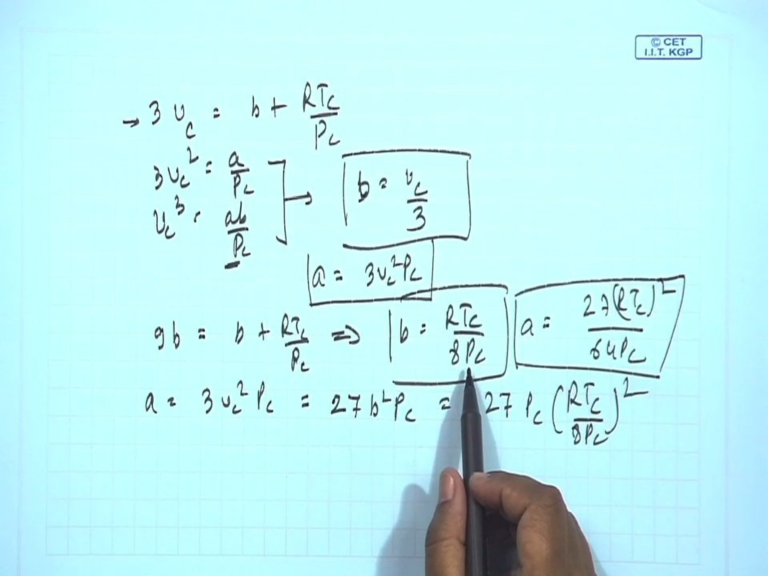

$$\rightarrow 3v_c = b + \frac{RT_c}{P_c}$$

$$\left.\begin{aligned} 3v_c^2 &= \frac{a}{P_c} \\ v_c^3 &= \frac{ab}{P_c} \end{aligned}\right] \rightarrow \boxed{b = \frac{v_c}{3}}$$

$$\boxed{a = 3v_c^2 P_c}$$

$$9b = b + \frac{RT_c}{P_c} \Rightarrow \boxed{b = \frac{RT_c}{8P_c}} \quad \boxed{a = \frac{27(RT_c)^2}{64P_c}}$$

$$a = 3v_c^2 P_c = 27b^2 P_c = 27P_c\left(\frac{RT_c}{8P_c}\right)^2$$

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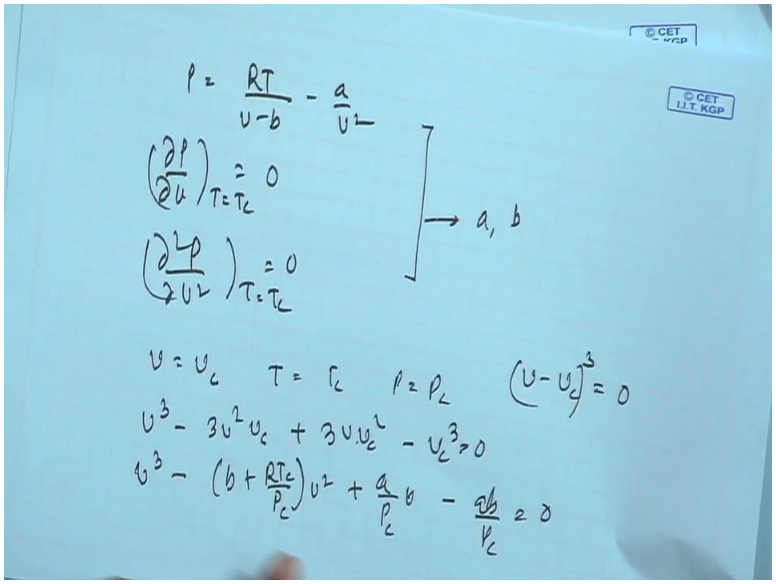

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\left.\begin{aligned} \left(\frac{\partial P}{\partial v}\right)_{T=T_c} &= 0 \\ \left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_c} &= 0 \end{aligned}\right] \rightarrow a, b$$

$$v = v_c \quad T = T_c \quad P = P_c \quad (v - v_c)^3 = 0$$

$$v^3 - 3v^2 v_c + 3v v_c^2 - v_c^3 = 0$$

$$v^3 - \left(b + \frac{RT_c}{P_c}\right)v^2 + \frac{a}{P_c}v - \frac{ab}{P_c} = 0$$

So therefore both these equations should be identical or in other words the constants should correspond to the constants should be equal in both the equations, so if we compare the constants, what do we get? By comparing the constants we get in fact we get three equations by comparing the constants, if you see we get 3 particular equations and we will find that 2 of the equations they are same, so we get 3vc equals to b plus RTc by Pc this is one.

And then the next equation which we get is 3vc square which is equals to a by Pc and Vc cube this is equals to ab by Pc, from these 2 equations immediately what comes out? From these 2 equations immediately we get b equals to vC by 3, right? We get this and if you substitute this particular equation in this term, then what do we get? We get a equals to 3vc square into Pc, right?

So we have expressed a and b in terms of critical constants but both of them contain Vc, at again I repeat the same thing that temperature and pressure they can be much easily measured, more accurately measured, more easily regulated so naturally we would like and it is much more easily evaluated.

As we have seen Tc and Pc is much easily evaluated as compared to vc for which usually we resort to the disappearing meniscus experiment which we have discussed in the last class. So therefore we would like to express a and b in terms of Tc, Pc instead of Vc. So for that what do we do? We take help of this particular equation and we know that in this equation we get 9b is equals to b plus RTc by Pc, right? 3vc from there we get which gives us b equals to RTc by 8Pc, fine.

And in the same way we can also say that a equals to 3vc square Pc we have found out again we can substitute vc equals to 3b we can substitute it here and finally from there what do we get? We get a equals to 27b square Pc which gives us as 27Pc RTc by 8Pc whole square, right? So therefore a is obtained as 27 RTc whole square by 64Pc, fine. So therefore we could express both of them in terms of Tc and Pc.

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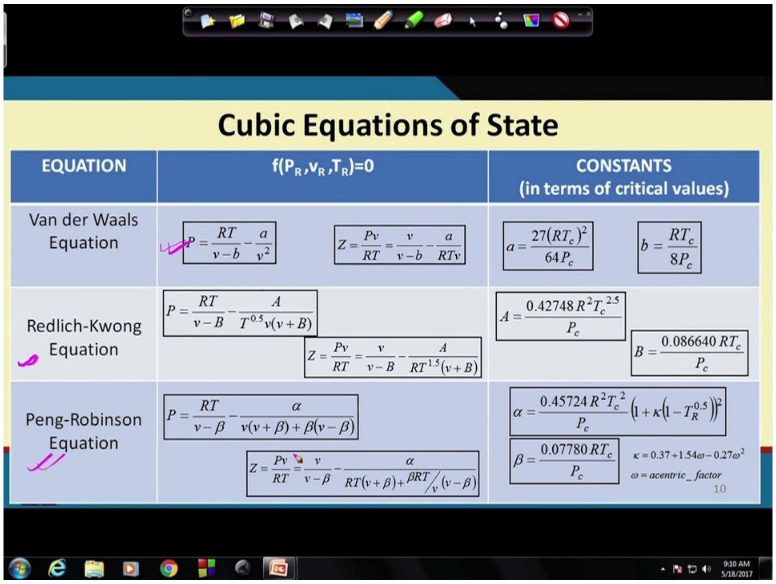

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Now this particular exercise I have shown to you for the Van der waals equation of state. You are advised to take up a similar exercise for both Redlich Kwong equation of state as well as for the Peng-Robinson equation of state and you will find that for both the cases proceeding in the similar manner we would be in a position to find out the empirical constants of the equation in terms of Tc and Pc. So therefore this has enabled us to reduce the number of constants that we required in order to characterize the properties of any particular gas but more important than this, this exercise brings up very interesting results let us see what is the result that this exercise brings up?

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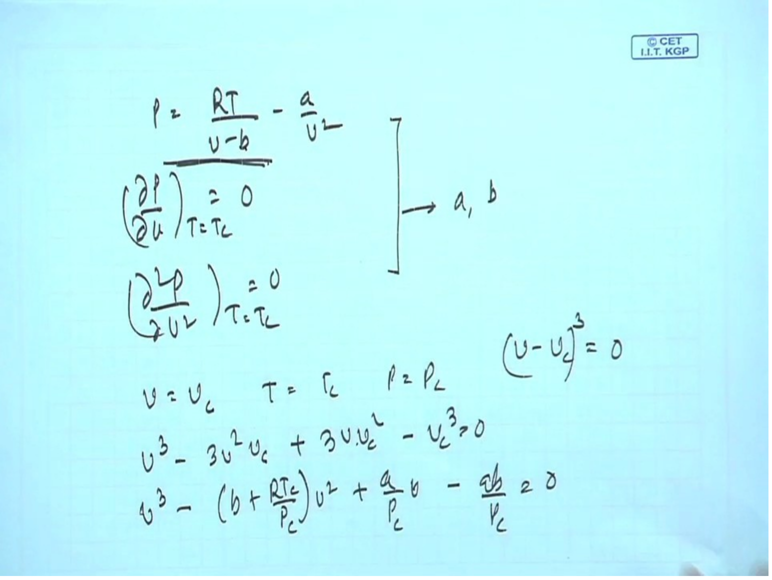

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$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\left.\begin{aligned}&\left(\frac{\partial p}{\partial v}\right)_{T=T_c} = 0\\&\left(\frac{\partial^2 p}{\partial v^2}\right)_{T=T_c} = 0\end{aligned}\right] \rightarrow a,\ b$$

$$(v - v_c)^3 = 0$$

$$v = v_c \quad T = T_c \quad p = p_c$$

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$$v^3 - \left(b + \frac{RT_c}{p_c}\right)v^2 + \frac{a}{p_c}v - \frac{ab}{p_c} = 0$$

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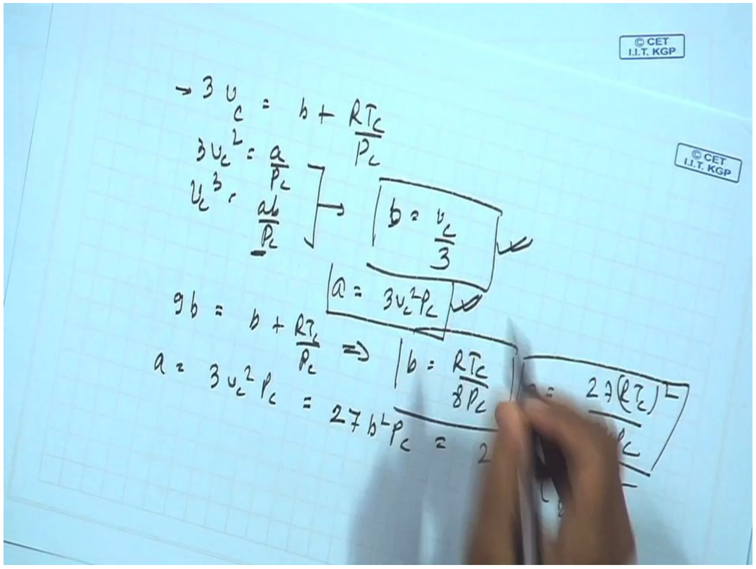

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$$\left.\begin{aligned}&\rightarrow 3v_c = b + \frac{RT_c}{p_c}\\&3v_c^2 = \frac{a}{p_c}\\&v_c^3 = \frac{ab}{p_c}\end{aligned}\right] \rightarrow \boxed{b = \frac{v_c}{3}} \checkmark \quad \boxed{a = 3v_c^2 p_c} \checkmark$$

$$9b = b + \frac{RT_c}{p_c} \Rightarrow \boxed{b = \frac{RT_c}{8p_c}} \quad \boxed{a = \frac{27(RT_c)^2}{p_c}}$$

$$a = 3v_c^2 p_c = 27 b^2 p_c = 2$$

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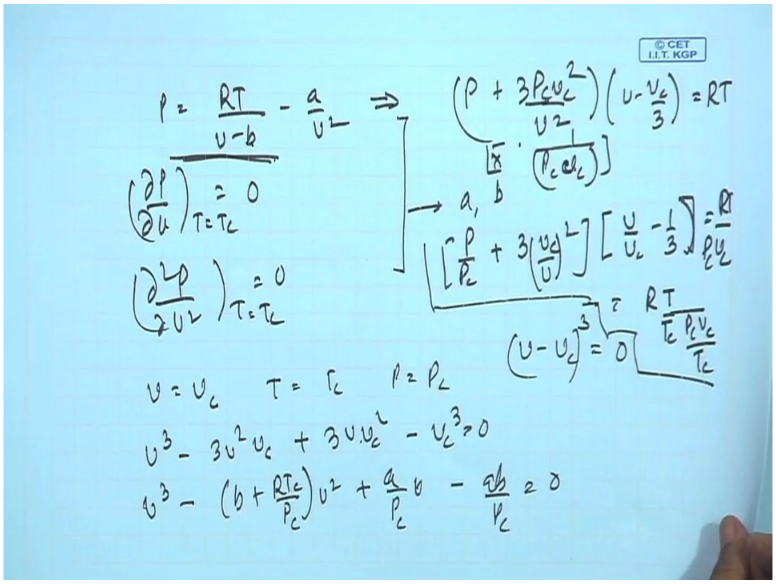

$$P = \frac{RT}{v-b} - \frac{a}{v^2} \Rightarrow \left(P + \frac{3P_c v_c^2}{v^2}\right)\left(v - \frac{v_c}{3}\right) = RT$$

$$\left[\times \cdot \frac{1}{(P_c v_c)}\right]$$

$$\left(\frac{\partial P}{\partial v}\right)_{T=T_c} = 0$$

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_c} = 0$$

$\rightarrow a, b$

$$\left[\frac{P}{P_c} + 3\left(\frac{v_c}{v}\right)^2\right]\left[\frac{v}{v_c} - \frac{1}{3}\right] = \frac{RT}{P_c v_c} = \frac{RT}{T_c \frac{P_c v_c}{T_c}}$$

$$(v - v_c)^3 = 0$$

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Again we start from the basic equation this is the basic equation, right? If we again start from the basic equation and we substitute a and b in terms of what we have obtained. We have obtained a and b if you substitute them in terms of this equation and this equation, what do you expect we are going to get? From here what do we get?

For the first equation substituting a and b we get P plus 3Pc vc square by v square into v minus Vc by 3 this is nothing but equal to RT, agreed? Now suppose I divide both the sides with PcVc or I multiply with 1 by PcVc, suppose I do this then what do I get? Just observe the equation very carefully I get P by Pc plus 3Vc by v whole square into V by Vc minus one third equals to RT by PcVc this RT can also be written down as RT by Tc Pc Vc by Tc, can I not write it down in this particular way?

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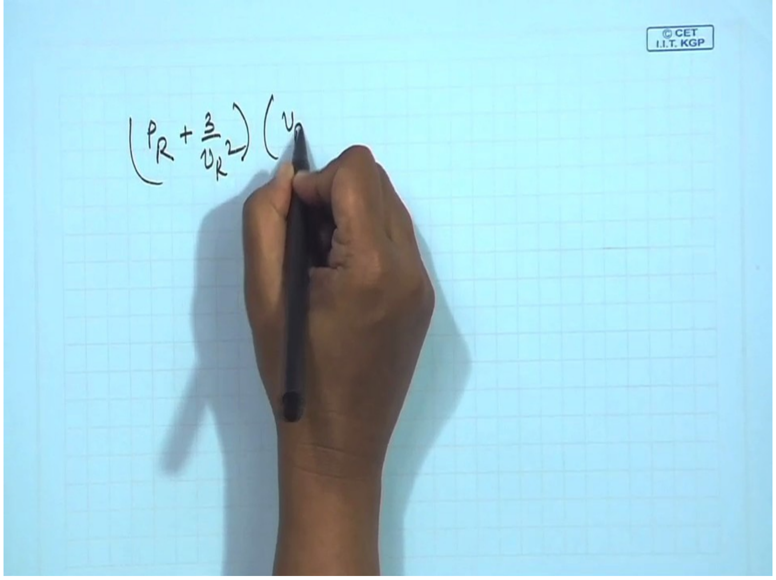

Just observe this part, what does this show? P by Pc is nothing but it is known as the reduced pressure, so therefore this can be when I rearrange this can be written down as PR plus 3 by vR square VR minus one third equals to 8TR by 3. Do you understand the importance of the equation that we have of obtained? Here what have we done?

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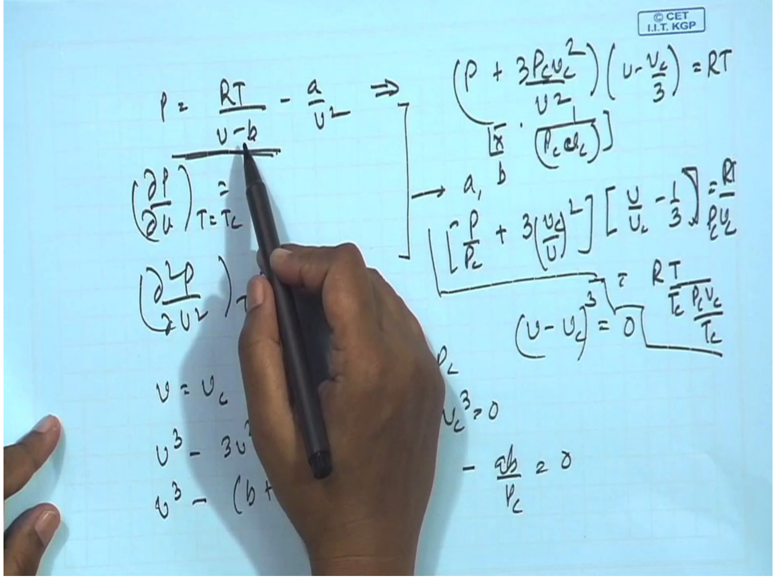

We had started just note with the Van der waals equation it had two constants characterizing the gas, so this equation should be having different forms for each and every gas that we take from here where have we arrived at?

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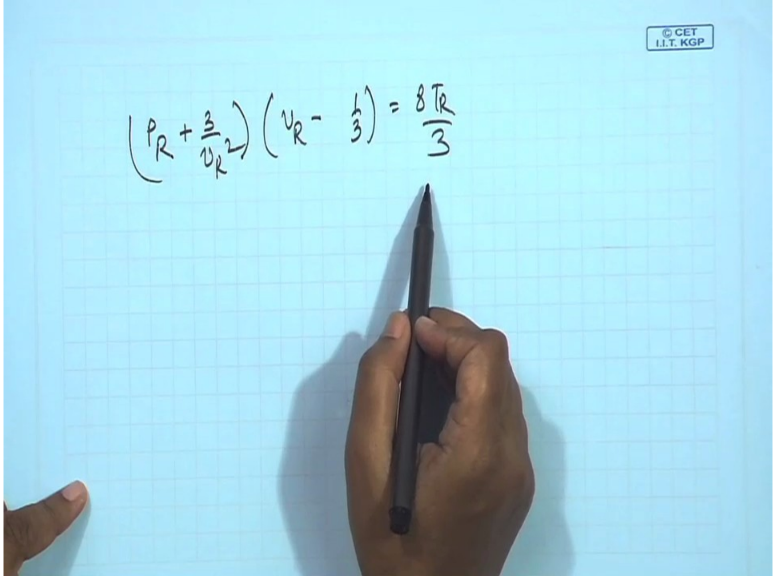

$$\left(P_R + \frac{3}{v_R^2}\right)\left(v_R - \frac{1}{3}\right) = \frac{8T_R}{3}$$

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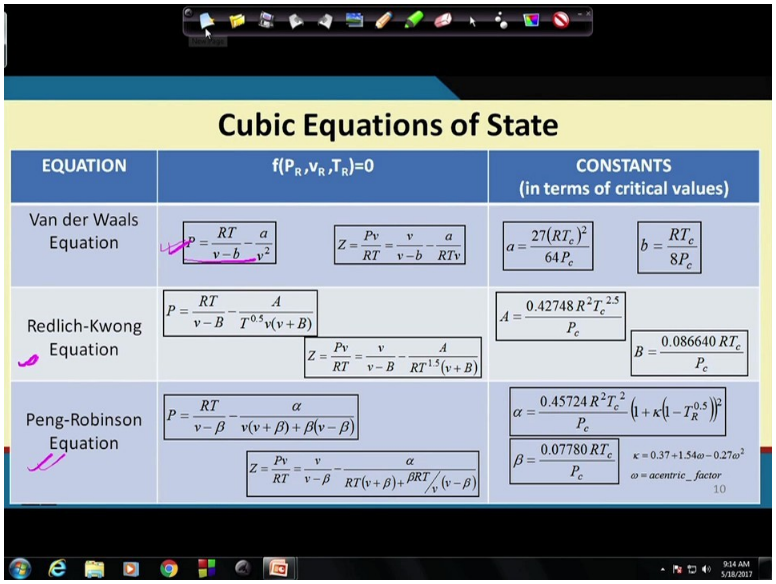

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This particular equation which has no constants characterizing a gas, it almost appears like the ideal gas equation Pv equals to RT where it is the same equation for any particular gas that we use. So therefore we find that starting from the basic Van der waals equation, for starting from this particular equation we find that we have arrived at a particular equation which is independent of any particular constant, right?

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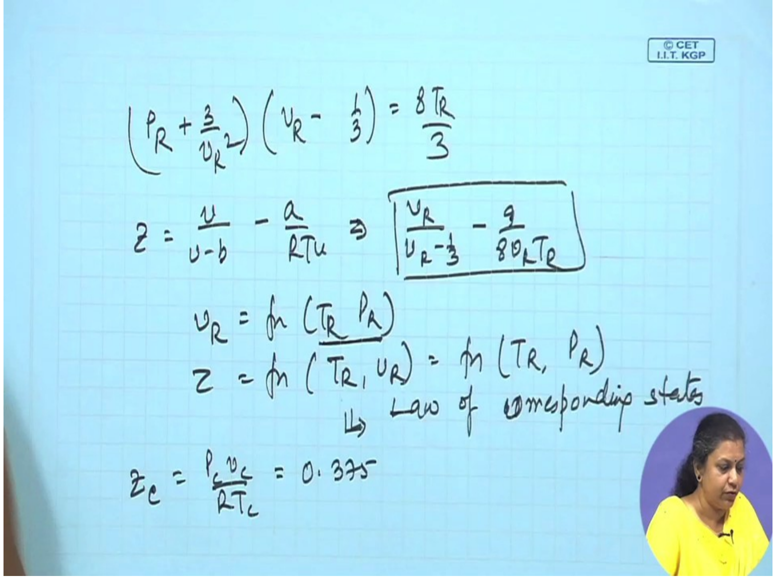

$$\left(P_R + \frac{3}{v_R^2}\right)\left(v_R - \frac{1}{3}\right) = \frac{8T_R}{3}$$

$$z = \frac{v}{v-b} - \frac{a}{RTv} \Rightarrow \boxed{\frac{v_R}{v_R-\frac{1}{3}} - \frac{9}{8v_RT_R}}$$

$$v_R = fn\,(\underline{T_R\; P_R})$$

$$z = fn\,(T_R, v_R) = fn\,(T_R, P_R)$$

↳ Law of corresponding states

$$z_c = \frac{P_c v_c}{RT_c} = 0.375$$

And then so therefore this particular is expected to be same for all the gases or in other words we can say that 2 gases when they are having the same Pr and Tr should have the same Vr, number 1. Number 2 suppose we try to calculate the z from here. From here if we calculate z, what is z equals to? Again starting from the basic Van der waals equation this is nothing but z can be expressed in this particular form and from here we find out again if you substitute a and b, what is the final form that we that we get?

From here the final form which we get is Vr by Vr minus one third minus 9 by 8 VR TR, so therefore what do we find? That z also at the same conditions of see what have they got? VR it is a function of TRPR, z it is a function of apparently it appears it is a function of TRVR. So therefore Z is also since VR is a function of TRPR z is also a function of TRPR so therefore we find that in this particular way if we start substituting different properties which we will be doing later we find that all of them are unique functions of TR and PR.

So what does this mean? This means that the gases when they are at the same conditions of TR PR they exhibit the same reduced other properties or rather they have the same reduced values for the different properties. Now this particular characteristic of the gas which has come out from the, this can be obtained from any cubic equation of state and this particular property of the gas it is known as the or it is given by the law of corresponding states this is known as the law of corresponding states that all gases under the same reduced conditions exhibits all same type of properties.

And from here we can also find out one more thing, what is ZC equals to can you tell me? ZC is equals to suppose you calculate ZC from here we find that this is equal to PCVC by RTc. You try to find it out from the basic Van der waals equation and we find that ZC for all gases it reduces to a unique value which I have mentioned here it's 0.375 as obtained from Van der waals equation it is sorry.

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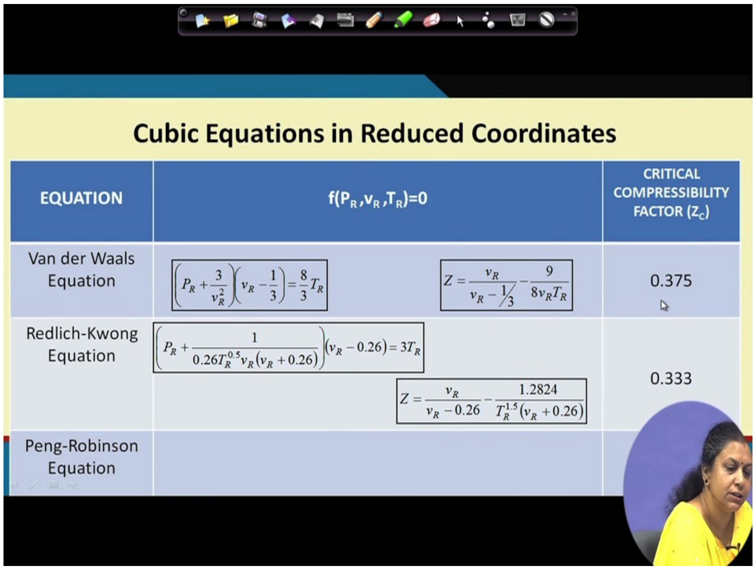

Cubic Equations in Reduced Coordinates

EQUATION	$f(P_R, v_R, T_R)=0$	CRITICAL COMPRESSIBILITY FACTOR (Z_C)
Van der Waals Equation	$\left(P_R + \frac{3}{v_R^2}\right)\left(v_R - \frac{1}{3}\right) = \frac{8}{3}T_R$ $Z = \frac{v_R}{v_R - 1/3} - \frac{9}{8 v_R T_R}$	0.375
Redlich-Kwong Equation	$\left(P_R + \frac{1}{0.26 T_R^{0.5} v_R (v_R + 0.26)}\right)(v_R - 0.26) = 3T_R$ $Z = \frac{v_R}{v_R - 0.26} - \frac{1.2824}{T_R^{1.5}(v_R + 0.26)}$	0.333
Peng-Robinson Equation		

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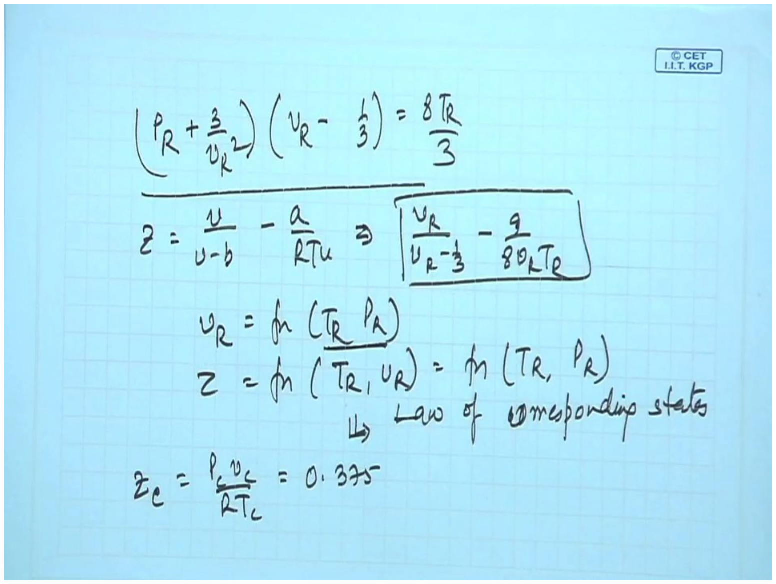

$$\left(P_R + \frac{3}{v_R^2}\right)\left(v_R - \frac{1}{3}\right) = \frac{8T_R}{3}$$

$$z = \frac{v}{v-b} - \frac{a}{RTv} \Rightarrow \boxed{\frac{v_R}{v_R - \frac{1}{3}} - \frac{9}{8v_R T_R}}$$

$$v_R = fn\,(T_R\ P_R)$$

$$z = fn\,(T_R, v_R) = fn\,(T_R, P_R)$$

↳ Law of corresponding states

$$z_c = \frac{P_c v_c}{RT_c} = 0.375$$

We find that the from the Van der waals equation if you are trying to do, we find that more or less we find that z is obtained as a unique value which is 0.375 for Van der waals equation 0.333 for Redlich Kwong equation 0.3 from the Peng-Robinson equation of state and the way I have obtained this particular equation which is written here the way I have obtained this particular equation and derived this equation it can be done in identical fashion for other cubic equations of state.

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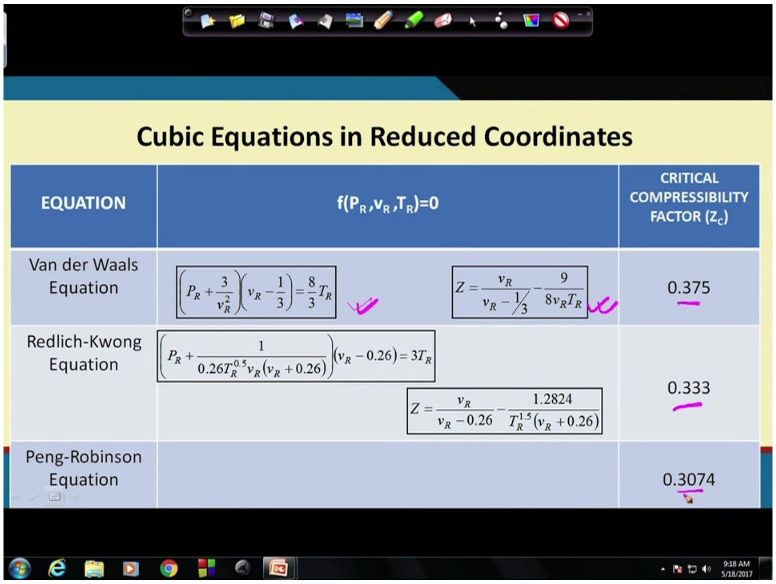

Cubic Equations in Reduced Coordinates

EQUATION	$f(P_R, v_R, T_R)=0$	CRITICAL COMPRESSIBILITY FACTOR (Z_c)
Van der Waals Equation	$\boxed{\left(P_R+\frac{3}{v_R^2}\right)\left(v_R-\frac{1}{3}\right)=\frac{8}{3}T_R}$ ✓ $\boxed{Z=\frac{v_R}{v_R-1/3}-\frac{9}{8v_RT_R}}$ ✓	0.375
Redlich-Kwong Equation	$\boxed{\left(P_R+\frac{1}{0.26T_R^{0.5}v_R(v_R+0.26)}\right)(v_R-0.26)=3T_R}$ $\boxed{Z=\frac{v_R}{v_R-0.26}-\frac{1.2824}{T_R^{1.5}(v_R+0.26)}}$	0.333
Peng-Robinson Equation		0.3074

Naturally the more complex the equation is what happens the more complex the reduced version will also be and the more difficult the deduction is going to be but with any cubic equation of state we find that we can finally express the behavior of any gas using the reduced cubic equation of state in terms of the reduced parameters and those equations that do not have any constants characterizing the gas and for each particular equation of state we find that they predict a unique value of the critical compressibility factor the values which we have got our different for different equations but each equation shows that ZC should be having a unique value provided the gases correspond to the 2 this is known as the 2 parameters law of corresponding states.

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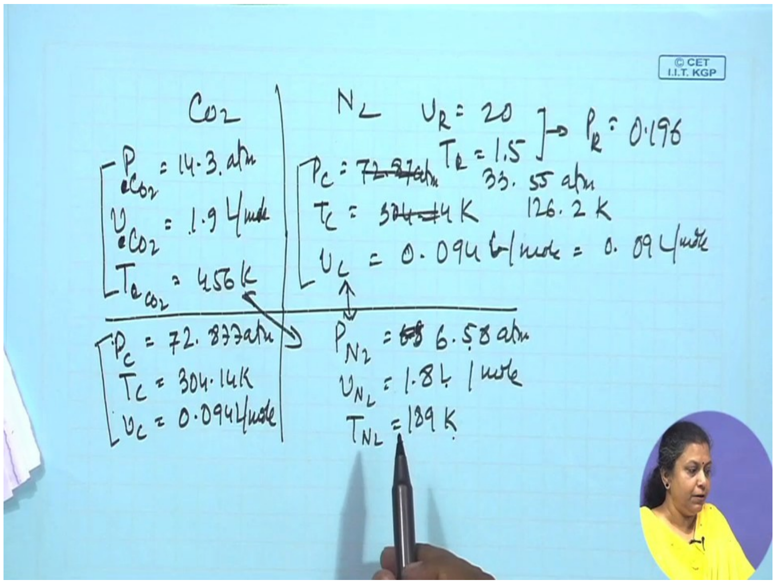

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CO_2

$P_{cCO_2} = 14.3$ atm

$v_{cCO_2} = 1.94$ l/mole

$T_{cCO_2} = 456$ K

N_2 $V_R = 20$, $T_R = 1.5$ → $P_R = 0.196$

$P_c =$ ~~72.87 atm~~ 33.55 atm

$T_c =$ ~~304.14~~ K 126.2 K

$v_c = 0.094$ l/mole $= 0.09$ l/mole

$P_c = 72.877$ atm

$T_c = 304.14$ K

$v_c = 0.094$ l/mole

$P_{N_2} =$ ~~68~~ 6.58 atm

$v_{N_2} = 1.84$ l/mole

$T_{N_2} = 189$ K

For example if I give you a very specific example say carbon dioxide and nitrogen, right? For these 2 gases I can very well tell you that just like all the gases for these 2 gases say suppose at a constant TR and say at VR equals to 20 TR equals to 1.5 both the gases they should have a PR which is equal to 0.196, right? Because they obey the law of corresponding states for carbon dioxide I know PCo2 equals to 14.3 atmospheres 1.9 liters per mole and TC, co2 actually this should be PC VC and TC Co2 this is 456 Kelvin.

For nitrogen also we find the PC is going to be 72.87 atmospheres TC is going to be 304.14 Kelvin, VC is going to be 0.094 liters per mole, right? So therefore from here and we know that at say VR equals to 20, TR equals to 1.5 they should have the same reduced pressure and this particular state it corresponds to say if you calculate it out, you will find that this corresponds to a pressure of nitrogen as 6.58 atmospheres, volume of nitrogen equals to 1.8 liters per mole, temperature of nitrogen 189 Kelvin.

And in this particular case your, for the PCo2 this corresponds to I'm sorry these were the corresponding states I am extremely sorry the PC these are the states which correspond to this and the PC VC for these are 72.877 atmospheres TC equals to 304.14 Kelvin, VC equals to 0.094 liters per mole, right? And for nitrogen the PC these are the 2 corresponding states I am extremely sorry. For nitrogen this is going to be 33.55, for nitrogen this is going to be 33.55 atmospheres VC is going to be 0.09 liters per mole and the TC is going to be 126.2 Kelvin, right?

And so therefore the corresponding states are what? Corresponding states this is PC VC for CO_2, this is PV VC for nitrogen and from there we find that carbon dioxide at 14.3 atmospheres 1.9 Litres per mole 456 Kelvin that corresponds to nitrogen at 6.58 atmospheres 1.8 liters per mole and 189 Kelvin. So from here what do we find? We find that for any particular gas if we know the critical constants then for that gas at any particular condition we should be able to find out the reduced parameters we know that even an unknown gas will have for the same reduced parameters, say for the same reduced TR and PR we will have all the other reduced properties the same.

So therefore what we can do from here we can find out the critical constants for any particular unknown gas from the known critical constants of a known gas. So therefore this shows that we have started from an approximate equation the cubic equation but the cubic equation what have they shown us? They show us that the properties of all gases are same if they are compared at conditions at same conditions related to the critical point or if they are compared under the same reduced parameters and this is a very powerful law this is given or this particular law of corresponding states is exhibited by all the cubic equations of states that we have the functional forms are of course different.

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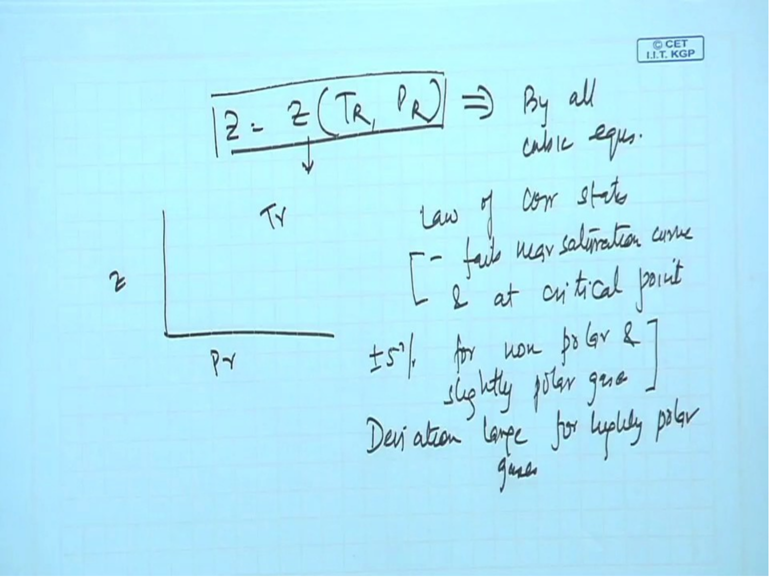

$\boxed{z = z(T_R, P_R)} \Rightarrow$ By all cubic eqns.

Law of corr states

[- fails near saturation curve
& at critical point

±5% for non polar &
slightly polar gases]

Deviation large for highly polar gases

So from there we got an idea that for all gases there should be a relationship as z is a function of TR PR this is given by all cubic equations of state but mind it the functional form which is given that is different but definitely that such a relationship exists this is evident from all cubic equations of state and the functional form for that we need to resort to experiments. So from resorting to experiments we found that this is indeed a characteristic which is shown by different particular gases and for all the gases what do we find?

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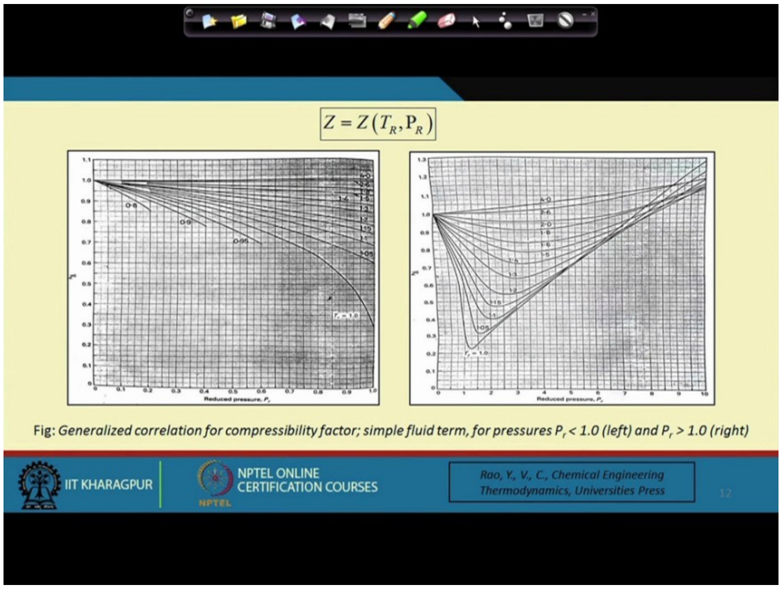

$$\boxed{Z = Z(T_R, P_R)}$$

Fig: *Generalized correlation for compressibility factor; simple fluid term, for pressures $P_r < 1.0$ (left) and $P_r > 1.0$ (right)*

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We find that indeed for all the gases we find that it's really true that z it has a unique relationship with PR and TR and for all for wide number of gases this particular graphs have been plotted these graphs are available in any book of thermodynamics and we find that from these particular graphs they have been drawn for 2 different conditions of PR one is for a lower PR and the other is for a higher PR, we find from these 2 equations indeed they have been tried for a number of gases and indeed z is a unique function of PR with TR as parameters and they hold very good for non-polar gases and for similar molecules of the gases these are extremely applicable.

Okay, more or less we find that this particular law, this law of corresponding states it appears to fail near saturation curve this is something important it appears to fail near the saturation curve and also at the critical point apart from these 2 places more or less we find that the predictions they are within plus minus 5 percent for non-polar and slightly polar gases. More or less we find that other than these 2 conditions they can predict the law of corresponding states holds within plus minus 5 percent for non-polar and slightly polar gases remember deviation is large for highly polar gases.

So therefore we find that this is a very powerful tool we have and people have often tried to use this when? In absence of any other additional data and it is also very important to note that cubic equations they are approximate in nature but the law of corresponding states holds for a large number of gases usually for most of the conditions, now before I end the class I would just like to place before you why this happens?

Why do gases obey the law of corresponding states the reason is if we take up any temperature scale even the Kelvin scale that has been made by us the gas does not know or the numerical value of the temperature which we assign to the gas is not known to the gas but the gas knows it's critical conditions, it knows how far removed it is from the critical conditions. So related to the critical condition it behaves identically, all gases behave in the same way at the critical point, right?

So when it is equally removed from the critical point it behaves identically and that is the reason why each and every gas it obeys the law of corresponding states now there is one thing I would like to mention that if suppose we can make a finding which is totally leak proof absolutely accurate then there is no need of any additional research that never happens whenever we find something we find that there are some fallacies, some limitations which have to be overcome, so therefore additional research goes on in this particular way, this case also it was the similar thing.

People found out that law of corresponding states is very powerful it can be applicable and the most of the conditions except the conditions which I have mentioned and we find that more or less the deviation it’s less than 5 percent for extremely non-polar molecules etc but it otherwise it can extend to 10 to 15 percent as well.

So naturally people wanted better predictions, so for better predictions what was needed? They thought that the deviation from the law of corresponding states it primarily arises because of the intermolecular interactions between the molecules. So therefore if a third parameter can be incorporated which would account for these intermolecular attractions or intermolecular interactions than possibly a better law of corresponding states can be proposed.

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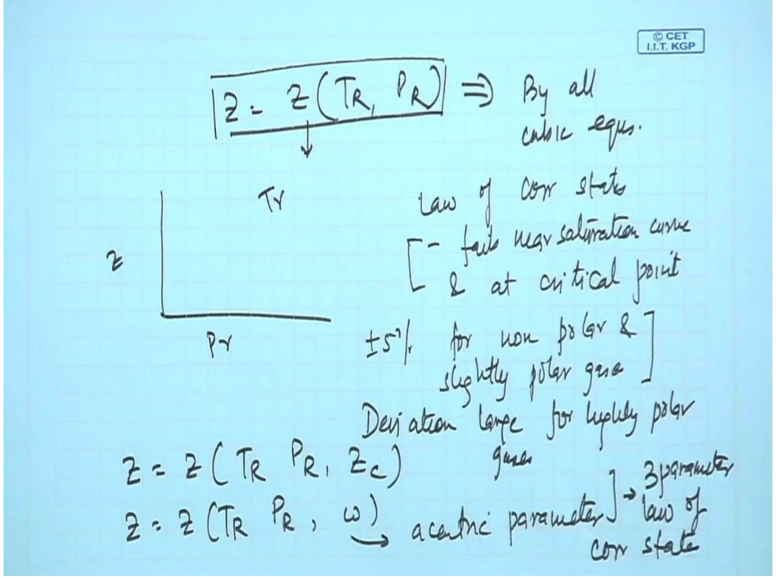

$\boxed{z = z(T_R, P_R)} \Rightarrow$ By all cubic equs.

$\downarrow$

law of corr state

[- fails near saturation curve
& at critical point

±5% for non polar &
slightly polar gase]

Deviation large for highly polar gases

$z = z(T_R\ P_R, z_c)$

$z = z(T_R\ P_R, \underset{\rightarrow}{\omega})$ acentric parameter] → 3 parameter law of corr state

Accordingly people started thinking about what can be the third parameter to be incorporated in the law of corresponding states? And 2 schools have come up with 2 particular third parameter for one particular school it was z it’s a function of TR PR and ZC. For the other particular school which is a much more popular thing apart from TR PR they have introduced something known as the acentric parameter. So in the next class what we do? We go for discussing the 3 parameter law of corresponding states and we find that the 3 parameter law is more accurate than the 2 parameter law of corresponding states. So we go for the 3 parameter law when we find that the 2 parameter law is not very accurate we continue in the next class.