Advanced Mathematical Techniques in Chemical Engineering

Prof. S. De

Department Of Chemical Engineering

Indian Institute Of Technology, Kharagpur

Lecture No. # 41

Fourier Transform

Good morning, every one. So, we are looking into the integral transformation for the solution of partial differential equation. In the last class, we completed the Laplace transform, and we remembered that, we can use the transform calculus only in case of, for the solution of linear partial differential equation. Now, in today's class, we will be looking into the Fourier transformation. In the case of Laplace transformation, it is basically used for the parabolic partial differential equations, which are typical for transient chemical engineering problem.

Now, in case of Fourier transform, that domain is generally minus infinity to plus infinity, not necessarily, you can take the sine Fourier transformation or cosine Fourier transformation and can bring down the problem into more amenable form.

The basic idea of using the linear mathematical transformations are that: they reduce the order of regority of the problem, they reduce the partial differential equation into an ordinary differential equation, and if the governing equation is ordinary differential equation, they reduce it into algebraic equation.

So, solution of ordinary differential equation is easier to solve compared to partial differential equation, and solution of algebraic equations are easier to solve compared to the ordinary differential equations. That is how one should use the transform calculus to reduce the regority of the solution of the problem.

Now, in today's class, what we will do, we have looked into detail of the Laplace transformation, their properties and the various methods to be used to solve the various problems using the Laplace transform. And in today, we will be looking into the Fourier transform; in the last class we have seen some of the properties of the Fourier transform, and definitions of Fourier transform, and then we will be, in this class we will be looking into some of the properties of the Fourier transform, and how they will be utilized to solve the partial differential equation.

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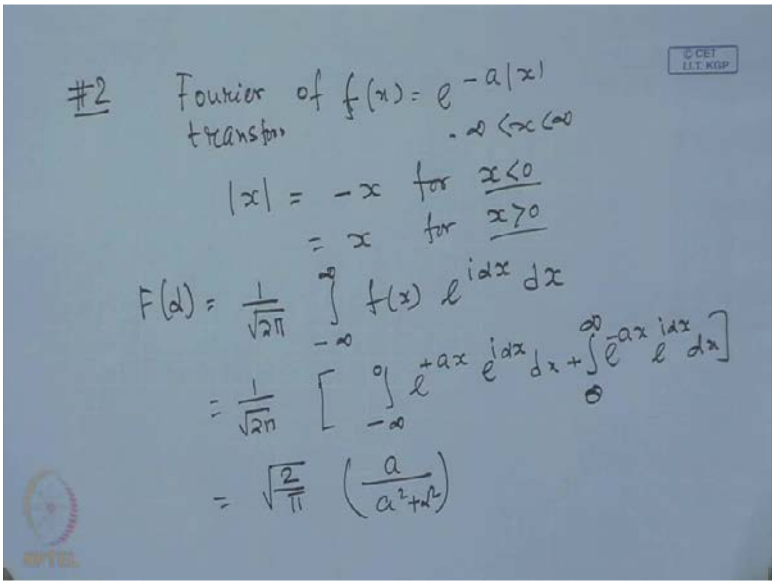

So, in the last class what we did, we looked into some of the functions, and had the Fourier transform of it. So, next we will be looking into the transform of another function, Fourier transform of, let us say, f of x, and f of x is given as minus e to the power of minus a mod of x, where x lying in between minus infinity to plus infinity. So, mod of x, if you look into the definition is that: minus x for x less than 0; and this is plus x for x greater than 0.

So, F of alpha, alpha is in the transform domain, if you write down definition of Fourier transform, it will be 1 over 2 pi minus infinity to plus infinity f of x e to be power i alpha x dx.

So, therefore, we will break down this since the definition of f x, and the f x is e to the power of minus a mod of x; so, definition of f x is different for these two domain; we will break down this integral into 2 parts, one is x is negative, that is minus infinity to 0, another is x positive, so, plus infinity to 0. So, it will be 1 over root over 2 pi minus infinity to 0, we will be putting e to the power minus minus plus ax e to the power i alpha x dx plus 0 to infinity e to the power minus ax e to the power i alpha x dx.

So, if you carry out these integrals, then, it will be e to the power a plus i alpha x dx, and then put the limits final results, you will be getting as root over 2 by pi a plus a square plus alpha square; so that is the Fourier transform of the function f x is equal to the e to the power minus mod x.

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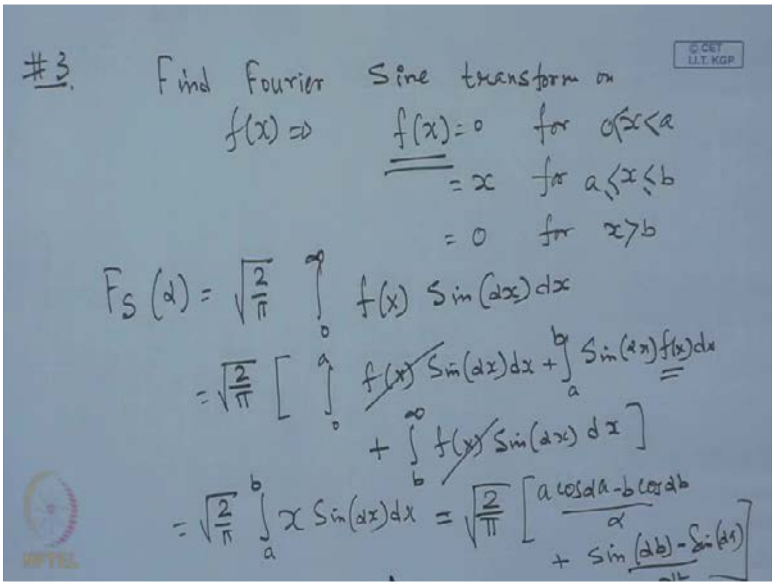

Then, we will take up one more example, that is Fourier sine transformation; let us find out, find Fourier sine transform on f x. And the definition of f x is, f x is equal to 0 for x lying in between the range 0 and a, this is equal to x for x lying in between the range a to b, and this is equal to 0 for x greater than b. So, we find out the Fourier sine transformation on this, so, F S alpha that is root over 2 by pi 0 to infinity f of x sine alpha x dx. So, this is a definition of Fourier sine transformation. So, again we break down this problem into three sub-problems, break down this integral into three sub integrals: one is from 0 to a f x sine alpha x dx; another is from a to b sine alpha x f x dx; and then another is from b to infinity f of x sine alpha x dx.

Now, if you look into the definition of f x, f x is equal to 0 for 0 to a, so, this term will be 0, because f x is 0 so anything multiplied by 0 will be 0; f x is 0 for x greater than b, b to infinity, so, this term will be also gone; and only one term will survive that is a to b where f x becomes x; so, this becomes root over 2 by pi integral a to b x sine alpha x dx, and this we can do integration by parts, so, this will be root over 2 by pi a cos alpha a minus b cos alpha b divided by alpha plus sine alpha b minus sine alpha a divided by alpha square. So, that is the Fourier transform, sine transform on f x.

So, likewise, if f x, any function is given, we can carry out the Fourier sine transformation, we can carry out the Fourier cosine transformation, you can find out the Fourier transformation as well on the function f x. So, this way by the definition of Fourier's transformation, one can find out the transform in the alpha domain from the x domain.

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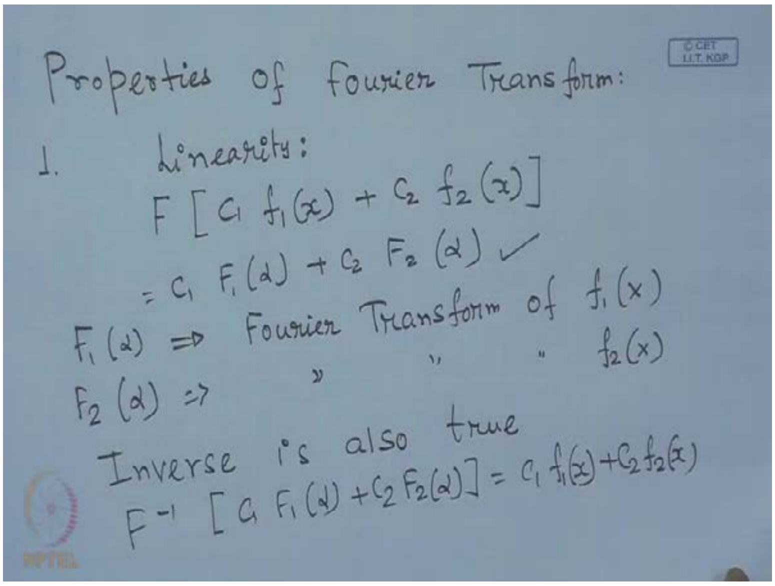

Next, we look into some of the properties of Fourier transform. First property is the linearity property. Fourier transform of two functions, which are linearly additive C 1 f 1 x plus C 2 f 2 x, this should be is equal to C 1 f of F 1 of alpha plus C 2 F 2 of alpha; F 1 alpha is Fourier transform of f 1 x and F 2 alpha is Fourier transform of f 2 x. Now, inverse of this relation is also true, that means, F inverse C 1 F 1 alpha plus C 2 F 2 alpha is equal to nothing, but C 1 f 1 x plus C 2 f 2 x.

So, the linear relationship of, if two functions are additive linearly, then Fourier transforms are also additive linearly with the same coefficient C 1 and C 2, and the inverse of these also, you will be get back the original functions with a linear form. So, that is the first property.

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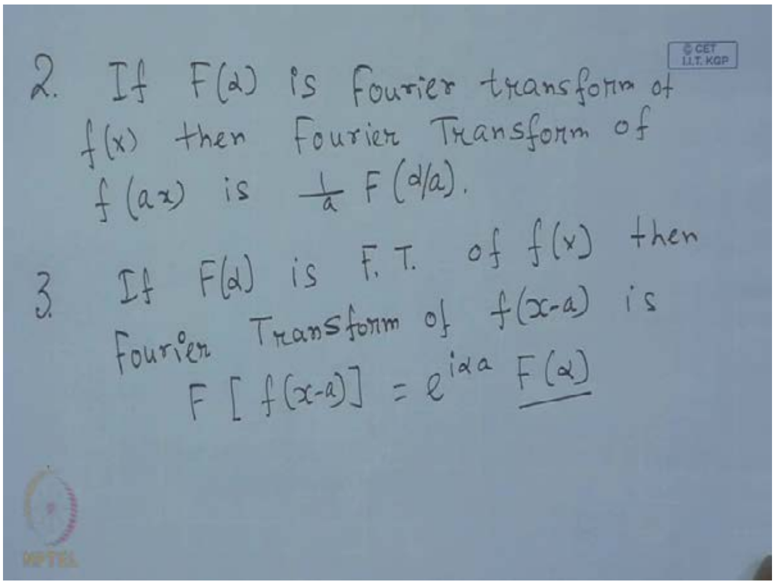

The second property is that: if F of alpha is Fourier transform of f of x, then Fourier transform of f of a x is 1 over a F of alpha by a.

We look into the third property: if F alpha is Fourier transform of f of x, then Fourier transform of f of x minus a is given as e to the power i alpha a times F of alpha.

So, these are the various properties of the Fourier transform. So, if F alpha is given by a Fourier transform of f x, then Fourier transform f x minus a will be nothing, but e to the power, this will be same as F alpha, but multiplied by e to power a alpha.

So, next we will look into some of the properties of the derivatives, and these are very important that, we have looked in to the Laplace transform of different derivatives in the earlier class; we will look in to the Fourier transforms of various derivatives.

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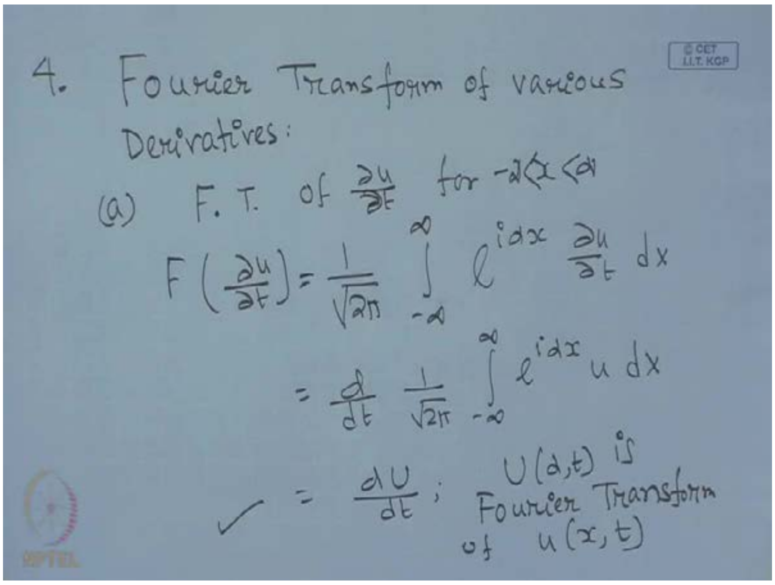

The first one is, let us look into the Fourier transform of del u del t for x lying between minus infinity to plus infinity. Then, we evaluate the Fourier transform of del u del t is, we put the definition of Fourier transform, this will be 1 over root over 2 pi minus infinity to plus infinity e to power i alpha x del u del t dx, because this integration is over x, so, this derivative does not matter; so, we take this derivative out of this integral, but in that case once we will be carrying out the integration over x, we are averaging out the x, therefore, his partial derivative becomes total derivative, if we take this derivative out; so, this will be d dt of 1 over root over 2 pi minus infinity to plus infinity e to the power i alpha x u dx, so, this becomes dU dt where U alpha t is Fourier transform of u as a function of x and t. So, that is the Fourier transform of the derivative del u del t.

Now, let us look into the Fourier transform of del u del x, and del square u del x square they are very important.

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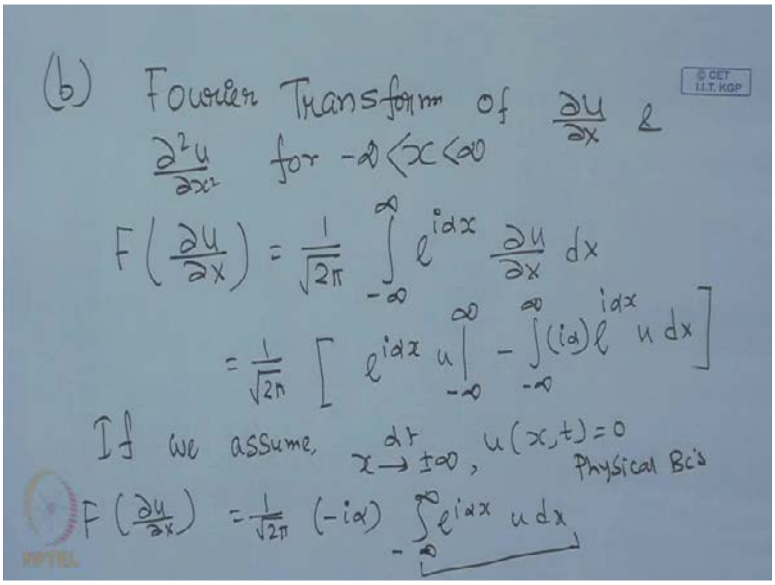

So, second property is Fourier transform of del u del x and del square u del x square for the domain x lying between minus infinity to plus infinity. So, let us find it out; Fourier transform of del u del x will be 1 over root over 2 pi minus infinity to plus infinity e to the power i alpha x del u del x dx. Then, what we can do, we can evaluate this integral, we carry out this integral by parts, so, 1 over root over 2 pi, so, this will be e to the power i alpha x u minus infinity to plus infinity minus minus infinity to plus infinity the first function integral of the second function minus derivative of first function, so, i alpha multiplied by the e to the power i alpha x integral of the second function u dx.

So, now, if we assume that limit x tends to plus minus infinity, u of x t is equal to 0, this assumption simply indicates that u is a bounded function at infinity; so it means that we are going to get a really feasible solution, we are not going to get an infeasible solution where the solution blows out at x is equal to plus minus infinity. So, in order to have a physical solution, we have to utilize these two physical boundary conditions, again, these are the examples of physical boundary condition; this boundary condition simply indicates that whatever we are going to do, the solution that we are going to have is a bounded solution, and it is a finite solution under any circumstances, it cannot go infinite in the domain of x, therefore, this is very important, and we have to take care of this.

So, we impose this boundary condition that at limit x tends to plus minus infinity, u as a function of x and t, it becomes 0, if that is the case, then, we solve lots of this problem.

So, F of del u del x is now, becomes 1 over root over 2 pi minus i alpha, so, this term will be 0, because u at infinity it is 0, u at minus infinity it is 0, so, the rest term is 1 over root over 2 pi minus i alpha e to the power i alpha x u dx e to the power i alpha x u dx minus infinity to plus infinity.

If you remember, what is this, this is nothing, but the transform Fourier, e capital u x alpha and t, so this is a Fourier transform of u; this expression is nothing, but the Fourier transform of u.

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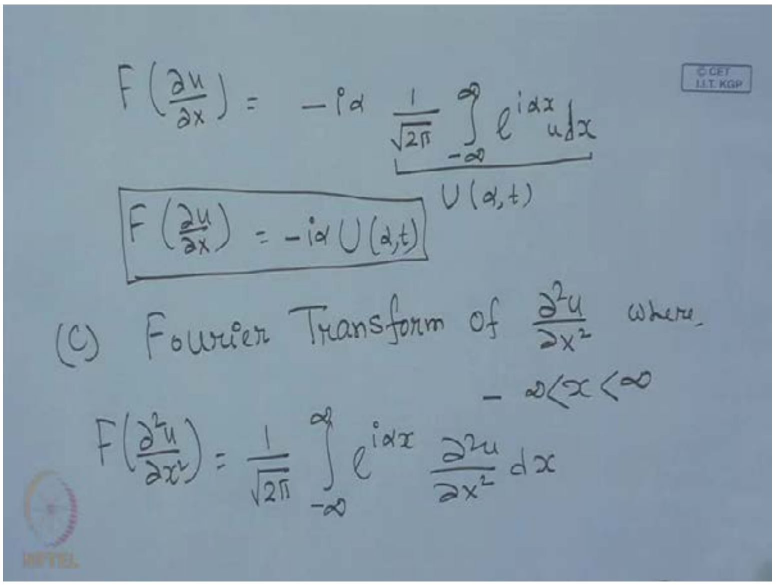

So, we write it down as F del u del x should be, is equal to minus i alpha 1 over root over 2 pi minus infinity to plus infinity e to the power i alpha x u dx, and this is nothing, but capital U alpha t; so, this is a Fourier transform of u, so this minus i alpha U alpha t.

Next property we see is the Fourier transform of del square u del x square where x lying in between minus infinity to plus infinity. So, let us take the Fourier transform of del square u del x square, that is, 1 over root over 2 pi minus infinity to plus infinity e to the power i alpha x del square u del x square dx.

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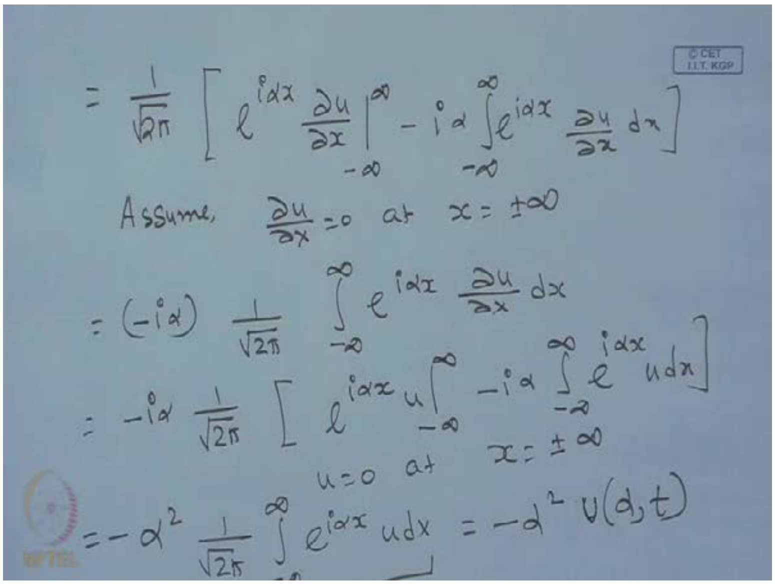

Now, again we do integration by parts, if we do integration by parts, let us see what we get; we will be getting 1 over root over 2 pi e to the power i alpha x del u del x, so, first function integration of the second function minus infinity to plus infinity minus i alpha integration e to the power i alpha x minus infinity to plus infinity del u del x dx.

In order to have a bounded solution, we assume del u del x is equal to 0 at x is equal to plus minus infinity; so, if we put this boundary conditions that, slope of u is also equal to 0 at x is equal to plus minus infinity, then this term will be vanishing, and you will be leaving with this one. So, it will be minus i alpha 1 over root over 2 pi e to the power minus infinity to plus infinity e to the power i alpha x del u del x dx, and again we do integration by parts, so, minus i alpha 1 over root over 2 pi, so, this becomes e to the power i alpha x u minus infinity to plus infinity minus i alpha minus infinity to plus infinity e to the power i alpha x u dx, and u is equal to 0 at x equal to plus minus infinity; that we have already assumed in the earlier problem as well. So, under this assumption, this term will vanish. So, we will be having i alpha square minus minus plus i alpha square, so, this becomes minus alpha square 1 over root over 2 pi minus infinity to plus infinity e to the power i alpha x u dx, and this becomes minus alpha square, this whole term becomes a Fourier transform of u, so, this becomes U capital U in alpha and t domain; so, that gives the Fourier transform of del square u del x square.

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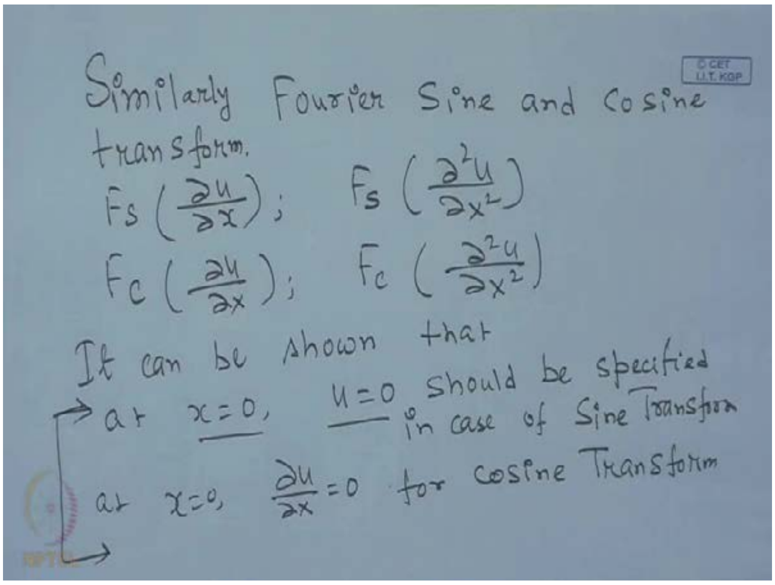

Similarly, we can carry out the sine and Fourier sine and cosine transformation. We can carry out the sine and Fourier sine and cosine transform, and it can be found out that for evaluation of sine transformation and cosine transformation of del square u del x square: we can find out the Fourier sine transformation of del u del x, we can find out the Fourier sine transformation of del square u del x square, we can carry out the Fourier cosine transformation of del u del x, Fourier cosine transformation of del square u del x square.

So, it can be shown exactly the same way we have done earlier that, at x is equal to 0, u must be equal to 0 in order to have a bounded solution should be specified in case of sine transform, and at x is equal to 0, del u del x must be is equal to 0 for cosine transform. That means, if we have a problem where this boundary condition appears at x equal to 0, u is equal to 0, we will be able to do a sine transformation, we can have a Fourier sine transform on both the sides for a problem where it is specified that at x is equal to 0, del u del x is equal to 0, then we can have a Fourier cosine transform.

So, these are very important, now, if these boundary conditions are specified, one can come to know that, which transforms one can apply, whether it is a sine transform or whether it is a cosine transform.

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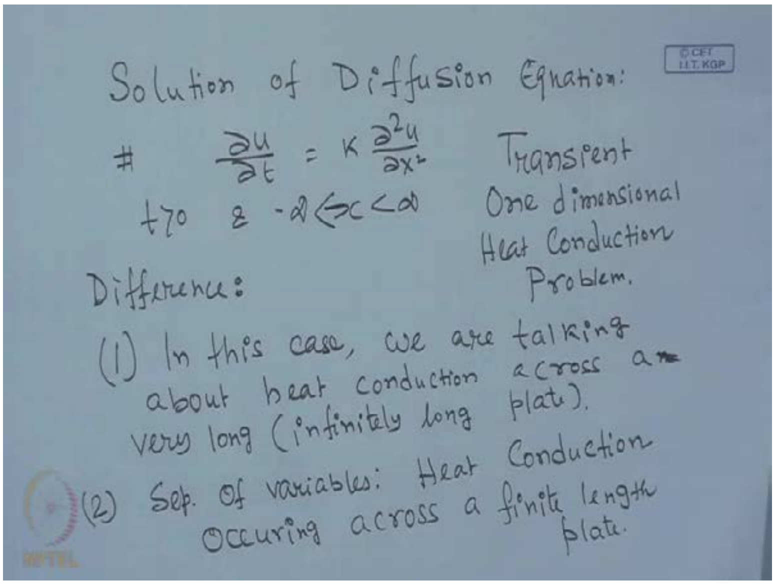

Next we do a problem of chemical engineering application; we look into the solution of diffusion equation. This problem says that del u del t is equal to K del square u del x square, but the domain of x is minus infinity to plus infinity.

So, we have a transient one dimensional heat conduction problem. Now, if you remember we have solved this type of equation using the separation of variable in the earlier classes; you must identify what is the difference between this problem and the separation of variable type of problem. The basic difference is, in this problem, this represents transient one dimensional heat conduction, but the x dimension is petty long.

So, you have a very long plate across which the heat conduction is taking place, on the other hand, if you remember the problem definition of the, whatever you have done earlier, one dimensional transient heat conduction problem, the definition remains the same, del u del t is equal to alpha del square u del x square where here alpha is equal to K, alpha is nothing, but the thermal diffusivity.

But, if you remember the boundary conditions, the boundary conditions where at x is equal to 0, u is equal to 0 and at x is equal l or a u is equal to(( )), so, it is a finite domain in the case where we have solve the separation of variables, here we are talking about an infinite domain.

So, that is the difference between the two. So, the difference let me write it down, then things becomes clearer: in one case, why we are taking Fourier transform, in another case, why you are not taking the Fourier transform, we are using the separation of variable

In this case, we are talking about heat conduction across a very infinitely long plate; on the other hand, the separation of variable type of solution, we had heat conduction occurring across a finite length plate. So, this is the difference between the two problems, let me write down the boundary condition for this problem and initial condition.

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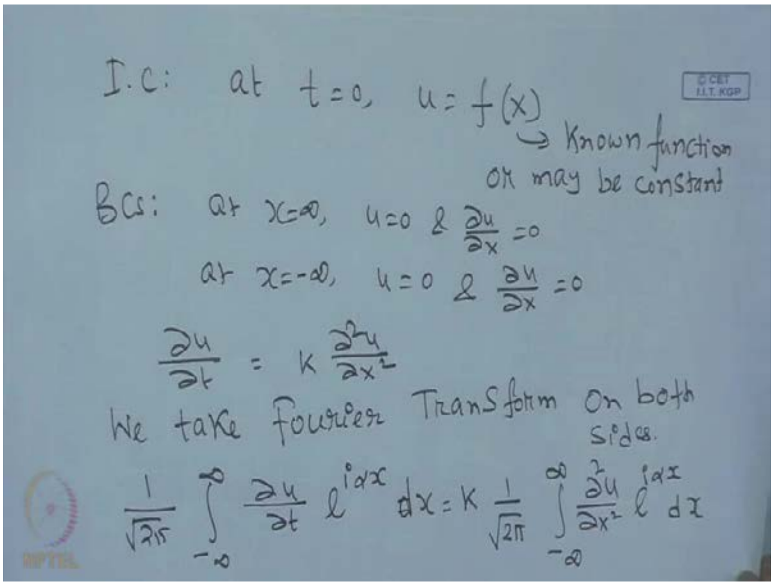

The initial condition is, at t is equal to 0, u is equal to f of x; so, f is a prescribed function, it is a known function, or it may be a constant as well. The boundary conditions are at x is equal to infinity, u is equal to 0 and del u del x is equal to 0, at x is equal to minus infinity, u is equal to 0 and del u del x is equal to 0.

So, therefore, at x is equal to plus minus infinity, both u and del u del x will be equal to 0. So, this means, the function temperature profile is a well behaved profile, it does not go to infinity at any place in the domain even at infinite distance, so, it will basically die down there, so, it is a well behaved problem.

So, we take the Fourier transform of partial differential equation, del u del t is equal to K del square u del x square; so, we take Fourier transform on both sides, so, 1 over root over 2 pi minus infinity to plus infinity del u del t e to the power i alpha x is equal to dx is equal to K 1 over root over 2 pi minus infinity to plus infinity del square u del x square e to the power i alpha x dx.

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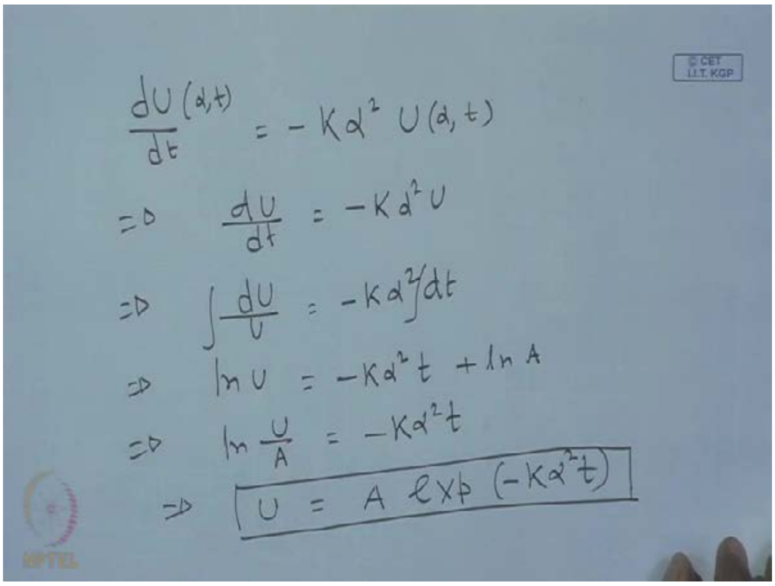

So, we have already looked into the Fourier transform of this derivative, del square u del x square; we have already looked into the Fourier transform of this derivative, del del t; and let us see what we get. So, this becomes dU alpha t dt is equal to minus K alpha square U as a function of alpha and t, so, ultimately you will be getting, dU dt is equal to minus K alpha square U, we separate the variables, this becomes dU by U is equal to minus K alpha square dt.

Now, one can carry out this integration, this integration will be ln U minus K alpha square t plus ln some constant, integration constant A, so, ln U over A is nothing, but minus K alpha square t, so, you will be getting U as a function is equal to A exponential minus K alpha square t; so, this becomes the solution of the transformed function.

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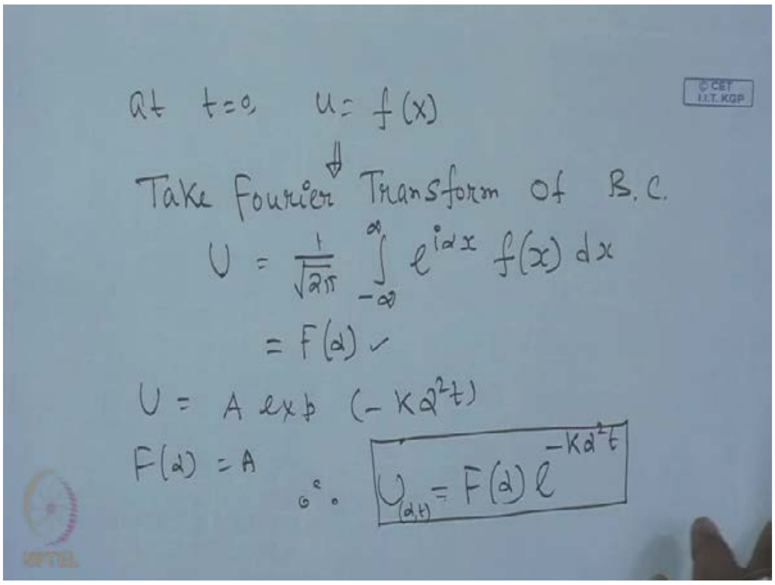

So, we write down the initial condition to solve this equation, because there is only one unknown here. So, at t is equal to 0, we ad u is equal to f of x. Now, we have since capital U is a Fourier transform variable, we have to take the Fourier transform of the initial condition as well; so, we take Fourier transform of this boundary condition, so, this becomes capital U is equal to 1 over root over 2 pi minus infinity to plus infinity e to the power i alpha x f of x dx.

So, this is the Fourier transform of f x, we call this as F of alpha; if f x is specified, then we can take the Fourier transform, suppose f x is x, then we can carry out the Fourier transform of that, if f x is different, let us say e to the power minus x square, then it will be different. So, depending on the expression of f x, we can have the Fourier transform of this equation; so let us call this in general that Fourier transform of this initial condition becomes F of alpha; so we put it here.

So, at the solution is equal to A exponential minus K alpha square t, so, F of alpha is nothing, but A we put t is equal to 0 there. So, the complete solution in the transform domain becomes F of alpha e to the power minus K alpha square t, is the solution in the transform domain, then what we can do, we can take the inverse transform and convert into x and t, so, this in the alpha and t domain, we can take the inverse transform, and convert into the x and t domain.

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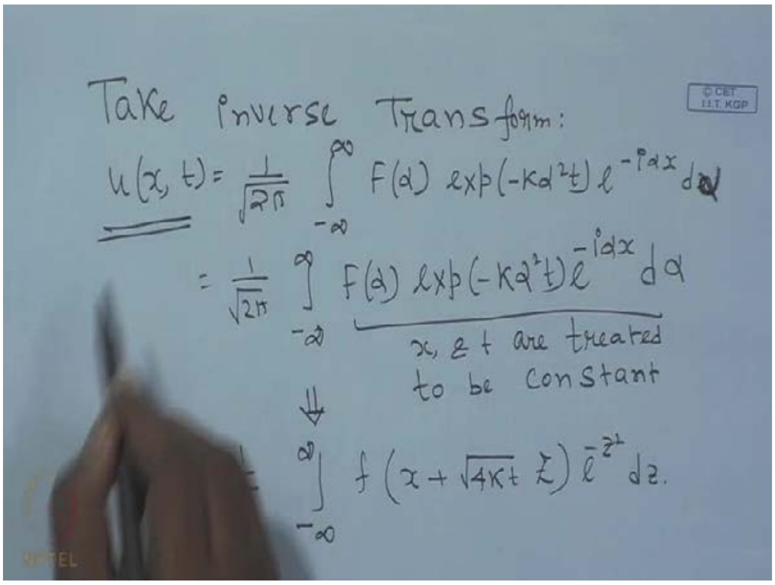

So, we take the inverse transform; so, you will be getting u as a function of x and t, is equal to 1 over root over 2 pi minus infinity to plus infinity F of alpha exponential minus K alpha square t e to the power minus i alpha x dx. So, when you take Fourier transform, we multiplied by integral i alpha x dx, and in this case, when you take the inverse Fourier transform, we multiply this minus i alpha x d alpha.

So, this will be 1 over root over 2 pi minus infinity to plus infinity F of alpha exponential minus K alpha square t e to the power minus i alpha x d alpha; in this integration both x and t need to be treated to be constant.

So, we will be getting the Fourier transform of inverse transforming x and t in fact, this turns out to be, after doing some mathematical manipulation, this becomes 1 over root over pi minus infinity to plus infinity f of x plus root over 4Kt Z e to the power minus Z square dZ.

So, it will be in that particular form, and one can get the inverse transform, and can obtain the solution u as a function of x and t.

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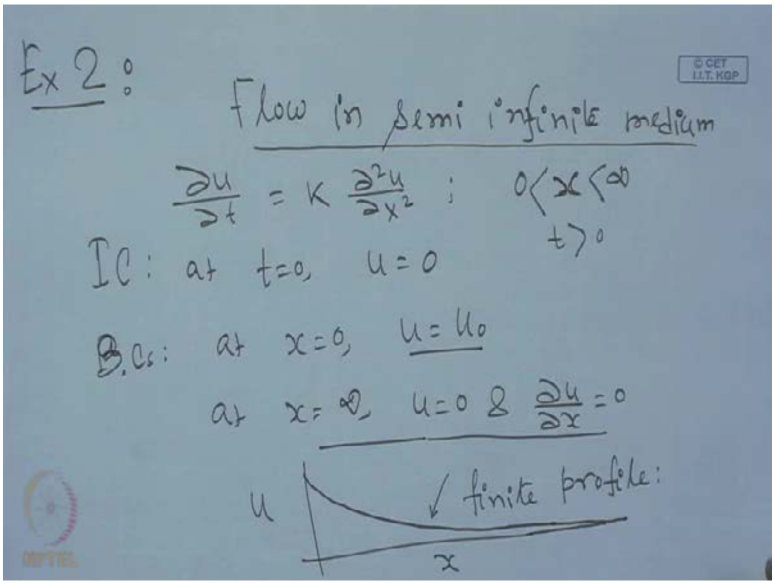

Next, we move over to another example. So, let us look into example 2. So, this is again a flow in a semi-infinite medium.

So, if del u del t is equal to K del square u del x square where the domain x lying in between 0 to infinity, and t greater than 0.

So, this is the problem. Now, let us set up the initial and boundary condition, the initial condition is: at t is equal to 0, u is equal to 0; boundary conditions are, at x is equal to 0, u is equal to u naught; at x is equal to infinity, u is equal to 0 and del u del x must be equal to 0.

Now, if you remember, since we have at x is equal to 0, we have the Dirichlet boundary condition, that is, u is equal u naught, you should take a Fourier sine transformation of this equation.

Now, this boundary condition that x equal to infinity, u is equal to 0 and del u del x is equal to 0, it indicates that, we have a finite solution even at infinite domain, and at x is equal to 0, u is equal to u naught; so, basically the variation of the dependent variable u is from u naught to 0, so, from infinite to 0.

So, if you look into the profile of u as a function of x, this looks something like this, so, it is always a finite one, finite profile.

So, now we take the Fourier, so, the finiteness of the profile is ensured by these two conditions, at, located at x is equal to infinity, both u is equal to 0 and del u del x will be equal to (( )), that means at infinity it will be asymptotically equal to 0.

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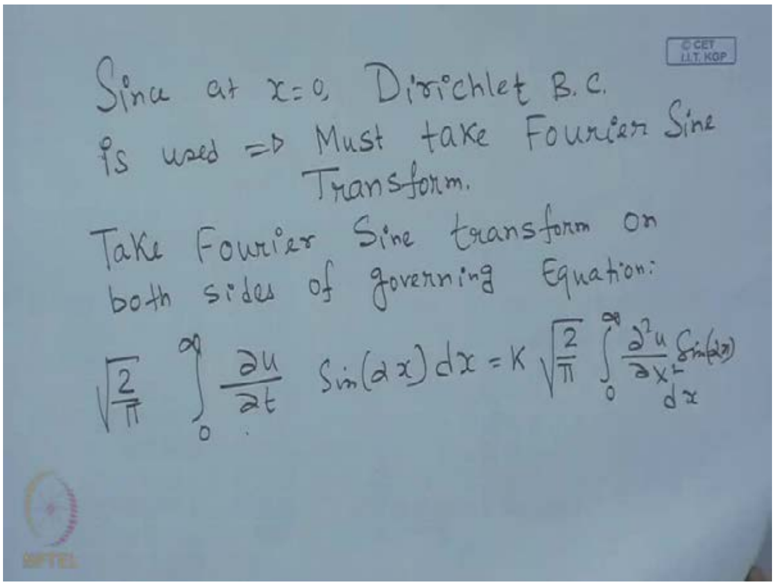

Let us look into this, come back to this problem, since we have the boundary condition at x is equal to 0, we have a Dirichlet boundary condition, Dirichlet boundary condition is used, we must take Fourier sine transform.

Now, you just take the Fourier sine transform of the governing equation, both sides of the governing equation, if you take that, then let us write down the equation, under root 2 by pi 0 to infinity del u del t sine alpha x dx is equal to K root over 2 by pi 0 to infinity del square u del x square sine alpha x dx; so we take the Fourier sine transform.

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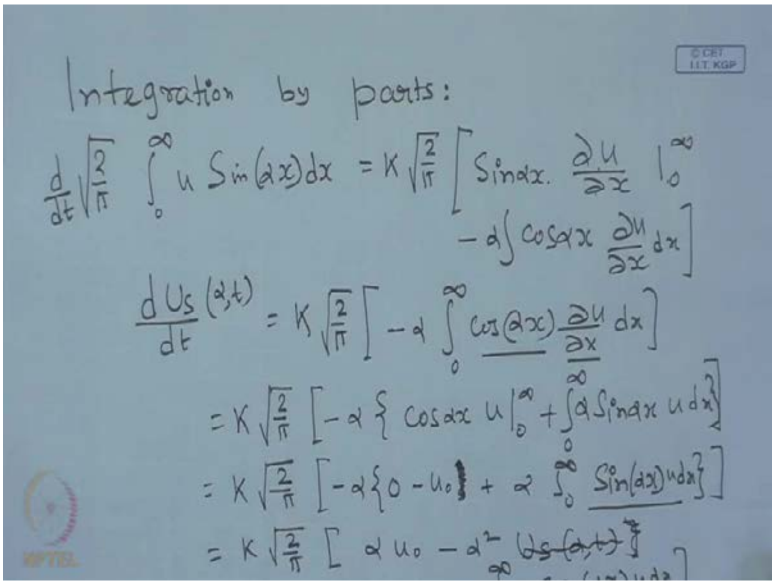

So, we evaluate both the integrals by parts and see what we get. Integration by parts, this will be root over 2 over pi del, del t can be taken out, so, it will be d dt, so, 0 to infinity u sine alpha x dx, so, that will be the Fourier sine transform, equal to K root over 2 by pi, so, sine alpha x first function, integration of second function will be del u del x evaluated from 0 to infinity minus differential of the integration of differential of first function, that means, alpha cosine alpha x, and integration of the second function, that is, del u del x dx.

So, let us see what we get on the left hand side. In the left hand side, we get the Fourier sine transform of u, so, it will be d U s dt where U s is function of alpha and t, and this will be K root over 2 by pi sine alpha x del u del x, so, when you put the infinity, then we had the condition del u del x at infinity equal to 0, when you put x is equal to 0, then sine 0 is equal to 0; so, this term will not be contributing anything, so, let us put these boundary conditions once again and explain.

Let us put at infinity, at infinity this will be sine alpha infinity del u del x evaluated at infinity, if you remember, we had the bounded boundary condition, finite boundary condition that at x equal to infinity u is equal to 0, and del u del x is equal to 0.

So, at x is equal to infinity we have del u del x is equal to 0, so, that term will be off, when you put the lower limit that at x is equal to 0, sine alpha x is equal to 0, sine 0 is 0; so, both these terms at these two limits will vanish, so, this will be contributing nothing.

So, we will be having minus alpha integral 0 to infinity cosine alpha x del u del x dx. Again, we do integration by parts, this is the first function, this is the second function, K root over 2 over pi minus alpha, second bracket, first function integral of the second function cosine alpha x u, so, this is from 0 to infinity minus integral derivative of first function, so, this will be minus alpha sine alpha x minus minus plus alpha sine alpha x, and integration of the second function, that is, u dx from 0 to infinity.

So, K root over 2 by pi, this will be minus alpha, when you put x is equal to infinity, then cosine alpha infinity, and u at infinity, but u at infinity, if you remember the boundary condition, that x at infinity it will be equal to 0, so, if you put the first boundary condition, this becomes 0, if you put the second boundary condition, this become cosine 0 is 1, and u at x is equal to 0, if you remember the boundary condition, u at x is equal to 0 is u naught.

So, we have this- there is a second bracket ends here- plus alpha integration at 0 to infinity sine alpha x u dx, second bracket ends here, then third bracket.

So, let us see what we get, we are getting as, K root over 2 by pi minus minus plus alpha u naught minus alpha square, and this will be sine 0 to infinity sine alpha x u dx this is the sine transform of U, so, these becomes U s alpha t.

So, still now it is not U s, when it will be multiplied by root over 2 by pi, so, I write this this integral here 0 to infinity sine alpha x u dx.

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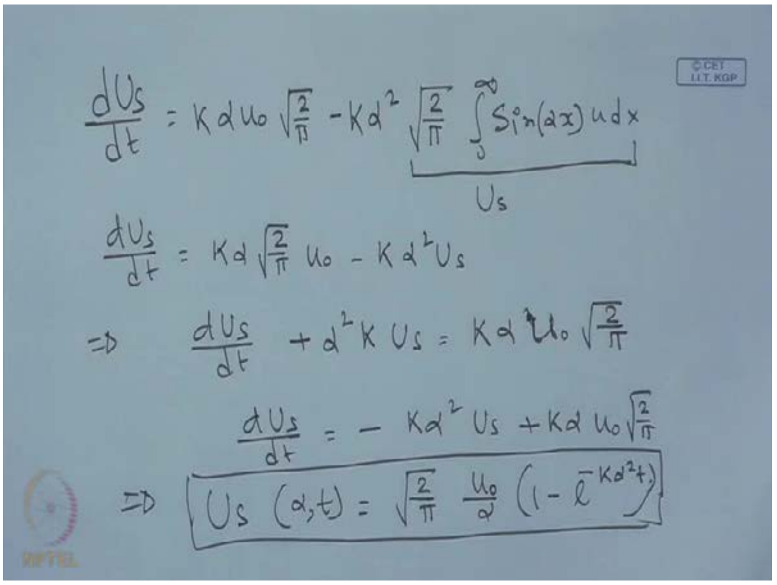

So, let us write it neatly. So, dU S dt is, K alpha u naught root over 2 by pi minus K alpha square root over 2 by pi 0 to infinity sine alpha x u dx, and this whole term is the Fourier sine transform of u.

So, dU S dt is equal to K alpha root over 2 by pi u naught minus K alpha square U S. So, you take it to the other side, this becomes dU S dt plus alpha square K U S is equal to K alpha square K alpha u naught root over 2 by pi.

Now, if you look into the solution to this problem, we know this solution can be obtained by breaking down the problem into the homogeneous part and non-homogeneous part; so, you can obtain this thing by, you can take it to the other side and can get it done.

So, dU S dt is equal to minus K alpha square U s plus K alpha u naught root over 2 by pi, bring it in the denominator and just integrate it out. So, you will be having U s as a function of alpha and t, so, it will be root over 2 by pi u naught by alpha 1 minus e to the power minus K alpha square t.

So, that is the solution of U S as the sine transform as a function of alpha and t.

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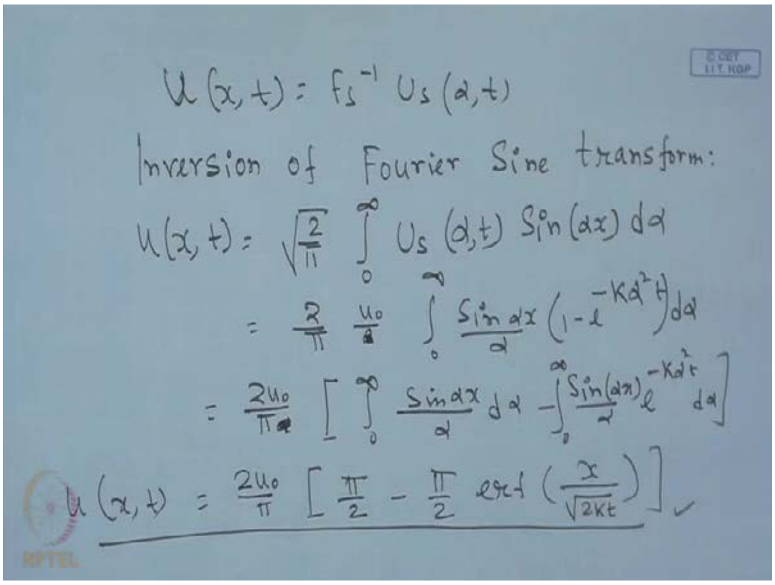

So, you will be getting U as a function of x and t, once you take the Fourier sine inverse of U S alpha t, so, inverse Fourier sine transform, now, let us do a little bit of inversion of Fourier sine transform; we will be getting u as a function of x and t is, root over 2 by pi 0 to infinity U S alpha t sine alpha x d alpha.

So, this will be 2 by pi u naught by alpha 0 to infinity sine alpha x divided by alpha U s, you just put the solution 1 minus e to the power minus K alpha square t d alpha.

Again, we do an integration by parts; so, this will be 2u naught by pi alpha, and this will be 0 to infinity sine alpha x by alpha d alpha minus sine- there will be no alpha here, so, there is no alpha- there sine alpha x by alpha e to the power minus K alpha square t d alpha from 0 to infinity.

So, we will be having 2u 0 divided by pi, this integral is pi by 2 minus, this integral is pi by 2 error function of x divided by root over 2Kt.

So, you can look into the table of Fourier sine transform, you will be seeing that this integral is nothing, but the error function of x by root over 2Kt. So, that gives the complete solution of u as a function of x and t using the Fourier transform.

So, let me conclude, whatever we have done, we have come to the end of our course; mainly we have looked into the two domains: one is the discrete domain another is the continuous domain.

In discrete domain, we represented all the equations in the form of matrix, we looked into several solutions of non-homogeneous partial non-homogeneous algebraic equations, homogeneous ordinary differential equations, non-homogeneous ordinary differential equations.

We looked into several tools, eigenvalues and eigenvector methods to evaluate them; we looked into the stability analysis of a chemical engineering process.

Then again we came down to the continuous domain; in the continuous domain we classified the partial differential equations, and looked into the various methods for the solution of partial differential equations. First, we studied in detail the separation of variable type of solution method, and we developed the theory for that, and used large, and used them for solution of problems for both homogeneous, non-homogeneous, all kinds of boundary conditions.

And then we looked into the non-homogeneous partial differential equations, but linear using Green's function method; again, we developed the theory of the Green's function, and applied them for the solution of parabolic partial differential equations and elliptical partial differential equations.

Then, we looked into the similarly transformation method, looked into the integral methods, then we looked into the integral transforms like: Laplace transform, Fourier transforms, and how they will be utilized for solution of chemical engineering problem, a realistic problem in multidimensional system.

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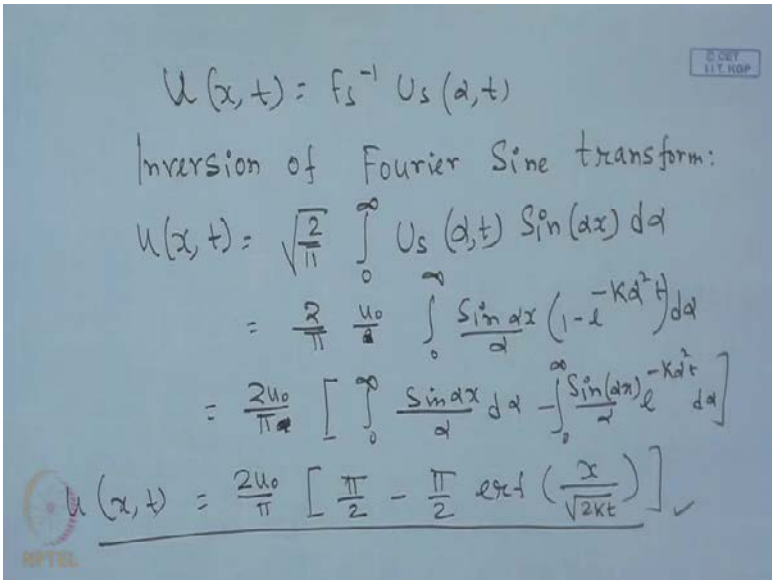

So, that gives a complete coverage of the syllabus, whatever we have done, we are supposed to cover in this course, and I hope that this course will be of use to you whenever you will be dealing with actual chemical engineering problems in future, and with that I just finish this course, and I wish best of luck to all of you.

Thank you very much.