

Adiabatic Two- Phase Flow and Flow Boiling In Microchannel
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Lecture- 21
Flow Pattern based Analysis in Micro Systems - Drift Flux Model

Hello everybody. So, moral is this is an extra class because I could not complete rather I felt that you need to a little more about the analytical models in order to get a complete idea regarding you are 2 phase flow in micro channels both for adiabatic as well as flow boiling conditions. So, in this particular class we are going to deal with flow pattern based systems.

Now, here I would like to mention that I had already rather we had already discussed the homogeneous flow model in my last few lectures right. What they what does the homogeneous flow model assume it assumes that the 2 phases they are flowing with the same velocity and there is no slip while they flowing to the conduit which automatically implies that the inlet and incentive compositions are the same. Now this is quite a drastic assumption because, whenever 2 phases are flowing they have they will interact at the interface and the lighter face will automatically try to slip fast the heavier one. So, therefore, there is going to be a relative motion between the phases based on this particular concept the very simple, but popular flow pattern based model is the drift flux model which assumes the drift between the phases.

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Drift flux model

Relative velocity between the phases is taken care of in an innovative manner

Volumetric Flux $j = \frac{Q}{A}$ $J_1 = (1-\alpha)u_1$ $J_2 = \alpha u_2$

Drift flux $J_{21} = \alpha(u_2 - j)$ $= u_2\alpha(1-\alpha) = J_2 - \alpha j$

$J_{11} = (1-\alpha)(u_1 - j)$ $J_1 = (1-\alpha)j - J_{21}$

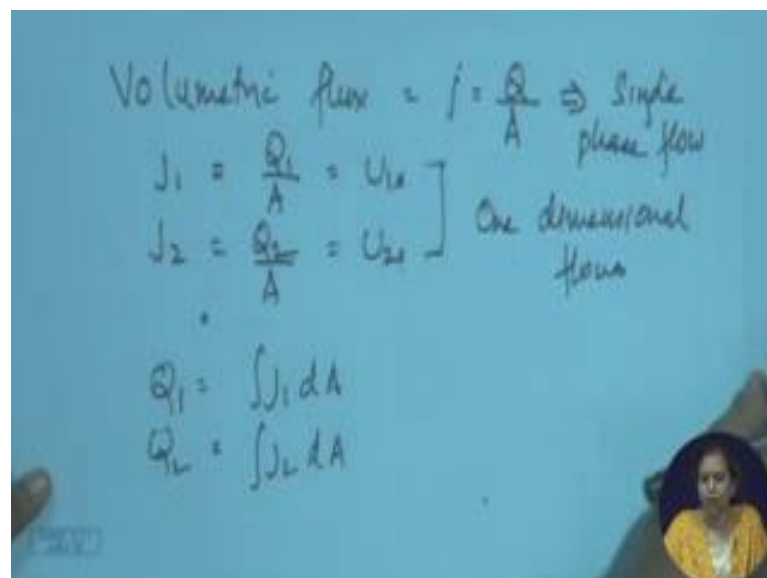
$J_2 = \alpha j + J_{21}$

$\alpha = \frac{J_2}{j} \left(1 - \frac{J_{21}}{J_2}\right)$

$\rho_m = \frac{J_1 \rho_1 + J_2 \rho_2}{j} + (\rho_1 - \rho_2) \frac{J_{21}}{j}$

Application- Bubbly flow, slug flow, drop regimes of gas-liquid flow as well as to fluidized bed

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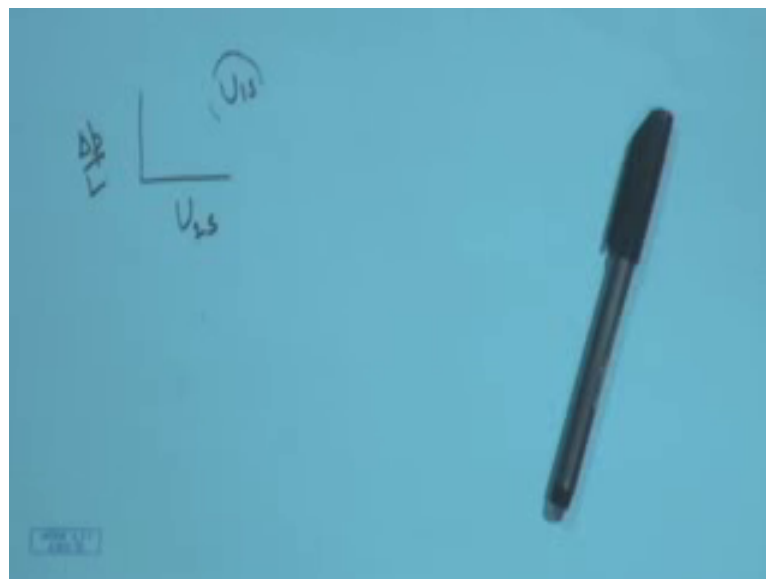


Now, in order to understand the drift flux model you would first like to define what a drift flux is. If you remember in the nomenclatures we had define something like the volumetric flux, which is nothing, but we define the volumetric fluid for unit area it is usually defined as j for single phase flows. So, in 2 phase flow we should have a J1 we should have a J2 J1 equals to Q1 by A and J2 equals to Q2 by A. Now this (Refer Time: 02:27) a very common question which I faced while I teach drift flux model in my classes that the mathematical expression of J1 is the same as the superficial velocity which I have talked about in the superficial velocity also I said the velocity which the

fluid would have had it fluid alone in the pipe. So, mathematically we find that this has a similar expression as U_{1s} and this as a similar expression as U_{2s} .

But let me tell you these equalities are applicable only for one dimensional flows right which means this scene shortly. Well the temporal variation or the special variation of velocity under that condition your J_1 and J_2 they vary across the cross section, but U_{1s} and U_{2s} are the inlet velocities they remain constant. So, suppose it happens that your volumetric flux it is varying along the cross section there is a radial variation of velocity in that case we find Q_1 is given by integral $J_1 d a$ Q_2 is given by integral $J_2 d a$ and then under this condition this J_1 cannot be equated to the superficial velocity. So, remember one thing since we are dealing with one dimensional flows the mathematical expression to find out volumetric flux and superficial velocity are the same, but this is just because we are dealing with 1 dimensional flows under when there is a temporal or a radial variation then genuinely they will not be the same, that is why we have defined volumetric fluxes all though earlier I had also defined superficial velocity.

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Now, before I proceed I would like to clarify one more point which I should have done while we were discussing the experimental results were I had try to predict your pressure gradient as a function of your say superficial velocity of phase 2 with phase 1 as a parameter. So, in that particular thing I forgot to clarify that when you are dealing with micro channels the superficial velocities do not usually correspond to the inlet velocities

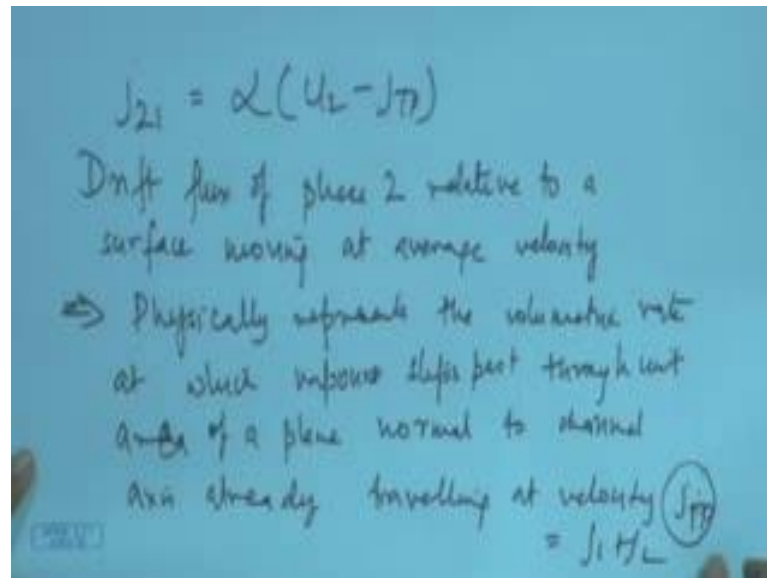
like 2 phase flowing micro channels. In 2 phase flow micro channels what you find is the superficial velocities of the fluids are nothing, but the inlet velocities because the pressure variation during flow is not very drastic. So we assume that there is the density change and the volume change is not very large when they are flowing to a pipe and therefore, the inlet and superficial velocities are the same this is not the case for micro channel flow.

Here as the pressure varies. So, therefore, the velocity also keeps on changing even when a single phase is flowing through a micro channel. So, in this particular case when I talk of superficial velocity, it means the velocity which the fluid would occupy had it flowed alone in the cross section in the view window where we are taking or in the measurement section where you are taking the measurements. The superficial velocity does not refer to the inlet condition it refers to the test window for the measurement has been taken. So, therefore, how to find out the superficial velocity in this particular case, we know the inlet velocity we can find out the outlet velocity we know the inlet pressure we can find out the outlet pressure we assume that the pressure varies linearly across the entire test channel.

So, accordingly we can find out the pressure in the measurement window based on that particular pressure we will try to find out the superficial velocity of the gas phase to corresponding to that pressure we find out the volumetric fraction of the gas phase and that volumetric fraction divided by the micro channel cross section area gives us this relevant superficial velocity, which should be used in the models and also were predicting the flow pattern maps.

When the pressure drop is not very significant during flow then the inlet and the superficial velocities can be taken to be equal. So, this was one point which I forgot to mention in while I was discussing the experimental results on what fractional pressure drop and you need to keep this in mind. While now coming back to the drift flux model, therefore, what we did we had defined the volumetric fluxes.

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And there is one other terms which is rather which should be defined when we are define rather for developing this particular model and that term is defined as J_{21} which is known as the drift flux of phase 2 relative to rather to a surface moving at average velocity.

So, therefore, suppose we assume that a surface is moving at a average velocity. So, therefore, in that case the drift of phase 2 relative to that is surface is known as drift flux or it is better defined as it physically represents the volumetric rate at which vapours slips past through unit area of a plane normal to channel axis already traveling at velocity j . So, what does it mean basically it means that the volumetric suppose say that we consider the vapour phase we find to try to find out the drift flux of the vapour phase. So, therefore, it represents the volumetric rate at fix this particular vapour is passing forward in up flow or it is flowing backward in down flow, through unit area of one particular plane which is perpendicular to the cross section and this particular velocity or the volumetric rate is with respect to the velocity of the channel axis. So, this channel axis is also traveling with an average velocity, what is the average velocity at which the channel axis is traveling it is nothing, but equal to J_{TP} the 2 phases volumetric flux which is equal to J_1 plus J_2 . So, relative to this to this particular J_{TP} the volumetric rate at which the vapour slips past.

So, accordingly this definition is given as this is the volumetric rate of flow minus the relative to the velocity of the center of the channel it is traveling at this particular velocity. So, accordingly the J_{21} is defined here and ones this J_{21} are defined next.

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$$J_{21} = \alpha(u_2 - J_{TP}) = \alpha\left(\frac{j_2}{\alpha} - J_{TP}\right)$$

$$= j_2 - \alpha J_{TP}$$

$$\alpha = \frac{j_2 - J_{21}}{J_{TP}} \quad \boxed{j_2 = \alpha J_{TP} + J_{21}}$$

$$\alpha_H = \beta = \frac{j_2}{J_{TP}} \quad \alpha_{DF} = \frac{j_2}{J_{TP}} \left(1 - \frac{J_{21}}{j_2}\right)$$

$$= \alpha_H \left(1 - \frac{J_{21}}{j_2}\right)$$

$$\langle j_2 \rangle = \frac{j_2 A}{J_{DA}} \quad \langle \alpha_{DF} \rangle = \frac{R_{DF} A}{J_{DA}}$$

So, therefore, what do we get we get the J_{21} this is nothing, but this is equal to u_2 minus J_{TP} . What is u_2 equal to in terms of j_2 this is equal to j_2 by α minus J_{TP} . So, therefore, this is nothing, but j_2 minus α into J_{TP} . So, therefore, from here what is the expression of α you tell me this should be equal to J_2 minus J_{21} divided by J_{TP} or in other words what is j_2 equals to this is αJ_{TP} plus J_{21} right. So, therefore, from here for do you get what was the α from the homogeneous value I had already mention that α equals to β which is nothing, but equal to j_2 by J_{TP} right. Now when we consider the drift flux we find that α_{DF} it is the homogeneous α into some particular correction factor which account rather it is the correction factor were it the factor by which α_H has to be corrected. So, that (Refer Time: 12:05) we get the actual value of the void fraction when drift flux is accounted for right.

So, therefore, what the drift flux basically does it correct the homogeneous void fraction data by taking to account the drift flux model and accordingly since it corrects the α accordingly it can correct other terms for example, it can correct the mixture velocity this is the homogeneous volume mixture velocity and this the correction terms which has been incorporated right. So, therefore, we find that for each and every term we can also

correct the momentum flux for, each and every term we find that we are correcting the homogeneous value with the correction term which is the function of j_{21} versus j_2 or j_{21} versus $J T P$ depending upon the condition, but correction factor correct the homogenous value by considering the drift flux.

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Corrections to the one dimensional model:

$$\langle j_2 \rangle = \langle j_{21} \rangle + \langle \alpha j \rangle$$

$$\langle \alpha j \rangle = \frac{\int (\alpha j) dA}{\int dA}$$

It may be noted that $\langle \alpha j \rangle \neq \langle \alpha \rangle \langle j \rangle$

Since $\langle \alpha \rangle \langle j \rangle = \left[\frac{\int \alpha dA}{\int dA} \right] \left[\frac{\int j dA}{\int dA} \right]$

C_0 is the ratio of average of product of flux and concentration to product of averages

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle} \quad \text{or} \quad C_0 = \frac{\frac{1}{A} \int (\alpha j) dA}{\left[\frac{1}{A} \int \alpha dA \right] \left[\frac{1}{A} \int j dA \right]}$$

Now, this was fine as well as reconsiders the 1 dimensional flow in 1 dimensional flow we assume that there is no cross section variation then it is fine, but if there is a cross section variation then actually we need to consider the cross section averaged values. What is the cross section average value for this cases this as we know from all fluid mechanics this can be given by a term of this sought? Same thing for this term also the correction factor should be this right and again for J_{21} also the correction factor it should be integral $J_{21} d A$ by integral $d A$ right. So, therefore, if there is a there is a variation across the cross section then this particular equation it should be written down in terms of proper average values and then what we do get at a result of that the expression should be something of this sought right.

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And in this expression if you observe we find that this is an average of alpha and J T P. So, in order to find out a correct average what we need to do we for each and every point in the cross section we need to find out alpha we need to find out J T P we need to multiply them and then we need to find we need to find out the total integration of this particular term over the area average gives you this term. Now we all understand that this is not at all a very easy thing, on the other hand finding out the void age profile and the velocity profile is going to be much easy. So, therefore, what we do we replace this term with the term rather we introduce a correction terms C_0 which can be defined as. So, what we do instead of alpha J T P average we replace it with C_0 into alpha and J T P average as the result of which the expression which comes C_0 alpha J T P plus J_{21} right.

And I would like to tell you that such type of correction factors are not new to you, you have come across such correction factors when we were dealing with single phase flow as well if you recollect will find that in the same way we have defined the kinetic energy correction term in Bernoullis equation, which was termed as alpha and we are also must have come across the momentum flux correction term. So, for those cases also if you remember we had introduce the correction terms right because in Bernoullis equation what we had we had a u square term and this particular u square term again, I would like to say that if its averaged over the entire cross section then actually what we would have to do we would have to measure velocity at each and every point squared the velocity. Find out the distribution if this squared velocity and then find out the proper average.

Instead of that if we can note up velocity profile and if, you can find out the velocity profile which is a much easier thing to do and we can square it up then it is going to be easier.

So, therefore, we had defined the alpha the kinetic energy correction term as u square by u square if you recollect in the same way using the same particular tone we had defined the distribution parameter in this particular case. So, therefore, this is not a very new thing that we have already done we are already come across such exercises in your single phase flows and they are always done in order to convert the average of products into product of averages. Because it is easier to find this than to find this same thing was to for the case of kinetic energy correction term same thing was also to for momentum energy correction term. So, from their what we do we express this particular term as I will be written it down we express this particular term as C0 into alpha into J T P plus J21.

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$$\frac{\langle J_2 \rangle}{\langle \alpha \rangle} = \frac{\langle J_{21} \rangle}{\langle \alpha \rangle} + C_0 \langle j \rangle$$

$$\frac{Q_2}{A \langle \alpha \rangle} = C_0 \frac{Q_1 + Q_2}{A} + \frac{\langle J_{21} \rangle}{\langle \alpha \rangle}$$

$$\langle \alpha \rangle = \frac{Q_2 - A \langle J_{21} \rangle}{C_0 (Q_1 + Q_2)}$$

For negligible slip between phases

$$\langle \alpha \rangle = \frac{Q_2}{C_0 (Q_1 + Q_2)}$$

Resembles Armand correlation with $K = (1 / C_0)$

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$$\langle \alpha \rangle = \frac{J_2 - \langle J_{21} \rangle}{C_0 J_{TP}} \quad Q_2/A$$

$$C_0 J_{TP} = \frac{Q_1 + Q_2}{A}$$

$$\langle \alpha \rangle = \frac{Q_2 - A \langle J_{21} \rangle}{C_0 (Q_1 + Q_2)} = \frac{Q_2}{C_0 (Q_1 + Q_2)}$$

$$J_{21} = \alpha (u_2 - j)$$

At $j=0$ $J_{21} = \alpha u_2$

$$\alpha = \frac{1}{C_0} \beta \Rightarrow \text{Armand type correlation with } \frac{1}{C_0} = K$$

So, from this particular expression how do we get Alpha then? So, from this particular expression we find that alpha it can be obtained as J2 minus J21 divided by C0 into J T P, is not it, where J T P is a known term it is nothing, but Q1 plus Q2 by a J2 is a known term it is Q2 by a and J21 is the relative drift flux of phase 2 with respect to phase 1.

So, if you would like to write this down in terms of volumetric flow rates then we find its already written down here, if you observe this particular equation it is Q2 minus A J21 by C0 into Q1 plus Q2. So, therefore, using this particular expression if you can find out J21 and if you can find out C0 then you can find out the corrected alpha and this corrected alpha is going to give us a corrected version of all the different or rather all the other average properties of the 2 phase mixture. And we find that J21 and C0 they are actually functions of the different flow patterns and we are different values or J21 and C0 for the bubbly slug etcetera. And an accordingly this drift flux model it takes into account the variation due to different flow patterns right.

Now, when we have come to the micro channels we find that for most of the cases J21 is negligible right. Because a little more detailed will tell you that there is in this particular case there is negligible slip between the phases or in other words J21 it is nothing, but as I have told you this J21 is nothing, but alpha into u2 minus j for j equals to 0 J21 should be equal to the alpha into u2, but we find that for j equals to 0 the droplet of the bubble that has to travel due to buoyancy at some particular velocity and we find that for micro

channel bubbles and slugs and drops they do not travel because the buoyancy is negligible in this particular case surface tension is dominating. So, therefore, when there no flows of the liquid phase the gas phase does not probably due to buoyancy.

As a result for most of the cases of multi flow phase systems we find that J_{21} is equal to 0 and alpha can be given in the form of Q_2 by Q_1 plus Q_2 . Which tells you that that this particular term this is nothing, but equal to beta. So, for this expression if we drop the averaging assuming that you are dealing with average things will find that C_0 equals to alpha equals to one by C_0 beta or on other words this says this explain why the Armand type correlation is applicable for micro channels of not very small dimension this is because it obeys the drift flux model and relative velocity is 0. So, therefore, we find that the drift flux model reduces to Armand type correlation with one by C_0 equals to the Armand constant K right.

(Refer Slide Time: 22:03)

Estimation of C_0

Assuming power law profiles for α and j

$$C_0 = 1 + \frac{2}{m+n+2} \left[1 - \frac{\alpha_w}{\langle \alpha \rangle} \right]$$

For fully developed bubbly flow (Ishii)

$$C_0 = C_0 \left(\frac{\rho_g}{\rho_l}, \frac{GD}{\mu_l} \right)$$

So, therefore, if we use this particular expression for micro channels the only challenges to find out C_0 .

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Flow situation	Co for air-water flow in macro systems	Correlations for macro channels
Round tube	1.2	Round tube $1.2 - 0.2\sqrt{\rho_g / \rho_l}$ Rectangular channel $(1.2 - 0.2\sqrt{\rho_g / \rho_l})(1 - e^{-18\alpha})$ For developing void profile ($0 < \alpha < 0.25$)
Co-current slug flow in vertical pipes	1.2	Round tube $1.35 - 0.35\sqrt{\rho_g / \rho_l}$ Rectangular channel $(1.35 - 0.35\sqrt{\rho_g / \rho_l})(1 - e^{-18\alpha})$
Counter-current slug flow in vertical pipes	1.2	
Rectangular channel	1.2-1.24	For boiling bubbly flow in an internally heated annulus
Annular geometry	1.3	$Co = \left[1.2 - 0.2\sqrt{\frac{\rho_g}{\rho_l}} \right] [1 - e^{3.12(-0.212)}]$
Triangular geometry	1.34	

Downward two-phase flow for all flow regimes
 $C_o = (-0.0214 <j^*> + 0.772) + (0.0214 <j^*> + 0.228) \sqrt{\frac{\rho_g}{\rho_l}}$ for $(-20) \leq <j^*> < 0$

$C_o = (0.2e^{0.00848[<j^*>+20]} + 1) - (0.2e^{0.00848[<j^*>+20]}) \sqrt{\frac{\rho_g}{\rho_l}}$ for $<j^*> < (-20)$
 Where, $<j^*> = \frac{j}{u_{fj}}$

Now, we find that for macro systems a large number of correlations have been suggested because this particular model is very popular for analyzing the bubbly and this slug flow patterns. So, therefore, for macro channel a wide range of CO has been propose and we find that the most of the cases the CO, it is greater than one it is around one point two for most of the cases, it suggest that there is some particular the velocity and void age profile are not flat and there is some particular rather than there is some sought of a code peaking tendency in these cases and that is expected.

Because when we are dealing with gas liquid flows or vapour liquid flows the gas or the vapour it flows at a faster velocity relative to the liquid and. So, naturally it will tend to be concentrated towards the central line for the velocity of the phase is the maximum and. So, naturally this tends to peek up the velocity and the void age profile and therefore, the CO is always greater than 1 for macro systems.

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Small Tube Correlations

$$C_0 = 1.2 + 0.510 e^{-0.692 D_H}$$

Expression shows C_0 decreases with decrease in hydraulic diameter
- Implies lower α at lower D H
- Consistent with experimental results
Same result for circular as well as rectangular channel

Attributed to

- Centralised void profile
- Laminar flow in narrow channels

For oil-water flows $C_0 = 1.06$ droplet flow
 $= 1.28$ slug flow Salim et al (2008)

Salim et al., The Canadian Journal of Chemical Engineering, 86, (2008), 978-987

Chung, P. M. Y., Kasaj, M. (2004). The effect of channel diameter on adiabatic two-phase flow characteristics in microchannels. Int. J. Multiphase Flow, 30, 735-761

Now, when we go for this small tube correlations we find that in this case the small tube rather for small tubs again I think this has been done by (Refer Time: 23:33) in this particular case the C_0 for the larger channel it has been modified by one particular additional term, were this term is a is a my function of the hydraulic diameter of the pipe. And this expression shows that C_0 decreases with decrease in hydraulic diameter and it automatically implies lower alpha at a lower hydraulic diameter. So, therefore, from here it is very evident that if C_0 decreases with hydraulic diameter. So, what does it imply it implies that when C_0 is a lower? So, naturally alpha becomes your alpha verses beta or rather the K it becomes higher is not it and. So, this implies that a lower alpha at a lower d_h and this is defiantly consistent with experimental an result which has been the same for circular as well as rectangular conduits and this particular thing that are C_0 it is lower at a lower d_h this stand attributed to 2 factors.

Well what are the 2 factors the first thing is that naturally it does a centralized void profile and therefore, the void would like to flow through the central region and the other thing is since there is laminar flow in the narrow channels? So, the naturally this also gives us a lower alpha at a lower d_h right and just as I was mentioning the other part on the other day what happens for a lower hydraulic diameter the gas it tends to flow through the center portion number 1 number 2 because its laminar flow. So, therefore, your alpha tends to be lower at lower d_h and this is what we have also obtained from a experimental results.

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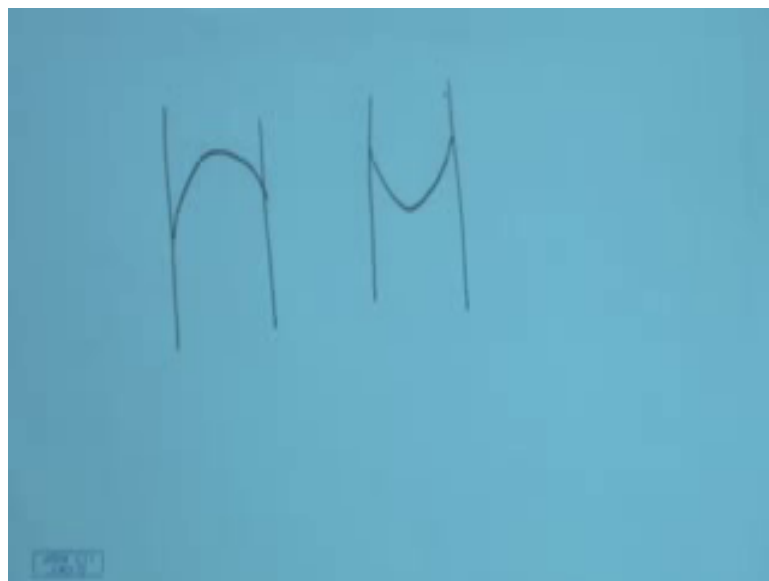
Void profile changes from concave to convex due to

- Wall nucleation and delayed transverse migration of bubbles towards centre
- Subcooled boiling regime
- Injecting gas into flowing liquid through porous tube wall
- Adiabatic flow at low void fraction when small bubbles tend to accumulate near the walls
- Droplet or particulate flow in turbulent regime

So, there are one more things which I would like to tell you that for most of the cases that we are dealt. So, for we have found that more or less C_0 it is greater than one which suggests that usually we have a concave profile.

Now, there are several cases particularly heated channel were the void profile actually changes from concave to convex or in other words instead of having a void profile something of this sort.

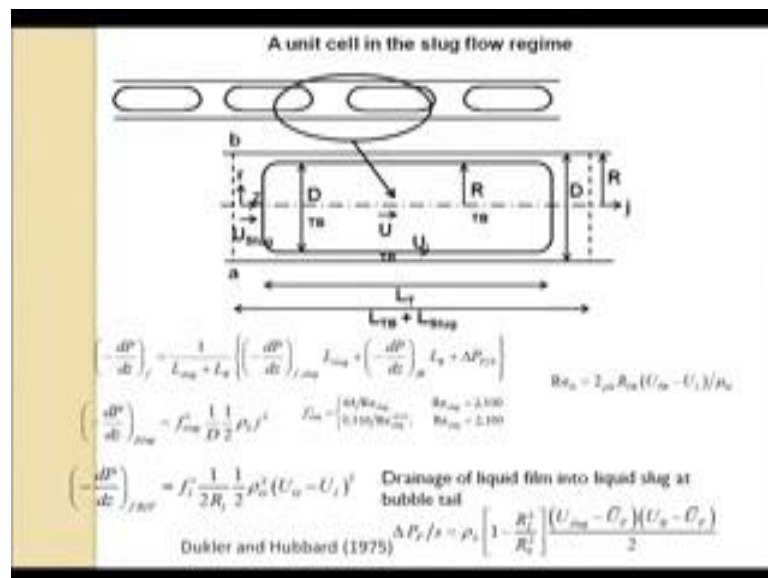
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There are cases where we can have a void profile something on this one. When does when do we have it we have it when there is wall nucleation and after nucleation the bubbles they are rather they stick to the wall till they grow to a sufficient size and till they migrate towards the center. So, therefore, the for the cases of for boiling 2 phase flow this is the case which happens this is particular to for sub cooled boiling regime and this can also happens for adiabatic flow at low void fraction when small bubbles tend to accumulate near the walls.

A similar situation can also occur when the inject gas into a flowing liquid through a porous tube wall. For these cases we can have a rather a convex instead of a concave profile and under this condition we actually get a C0 which is less than 1.

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So, with this I complete the drift flux model and in the next class we will be shifting to the rather we will be taking up one particular typical model for this flux flow pattern just because flux flow is. So, very common and we encounter flux flow. So, I would like to discuss a typical model develop on the bases of drift flux for the flux flow pattern.

The usual approach for doing this in macro system is to consider a unique cell approach were each unit cell comprises of a gas slug and its adjoining liquid slug. The analysis is done for one particulars slug unit and it assume that this the pressure drop, which has been predicted for one slug unit when multiplied by the number of slug units available in the flow passage is going to give us an idea regarding the void fraction or rather the

pressure drop over the entire tube. So, in the next class we discuss the slug flow pattern based model in the case of micro channels.

Thank you very much.