Advanced Mathematical Techniques in Chemical Engineering

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Lecture No. # 38

Similarity Solution (Contd.)

Good morning, everyone. So, we were looking into the similarity solution method for the solution of partial differential equation. In the last class, we elaborated about when a similarity solution method can be utilized for chemical engineering problems; typically, whenever you will be having a self similar profile in the definition of the problem, then we may be having a similarity solution method-by self-similar profiles means we are talking about something like boundary layer profile.

If you remember, that the boundary layer profiles like, velocity boundary layer or mass transfer boundary layer or thermal boundary layer, the concentration profile and temperature profile in case of thermal boundary layer, they will be self similar in nature. So, one can have a similarity solution method in those cases.

Now, what is the advantage of similarity solution method? In the similarity solution method one can reduce the number of variables from two to one. So, therefore if you have an original problem, which is having two independent variables, that will be basically giving the governing equation of partial differential equation by similarity solution method will be defining a combined variable, which will be a combined variable of the two independent variables, so, and the dependent variable can be expressed as a function of this parameter only; this is known as the similarity parameter or combined variable parameter.

So, therefore, this equal, the governing equation now, has become ordinary differential equation. So, from partial differential equation it will be reduced to ordinary differential equation; and the solution of ordinary differential equations are always simpler and easier compared to the partial differential equation.

You also looked into two thumb rules where the similarity solution will be applicable: first thumb rule is that, the partial differential equation must be a parabolic partial differential equation; and secondly, one of the boundary condition must be residing at infinity; if these two conditions are satisfied, then one must be, may be having a similarity solution or may not be having a similarity solution. So, these are the sufficient conditions, that means, if the similarity solution is there, these two thumb rules have to be satisfied. Next, we took up a problem, we could not finish the problem in the last class, the problem was stokes first problem.

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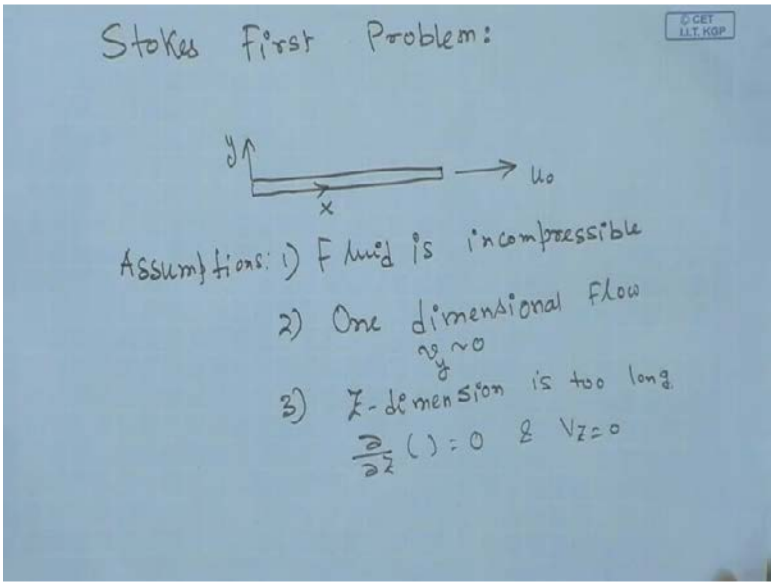

Let us define it once again. What is stokes first problem? In case of stokes first problem, we have a pool of liquid and it was stationary in nature initially. Now, we place a plate at time t is equal to 0 in the stationary pool of liquids, and at t equal to 0 plus this plate starts moving in the forward direction with a velocity u naught- so, we fix up your coordinate system here, y, this will be in the direction of x- so at t is equal to 0 the plate starts moving in the forward x direction with a constant velocity, uniform velocity u naught.

So, the fluid particles close to the plate, they will be having the similar type of velocity u naught in the forward x direction because of no slip boundary condition; as you go away from the plate in the y direction, the fluid, because of the viscous forces, the velocity of the fluid particle in the x direction start decreasing.

So, what will happen after that? After that, when beyond a particular point, the fluid particle will be having, the velocity will be decreasing and it will be decreasing to the 0; so, the beyond that particular point the fluid particle does not experience the presence of this moving plate located at the origin.

So, we, let us write down the several assumptions that we require to solve this problem. So, these assumptions, let me lease down this assumption: fluid is incompressible, that means, the density is constant; second assumption is, it is a one dimensional flow, that means, y component velocity is not equal to, it is not present there; and third assumption is that, Z-dimension is too long, that means, the variation del del Z any derivative with respect to Z is equal to 0, as well as the velocity component v Z is equal to 0- this v y is equal to 0, one dimensional flow- for Z-dimensional too long, we have v Z is equal to 0.

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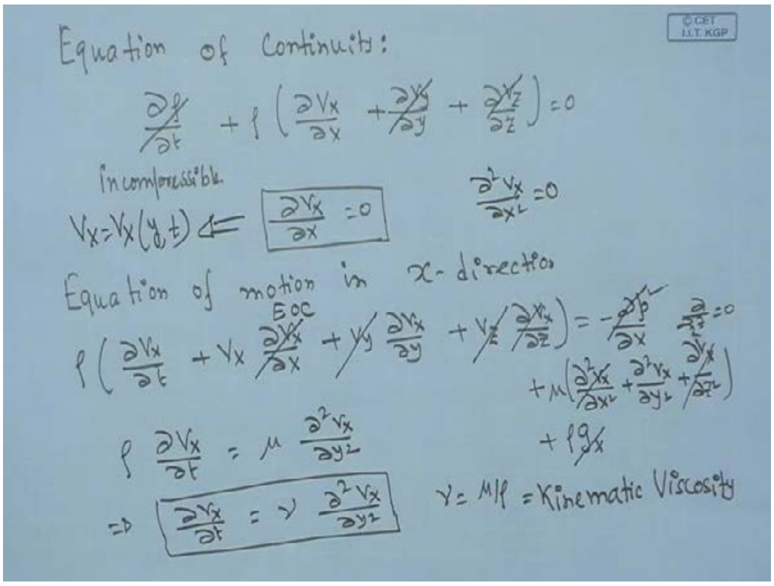

Now, once we have this, let us write down the equation of continuity at equation of motion in the x direction, so, if you do that, the equation of continuity will be giving you the following expression: We write down equation of continuity in its full form- del rho del t plus rho del v x del x plus del v y del y plus del v Z del Z is equal to 0;now, this term will be gone because the fluid is incompressible, and del v y del y v y is not present there, so, v y is equal to 0, and this last term will be equal to 0 because del del Z of anything is 0 or v Z is equal to 0. So, what we will get from equation of continuity is that, del v x del x is equal to 0- that is a very important relationship which may be utilized for the simplification of equation of motion.

Let us write down the equation of motion in x direction, let us write down the full form of equation of motion- rho del v x del t plus v x del v x del x plus v y del v x del y plus v Z del v x del Z is equal to minus del p del x plus mu del square v x del x square plus del square v x del y square plus del square v x del Z square plus rho g x, g x is the x component of the gravity force.

So, let us look into the various terms of the Navier-Stokes equation in the x direction and see what these terms they represent. The first term on the left hand side, it represents the transient term or local derivative term, the rest three terms on the left hand side, they replace the convective term or the inertial term the inertia motion, and on the right hand side, the first term is the pressure gradient term, the second term is mu times all these three terms, they are basically the viscous terms, and plus rho g x is the body force where g x is the x component of the gravity.

So, now, let us simplify this equation, this term will be remaining as it is, then del v x del x is equal to 0, this is because of equation of continuity, v y is equal to 0 because our assumption that in the y direction the velocity component is negligible, v Z is equal to 0 or del del Z is equal to 0 because Z component does not exist; the first term on the right hand side, del p del x is equal to 0 because there is no pressure gradient in this system- the motion is because of the quite flow- because of the moment of the wall there is no pressure gradient present. Therefore, del p del x is equal to 0.

Now, del v x del x is equal to 0, you differentiate this equation with respect to x once again, so, this becomes del square v x del x square is equal to 0, so, first term will be off, then del square v x del y square, this term will be there, and del del Z square is equal to 0 because del del Z of anything is equal to 0, and there is the x component g will be equal to 0 because g is only in the minus y direction.

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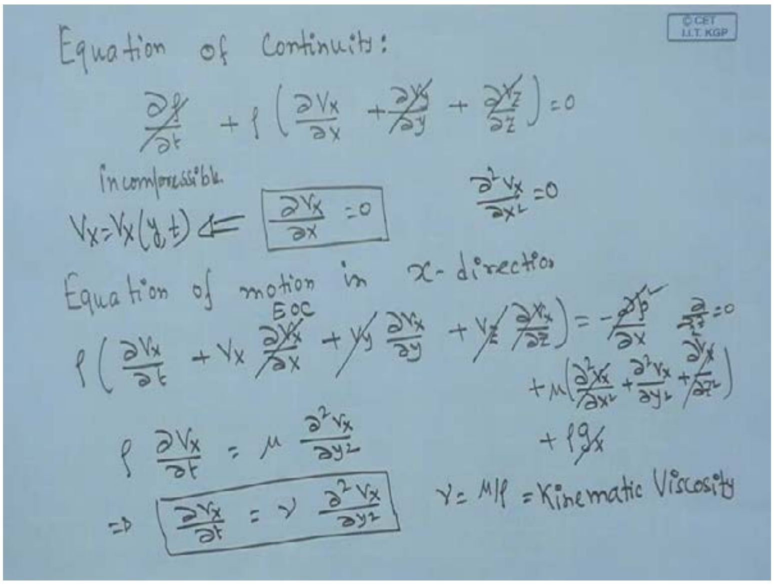

So, we do not have any x component of g, therefore, what we have is, rho del v x del t is equal to mu del square v x del y square; now, del v x del t is nu by rho, that is, nu del square v x del y square. So, this becomes the governing equation; nu is nothing but mu by rho this is known as kinematic viscosity.

So, this becomes the governing equation. Now, if you see, this equation v x is a function of time and y; so, that can be interpreted from equation of continuity as well; if you look into the equation of continuity, del v x del x is equal to 0, this simply means v x is equal to a function of y and t.

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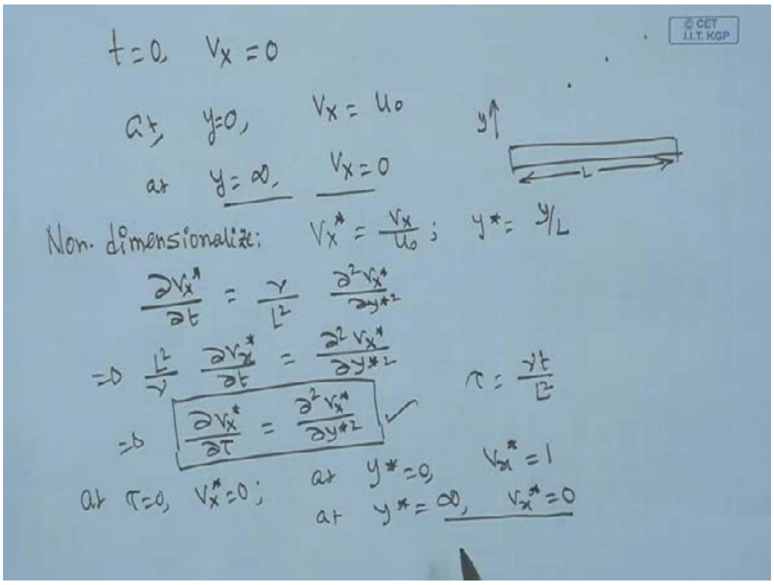

Now, let us write down the, set up the initial condition and boundary condition for this problem. The initial condition is that, at time t is equal to 0 v x is equal to 0, at y is equal to 0 we have v x is equal to u naught, that is the uniform velocity, at y is equal to infinity v x is equal to 0.

Now, let us discuss this boundary condition. So, this is the plate, this is y direction, so, the velocity becomes 0 when you go away some point in your system, in the y direction, but what is that point, but definitely, so, it may be some point here, it may be some point there, it may be some point there, but you do not know approrary at what point, at what y location v x is equal to 0.

So, therefore, the convenient boundary condition for this problem is at infinite distance in the y direction, from the plate velocity will be equal to 0, so, we put this boundary condition at y is equal to infinity v x is equal to 0.

Now, let us non-dimensionalize this problem. We write v x star is equal to v x divided by u naught, we write y star is equal to y by L, if the length of this plate is L, and we will be automatically getting the non-dimensional time, so, if you put all this in the del v x, so, this becomes del v x star del tau del t in the governing equation, nu L square del square v x star del y star square, so, this becomes L square by nu del v x star del t is equal to del square v x star del y star square, so, this becomes del v x star del tau is equal to del square v x star del y star square.

So, tau is non-dimensional time is nu t over L square. Now, let us set up the non-dimensionalize boundary condition- at tau is equal to 0 v x star is equal to 0, at y star is equal to 0 v x star is equal to 1, at y star is equal to infinity v x star is equal to 0; so, these are the boundary condition of this problem. So, if you look into the governing equation and the boundary conditions, this equation is purely a parabolic partial differential equation, and one of the boundary condition is residing at infinity, that means, this problem satisfies the thumb rules of a problem to be, which will be admitting a similarity solution.

So, therefore, we can have a similarity solution for this particular problem, but before going for the solution one has to get what will be the similarity parameter or the combined variable parameter so that we can make this equation to be expressed in that parameter only.

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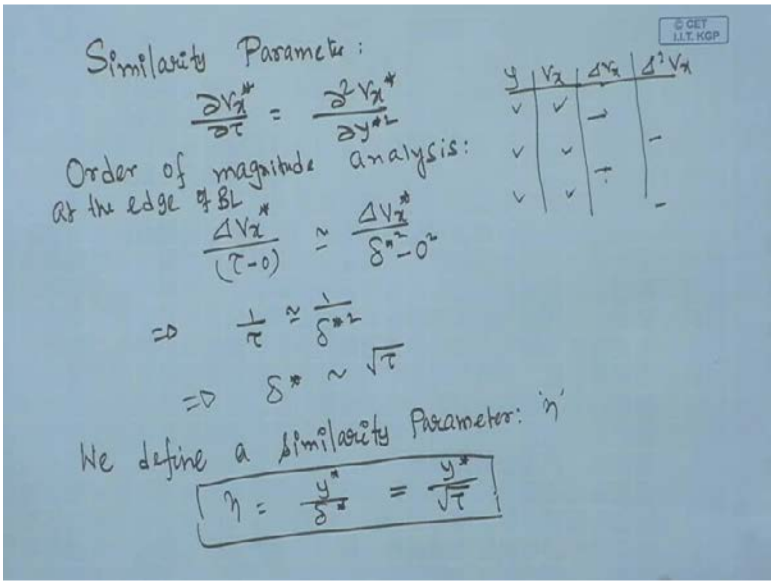

So, next exercise is getting the similarity parameter. So, for obtaining the similarity parameter, what we do? We evaluate the governing equation, we do an order on magnitude analysis, del v x is, let us say, the change is delta v x star, and del tau let say, from time t is equal to 0 the change is tau minus 0, that will be roughly equal to del square v x del square v x; if you remember that, if you remember the numerical stuff, the del square v x nothing but some kind of delta v x, it is second difference, that means, if you have a discrete variable let say, y and v x, then let say, you have the terms like here, so, delta v x will be given by this minus this, so, you will be getting this difference, this minus this, so, you will be getting a difference in between, and delta square v x is the difference between the two, so, it is being there, then you will be getting a difference between the two.

So, it is also some kind of delta v x star, and delta y star square is at the edge of the boundary layer, so, it will be delta star square minus 0 square; so we do an order on magnitude analysis at the edge of boundary layer, so, what will be getting there ? 1 over tau is equal to 1 over delta star square and delta star is nothing but root over tau.

So, if you remember, in the last class we have talked about, if we write down the similarity parameter in terms of delta star divided by independent variable, then all the curves will collapse on single curve. So, we define a similarity parameter, in this case, parameter eta, and then eta becomes y star y delta star, therefore, this eta becomes y star divided by root over tau.

So, this becomes a similarity parameter, and we can express now the similarity parameter, the various differentials of the governing equation in terms of similarity parameter; in fact, this is one of the method how the similarity parameter is defined in case of boundary layer analysis.

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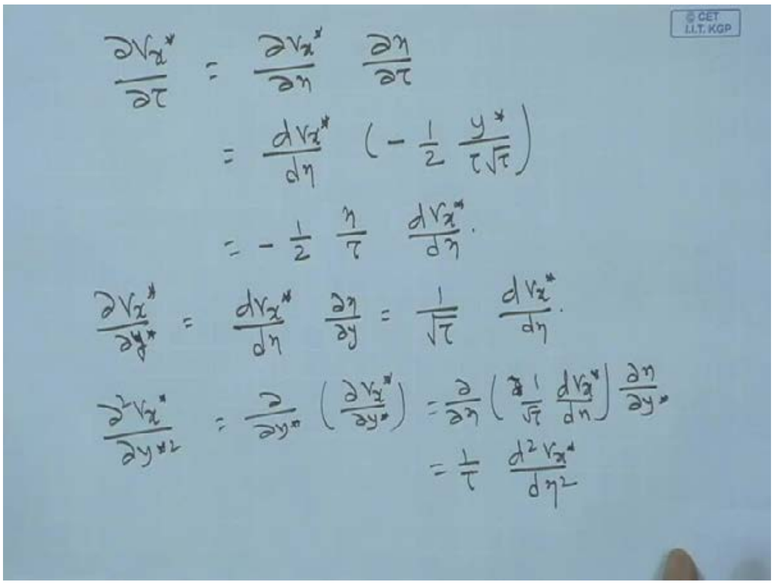

So, we expressed the partial derivative of the governing equation in terms of eta so that the partial differentials become total derivatives. So, del v x star del tau will become del v x star del eta and del eta del tau by the chain rule, and this becomes- since, v x star is now a function of eta only- so this becomes a total derivative dv x star d eta, and del eta del tau will be nothing but minus half y star tau root tau, which become basically tau 2 to the power 3 by 2, so, it will be minus 1 by 2- this y star divided by root over tau is nothing but eta- so, this becomes half eta by tau dv x star d eta.

Similarly, we will be getting del v x star del y star is nothing but dv x star d eta and d eta d y by the chain rule, so, this becomes 1 over root over tau dv x star d eta, and we do the second derivative, one more derivative of that, del square v x star del y star square, so, this is nothing but del y star del v x star del y star.

So, del del eta del v x star del del y star, we write the this expression 1 over root over tau dv x star d eta del del eta del eta del y star, del eta del y star is again 1 by root over tau, so, this becomes 1 over tau d square v x star d eta square; so, we get the different derivatives of the governing equation in terms of the combined parameter at the similarity parameter eta.

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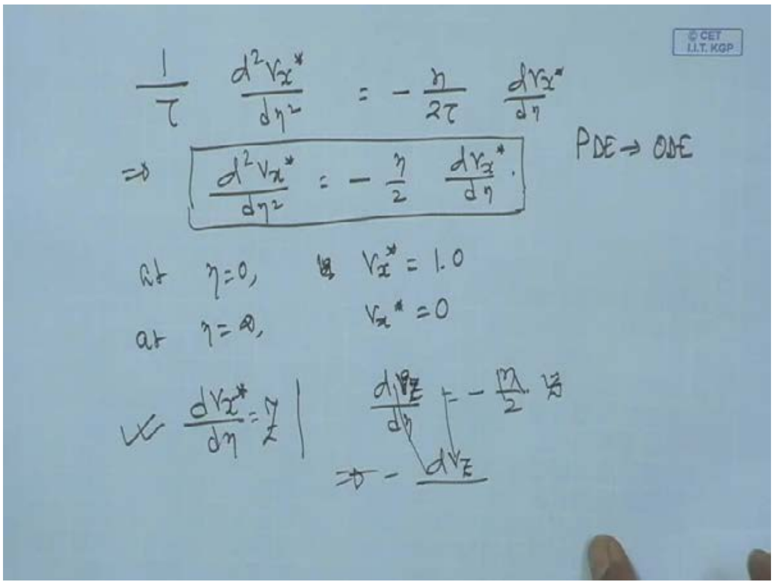

So, let us see what is the form of our governing equation. If you write down in the form governing equation, this becomes 1 by tau d square v x star d eta square is equal to minus eta by 2 tau dv x star d eta- tau will be cancelling out- so, you will be getting d square v x star d eta square is equal to minus eta by 2 dv x star d eta.

Now, this is the form of the governing equation in terms of the similarity parameter eta, and you check please that partial differential equation has now boiled down to ordinary differential equation.

Let us set up the non-dimensional boundary condition in terms of eta at y star is equal to 0, that means, at eta is equal to 0 your v x was equal to u naught, so, it will be v x star is equal to 1, and at eta equal to infinity your v x star is equal to 0.

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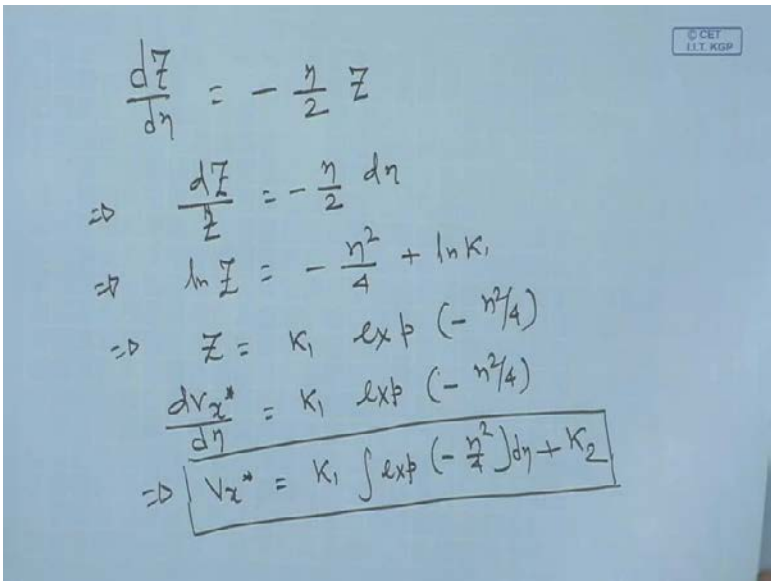

Now, this problem can easily be solved now. So, we have to define, let say, dv x star d eta is equal to Z, if you define this thing, then you will be getting dv Z d eta is equal to minus eta by 2 Z- so bring it to the other side- so, dv Z, this becomes d Z, dv x star d eta equal to Z, that means, dv x star d eta square will be nothing but d Z d eta, we will write it down. So, we define del dv x star d eta is equal to Z and expressed this equation in terms of Z. So, if you do that what will be getting is, we will be getting dZ d eta is nothing but minus eta by 2 Z, so, dZ by Z is nothing but minus eta by 2 d eta- integrate this out- so, this becomes ln Z is equal to minus eta square by 2, so, it will be eta square by 4 plus some constant, let say that constant is ln K 1, so, Z becomes K 1 exponential minus eta square by 4, and Z itself is dv x star d eta is equal to K 1 exponential minus eta square by 4.

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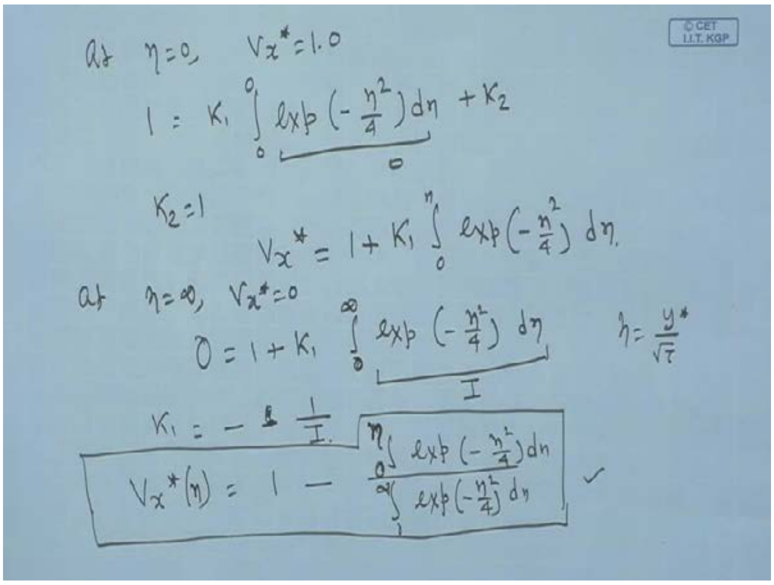

So, v x star becomes K 1 integral exponential minus eta square by 4 plus another constant K 2. So, this gives the profile of velocity within the boundary layer, and there will be d eta there. So, you have two constants of integration, K 1 and K 2; these two constants have to be evaluated, but that, by that two boundary conditions; if you put the boundary condition at eta is equal to 0, v x star is equal to 1, so, that means, 1 is equal to K 1, and this integral from 0 to eta, so, it will be effectively 0 to 0 exponential minus eta square by 4 d eta plus K 2.

So, when you evaluate this integral from the same limit the whole integral becomes 0, so you will be getting K 2 is equal to 1. So, once you get K 2 is equal to 1, let us look at what is the form of profile; so, this becomes 1 plus K 1 0 to eta exponential minus eta square by 4 d eta.

Now, we have evaluated one constant we have to evaluate the other constant K 1. Let us put the condition that is, at eta is equal to infinity v x star is equal to 0, so, there was 0 is equal to 1 plus K 1 0 to infinity exponential minus eta square by 4 d eta.

Now, if you look into this integral, this integral is a definite integral because the values from 0 to infinity, so, how to evaluate this integral? You can evaluate this integral by doing a numerical integration, by may be using a trapezoidal method.

So, the infinity you can take some high value may be 5 or 6; so, if you put the upper limit as 5 and evaluate this integral, then change the upper limit from 5 to 10, if the integral does not change in the significant places, may be 3 or 4 decimal place, then you can say 5 is the infinity for this particular problem.

So, we call this definite integral as I, so, K 1 is nothing but minus 1 over I. So, the profile of v x star as a function of eta will be nothing but 1 minus 0 to infinity exponential minus eta square by 4 d eta and 0 to eta exponential minus eta square by 4 d eta. So, this is the form of the velocity profile in terms of the combined variable, now, this will be from 0 to eta.

So, if anyone wants to plot in terms of x and y star and tau can free to do because eta is known as y star divided by root over tau; for a fixed value of y star one can plot v x star as a function of time. If you would like to plot what is the velocity profile at y star is equal to 0.1, put the value of y star as 0.1, then put a do loop where the tau will be changed from 0 to some, in a loop, and then we will be computing this equation on the right hand side and you will be getting a profile of v x star as a function of tau.

Similarly, if you want to put what is the profile of v x star at different points of tau as a function of y star, then what we do? So, for a fixed value of tau let say 0.5, we put the value of 0.5 and eta becomes y star root over 0.5 and evaluate this integral under a loop of y star so that you can get a profile of v x star as a function of y star.

So, that is how the similarity parameter can reduce the partial differential equation into an ordinary differential equation, and you will be almost getting an analytical solution in this case.

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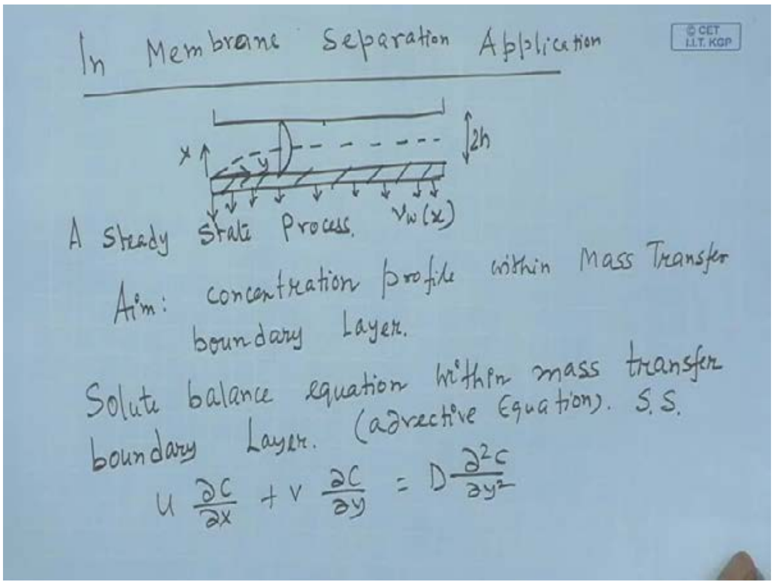

Next, we will take up one more example of similarity solution, that is, in membrane separation application. Suppose you have a membrane here, which is porous in nature and the flow is occurring into a thin channel of, let us say, the full height is 2h, the half height is h, so, we fix our coordinate system y here, x there. We assume a steady state process, so, in this case the solute particle is... the whole operation is under high pressure, because of the pressure the solute particles will be depositing over the membrane surface forming a thin concentration boundary layer. So, this is a thin mass transfer of concentration boundary layer.

We assume, in fact, the permeate flux, the permeation velocity will be maximum here, because the thickness of this boundary layer will be governing the resistance against the solvent flux, so, lower the thickness then the higher be the permeate flux, then this flux will reduce and then it becomes almost constant. So, the permeate flux, v w, is a function of x.

Next, we assume that the axial velocity is in order of magnitude higher compared to the permeation velocity; therefore, we assume that the laminar velocity profile remains undistorted because of the suction at the wall.

So, therefore, we can we can say that this profile remains still laminar. And what we do next is that, we would like to find out what is the concentration profile in this mass transfer boundary layer or concentration boundary layer.

So, our aim is to obtain concentration profile within mass transfer boundary layer. So, in order to do that let us write down the velocity, the concentration field within the boundary layer 0 to delta. So, let us write down the solute balance equation within mass transfer boundary layer.

If you write down the solute balance equation, this is also known as the advective equation, so, it becomes u del C del x, and we assume the system is operating under the steady state, so, u del C del x plus v del C del y is equal to D del square C del y square. So this is the advective equation we would like to solve within the mass transfer boundary layer. Now, once, we will able to solve this equation once we get the velocity field u and v appropriately within the mass transfer boundary layer.

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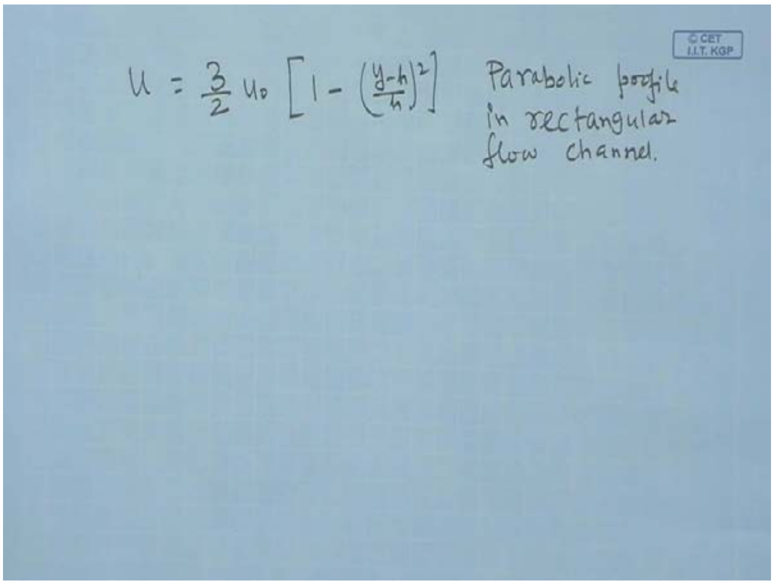

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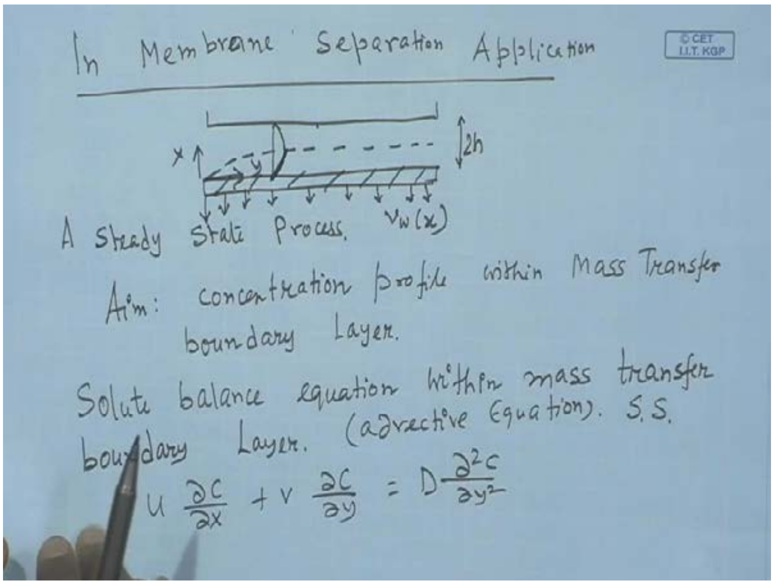

So, if you look into this, we write down the velocity field, and we have assumed that the flow is laminar, and the parabolic velocity profile remains undistorted. So, 3 by 2 u naught into 1 minus y minus h over h square- this is the parabolic velocity profile in a rectangular thin channel. Now we are going to solve the concentration profile within the thin boundary layer. So, you can see that we need not do, beyond the concentration boundary layer the concentration remain same, so, we need not to have the full profile, we need to have the profile within the mass transfer boundary layer; if you see the profile of the velocity within the mass transfer boundary layer it becomes a linear one.

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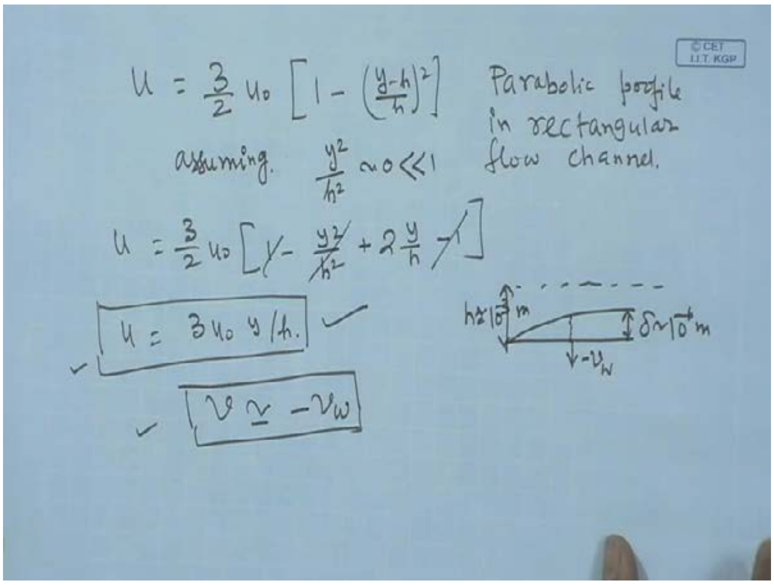

So, therefore, we linearize the velocity field assuming y square h square and higher order terms will be negligibly small, they will be much less than 1; so, with these you will be having 3 by 2 u naught 1 minus y square by h square minus minus plus 2 y by h minus 1- so, 1 1 will be cancelling out, y square by h square is small compared to y by h- then you will be having 3u 0 y over h- so that is the linear velocity profile within the mass transfer boundary layer. So, this is the profile of u, that is, the v component, v comes, since the mass transfer boundary layer is very thin so probably this will be in the order of 10 to the power minus 6 meter, and half channel height will be in the order of 1 millimeter, this will be in the order of 10 to the power minus 3 meter.

So there are three order of magnitude differences between delta and h, so, with in this thin boundary layer, we assume that whatever the y component velocity is present in the wall, that will be remaining same within the mass transfer boundary layer as well.

So, we assume that velocity is nothing but minus v w at the wall. So, with the help of two simplified velocity profiles we can solve the concentration profile in the advective equation within the mass transfer boundary layer.

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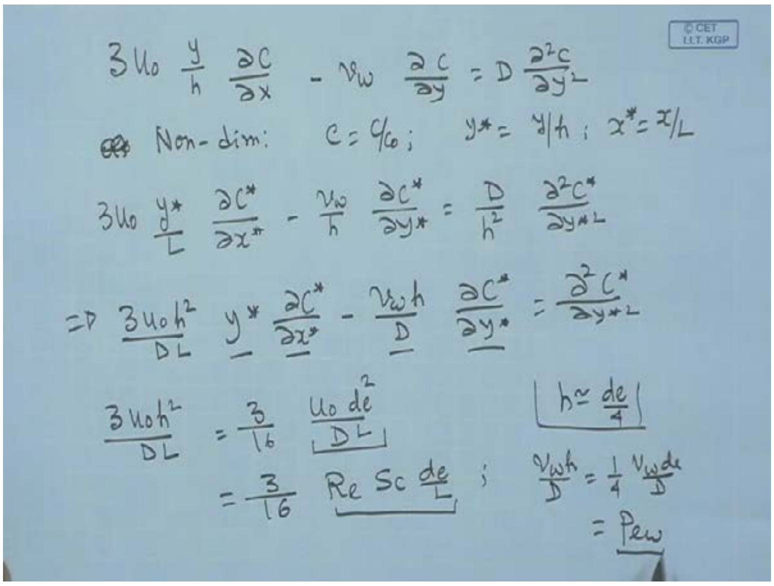

So, if you insert these two velocity profile in the governing equation, let us see what is the form of the governing equation. This becomes 3u naught y over h del C del x minus v w del C del y is equal to D del square C del y square.

Now, let us make this non-dimensionalization as follows: C is equal to C by C naught; y star we write it as y by half height of the channel, and the x star is nothing but x by L, L is the length of the channel; h is the half height at the channel in the y direction and L is the length of the channel in the x direction. So, if you put everything in the non-dimensional form, so, y star this will be del C star del x star minus v w over h del C star del y star is equal to D h square del square C star del y star square; then we multiply both side by h square by D h square by D and see what we get; 3u 0 h square by D times L y star del C star del x star minus v w h over d del C star del y star is equal to del square C star del y star square.

The right hand side is completely non-dimensional, there are two terms on the left hand side; if you look into the first term, this term is a product of three quantities, the second one is non-dimensional, the third one is non-dimensional, that means the other one has to be a non-dimensional term. Similarly, the second term on the left hand side, the second term is entirely in non-dimensional that means, this has to be a non-dimensional quantity as well.

So, if you look into the 3u 0 h square by DL this becomes- you just express h, the half height will be in terms of equivalent diameter, so, this will be de by 4- so, this will be 3 by 16 u 0 de square by DL; now, this term, if you break it down in Reynolds and Schmitt this becomes 3 by 16 Reynolds Schmitt de by L, and if you look into the v w by h the v w h by d is nothing but 1 by 4 v w de by D, and this will be nothing but non-dimensional wall velocity, we call it non-dimensional pickle number. So, you call it P ew pickle at the wall, so, this will be a constant, and the v w h by D this will be a function of x because as you go down the length of the channel the permeation velocity decreases, so, this will be typically a function of x.

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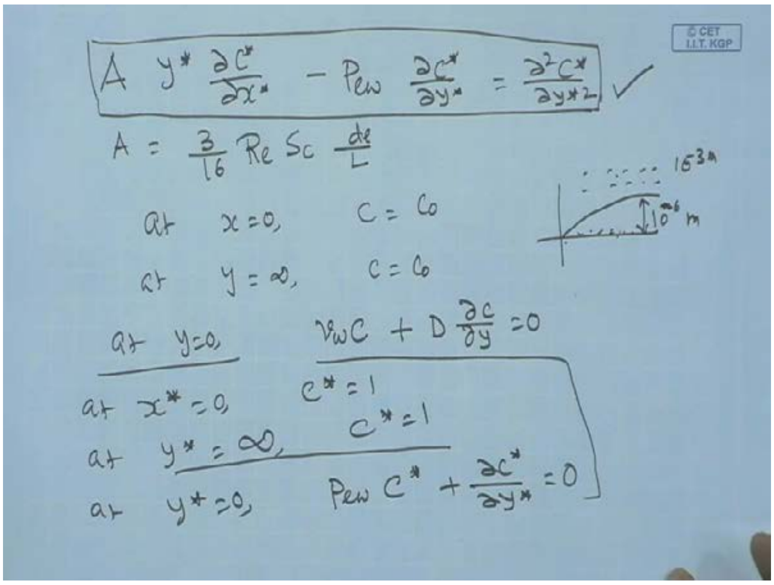

Now, let us put these equation in the governing equation; so, this becomes A y star del C star del x star minus P ew del C star del y star is equal to del square C star del y star square, so, where A, the constant term becomes 3 by 16 Reynolds Schmitt de by L, therefore, this becomes a constant and will be having the governing equation like this.

Now, let us write down the boundary condition, at x is equal to 0 C was equal to C naught, the feed concentration, at y equal to infinity C is equal to C naught. So this is the boundary layer, and this thickness of the boundary layer will be in the order of 10 to the power minus 6 meter, and the half height will be 10 to the power minus 3 meter.

So, it is obvious that anything here will be having a 0, will be having the same concentration as C naught, so, assume that, if any place within the half height will be 10 to the power 3 times compared to the thickness of masters of boundary layer, so, we call that as infinity; so, at y equal to infinity C is equal to C naught at y is equal to 0 we have the mixed boundary condition, that is, the convictive term of the solute, whatever solute been convicted to towards the membrane surface will be is equal to the diffusion from the surface towards the bulk; so, D del C del y will be is equal to 0 that means, at the membrane surface the solute concentration will be more, and because of that it will set up a backward diffusive flux from higher concentration to the lower concentration because the concentration of the solute at the wall will be more compared to the bulk, so, it sets up a backward diffusion because of the fixed first law.

Now, let us put these equations non-dimensional, so, at x star is equal to 0 your C star is equal to 1, at y star is equal to infinity your C star is equal to 1, at y star is equal to 0- we make a non-dimensional- so, this becomes P ew C star plus del C star del y star is equal to 0. So, these three are non-dimensional boundary condition of this particular problem.

Now, if you look into the governing equation, the governing equation is a parabolic partial differential equation, if you look into one of the boundary conditions, one of boundary conditions is residing at infinity.

So, this problem is a suitable candidate to admit a similarity solution, but for progressing further with the similarity solution we have to identify what is the similarity parameter, so for that what we do? We do an order on magnitude analysis on the governing equation at the edge of the boundary layer.

So, Ay star del C star del x star minus P ew del C star del y star is equal to del square C star del y star square, do an order on magnitude analysis at the edge of boundary layer; and we have seen in the last class that the edge of the boundary layer, these two conditions are always satisfied, C star is equal to 0 and del C star del y star is equal to 0 at the edge of boundary layer.

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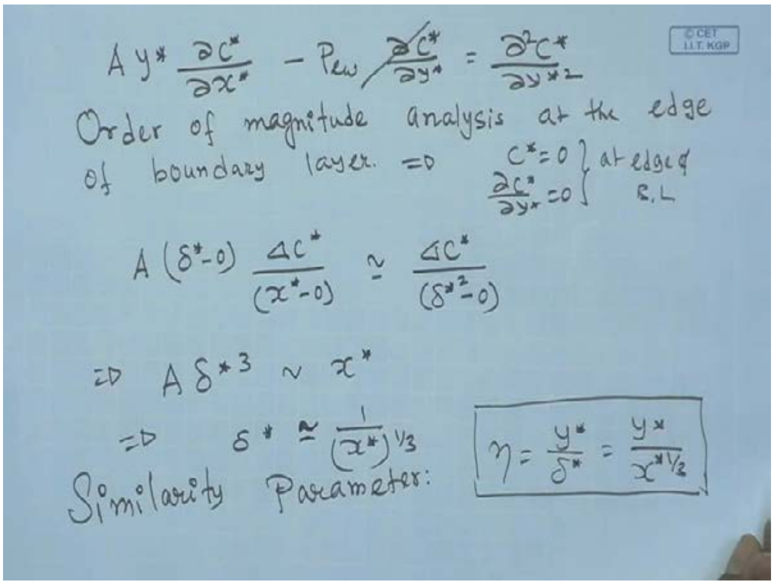

So, this term will be off, and y star will be delta star minus 0, so, this becomes delta star minus 0 del C star is delta C star, and del x star is nothing but x star minus 0, this will be roughly, del square C star we have already seen, this is nothing but some kind of delta C star or same order of the magnitude, and delta y star square is nothing but delta star square minus 0.

So, this becomes A delta star cube, del C star del C star will be canceling out, so these will be x star; therefore, we write delta star cube delta star is nothing but 1 over x star to the power 1 upon 3. So delta star will be nothing but x star raised to the power 1 upon 3- so that is the form of delta star, how delta star varies as x star, so, it is varying as x star to the power 1 upon 3.

So, we can now in a position to define the similarity parameter. The similarity parameter now becomes eta, eta becomes y star by del star, so, this becomes y star y x star to the power 1 upon 3. So, that becomes the similarity parameter in this particular problem; if you look into the earlier problem, the earlier problem similarity parameter was y star by tau to the power half, but in this problem it becomes y star divided by x star to the power 1 upon 3.

So, we identified what is the similarity parameter, so, what is next is that, to express all the partial differential of the governing equation in terms of similarity parameter so that the partial derivative becomes the total derivative.

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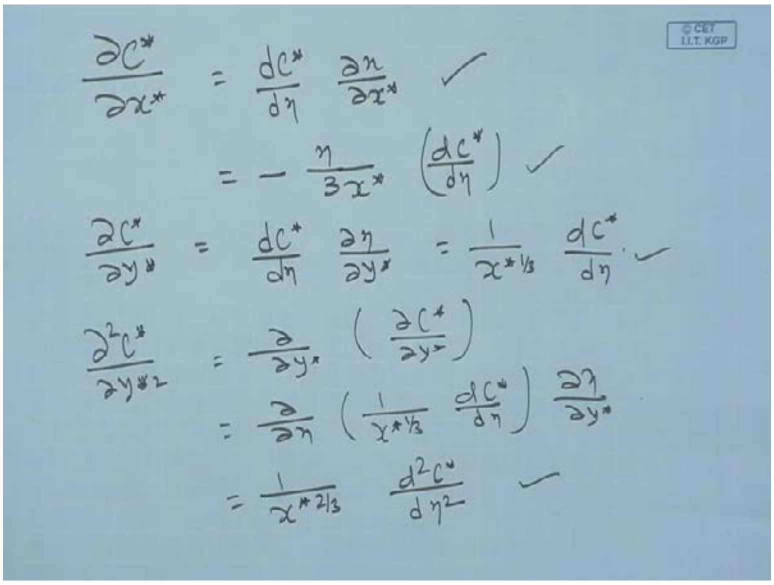

So, we defined del C star del x star; del C star del x star is nothing but dC star d eta, and by using the chain rule del eta del x star, so, this becomes minus eta by 3 x star dC star d eta, and del C star del y star is equal to dC star d eta del eta del y star; so, this becomes 1 over x star to the power 1 upon 3 that is del eta del y star, this is dC star d eta; and del square C star del y star square becomes del del y star of dC star del C star del y star, and this becomes del del eta 1 over x star to the power 1 upon 3 dC star d eta del eta del y star; so, this becomes 1 over x star to the power 2 by 3 d square C star d eta square. So, we get the three partial derivatives in terms of total derivative of dC of eta.

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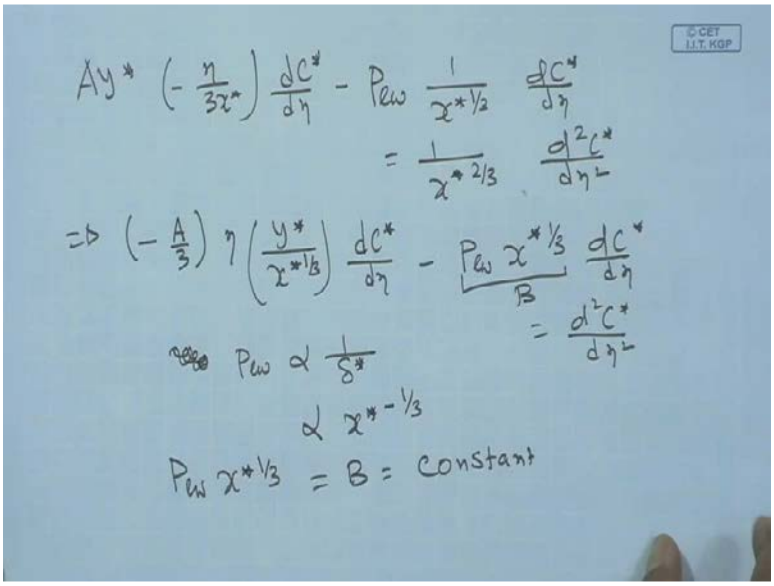

So, we substitute this derivative in the governing equation and see what we get. If we substitute these three derivatives in the governing equation, we will be getting Ay star minus eta by 3x star dC star d eta minus P ew 1 over x star to the power 1 upon 3 dC star d eta is equal to 1 over x star to the power 2 by 3 d square C star d eta square-so, you multiply both side by x star to the power 2 by 3- so, this becomes minus A by 3 eta y star divided by x star to the power 1 upon 3- if you remember this term itself becomes, the term in the bracket becomes eta- so, dC star d eta minus P ew x star to the power 1 upon 3 dC star d eta is equal to d square C star d eta square.

Now, we have seen that v w, the permeate flux, let us say non-dimensional permeate flux, is inversely proportional to thickness of boundary layer, more be the thickness of boundary layer less flux you are going to get, but delta star is proportional to 1 over x star to the power 1 upon 3 so this will be proportional to 1 over x star to the power 1 upon 3, therefore, P ew x star to the power 1 upon 3 is a constant and that constant is let say, this is a constant called B, so, the whole thing becomes a constant B, and let us see what we get.

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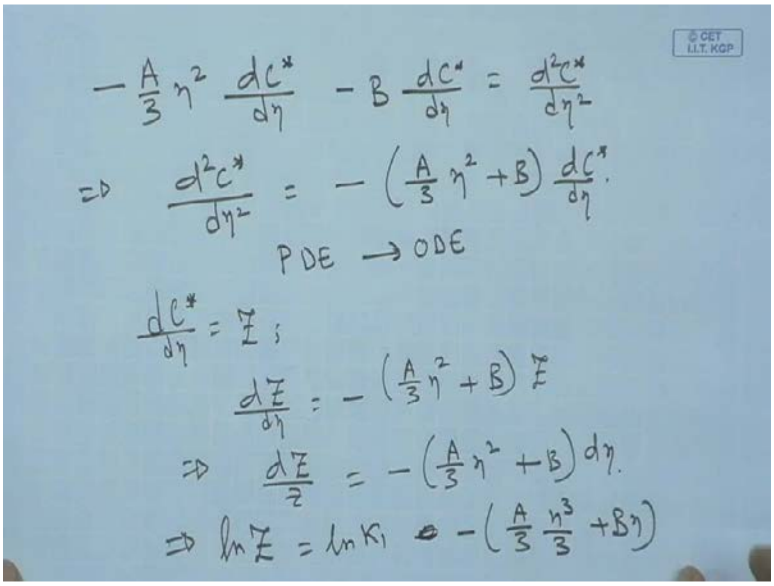

So, minus A by 3 eta square dC star d eta minus B dC star d eta is equal to d square C star d eta square, therefore, we change sign, so, d square C star d eta square is nothing but minus A over 3 eta square plus B dC star d eta.

So, the partial differential has become an ordinary differential equation; so, the ordinary differential equation is easy to solve, so, you define dC star d eta is equal to Z, therefore, we have dZ d eta is equal to minus A by 3 eta square plus B Z- so we change sign- so dZ by Z is nothing but minus A over 3 eta square plus B d eta.

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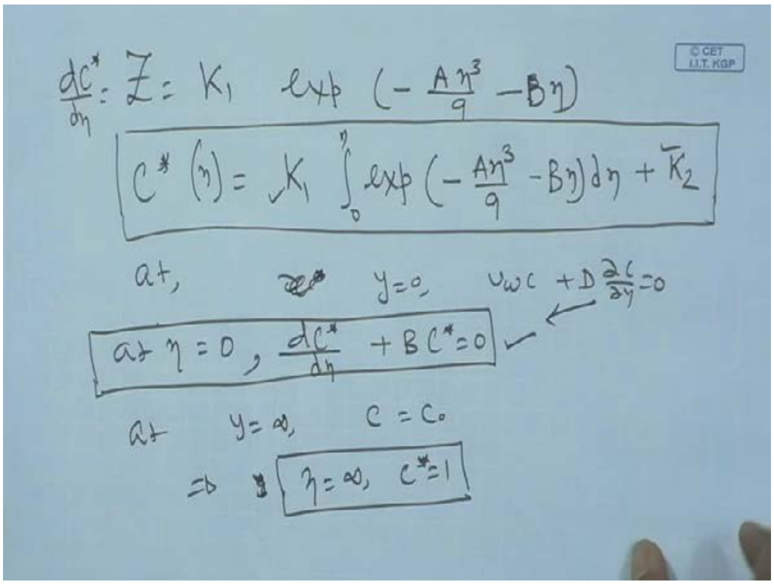

Now, we integrate it out, so, this becomes ln Z is equal to ln K 1 minus A by 3 eta cube by 3 plus B eta, therefore, this becomes Z is equal to K 1 exponential minus A eta cube by 9 minus B eta. Now, what is Z? Z is nothing but dC star d eta, so, one more integration will give you C star as a function of eta, this one will be 0 to eta exponential minus A eta cube by 9 minus B eta d eta plus another constant of integration that is K 2.

So, this gives the complete solution of concentration profile. Now this K 1 and K 2 have to be evaluated from the boundary condition. Let us look down the boundary condition, at x star is equal to 0, at y is equal to 0 we had v w C plus D del C del y is equal to 0, in the non-dimensional form this becomes dC star d eta plus BC star is equal to 0, to bring this, so, this is at eta equal to 0, so, to get here,, to here just express del C del y in terms dC star d eta, and I omitted couple of steps here to get this equation; and at y is equal to infinity we had C is equal to C naught, so, at y star, at eta is equal to infinity we had C star is equal to 1, so this is boundary condition 1, and this is the second boundary condition at eta is equal to 0.

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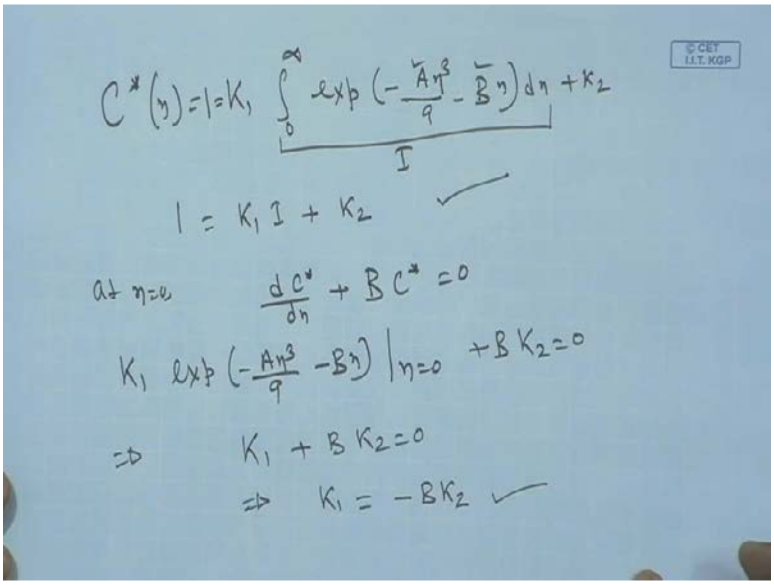

Now, with the help of these two one can evaluate the constants K 1 and K 2. So, if you do that, so, this become C star eta is equal to 1, so, this becomes K 1 0 to infinity exponential minus A eta cube by 9 minus B eta d eta plus K 2; so this is a definite integral because A is constant B is constant, so, we call this constant as I, so, 1 is equal to K 1 I plus K 2.

Now, if you put the other boundary condition that, dC star d it at eta is equal to 0, dC star d eta plus BC star is equal to 0, so, what is dC star d eta, dC star d eta is nothing but K 1 exponential minus A eta cube by 9 minus B eta.

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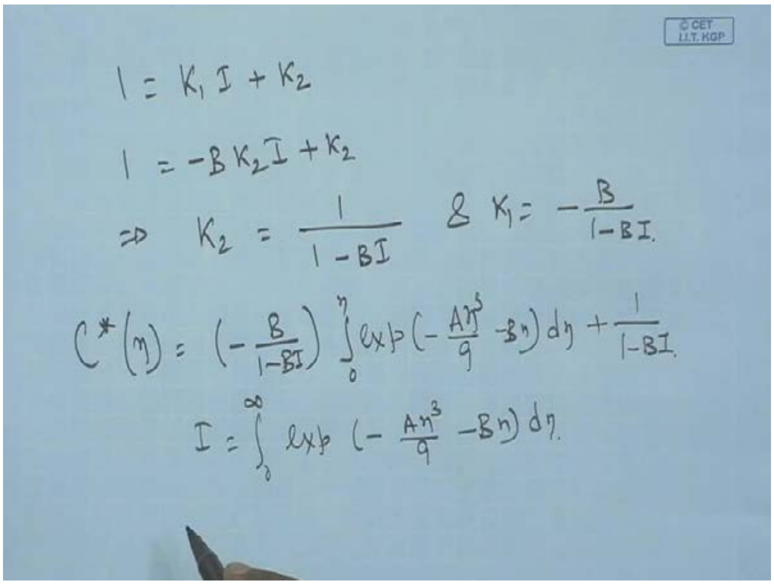

So, this will be K 1 exponential minus A eta cube by 9 minus B eta evaluated at eta is equal to 0 plus BC star evaluated at eta is equal to 0 that means, this will be 0 to 0, so, the first term will be gone, so, this will be B K 2 is equal to 0; and at eta is equal to 0 exponential 0 will be 1, so, K 1 plus B K 2 is equal to 0, so, we will be having K 1 is equal to minus B K 2. And now, we have two equations and two unknown in K 1 and K 2, you can solve them, so, if you can solve them, this becomes 1 is equal to K 1 I plus K 2, and put K 1 is equal to minus B K 2, so, this becomes minus B K 2 I plus K 2, so, you can get K 2 as an (( )) expression 1 minus BI, and K 1 is equal to minus B K 2, so, it will be minus B 1 minus BI; so, you get the total concentration profile as a function of eta, so, this becomes minus B 1 minus BI 0 to eta exponential minus A eta cube by 9 minus B eta d eta plus K 2, K 2 is nothing but 1 minus BI, where I is the definite integral 0 to infinity exponential minus A eta cube 9 minus B eta d eta.

So, once we know the value of B, we know the value of A, we can complete plot C star as a function of eta, one can also plot C star as a function of y and x by a combine variable because we know the form of eta is nothing but y star by x star to the power 1 upon 3; for a particular y value we know the value of eta as a function of x star, and we evaluate this profile in a lope of x star so you can plot C star as a function of x star. Similarly, if you would like to plot the C star as a function of y star at fixed value of x star one can do that as well the same way we have done for the profile of y star.

So therefore by using similarity solution method one can solve the partial differential equation by boiling into ordinary differential equation, and the solution of ordinary differential equation is very simple. So I stop the whole, this class at this point. In the next class I will take up one more solution technique partial differential equation that is the integral method of solution. Thank you very much.