Advanced Mathematical Techniques in Chemical Engineering

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Module No. # 01
Lecture No. # 34

Solution of non-homogeneous Parabolic PDE

Good afternoon, everyone! So, we are starting this class with the solution of non-homogeneous partial differential equation using the Green's function method. In the earlier class we have developed the theory of Green's function method, and we looked into several steps of a solution. First, we construct the causal Green's function, then we will be looking into the adjoint Green's function, then we connect the adjoint Green's function with the original problem.

Now, we develop the various methods, how to obtain the Green's function depending on the source of the point where we are applying the unit impulse or unit step function, and we will be having a lower half solution and the upper half solution; and we have seen several conditions which will be coming out, for example, continuity of Green's function, jump discontinuity condition and the actual boundary conditions prevalent at that two boundaries, so, we will be having four constants, and four conditions to evaluate these four constants.

So, we are able to completely define the Green's function. Once we defined the Green's function, then we looked into, took up an actual problem in ordinary differential equation and demonstrated our method, how to utilize this method, how to solve this problem with the Green's function method.

Next we took up another problem, which was basically an ordinary differential equation; there is non-homogeneity in the governing equation as well as the non-homogeneity in the boundary condition.

Again, we solve that problem almost completely by using Green's function method, to demonstrate our Green's function method in order solve the ordinary differential equation.

However, the Green's function method need not to be used for the solution of ordinary differential equation which are non-homogenous, there are more elegant and direct methods available, but we have just taken these methods these examples in order to demonstrate. Now, these methods become very useful and handy whenever we will be moving into the partial differential equations.

So, first example I will be talking about, I will be talking about a parabolic partial differential equation which is non-homogeneous in nature, and these equations are quite common in chemical engineering applications because they represent a transient state.

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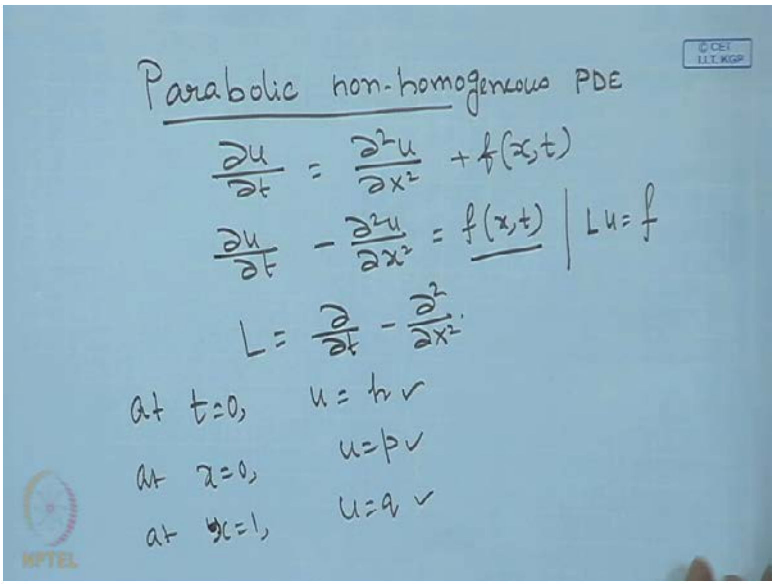

So, we will formulate this problem, go through the theory develop it and try to solve this problem completely. So, the governing equation of this problem is, del u del t is equal to del square u del x square plus some non-homogeneous term, which is a function of x and t both; so, del u del t minus del square u del x square is equal to f of x and t, so, this is in the form of L u is equal to f, where the operator L is del del t minus del square del x square.

On the other hand, the boundary conditions and initial conditions defined at t is equal to 0, u equal to h- it may be a function of x or it may be a constant; at x is equal to 0, u is equal to p; at x is equal to 1, let us say, u is equal to q.

So, we consider the problem completely, non-homogeneous, non homogeneity at the governing equation, non-homogeneity at the boundary conditions as well. So, there are four sources of non-homogeneity in this problem.

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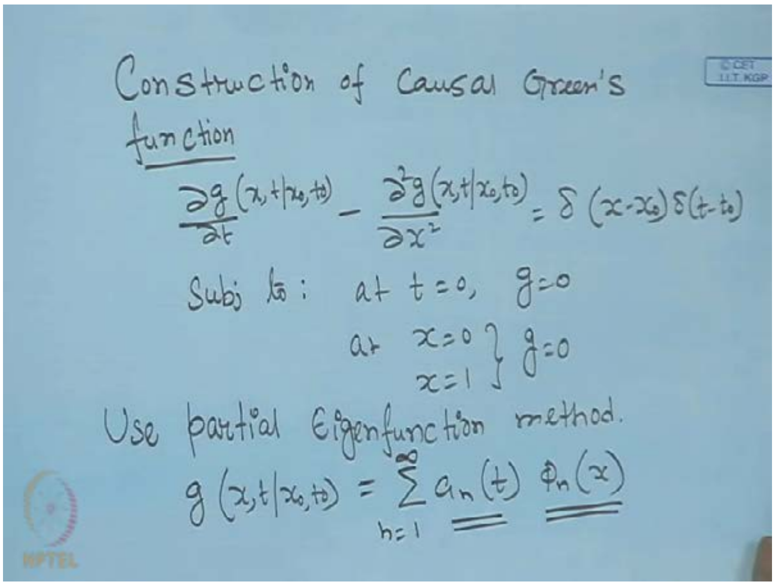

So, if you remember, the first step construction of causal Green's function, so, we do that construction of causal Green's function…So, what is the construction? So it is a two-dimensional problem; so, we used to have a unit impulse at the location x naught t naught that is number one.

Secondly, the unit impulse, the non-homogeneous term in the governing equation must be replaced by the unit impulse. So, therefore, the construction is del g x t x 0 t 0 del t minus del square g x t x 0 t 0 del x square is equal to delta x minus x naught delta t minus t naught.

And we forced all the boundary conditions of the original problem to be homogeneous. So, that was the condition of formulation of Green's function- at t equal to 0 my g is equal to 0; at x is equal to 0; and at x is equal to 1 our g is equal to 0.

Since we had initial conditions as Dirichlet, boundary conditions as Dirichlet boundary conditions, but non-homogeneous in the original problem, the boundary conditions are made to be homogeneous- the initial condition was non-homogeneous, so it has to be made homogeneous as well.

Now, what we do? We use partial Eigenfunction method to solve this problem. What is the partial Eigenfunction method?

The partial Eigenfunction method is that, the solution, that means g, the solution, g, is supposed to be it is, it is a function of a standard Eigenvalue problem, because this boundary conditions are homogeneous, so, this function can be expressed as a linear combination of eigenfunctions that is present in the corresponding eigenvalue problem. So, therefore, g x t x naught t naught is a function of a n, which is a function of time, and phi n which will be function of space; so, n is equal to 1 to infinity, this is called an Eigenfunction method.

So, what a phi n are? Phi n are the eigenfunction corresponding to the eigenvalue problem. And what are the corresponding coefficients? The corresponding coefficients are basically some kind of coefficients which are a function of time, then only g will be function of x and time.

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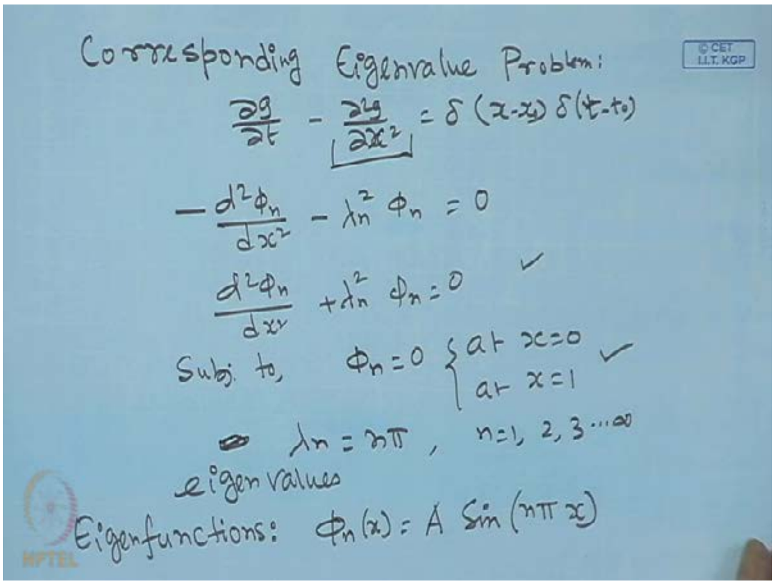

So, let us formulate the corresponding eigenvalue problem. So, corresponding eigenvalue problem now becomes, if you look into the governing equation del g del t minus del square g del x square is equal to delta x minus x naught delta t minus t naught; so, if you look into the space boundary conditions in space, it has 2 boundary conditions and these two boundary conditions are homogeneous, so, this corresponding eigenvalue problem is nothing but minus d square phi n d x square minus lambda n square phi n is equal to 0.

So, this is the corresponding eigenvalue problem; so, d square phi n d x square plus lambda n square phi n is equal to 0, and the boundary conditions must be satisfied by the boundary condition of the original problem; the original problem is basically g problem- if you look into the boundary conditions of g, both the boundary conditions are homogeneous. Therefore, we have phi n is equal to 0 both at x is equal to 0 and at x is equal to 1.

So, we know the solution of this problem, the eigenvalues are n pi- so, these are the eigenvalues n is equal to 1, 2, 3 up to infinity; and eigenfunctions are sin functions- so these eigenfunctions are a sin n pi x.

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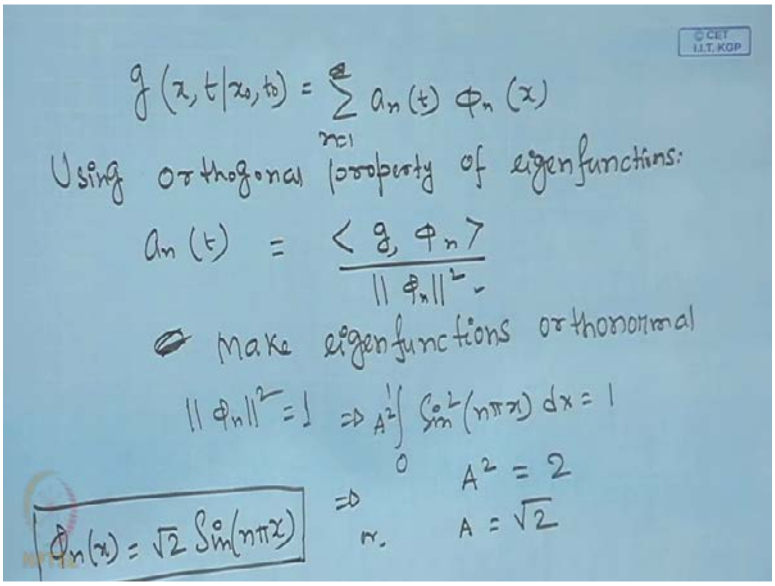

Therefore, if we look into the solution of g, which is constructed by partial eigenfunction expansion method, this will be nothing but n is equal to 1 to infinity a n t times phi n x. So, we know that eigenfunctions are orthogonal to each other, so, using orthogonal property of the eigenfunctions, we can evaluate the constant a n; a n will be nothing but inner product of g and phi n divided by norm of phi n square.

So, that means what we have done here: we multiply both side by phi m x d x and integrate over the domain of x; in the right hand side, if you open up the summation series, you will be getting only one term that will be surviving, that will be phi n- when n is equal to m all the other terms will vanish because of the orthogonal property of the eigenfunctions sin functions. So, you will be getting a n into phi n norm of phi n on the right hand side and then you will be getting on the left hand side integration of g phi n d x, so, it will be inner product of g n phi n.

Now, what we do? We make the eigenfunctions orthonormal. So, we make eigenfunctions orthonormal. So, norm of phi n square should be is equal to 1, so, integral 0 to 1 sin square a square phi n square – so, there will be a, square is there, it is constant, so it will be taken out of the integral sign- sin square n pi x d x is equal to 1 makes it orthonormal.

Now, integral sin square n pi x d x is half, that we have already proved earlier, so, a square is equal to 2 or a is equal to root 2. So, therefore, the eigenfunctions are phi n x is equal to root over 2 sin n pi x. If we formulate the eigenfunctions like this, then the eigenfunctions phi n becomes orthonormal, and the denominator of this equation will be is equal to 1.

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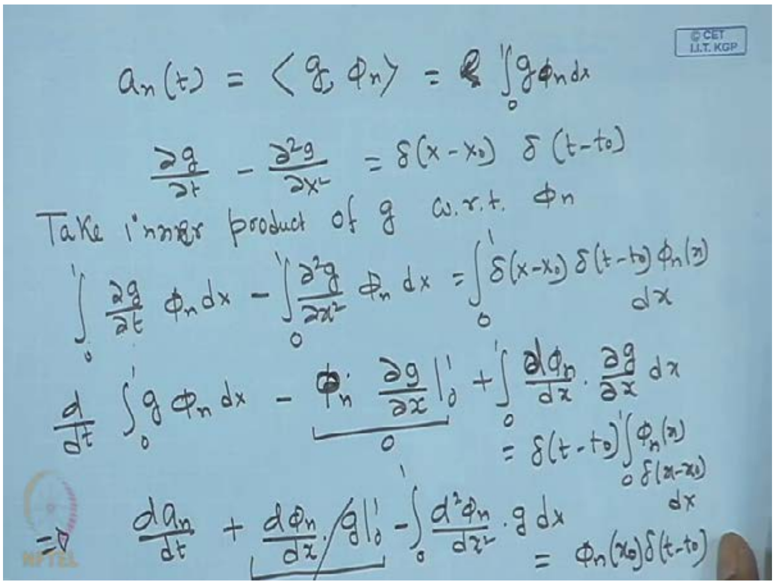

So, therefore, we will be getting a n, which is a function of time, is equal to inner product of g and phi n.

Next, we write del g del t minus del square g del x square delta x minus x naught delta t minus t naught, the governing equation of g, we take inner product of this equation of g with respect to phi n.

So, we take integral del g del t phi n d x minus del square g del x square phi n d x is equal to integral delta x minus x naught delta t minus t naught phi n x d x, from domain of x 0 to 1, 0 to 1.

So, we, since this derivative with respect to space, this respect to del g del t, this derivative with respect to time and this integral with respect to space, so, I can do the first integration and then I can take the differentiation- absolutely no problem, because derivative is not with respect to x, it is with respect to t whereas, the integration is with respect to space, that is x. So, I take del del t out; so, when you bring it outside, this becomes d d t, because integral 0 g dot phi n d x that will be independent of x after putting the different integral from 0 to 1.

So, this becomes d d t of integration g phi n d x minus, we do it by parts, first function, we take the first function as phi n, integration of second function del g del x from 0 to 1, minus minus plus, differential of first function, d phi n d x, multiplied by integration of the second function, del g del x d x.

So, since phi n is a sole function of x therefore, this differential with respect to x becomes a total differentiation, and del g del x remain as it is, then we write 0 to 1.We integrate this out; so, this integration is with respect to x, so, delta t minus t naught can be taken as common, and it will be out of the integral side, so, it is becomes phi n x delta x minus x naught d x 0 to 1.

So, integration of g phi n is nothing but inner product of g n phi n, so, this will be nothing but integration g phi n d x over 0 to 1. So integration of g phi n is nothing but a n t; so, since a n is a sole function of t, so, we will be having d a n d t minus- phi n at x is equal to 1 is 0 and phi n at x is equal to 0 is also 0, because the boundary condition on phi n and homogeneous therefore, after putting the limits we will be getting a 0 contribution out of this, then again, we divide this whole thing becomes 0, we do integration by parts once again, first function integration of second function- so, this plus first function integration of the second function from 0 to 1 minus differential of first function, that is, d square phi n d x square integration of second function g d x from 0 to,1 so, this becomes integral phi n x delta x minus x 0, this becomes phi n x 0 delta t minus t naught.

Now, let us see what we get? If we open up this part of the bilinear concomitant, we find phi n prime at 1 g at 1 minus phi n prime at 0 g at 0, but g is having homogeneous boundary condition both at x is equal to 1 and x equal to 0; so this part will also be equal to 0.

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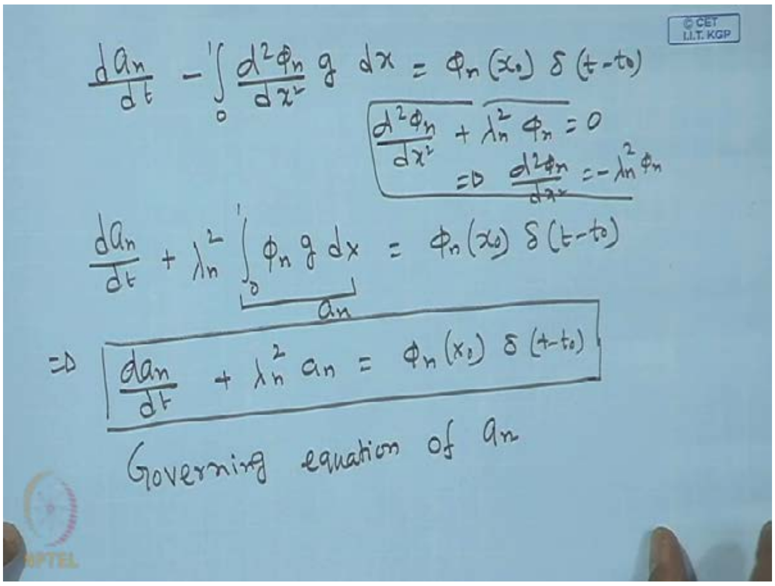

And, so, what will be getting ultimately out of this equation is, del d a n d t minus d square phi n d x square times g d x 0 to 1 is equal to phi n x naught delta t minus t naught; and d square phi n d x square plus lambda n square phi n will be is equal to 0, so, d square phi n d x square is nothing but minus lambda n square phi n. We utilize this in the governing equation. This becomes d a n d t minus, we put, minus minus plus, so, lambda n square, it is a constant, it will be out, so, it will be integral 0 to 1 d square phi n replace phi n, so, integral phi n, and g d x is equal to phi n x naught delta t minus t naught.

So, therefore, again, integral of phi n g is nothing but a n, so, d a n d t plus lambda n square a n is equal to phi n x naught delta t minus t naught; see the development, these gives the governing equation of the coefficient that we have defined whenever, we define the Green’s function g. So this gives the governing equation of coefficient a n, which is a function of time.

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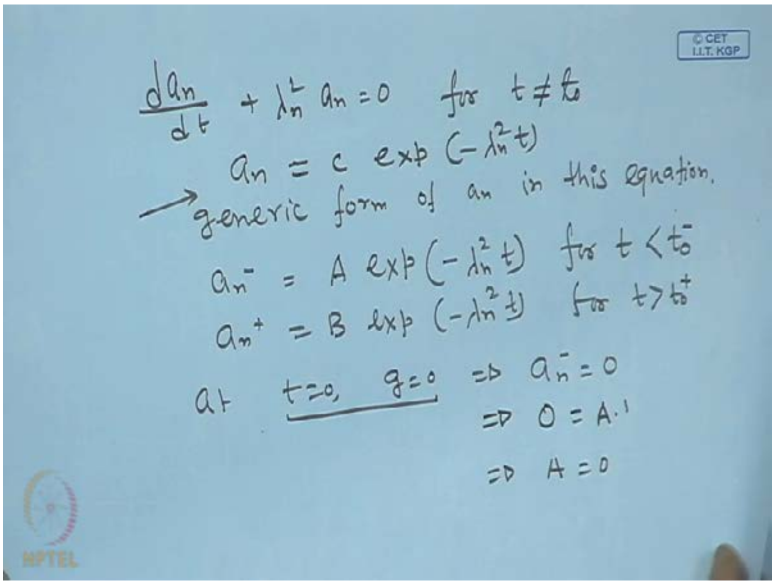

Next, we solve for a n. If you look the way we have done earlier, d a n d t plus lambda n square a n is equal to 0, for any value of t which is not equal to t naught; so, except that point, d a n d t plus lambda n square n equal to t.

Therefore, the solution of this equation will be is equal to- so, the Dirac delta function, the right hand side will be equal to 0 for any value except the point of application of unit impulse, that is, t is equal to t naught- so, a n will be having a generic form c exponential minus lambda n square t; so, this is a generic form of solution a n in this equation.

(()) a n minus becomes a exponential, so, you can have the lower half and upper half solution, minus lambda n square t for t less than t naught minus, and is equal to b exponential minus lambda n square t for t greater than t naught plus. So, we will be having a lower half solution, and this is having a upper half solution, the lower half is less than t naught, the upper half is above of t naught, at t is equal to 0, g is equal to 0, so, therefore, a n minus is equal to 0; this is the condition on g, so lower half of solution must be satisfying that- so, a n should be equal to 0 in order to make g is equal to 0.

So, what is left is, a n minus is equal to 0; if a n minus is equal to 0, what is left is, this is 0 and a e to the power put t is equal to 0, so, it will be 1, so, therefore a is equal to 0.

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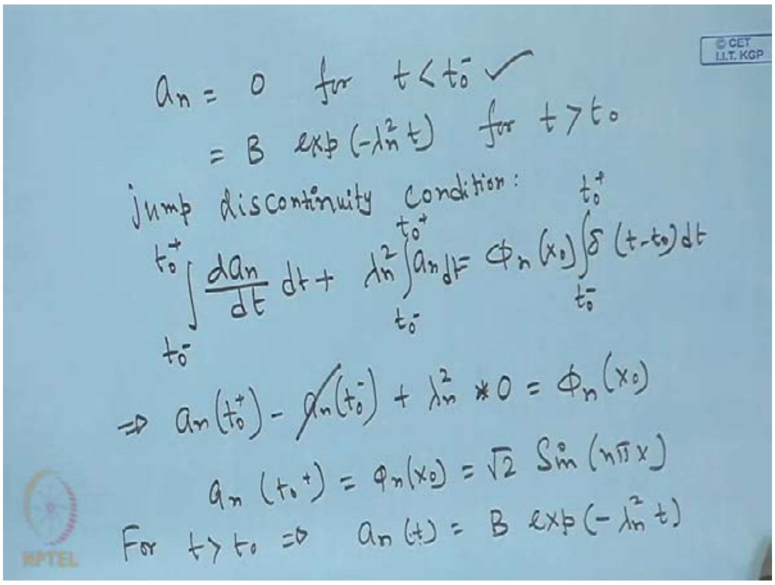

So, we can get an expression of a n now. The expression of a n is, a n is equal to 0 for t less than t 0 minus, and this is equal to b exponential minus lambda n square t for t greater than t 0.

Now, we use the jump discontinuity function condition. So, by jump discontinuity condition, let us evaluate it the governing equation, d a n d t plus lambda n square a n is equal to phi n x naught delta t minus t naught; so, that was the jump discontinuity, that was the definition of the original problem. Now, we multiply both side by d t and integrate over across the point of application of impulse t 0 minus to t 0 plus.

So, this becomes a n t 0 plus minus a n t 0 minus plus lambda n square- a n is a some function of t, after multiplying with d t, it will be again be some function of t, that function will be evaluated as t 0 plus and t 0 minus, where it will be t 0 plus epsilon and t 0 minus epsilon ,and when epsilon tends to 0, you will be basically getting same value minus same value, after putting the boundary limits- so, this will be equal to 0, is equal to phi n x naught delta t minus t 0 d t integral over t 0 minus to t 0 plus, it will be having a value 1 because it is unit impulse, and a n t 0 minus, we have already seen this is equal to 0, so, this value will be equal to 0; so, a n t 0 plus should be is equal to phi n x naught, so, that will be root over 2 sin n pi x. So, for t greater than t 0, we have already seen that, a n t is nothing but b exponential minus lambda n square t; so, therefore, we combine these two equation and see what we get.

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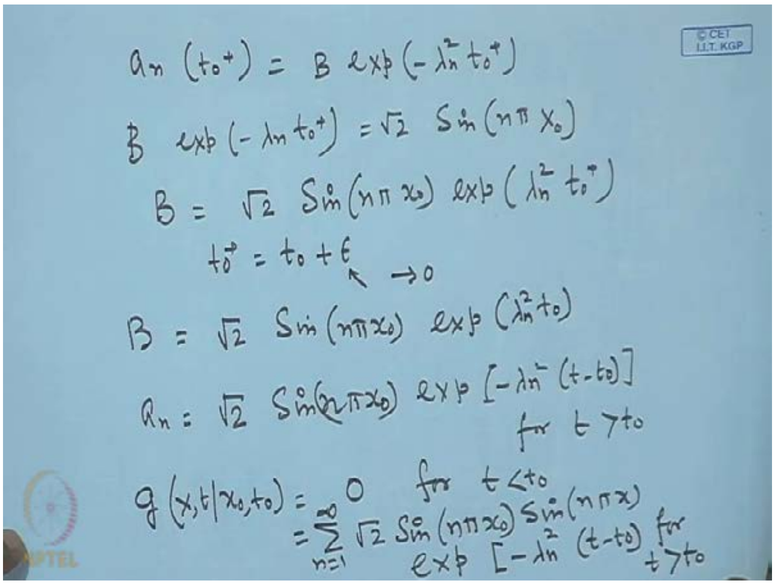

By combining these two equation we will be getting, a n t 0 plus is equal to b exponential minus lambda n square t 0 plus, so, we will be getting, b exponential minus lambda n t 0 plus is equal to root over 2 sin n pi x naught.

So, we equate these two, and whatever we will be getting is that, we will be getting the constant B now. B will be root over 2 sin n pi x naught exponential lambda n square t 0 plus, and if we put t 0 plus is equal to t 0 plus epsilon, where epsilon tends to 0, then b becomes root over 2 sin n pi x naught exponential lambda n square t 0.

So, a n will be nothing but root over 2 sin n pi x naught exponential minus lambda n square t minus t naught for t greater than t 0; and therefore, now, we are in a position to write down the expression of Green's function.

We write down the expression of Green's function as: g x t x naught t naught is equal to 0 for t less than t naught; and this is equal to summation n is equal to 1 to infinity root over 2 sin n pi x naught sin n pi x exponential minus lambda n square t minus t naught for t greater than t naught.

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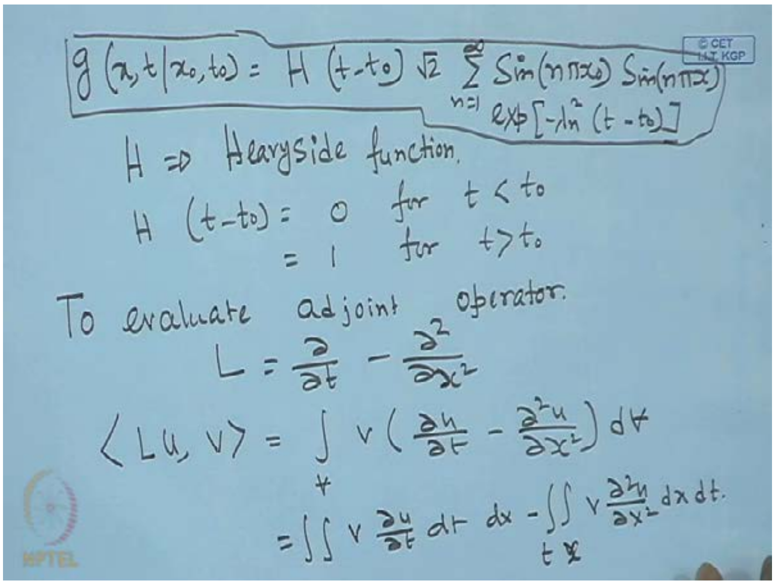

So, we write it down, the value, the expression of g in a more compact form. If we write it down in a more compact form, this becomes g x t x naught t naught is equal to H t minus t naught root over 2 summation sin n pi x naught sin n pi x exponential minus lambda n square t minus t naught, where n is equal to 1 to infinity.

So, where H is the heavy side function, and it is definition is, H t minus t 0 is equal to 0 for t less than t naught, and this is equal to 1 for t greater than t naught. So, that is the definition of the heavy side function, and this gives the solution of Green's function.

So, we get the complete solution of Green's function for the parabolic operator. Next we check whether the adjoint operator is same as the operator we are dealing with or it is different, and if it is different, what is that? So, next step is to evaluate adjoint operator; if you look into the operator, this will be del del t minus del square del x square.

So, let us find out what the operator is; inner product, same way we have done earlier in the case of ordinary differential equation, so, inner product of L u and v will be nothing but integration over volume v del u del t minus del square u del x square d v; so, it will be double integration- because this d v is, it is a two-dimensional problem, it will d t d x, so, it will be double integration one over t another over x- so, it will be v del u del t d t d x minus integration over t integration over x, so, this becomes v del square u del x square d x d t. So, we carry out the integration by parts.

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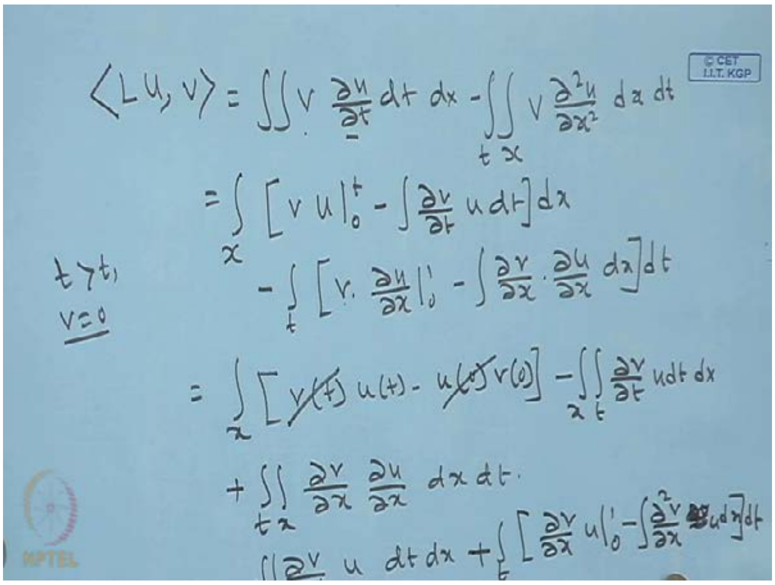

So, inner product of L u and v will be, let us write it down, v del u del t minus d t d x minus double integral one over t another over x v del square u del x square d x d t. So, first function, second function.

So, integration over x remaining first function integral of the second function over 0 to t minus differential of first function del v del t and integration of the second function, that is u d t then d x minus integration over t first function differential of second function del u del x 0 to 1 minus differential of first function integration of the second function del u del x d x d t.

So, what we are getting that, at v is equal to, for t greater than t naught, we have already obtained that t greater than t 1 v is equal to 0, so, we put it, v t u at t minus u at 0 v at 0, that is, 1 minus integration over x over t del v del t u d t d x, that is, the first one minus integration over t- now, since we have the homogeneous boundary condition, so, v at x is equal to 1, v at x is equal to 0 both of them will be 0, so, these will be 1, so, this will be contributing 0- so, minus minus plus, you will be having, del v del x del u del x d x d t, and in this case the homogeneous boundary condition at t is equal to 0, so, u will be is equal to 0, and v is equal to 0 for any t greater than 0, so, this term also be off.

So what will be having is, minus double integral del v del t u d t d x, and one more integration here, first function integration of the second function plus first function del v del x integration of second function u from 0 to 1 minus differential of second function, that is, del square v del x square integration of the first one that will be u d x, then d t.

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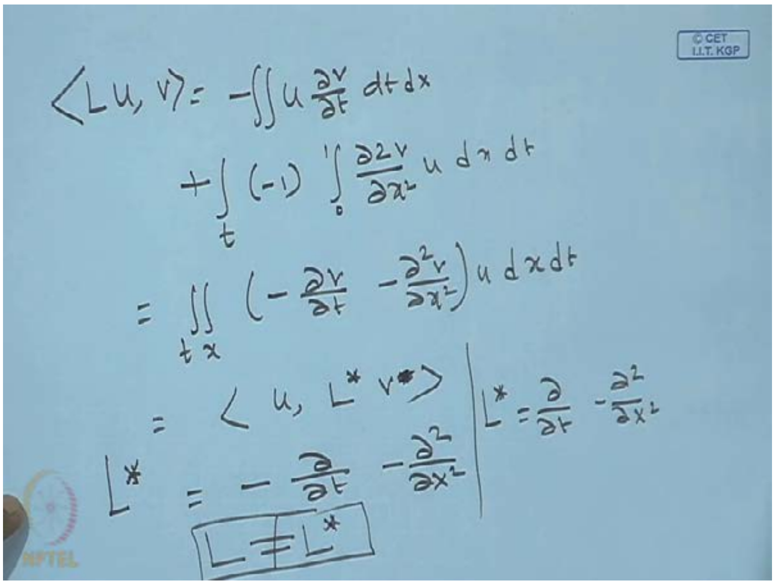

So, let us see what we get. We get L u and v should be is equal to minus double integral u del v del t d t d x plus double integral integration over t and then u at x is equal to 1 and u at x is equal to 0 will be 0 so there will be a minus sign there and it becomes integral del 0 to 1 del square v del x square times u d x d t

So, ultimately, what we will be obtaining is that double integral 1 over t another over x minus del v del t minus del square v del x square u d x d t

So, you will be getting inner product of u L star v star so if you L star v u L star v so if you look into the L star the adjoint operator is minus del del t minus del square del x square and if you see the actual operator the original operator it was del del t minus del square del x square

So, therefore, L is not is equal to L star, so, L is not equal to L star, and it, the problem is not a self adjoint problem. So, therefore, we have to evaluate the adjoint Green's function, then connect the adjoint Green's function with the original problem, u.

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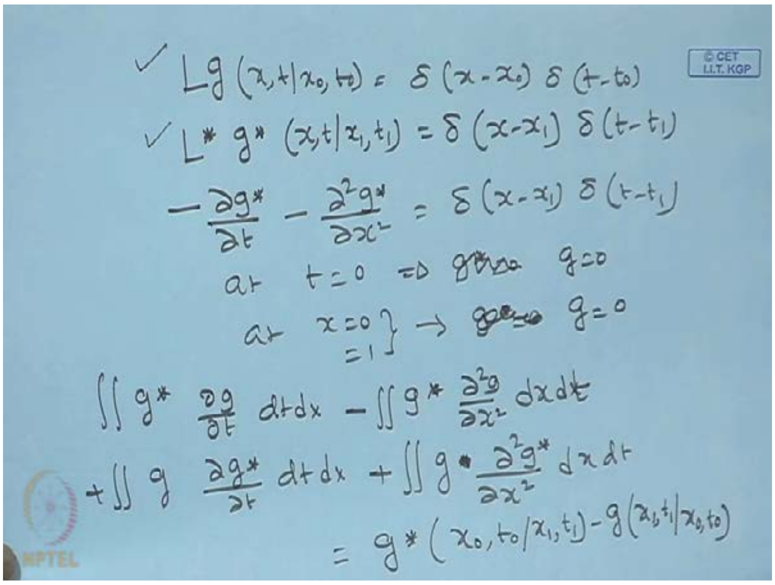

So, next step is obtaining the adjoint Green’s function. This was the original Green’s function, casual Green’s function; so, this will be delta x minus x naught delta t minus t naught and adjoint Green’s function is L star g star x t x naught x 1 t 1 is equal to delta x minus x 1 delta t minus t 1.

So, if you write down the adjoint operator, L star, this becomes minus del g star del t minus del square g star del x square is equal to delta x minus x 1 delta t minus t 1, and at t is equal to 0, and g star is equal to 0 at x is equal to 0, and 1 your g star is equal to 0. Similarly, so, these are the conditions, this is the condition on g, at least on g.

We know these are the conditions on g, now, let us find out the conditions on g star. For that what we have to do? We have to take the inner product with this equation with respect to g star, we have to take the inner product of this equation with respect to g and then we will subtract, if you do that what will be getting is that, inner product double integral g star del g del t d t d x minus double integral g star del square g del x square d x d t plus, minus minus plus, so, it will be, g del g star del t d t d x , double integral g del square g star del x square d x d t, and this will be is equal to g star x naught t naught slash x 1 t 1 minus g x 1 t 1 slash x 0 t 0.

So, if we do that, we combined these two part and combined these two part, we have already seen, if we differentiate with respect to, if we integrate this thing by parts, then we will be getting the term that will be equal to 0. Similarly, if we evaluate this integration by parts again, we will be getting a 0.

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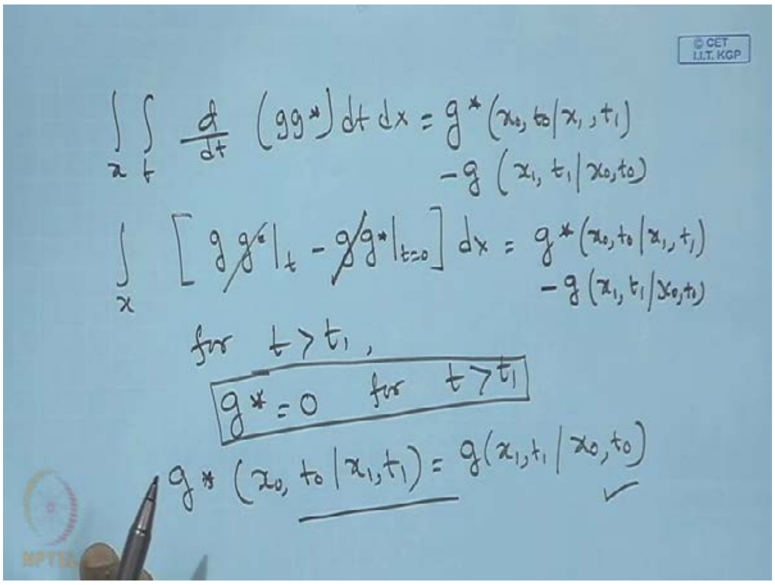

So, ultimately, whatever we will be getting is double integral x and t d d t of g and g star, when you combine these two, d t d x will be nothing but g star x naught t naught x 1 t 1 minus g x 1 t 1 x 0 t 0. Now, integration over x, which is g g star at t minus g g star at t is equal to 0 d x that should be is equal to g star x 0 t 0 x 1 t 1 minus g x 1 t 1 x 0 t 0.

So, we know that g at time t is equal to 0 equal to 0, so, this part will be gone, and to make this bilinear concomitant term to be 0, we put for t greater than t 1 g star is equal to 0 for t greater than t 1.

If we have this boundary condition, then this will be gone and we will be getting the conventional nomenclature that is, g star x naught t naught slash x 1 t 1 should be equal to g x 1 t 1 slash x naught t naught.

Now, we are in a position, since we have obtained this relationship, and we have already obtained the expression of g, we can get the expression of g star- you need not solve the partial differential equation once again; if we do that what will be getting is that as follows:

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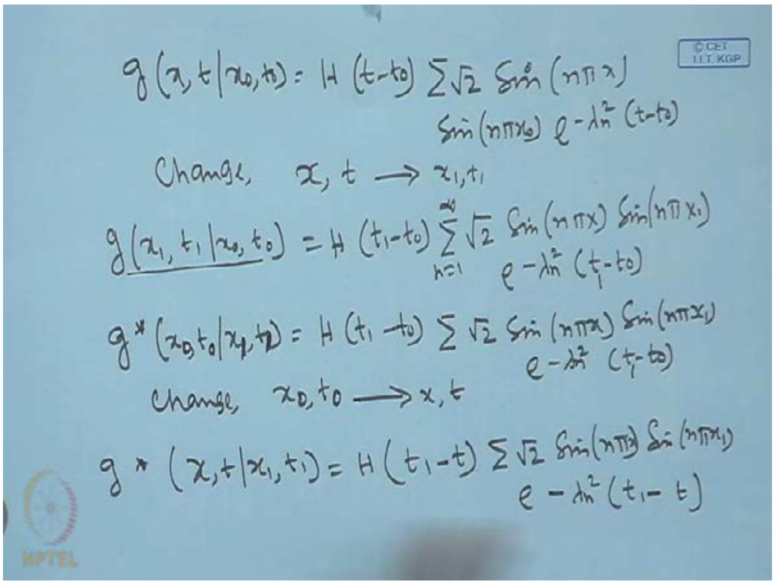

If you remember, the expression of g is equal to H t minus t naught summation root over 2 sin n pi x sin n pi x naught e to the power minus lambda n square t minus t naught.

Now, we change the nomenclature x t to x 1 t 1. So, this gives g x 1 t 1 slash x 0 t 0 is equal to H t 1 minus t 0 root 2 sin n pi x sin n pi x naught e to the power minus lambda n square t minus t naught n is equal to 1 to infinity; now, g x 1 t 1 x naught t naught is nothing but g star, so, this becomes g star x 1 t 1 slash x 0 t 0 is equal to H t 1 minus t 0 summation root 2 sin n pi x sin n pi x 1 e to the power minus lambda n square t minus t 0.

Now, what we do? We change- so, this is nothing but g star x 0 t 0, this is same as g star x 0 t 0 and x 1 t 1, so, we change the nomenclature x 1 t 1 to x and t; so, this becomes g star x 0 t 0 to x and t, so, we get g star x t x 1 t 1 H t 1 minus t summation root over 2 sin n pi x sin n pi x 1 e to the power minus lambda n square t 1 minus t, so, this becomes t 1

When you change x and t to x 1 t 1, this becomes t 1, this remains t 1, this remains t 1 and then we substitute, we replace t 0 by t. So, this is the governing equation.

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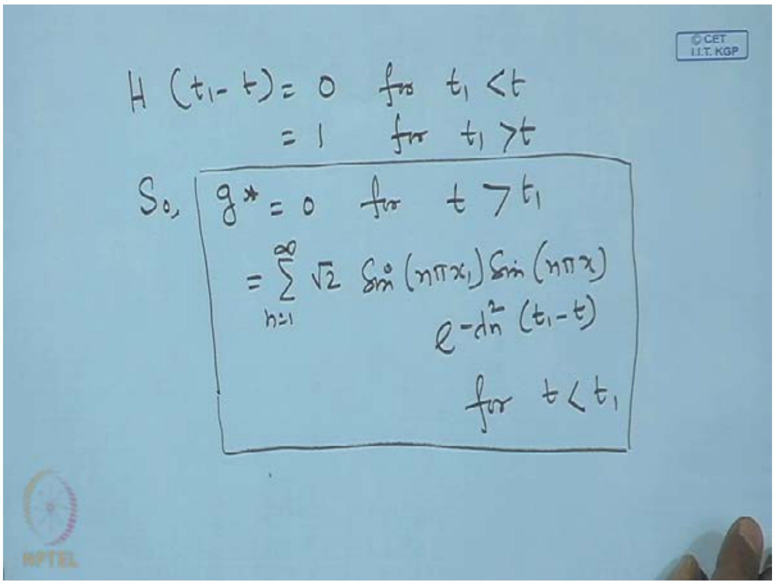

Now, if you remember, what is the definition of t 1 minus t heavy side function... so, the heavy side function will be like this: the heavy side H t 1 minus t is equal to 0 for t 1 less than t, and it is 1 for t 1 greater than t. So, g star is equal to 0 for t greater than t 1, and g star is equal to summation root over 2 n is equal to 1 to infinity sin n pi x 1 sin n pi x e to the power minus lambda n square t 1 minus t for t less than t 1.

So, that is the expression of the adjoint Green's function, we have derived in this expression, in this equation. So, if you remember, whatever we have done, we really did not solve the governing equation of g star, we solved the governing equation of g, and obtained the definition of expression of g star, then by utilizing the property, g is equal to g star, we change the subscripts of the variables x and t, and ultimately we got an expression of g star.

And we have seen for t greater than t 1, g star is equal to 0, and for t less than t 1, g star is not equal to 0, and that will be given by this expression root over 2 sin n pi x 1 sin n pi x e to the power minus lambda n square t 1 minus t.

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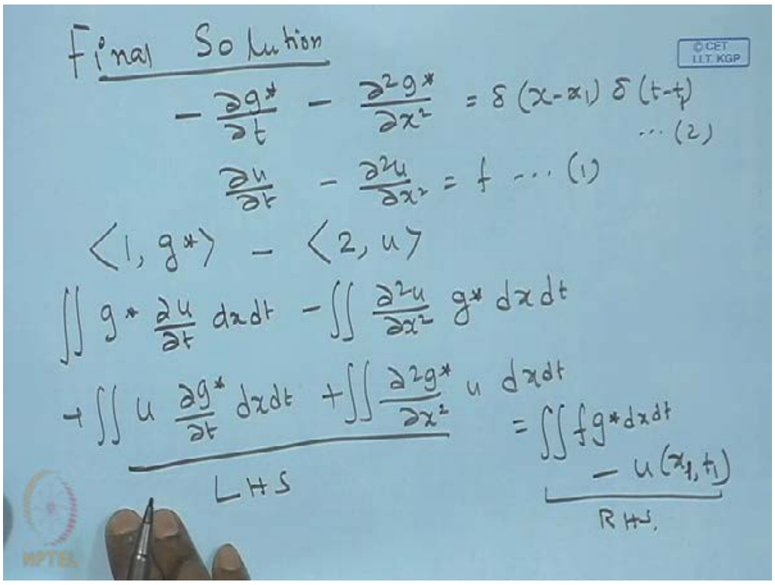

Now, what is left is, the left is, the connection of adjoint Green's function and the original problem u, by doing some inner product method that is required. So, let us look into the final solution; we are almost at the final step, the final solution is, let us write down the governing equation of, del g star, of adjoint Green's function.

So, this will be minus del g star del square g star del x square is equal to delta x minus x 1 delta t minus t 1- this is equation number two; and del u del t minus del square u del x square is equal to f- this is equation number on.

Then, what we do, we take inner product with g star and minus inner product of 2 with u and then subtract, so, if you do that what you will be getting is- g star del u del t d x d t minus double integral del square u del x square g star d x d t plus double integral u del g star del t d x d t, minus minus plus, so, it will be del square g star del x square u d x d t.

On the right hand side you will be having integration double integral f g star d x d t minus u as a function of x 1 t 1, because the it is integral of u delta x x minus 1 delta t minus delta x x minus 1 delta t minus t 1 and d x d t, so, it will be u of x 1 t 1.

So, therefore, what will be getting is, is this equation. Now, what we do, we look into the left hand side of the equation- so, this is the left hand side, this is the right hand side- if you look into the left hand side, let us see what we get out of this.

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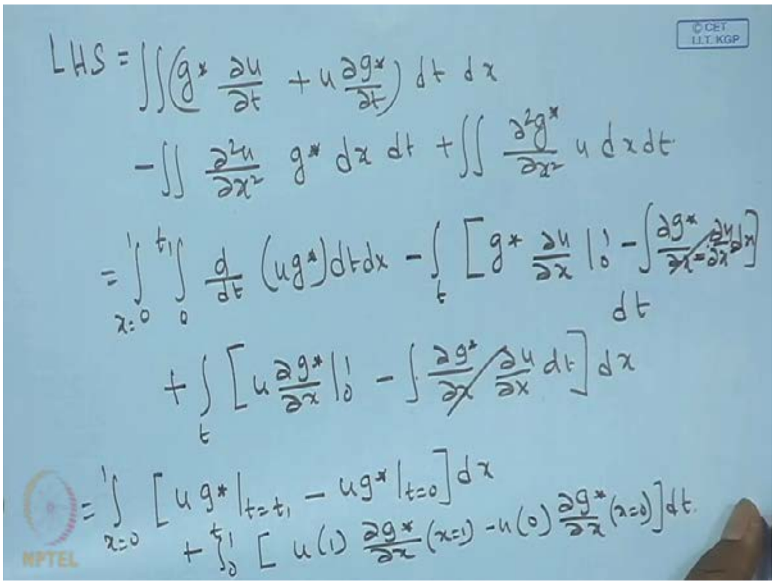

The left hand side becomes very complicated, this is inner product of double integral g star del u del t plus u del g star del t d t d x minus double integral del square u del x square g star d x d t plus del square g star del x square u d x d t

So, it will be 0 to 1 over x, and 0 to t 1 over t, and it will d d t of u g star d t d x- because g star, this limit over t is, to simply say that confirm g star is not equal to 0 for t less than t 1, and g star, anything above t 1 it will be equal to 0, therefore, we have to put the limit from 0 to t 1 because g star is not equal to 0 within that domain, it exists there- minus over t first function by integration by part first function integral of second 1 from 0 to 1 minus del g star del x star del u del x del g star del x del u del x , d x times d t plus we have integration over time, so, it becomes u del g star del x from 0 to 1 minus integration del g star del x del u del x d t d x; so, if you look into this equation this is minus minus plus del g star del x del u del x d x d t, and this is minus del g star del x del u del x d x d t; so, this will be plus, this will be minus, so these two will be cancelled out.

So, we have x is equal to 0 to 1 and then integration of this u g star evaluated at t is equal to t 1 minus, u g star evaluated at t is equal to 0 d x, that is the first part.

And now, we put the second one, plus 0 to t 1 u at 1 x is equal to 1 del g star del x at x is equal to 1 minus u at 0 x is equal to 0 del g star del x x equal to 0 bracket d t.

The other parts of bilinear concomitant like g star at 1, and g star 0 will be 0, and u at 1, and u at 0, they are not equal to 0, they are non-homogeneous; so, therefore, we put this term there.

So, now, let us see what we get out of this. And at t is equal to t 1, g star is equal to 0, for that this term will be off, so, whatever is left on the left hand side is this:

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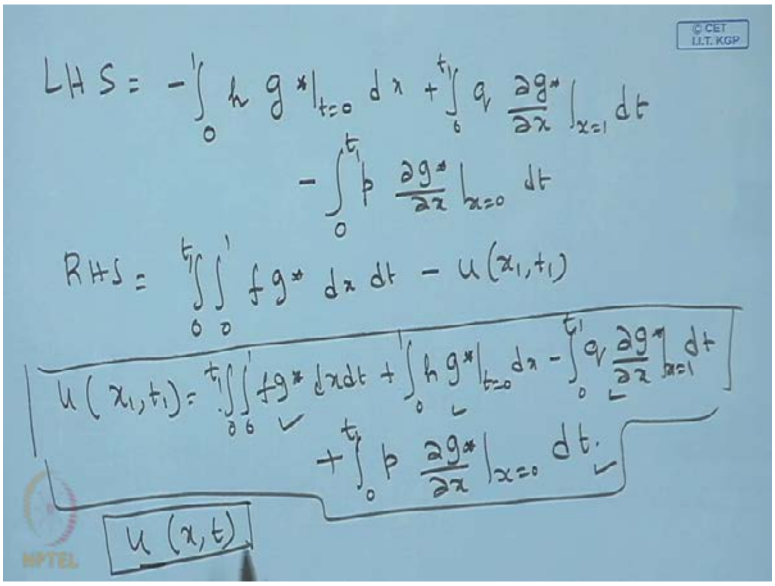

So, left hand side, we have minus 0 to 1 and g star at 0 to (( )), and u at 0 to u at 0 becomes h so h g star at t is equal to 0 d x plus 0 to t 1 q del g star del x x is equal to 1 d t minus integration p del g star del x at x is equal to 0 d t, so, this will be over 0 to t 1, this will be over 0 to t 1.

So, if you look into the right hand side, the right hand side is double integral 0 to t 0 to 1 0 to t 1 0 to 1 f g star d x d t is equal to minus u x 1 t 1.

So, now, if we take u on the right hand side, u becomes, u x 1 t 1 becomes 0 to t 1 0 to 1 f g star d x d t plus 0 to 1 h g star evaluated at t is equal to 0 d x minus 0 to t 1 q del g star del x x is equal to 1 d t, minus minus plus, integral 0 to t 1 p del g star del x at x is equal to 0 times d t.

And this h q and p, if they are constant, we just take them outside of the integral and carry out this integration. Now, if you see on the right hand, there are four terms, there are three sources of non-homogeneity in the governing equation u at t is equal to 0; so, this corresponds to that non-homogeneous term at x is equal to 0, u was equal to p, so, this correspond to that non-homogeneous term at x is equal to 1 u was equal to q.

So, this corresponds to that non-homogeneous term, and this non-homogeneous term occurs because of the non-homogeneous term appearing in the governing equation; if you see, there are two boundary conditions on x, those are non homogeneous, so, you have line integral over them, and they will be integrated over time.

There was one non-homogeneous term in the initial condition, so, this corresponds to that one line integral over x corresponding to the one non-homogeneous term in the initial condition, there was one non-homogeneous term in the governing equation, since it is valid throughout the whole control volume, we will have a double integral corresponding to the whole control volume. So, there will be double integration over f and g star in the whole control volume.

So, since there are four sources of non-homogeneity in the governing equation, one non-homogeneity the main equation and three non-homogeneities in the initial and boundary conditions, there are four terms appearing on the right hand side.

And next, we carry out this integration over t 1 and x, and we change the index from, the running index from, running variables x 1 t 1 to x and t, we will be getting the solution of u as a function of x and t.

So, I stop here in this class. In tomorrow's class, what I will do, I will really evaluate these integrals one after another and will obtain the complete solution of u as a function of x and t for a parabolic partial differential equations, which is non-homogeneous using the Green's function method. Thank you very much.