Advanced Mathematical Techniques in Chemical Engineering
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Lecture No. # 33
Solution of non-homogenous PDE (Contd.)

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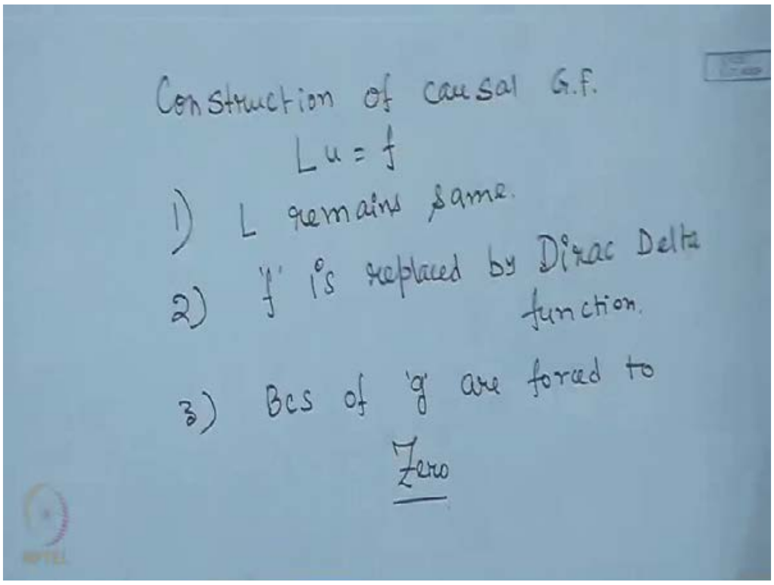

Welcome to the session. We are discussing about the construction of causal Green’s function. Now, if the actual problem is given as Lu is equal to f, in the case of construction of causal Green’s function, we have to keep 3 things in mind. One is the L remains same; second: f is replaced by a Dirac delta function or unit step and three is that boundary conditions of g, the causal Green’s function are forced to 0.

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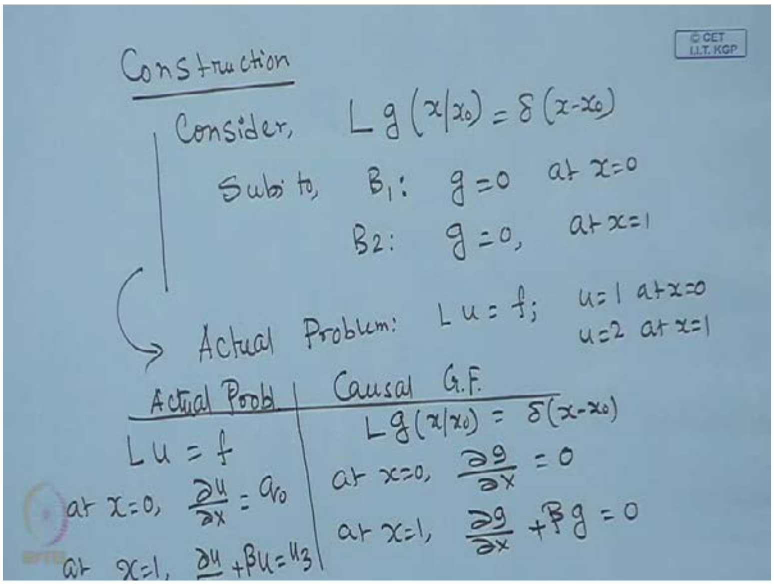

So, with this, let us go ahead with the formulation of construction of causal Green's function. Consider Lg x slash x naught is equal to delta x minus x naught. That is, L we will be obtaining from the actual problem. Lu is equal to f subject to boundary condition, B 1: g is equal to 0 at x is equal to 0, subject to boundary condition 2: that is, g is equal to 0 at x is equal to 1.

Now, we consider the actual problem is having the Dirichlet boundary condition at x is equal to 0 and x is equal to 1. That means, what is the actual problem? We can construct this causal Green's function, if we have the actual problem as something like this. L of u is equal to f u is equal 1; at x is equal to 0, u is equal to 2; at x is equal to 1, u may be 1. u may be homogeneous or u may be non-homogeneous, but they are Dirichlet boundary conditions. So, there are Dirichlet.

Now, I will give some of the example on how to construct. Now, this is the actual one - actual problem. This is the causal Green's function. In the actual problem, we have let us say Lu is equal to f; at x is equal to 0, del u del x is equal to let us say, q 0; at x is equal to 1, del u del x plus beta u is equal to u 3. If this is the actual problem, then causal Green's function should be Lg x slash x naught is equal to delta x minus x naught; at x is equal to 0, identical boundary condition. We force the boundary condition to be homogenous. So, del g del x is equal to 0. At x is equal to 1, we keep this form of boundary, but force it to be homogenous; that is all we do. So, del g del x plus beta g should be is equal to 0. So, we make them homogeneous.

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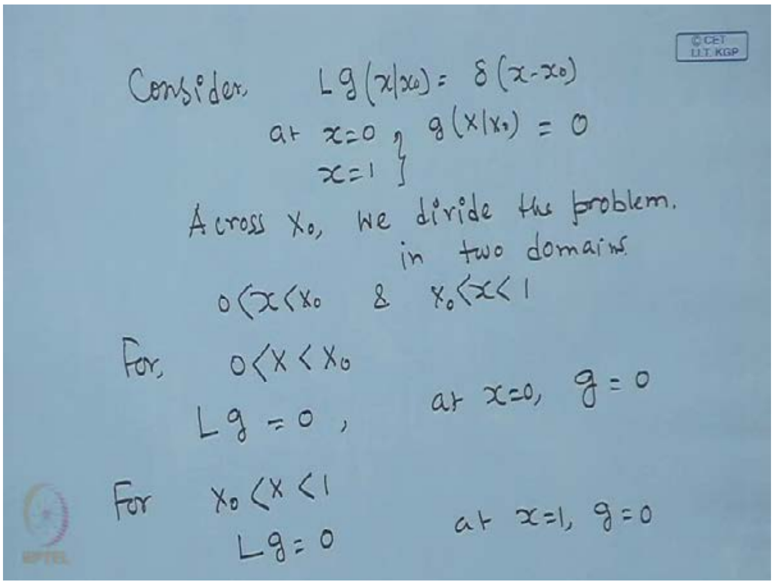

Now, let us typically formulate the problem. So, we consider Lg x slash x naught should be is equal to delta x minus x naught, subject to at x is equal to 0 and at x is equal to 1, g of x slash x naught; these are equal to 0.

Let us say, we have the Dirichlet boundary condition in the original problem. So, we divide the problem into 2 sub-parts. Across x naught - the point at which we are applying the unity impulse; across x naught, we divide the problem in 2 domains.

One is x lying between 0 and x naught and another is x lying in between x naught and 1. Now, for the domain of x lying in between 0 and x naught, we have Lg equal to 0 and boundary condition of this will be at x is equal to 0, we have g is equal to 0; for x lying in between x naught and 1, we have Lg is equal to 0; at x is equal 1 g is equal to 1.

Just see the formulation, Lg x slash x naught is equal to delta x minus x naught. That means your Dirac delta function is imposed at x is equal to x naught. If you look into the property of the Dirac delta function that at x is equal x naught, it is having a value 1. For every other x, which is not equal to x naught, its value is 0. So, that is the property of how Dirac delta function is defined at x is equal to x naught, its value is 1 - unit step function and every other value of x except x is equal to x naught, its value is 0.

So, the way, we have defined it is we have not included x is equal to x naught. We have included the lower domain x from 0 to x naught, which is less than x naught. It is not x naught; it is x naught minus x epsilon and is less than x naught.

So, the Dirac delta function in that domain will be equal to 0 because its value is 1 exactly at x is equal to x naught, but its value is equal to 0 everywhere else.

So, Lg is equal to 0 and we have the homogenous boundary condition at x is equal to 0, g is equal to 0. Similarly, the upper half solution between 1 and x naught, but we are not including x naught is less than equal to x. We are not including x is equal x naught; we are excluding x equal to x naught.

So, for the upper half solution, x lying between x naught and 1, the Dirac delta function is 0. So, we will be having Lg is equal to 0 and at x is equal to 1, we have the boundary condition at g is equal to 0.

Now typically, the x varying part is one will be landing up with an equation, which will be having order 2. So, for the order 2, we break down this problem into 4 sub-problems. Each of the sub-problem will be having a lower half solution; we call this a lower half solution; this is an upper half solution.

Since L is an operator of order 2, we need to specify 2 boundary conditions on Lg is equal 0 in order to solve this equation completely. On the other hand, even in the upper half plane, we need to have 2 boundary conditions to be used in order to solve this problem completely.

So, we need to have 4. Since we have 2 equations and 4 unknown, we need to have 4 boundary constant boundary conditions, but we have only 2 boundary conditions. So, we need to search 2 more boundary conditions and see what we get.

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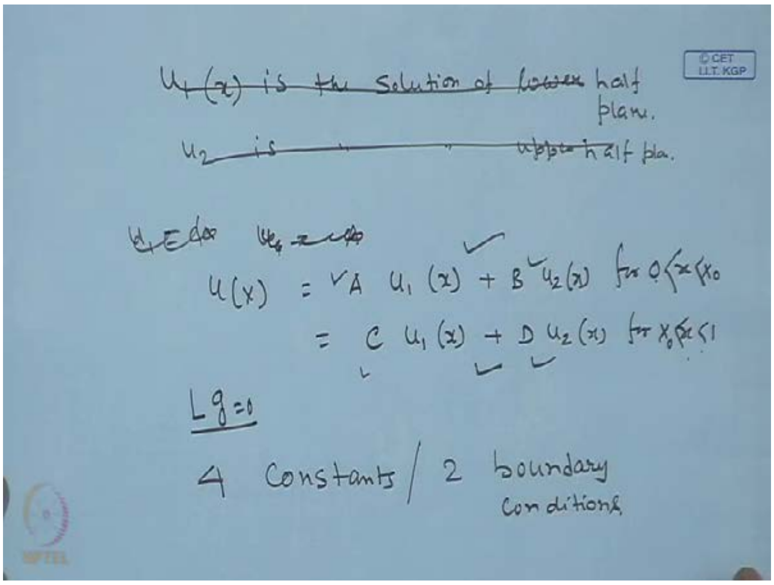

So, u 1 x is the solution of lower half plane and u 2 is the solution of upper half plane. Therefore, u 1 is nothing, but A some solution let us say, it will be. Only the u of x will be constructed of 2 solutions, A u 1 x plus B u 2 x, for x lying between 0 and x naught, is equal to C u 1 x plus D u 2 x, for x lying between x naught and 1.

This u 1 x is not the upper half solution; this is the lower half solution; this is the upper half solution and what is this u 1 and u 2? u 1 and u 2 are basically L of g is equal to 0.

So, the general solution, you will be getting 2 parts. u 1 and u 2 and this u 1 and u 2 will be related to u; only the constants will be varied.

So, therefore, you will be having 4 constants A, B, C, D and you need to have 4 boundary conditions to be specified. We have the boundary conditions specified at x is equal to 0; we have boundary condition specified at x is equal 1.

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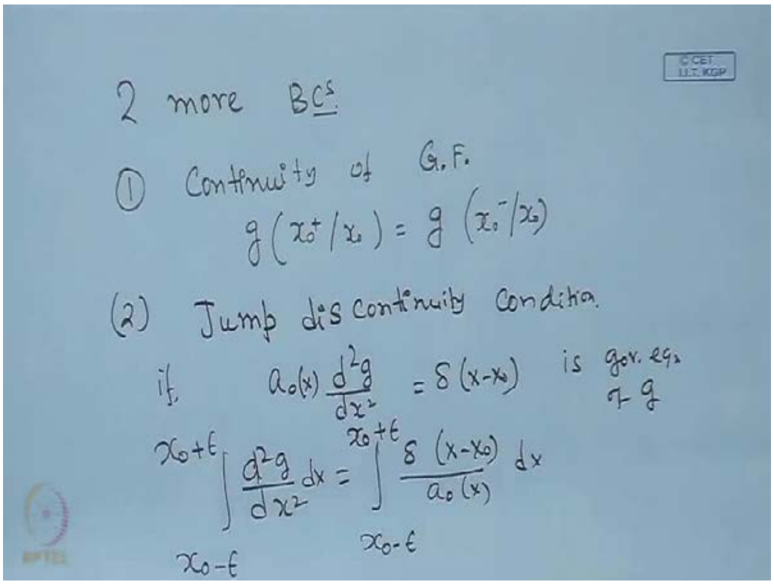

So, 4 constants and we have 2 boundary conditions. Anyway, I will be taking up one example to demonstrate whatever I am saying. Rest 2 boundary conditions, we need to find out. So, 2 more boundary conditions, we need to find out. The first boundary condition will be - first extra boundary condition will be continuity of Green's function. This will be g x naught plus slash x naught is equal to g x naught minus slash x naught.

Across x naught, Green's function is continuous. Second one is called a jump discontinuity condition. In this case, if a 0 d square g dx square is delta x minus x naught is the governing equation for g, where a 0 can be in general function of x, then let us integrate across x naught minus epsilon to x naught plus epsilon.

We multiply both side by dx and integrate it out. So, d square g dx square is nothing, but delta x minus x naught divided by a 0 as a function of x. Multiply both side by dx and integrate across x naught - so, x naught minus epsilon to x naught plus epsilon and here also, x naught minus epsilon to x naught plus epsilon.

So, if you carry out this integration, what you will be getting is that, the right hand side is very simple. We use the shifting property of the Dirac delta function or the unit step function. It will be integration of f of x multiplied by delta x minus x naught dx across x naught is nothing, but f of x naught.

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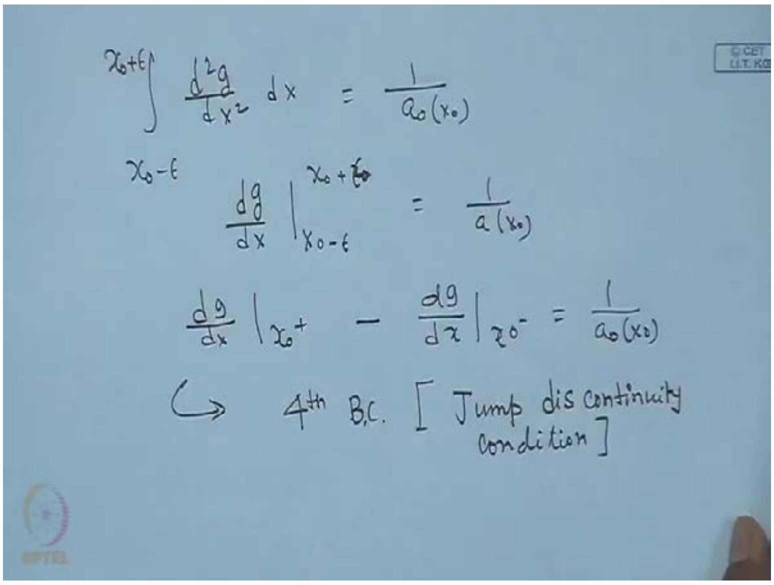

So, what we will be getting is d square g dx square dx from x naught minus epsilon to x naught plus epsilon is equal to 1 over a 0 x naught.

Now, integration of this will be giving you dg dx from x naught minus epsilon to x naught plus epsilon is equal to 1 over a 0 x naught. So, dg dx evaluated at x naught plus minus dg dx evaluated at x naught minus is equal 1 over a 0 x naught.

This gives the fourth boundary condition and also this is known as jump discontinuity condition. that means How will you evaluate?

So, we will be having 2 solutions of g: one is the lower half solution; another is the upper half solution. del g del x evaluated at x naught plus means, we evaluate the upper half solution and evaluate it at x naught plus epsilon and making epsilon tends to 0.

On the other hand, this simply means, we have the lower half solution, we take the x derivative of lower half solution, evaluate it at x naught minus epsilon, and then allow epsilon to tend to 0.

We will be computing this term and that will be equal to 1 over a 0 x naught. So, this will be giving you one more equation in order to solve the fourth boundary condition.

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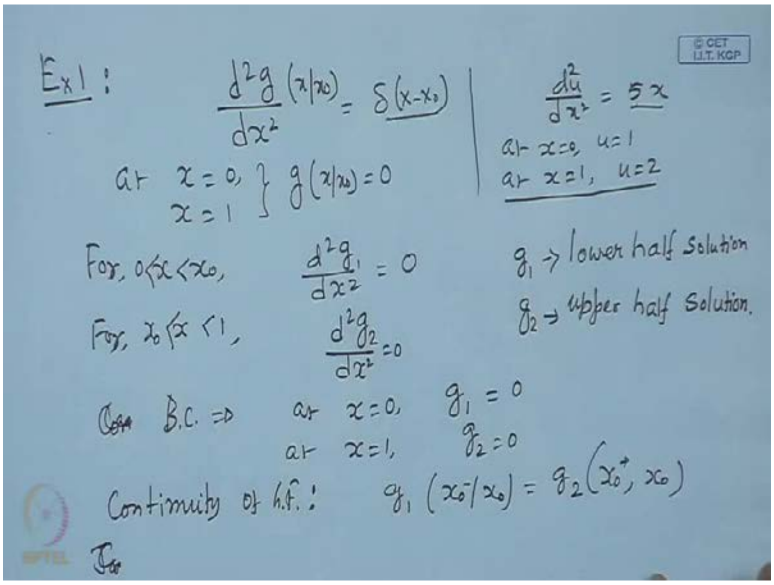

Let us take up one example and demonstrate how to get the Green's function. The first example, we will be taking is d square g. So, it is 1-dimensional problem. d square g dx square x slash x naught is equal to delta x minus x naught.

So, original problem should have been d square u dx square is equal to let us say, 5x; that is the non-homogenous term. We are replacing the non-homogenous term by the Dirac delta function and subject to at x is equal to 0 and at x is equal 1, my g is equal to 0. That means correspondingly, we had at x is equal to 0, u may be equal to 1; at x is equal to 1, u may be equal to 2, but in this case, both of the boundary conditions are Dirichlet and they may be non-homogeneous, may be homogeneous, but in the formulation of Green's function, we force the non-homogeneous boundary conditions to be homogeneous.

Now, let us try to solve this problem. We divide this problem into sub-problem: for x lying in between 0 and x naught, we have d square g dx square let us say, d square g 1 dx square is equal to 0; g 1 is the lower half solution and for x lying between x naught and 1, we have d square g 2 dx square is equal to 0.

So, g 1 is lower half solution and g 2 is upper half solution. and we have the continuity of Green's function. through the boundary condition. Let us fix up the boundary conditions. At x is equal to 0, g 1 is equal to 0; at x is equal 1, g 2 is equal to 0 because g 2 is the upper half part. Continuity of Green's function, you have the third boundary condition.

Green's function, this will be g 1 x naught minus slash x naught is equal to g 2 x naught plus comma x naught.

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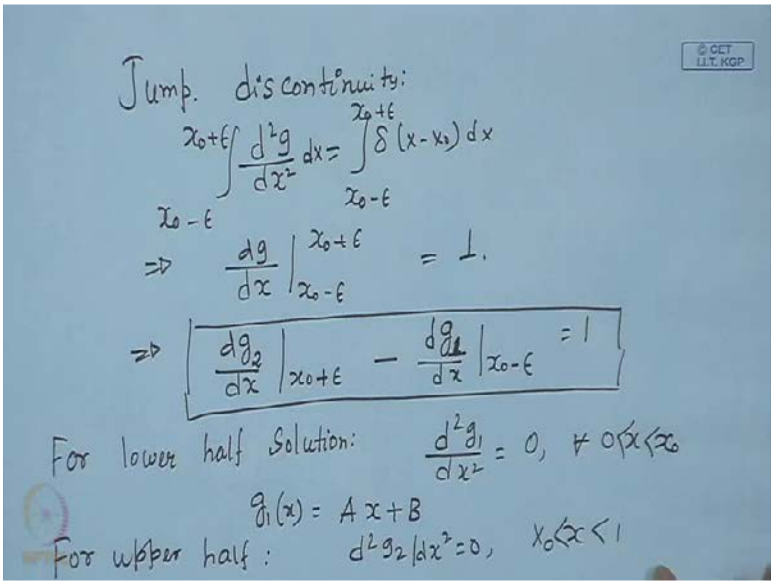

The jump discontinuity condition: this will give you d square g dx square, we start with the original problem, is equal to delta x minus x naught; we multiply both side by dx; integrate over x naught minus epsilon to x naught plus epsilon. So, this will be dg dx evaluated at x naught minus epsilon to x naught plus epsilon and that will be equal to 1. What is dg dx? This will be dg 2 dx, evaluated at x naught plus epsilon. So, it is the upper half solution, differentiation of that evaluated at x naught plus epsilon minus lower half solution, differentiation of that dg 1 dx x naught minus epsilon is equal to 1.

So, this gives the fourth boundary condition or the jump discontinuity condition. Now, let us solve the problem. Let us look into the lower half solution. We had d square g 1 dx square is equal to 0 for x lying in between 0 and x naught. Therefore, what is the solution of this? Solution is nothing, but Ax plus B and for upper half solution, upper half we have d square g 2 dx square is equal to 0, where x lying in between x naught and 1.

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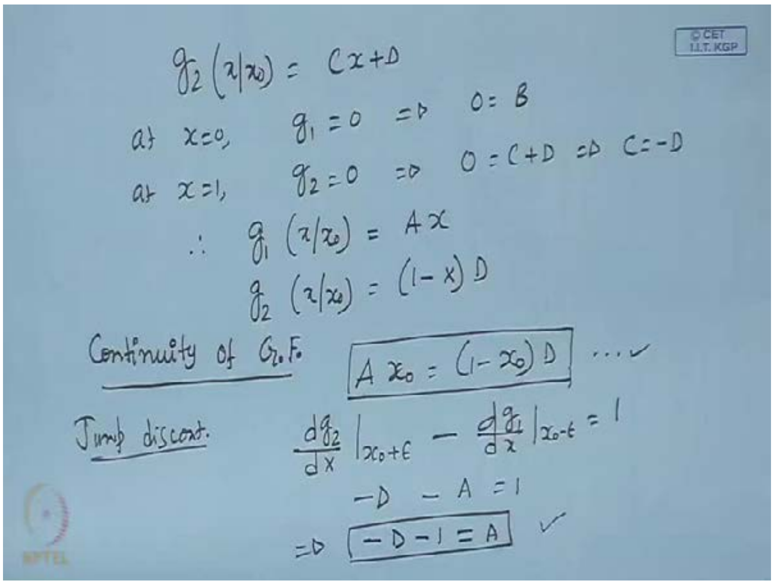

So, g 2 will be equal to Cx plus D.

So, we have 4 constants A, B, C, D. Now, at x is equal 0, we have g 1 is equal to 0; therefore, 0 is equal B. At x is equal 1, we have g 2 is equal to 0. That means, 0 is equal to C plus D; therefore, C is equal to minus D.

So we have g 1 x slash x naught is nothing, but Ax, g 2 x slash x naught this will be nothing but, 1 minus x times D. Next, we do the continuity of Green's function. Continuity of Green's function is across x naught, g 1 and g 2 are equal. So, we will be getting Ax naught is equal to 1 minus x naught times D.

So, that is the one and next, we will be getting the jump discontinuity condition for this particular problem. This will be dg 2 dx evaluated at x naught plus epsilon minus dg 1 dx evaluated at x naught minus epsilon should be equal to 1. What is dg 2 dx? dg 2 dx is nothing, but minus D; so, this will be minus D. minus dg 1 dx is nothing, but A, should be is equal to 1. So, A is equal to minus D minus 1. That is the definition of A. [A will be the line will be minus.]

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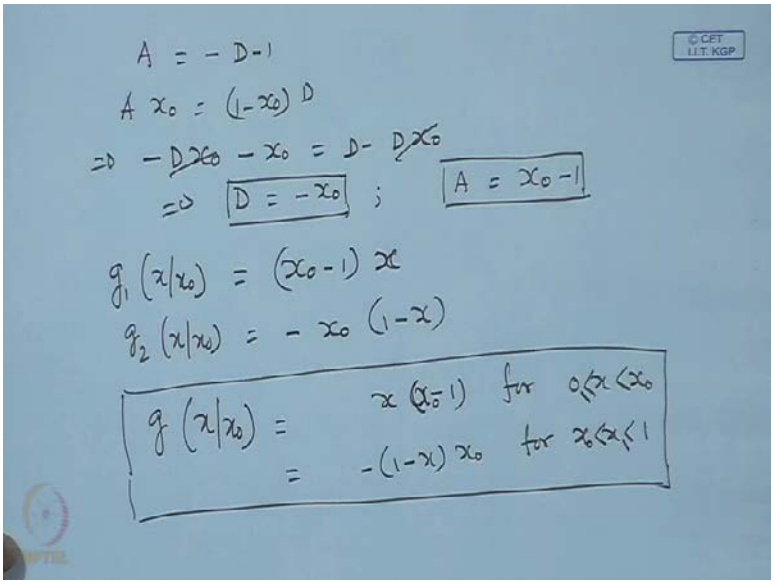

Now, we can solve these two equations and see what you get from these two equations. We will be getting the constants A and D and we can evaluate these two. So, I will just solve it. A is equal to minus D minus 1; the other equation is Ax naught is equal to 1 minus x naught times D. So, I just put A is equal to minus D minus 1. minus Dx naught minus x naught is equal to D minus Dx naught. So, Dx naught Dx naught will cancel out. It will be D is equal to minus x naught. So, that will be 1 and similarly, one can get A is equal to minus D minus 1; so minus, minus - plus x naught minus 1. Therefore, this is the last other constant A; C and B we have already developed.

So, therefore, g 1 x slash x naught is equal to Ax. So, x 0 minus 1 times x and g 2 x slash x naught is equal 1 minus x times D; D is minus x naught into 1 minus x. Therefore, we can write down the expression of Green's function as: lower half solution is g 1 x into x naught minus 1 for x lying in between 0 and x naught; the upper half solution is that minus 1 minus x times x naught.

So, it will be for upper half solution. So, we divide it into 2 parts, excluding x naught. So this gives the general expression of g for the 2 parts.

That is how, one will be able to solve the Green's function in 1-dimensional form. Now, let us try to solve one problem using Green's function method and we demonstrate this method for 1- dimensional problem only. I recover this solution later on.

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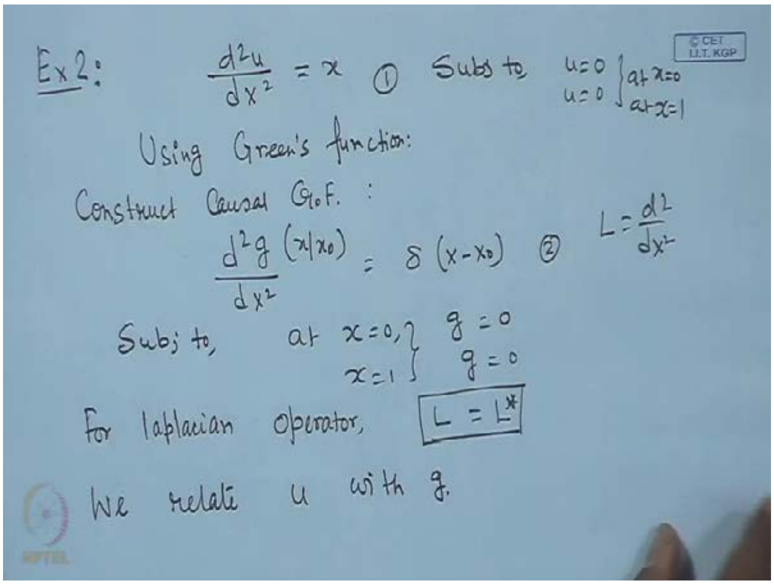

So, let us get into the second example. This example is we like to solve d square u dx square is equal to let us say, x subject to u is equal to 0, at x is equal to 0 and u is equal to 0, at x is equal to 1.

Let us try to solve this problem using Green's function method. We have considered the boundary conditions are Dirichlet and homogeneous. Now, construct causal Green's function, that will be simply d square g x slash x naught dx square is equal to delta x minus x naught. We replace the non-homogeneous part of the governing equation by a Dirac delta function, subject to at x is equal to 0, g is equal to 0 and at x is equal to 1, g is equal to 0.

Next, what we do? We assume for the time being and we can prove it later on that Laplacian operator is a self adjoint operator, we have already proved earlier. Probably, in the earlier classes, we have proved for Laplacian operator is a self adjoint operator; L is equal L star. We have already proved this thing earlier. Therefore, since Laplacian operator is a self adjoint operator, we need not to find an adjoint Green's function in this problem because the operator itself is self adjoint.

So, therefore, if you remember the steps of solving the equation, first you construct the causal Green's function. We solve the causal Green's function, then we go for the adjoint Green's function and then we go for the connection of relation of adjoint, relate the adjoint Green's function with the original problem.

But if you look into this problem since this Laplacian operator L is d square dx square, in this case and we have already proved earlier that laplacian operator is a self adjoint operator, we need not go for adjoint operator and adjoint Green's function.

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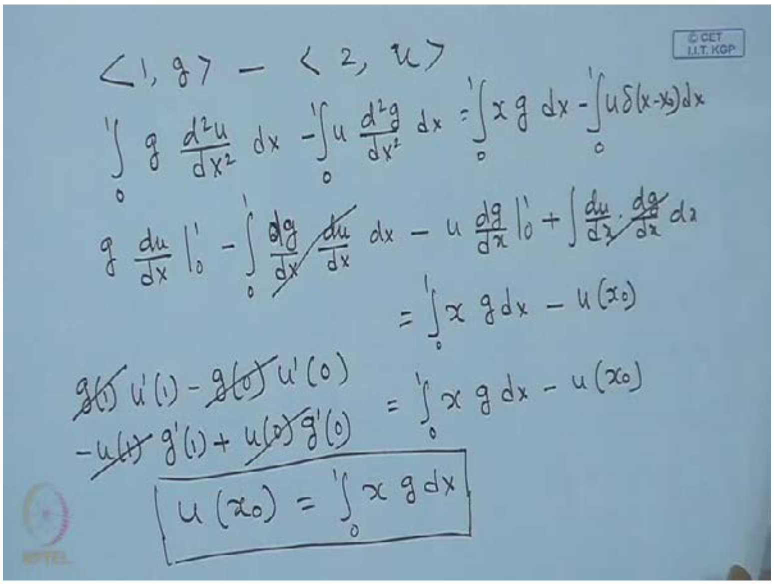

We can directly connect Lg is equal to delta to Lu is equal to f directly because the operator is itself a self adjoint operator. So, we connect or we relate u with g. How to do that? We multiply both side of 1 with g and integrate it - that means, we take the inner product of 1 with respect to g and integrate it; we take the inner product of 2 with respect to u and then we subtract and see what we get - that means, inner product of 1 with respect to g minus inner product of 2 with respect to u and see what you get. So, if you do that inner product in continuous function is nothing, but integration - g times d square u dx square dx minus u d square g dx square dx is equal to integration x times g dx minus u delta x minus x naught dx; all this integration is over the domain of x from 0 to 1.

Now, let us see what we get out of it. Now, we integrate it by parts. g first function integration of second function is du dx from 0 to 1 minus differential of first function dg dx, integration of the second function du dx dx 0 to 1 minus first function integration of second function u dg dx from 0 to 1; minus, minus - plus integration differential of first function du dx, integration of second function dg dx dx is equal to 0 to 1 x g dx minus, use the shifting property of Dirac delta function so this will be nothing, but u of x naught.

Now, these two will be simply cancelling out; they are equal and opposite in side. So, what we will be getting is g at 1, u prime at 1 minus g at 0, u prime at 0 minus u at 1, g prime at 1, minus, minus - plus u at 0, g prime at 0 is equal to integral 0 to 1 x g dx minus u of x naught. Now, the boundary condition of the original problem was u equal to 0 at x equal to 0, u was equal to 0; so, this term will be off; boundary condition in the original problem at x is equal to 1, u is equal to 0; so, this will be gone and the boundary conditions on g they are always homogeneous. This is gone and this is gone. So, the whole left hand side will be equal to 0. So, what we can have, we have u of x naught is nothing, but x g dx from 0 to 1.

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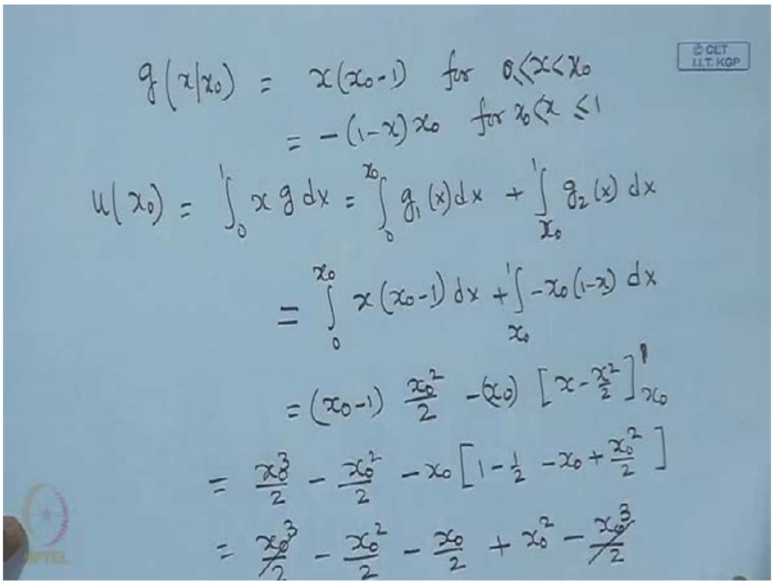

But the point is we do not have a continuous expression of g throughout the domain 0 to 1. What we have done? We have broken down the problem into 2 sub problems across x naught, if you remember. What we have done? We evaluated g in the earlier problem for the same operator. We evaluated that g of x is nothing, but x multiplied by x naught minus 1 for x lying between 0 and x naught and this is equal to minus 1 minus x x naught for x lying in between x naught and 1.

So, what we do, we put this expression and we break down this integration on the right hand side - x g dx into 2 parts: one from 0 to x naught, we write the solution g 1 x dx plus x naught to 1 g 2 x dx. So, 0 to x naught, we put the lower half solution, integration

0 to x naught x into x naught minus 1 dx plus x naught to 1, g 2 is minus x naught 1 minus x dx.

Now, this is an integration where the running variable is x. So, x naught minus 1 is constant. So, x naught minus 1 integral x dx that is, x square by 2, put the limits, it becomes x naught square by 2.

Similarly, here minus x naught is constant; it is taken out, then it is 1 minus x dx. So, we put it x minus x square by 2 and then evaluate between x naught and 1. If you do that, let us see what you get? This is x naught cube by 2 minus x naught square by 2 minus x naught; put the limit, 1 minus half minus x naught plus x naught square by 2.

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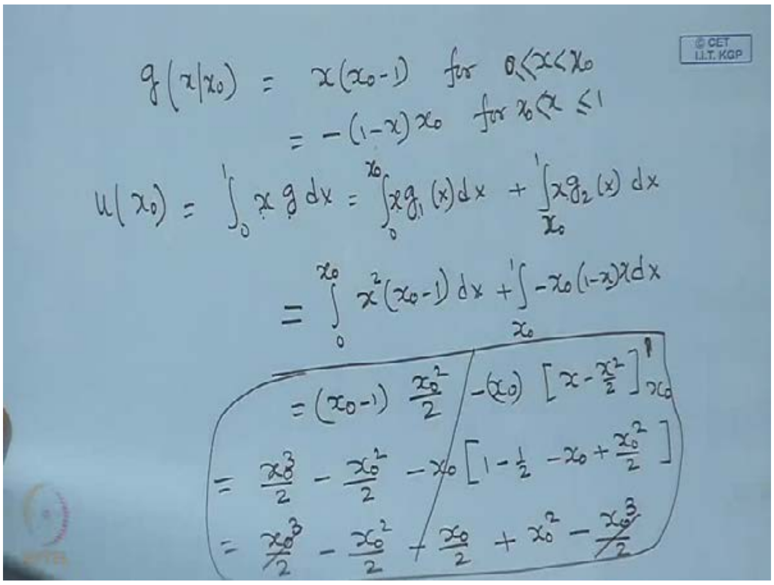

So, we will be getting x naught cube by 2 minus x naught square by 2 minus, this will be half, so, x naught by 2, minus, minus - plus x naught square, minus, plus - minus x naught cube by 2; x naught cube will be cancelling out. So, this will be gone. and then what we will be having is that u of x naught is equal to then There is a x there. So, it should be multiplied by x here; it should be multiplied by x there.

So, we put g 1 is equal to x into x naught minus 1, x should be multiplied. So, there will be x square here; it should by multiplied by x. So, I redo the rest of the calculation once again.

This is not correct. So, we do it once again. I missed a x here. So x should be Because this x is there; x times g. When I put the lower half solution, it will be x g 1, x g 2. g 1 – so, x into x naught minus 1. So, it will be x square into x naught minus 1, x into g 2 – so, x naught into 1 minus x multiplied by x.

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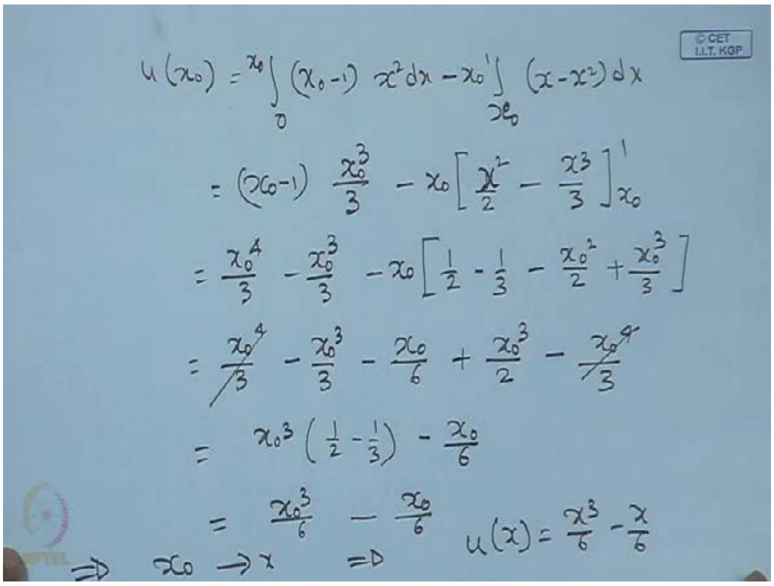

So, we redo the calculation. u of x naught is equal to integral 0 to x naught x naught minus 1 x square dx minus x naught x naught to 1, this will be, x minus x square dx. Therefore, x naught minus 1 can be taken out. This is x square dx; so, it will be x naught cube by 3. That is how, one will be getting it; minus x naught to be treated as common. It will be 1 by x square by 2 xdx minus x square dx which is x cube by 3, evaluated from x naught to 1.

So, this becomes x naught to the power 4 by 3 minus x naught cube by 3 minus x naught. Put it 1, 1 by 2 minus 1 upon 3 minus x naught square by 2, minus, minus - plus x naught cube by 3. So, it will be x naught to the power 4 by 3 minus x naught cube by 3 minus 1 by 2 minus 1 upon 3 it will be 1 upon 6 so x naught by 6, minus, minus - plus x naught cube by 2 and this will be minus, plus – minus and so, it will be x naught 4 by 3.

So, we cancel them out. What we get is 1 by 2 minus 1 upon 3 x naught cube, 1 by 2 minus 1 upon 3 minus x naught by 6. So, we will be getting x naught cube by 6 minus x naught by 6.

Now, change the running variable from x naught to x. So, you will be getting u of x is equal to x cube by 6 minus x by 6. So, that gives a demonstration how the problem can be solved. Next, I will take up one example, where both the boundary conditions are not homogeneous; they are non-homogeneous. Let us see what we can do about it.

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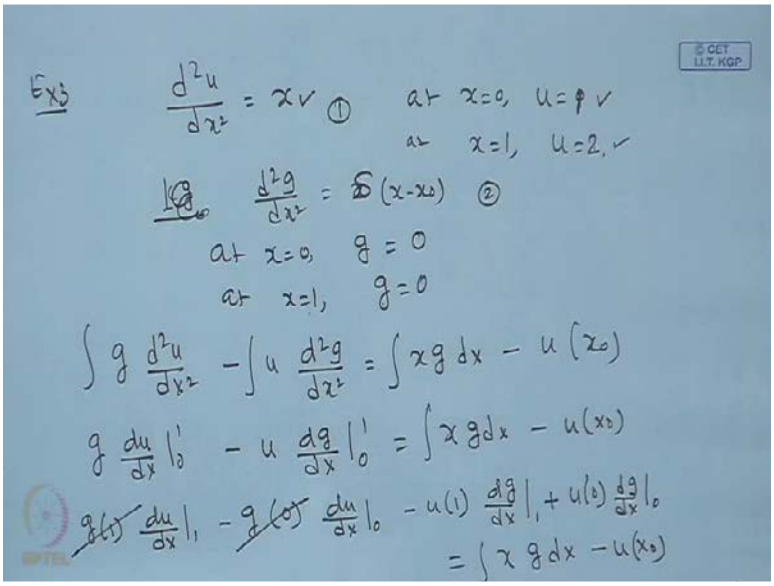

d square u so This is example 3. d square u dx square is equal to x, let us say and at x is equal to 0, u is equal to 1; at x is equal to 1, u is equal to 2.

Now, we formulate the corresponding Green's function d square g dx square is nothing, but delta x minus x naught and at x is equal to 0, we have g is equal to 0; at x is equal to 1, we have g is equal to 0.

Now, there are 3 sources of non-homogeneity in this equation; one is the boundary condition, another is another boundary condition and then one is the governing equation.

If we look into the solution, now, we connect 1 and 2. So, you multiply g with the first equation - g d square u dx square minus u d square g dx square is equal to integral x g dx minus u x naught.

This will be first function. Integral of the second function g du dx from 0 to 1 minus in fact, that will be cancelling out later on. So, you will be getting one more bi-linear concomitant part from here. u dg dx at 0 to 1 is equal to integral x g dx minus u of x naught.

Now, we evaluate it g at 1 du dx at 1 minus g at 0 du dx at 0 minus u at 1 dg dx at 1 minus, minus - plus u at 0 dg dx is equal to at 0 integral x g dx minus u of x naught.

So, we have g is equal to 0 at x is equal to 0; so, this is gone; g is equal to 0 at x is equal to 1; so, this is gone; g is equal to 0 at x is equal to 0; this is gone.

u at 1 is 1; so, we put it at 1, u at x is equal to 1 is 2; so you put at 2 there. u at x is equal to 0 is 1; so, you put a 1 here. Let us see what you get.

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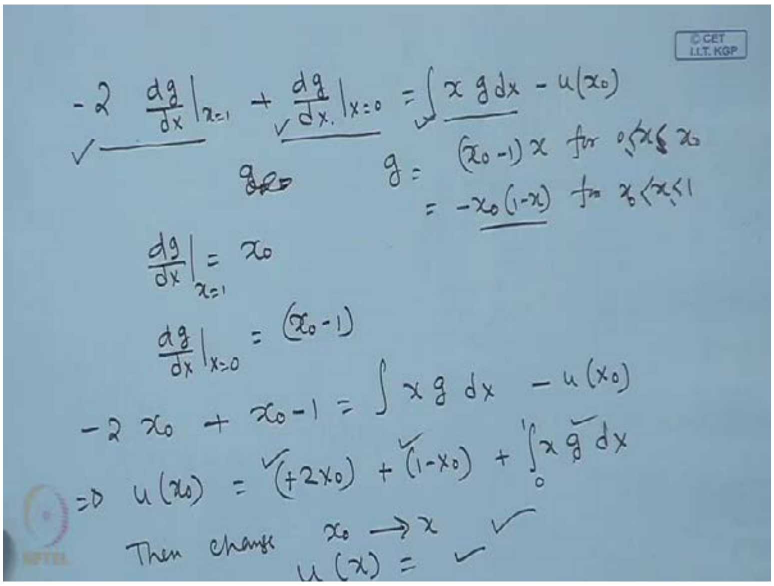

So, u of 1 at x is equal to 1 is 2; so, it is 2, minus, dg dx evaluated at x is equal 1 plus u at x is equal to 0 is 1; so, 1 multiplied by dg dx evaluated at x is equal to 0 is equal to integral x g dx minus u of x naught.

Now, if you remember, g 1 is equal to the expression of g; g is equal to x naught minus 1 times x, for x lying in between x naught and 0 and is equal to minus x naught into 1 minus x, for x lying in between x naught and 1.

What is dg dx evaluated at x is equal to 1? That means we are talking about the upper half plane. dg dx is nothing, but differential of this with respect to x, minus, minus - plus x naught and evaluated at x is equal to 1; dg dx, x equal to 0 - dg dx evaluated at x is equal to 0; that means, we are talking about the lower half solution. So, it will be x naught minus 1.

We write here minus 2 dg dx at x is equal 1 is x naught plus dg dx x is equal to 0 is x naught minus 1 is equal to integral x g dx minus u of x naught. So, if you can observe that there are 3 sources of non-homogeneity in this governing equation of u; there were 2 non-homogeneities present at the 2 boundaries and one non-homogeneity is present in the governing equation.

So, while 2 terms of the bi-linear concomitant is not 0 and since you will be having a non-homogeneity the governing equation therefore, will be having term which will be an integral; integral means, it is not on the surface; it is occurring throughout the whole bulk of the control volume and there will be 2 non-zero terms; they will be appearing on the surface.

So, 3 sources of non-homogeneity. There will be 3 terms; one can get as a solution of u of x naught. You will be getting u of x naught on the other side; so, we will be getting minus 2x naught. This will be plus. You bring it to the other side. minus 1 plus x naught; so, it will be plus. So, 1 will be on the other side it will be plus, x naught will be on the ( ) minus plus that remains same x g dx from 0 to 1.

So, 3 sources of non-homogeneity; we will be having 3 terms here: one term corresponding to one non-homogeneous boundary condition; another term corresponding to another non-homogeneous boundary condition; then the third term which will be corresponding to the integral term which will be corresponding to the non-homogeneous term in the governing equation. So. it will be an integral.

Now, exactly the same way whatever we have done earlier, we break down this problem into 2 sub-problems. We break down this integral into 2 sub integrals; 0 to x naught and x naught to 1 and then put the corresponding equations of x at g and integrate it out. So, you will be getting everything on the right hand side in terms of x naught. Then change the running variable x naught to x and you will be getting a complete solution in terms of u as a function of x. So, that is how the Green's function method is utilized for the solution of partial differential equation and you will be getting the solution soon.

So, these two examples I have taken up the ordinary differential equation for demonstration purpose. Next, I will be taking up the partial differential equation. How to utilize these concepts to get the solution of the partial differential equation, using the non-homogeneous partial differential equation using the Green's function technique. We have seen and applied these techniques for the ordinary differential equations.

Now, nobody will solve the ordinary differential equation by this method, but it is for demonstration purpose and next, we will be taking up the partial differential equation and apply the Green's function method in order to solve them.

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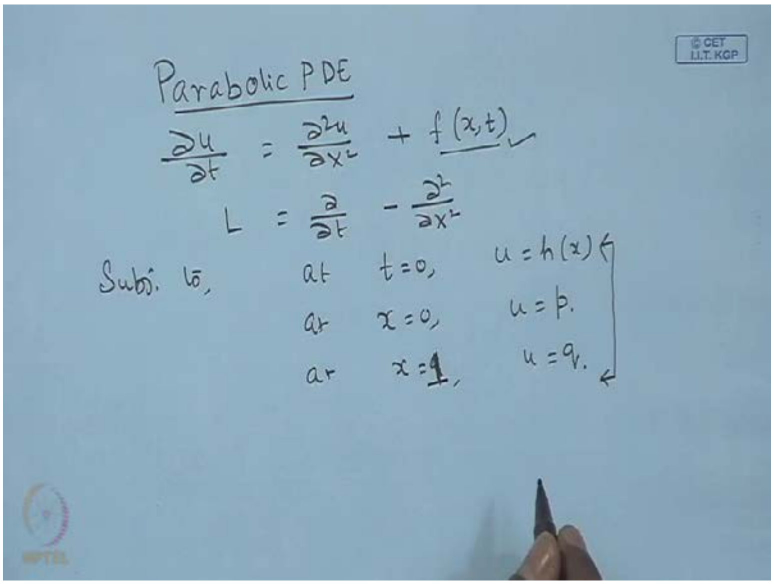

Now, let us take up one example. We will be taking up 2 examples: one is the parabolic partial differential equation and another is the elliptic partial differential equations because these types of PDEs are most common in chemical engineering application.

Let us take up a parabolic partial differential equation first. It is del u del t is equal to del square u del x square plus f of x t.

So, the operator here is del del t minus del square del x square. Now, boundary conditions subject to at t is equal to 0, we have u is equal to h x; at x is equal to 0 we have u is equal to p; at x is equal to 1, we have u is equal to q.

So, we have taken a general parabolic partial differential equation in 2-dimensional: 1-dimension in time, another dimension in space. del u del t is equal to del square del x square; this is the non-homogeneous term and if you look into the initial and the boundary condition at t is equal to 0, you have taken everything a non-homogeneous term and non-homogeneous boundary condition. It is a more general boundary condition, but we have taken the Dirichlet boundary conditions. So, at t is equal to 0, u is equal to h x; at x is equal to 0, u is equal to p; at x is equal to 1, u is equal to q

Now, we solve this problem by a method called partial eigenfunction expansion method. I will stop here in this class because this requires a lot of development and some theoretical work should be done before attempting this problem. So, we will be developing the theory of solution of non-homogenous partial differential equation using Green's function method. We develop the theory and then we apply it to the solution of the actual problems.

So, I stop here in this class. Then we take up this problem in the next class for the solution of parabolic partial differential equation by using Green's function method. Thank you very much.