Advanced Mathematical Techniques in Chemical Engineering

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Module No. # 01

Lecture No. # 29

Solution of Elliptic and Hyperbolic PDE

Welcome to this session of the class. As we have discussed in the last class, we are formulating an elliptic partial differential equation.

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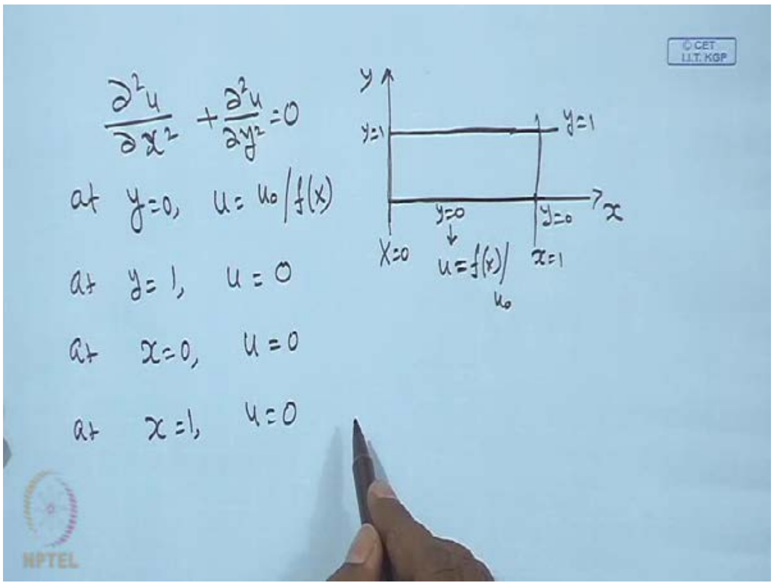

So, we were looking into a domain in x and y, from x is equal to 0 to x is equal to 1 and from y is equal to 0 to y is equal to 1. This surface is located at X is equal to 0, this surface is located at x is equal to 1, this surface is located at y is equal to 0 and this surface is located at y is equal to 1 (Refer Slide Time: 00:45). There are four surfaces, so the governing equation to this problem is del square u del x square plus del square u del y square is equal to 0.

Now, we put the initial boundary condition, we assume that at x is equal to at y is equal to 0. So, this boundary is located at y is equal to 0; at y is equal to 0 we have u is not 0 and it is some general function of f x or it may be constant.

At y is equal to 0 u is equal to u naught or constant or some function of f and at y is equal to 1, you have u is equal to let us say 0; at x is equal to 0 we consider u is equal to 0; at x is equal to 1 we have u equal to 0. So, if we have all the four boundary conditions to be homogeneous then, we are going to get a trivial solution.

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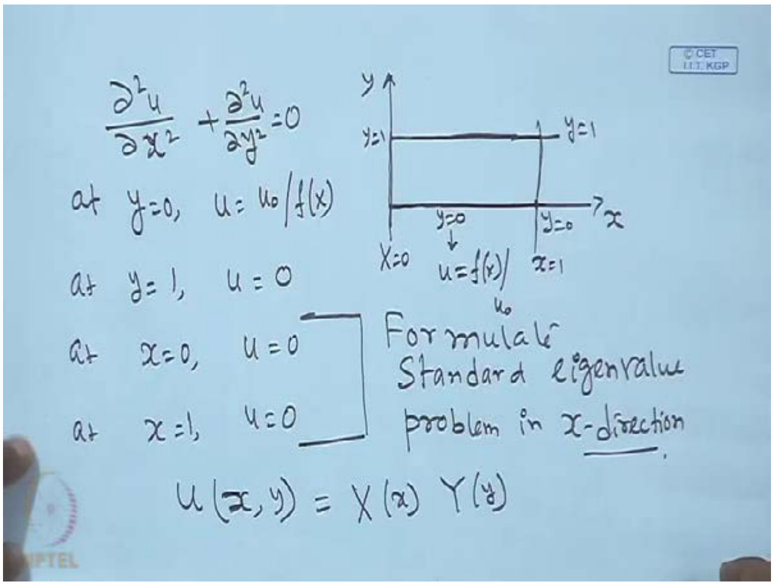

Therefore, the particular solution or the non-trivial solution exists if and only if one of the boundary condition becomes non-homogeneous. Now in this case, if you examine these four boundary conditions, the boundary conditions on x are all homogeneous, both the boundaries on x are homogeneous.

So you will be having a standard Eigen value problem, you formulate a standard Eigen value problem in x direction, ok. What we do? We again do a separation of variable type of solution because the all the governing equation is linear and the boundary conditions are all linear. Therefore, we formulate u as a function of x and y should be a product of two terms; one is a sole function of x, another is a sole function of y.

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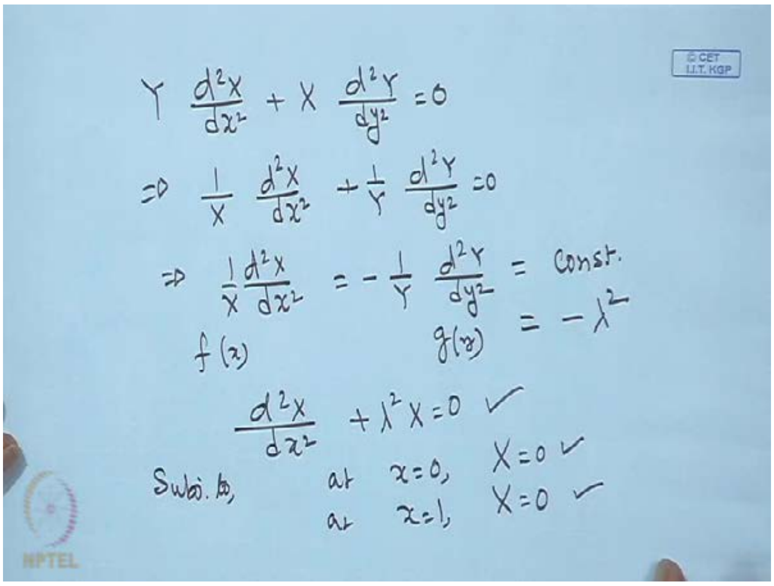

Now this has to be substituted into the governing equation, let us see what you get. We get Y d square X d x square plus X d square Y d y square must be equal to 0. You divide both sides by x y, so you will be getting 1 over X; d square X d x square plus 1 over Y d square Y d y square is equal to 0. So, we will be getting d square X d x square 1 over X is equal to minus 1 over Y d square Y d y square.

Now, the left hand side is a function of x alone, the right hand side is a function of y alone, they are equal and they are equal to some constant. We have already examined the boundary conditions, we have observed that boundary conditions on x varying part are homogeneous in both the boundaries but, the boundary conditions in y are not homogeneous on both the boundaries.

Therefore, the important thing is how we will be solving, how we will be formulating the standard Eigen value problem? We will be formulating the standard Eigen value problem in the x direction only because the x direction boundaries have homogeneous boundary conditions. So this constant, we formulate the standard Eigen value problem in the x direction but, not on the y direction because one of the boundary conditions in y direction is not homogeneous.

Therefore, this constant can be 0, can be positive or can be negative; so if this constant is 0 and positive, we have already seen that will be landing up with a trivial solution. So, this constant has to be a negative constant. I will be getting d square X d x square plus lambda square X is equal to 0.

Now, we should write down the boundary conditions on x subject to at x is equal to 0, we have capital X is equal to 0. At x is equal to 1, we have capital X is equal to 0 simply because the original problem both the boundaries on x is equal to 0 and x is equal to 1 they were homogeneous. So in this problem also, they will be having the homogeneous boundary conditions on x.

Now, we have already seen the solution to this problem with the set of boundary conditions, this is a standard Eigen value problem with the Eigen values lambda n is equal to n pi and Eigen functions or the sine functions they are sin lambda n x or sin n pi x.

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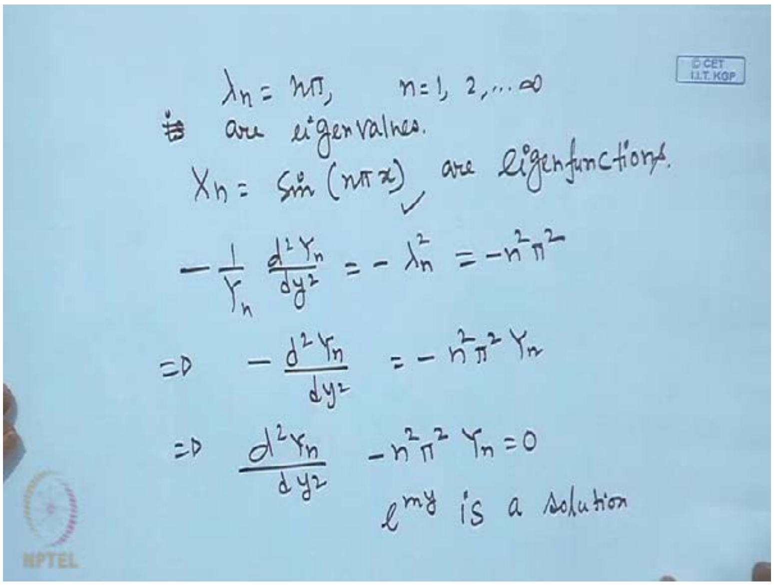

I am not going to solve this problem therefore, I will be writing down the solution. So lambda n is equal to n pi, where the index n transform 1 to infinity are the Eigen values and sine functions they will be sin n pi x are the Eigen functions.

We get the x varying part; let us look into the solution of y varying part. So 1 over minus 1 over Y d square Y d y square should be is equal to minus lambda n square, so this will be minus n square pi square. We write down the Y as a subscript n corresponding to the solution of nth Eigen value n pi.

So if we multiply both sides by Y n, what will be getting is that d square Y n d y square is equal to minus n square pi square Y n. Take it on the other side, this is becomes d square Y n d y square minus n square pi square Y n is equal to 0.

Now, if you look into this differential equation, this is a second order ordinary differential equation and if you look into this equation, if you rebrush our earlier fundamentals, the form e to power m y is a solution; that means solution is of the form of e to the power m y.

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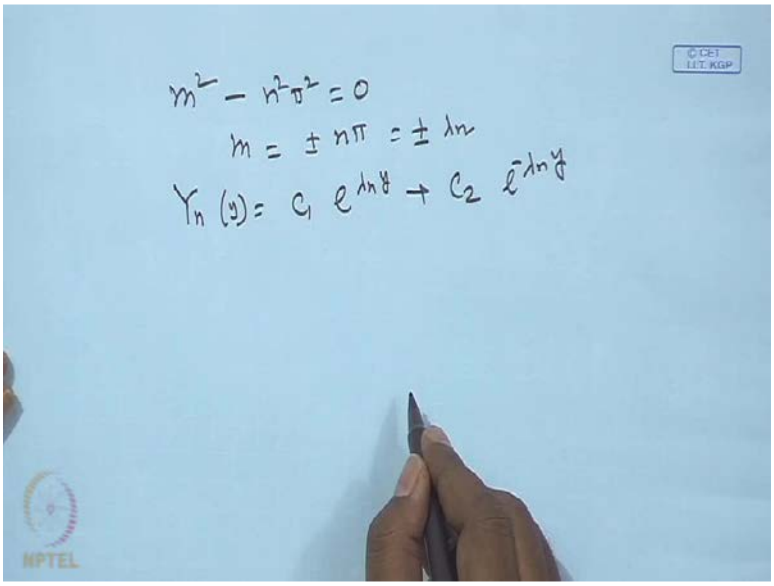

If we put it there, then it becomes m square minus n square pi square is equal to 1 that gives me the characteristic equation. So m will be is equal to plus minus n pi or plus minus lambda n. Therefore, we will be getting d square; the solution will be constituted of C 1 e to the power lambda n y plus C 2 e to the power minus lambda n y.

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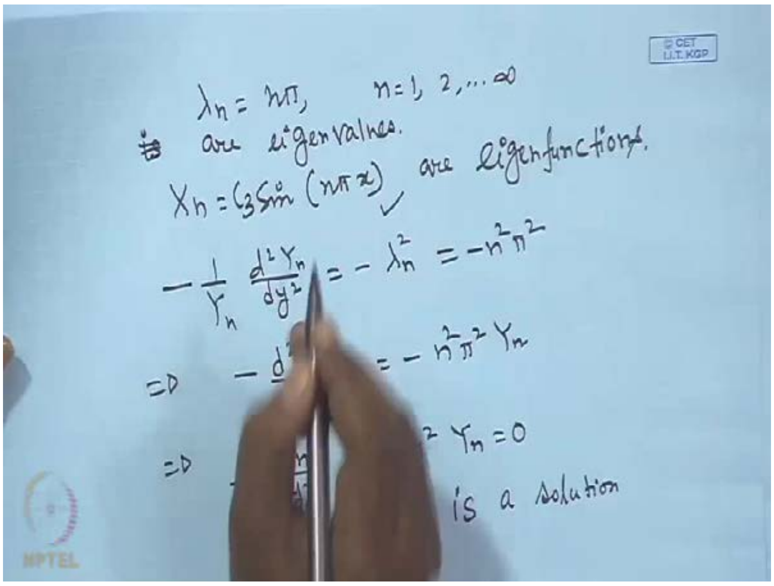

So this is the solution; we have already written down x n is equal to sin n pi x should be multiplied by some constant, let us say C 3 sin n pi x.

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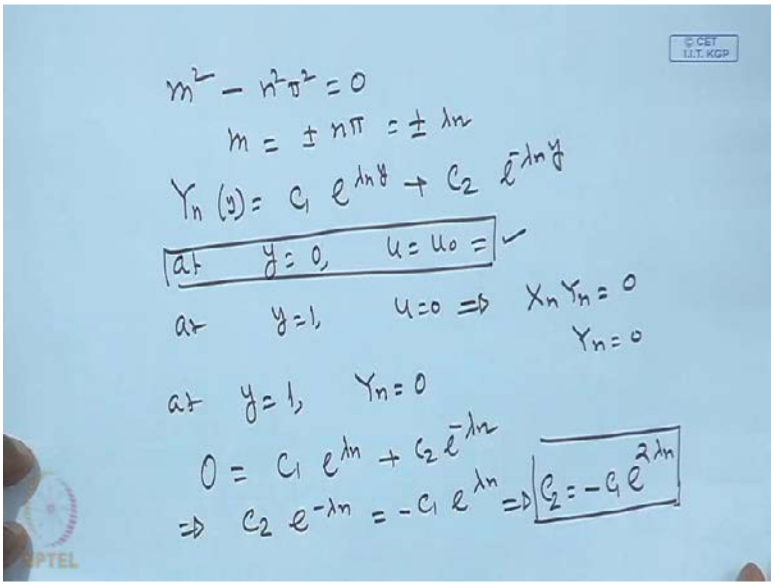

So the value variation, the function or solution of Y n is given as C 1 e to the power lambda n y plus C 2 e to the power minus lambda n y.

We have already utilized both the boundary conditions of x; now, if you remember that there are 2 boundary conditions on y. One boundary condition is at y is equal to if you look into the original problem at y is equal to 0, u is equal to u naught and at y is equal to

1 we have u is equal to 0. Therefore, we will not utilize this one at y is equal to 1, u is equal to 0 from here what you will be getting is that our X n and Y n they should be is equal to 0 and at y is equal to 1, the x varying part cannot be is equal to 0 therefore, Y n must be is equal to 0.

So, we will be getting the boundary condition at y is equal to 1, Y n is equal to 0. We utilized this boundary condition and see what we get? If we utilized we cannot use this boundary condition right now, we will be utilizing this boundary condition earlier; we will be using this boundary condition later on to evaluate the final constant.

Let us use this boundary condition, so that we can evaluate one of these two constants in terms of the other (Refer Slide Time: 11:19). So, if you utilize this boundary condition, this becomes 0 C 1 e to the power lambda n plus C 2 e to the power minus lambda n. We can get C 2 e to the power minus lambda n is nothing but, minus sign C 1 e to the power lambda n, so we evaluate C 2, C 2 is nothing but, minus C 1 e to the power 2 lambda n.

So that is how the C 2 is relate to C 1, we are going to substitute this here and get Y n as a function in terms of only one constant.

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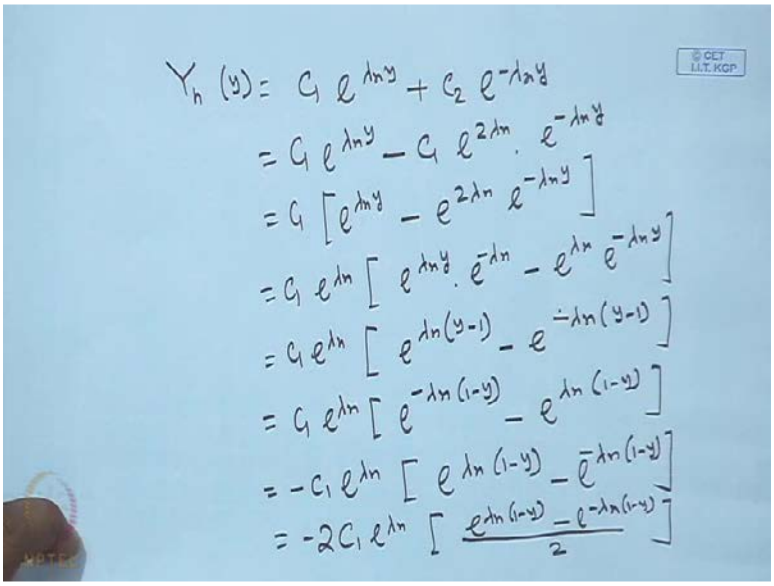

If you do that Y n becomes C 1 e to the power lambda n y plus C 2 e to the power minus lambda n y and C 2 is minus C 1 e to the power 2 lambda n into e to the power minus lambda n y. You take C 1 common, so C 1 will be e to the power lambda n y minus e to the power 2 lambda n, e to the power minus lambda n y.

Now we take e to the power lambda n common, so it will becomes C 1 we take e to the power lambda n common, so it becomes e to the power lambda n y into e to the power minus lambda n minus e to the power lambda n e to the power minus lambda n. So, this becomes C 1 e to the power lambda n, e to the power lambda n this becomes y minus 1 minus e to the power minus lambda n, this becomes y minus 1.

We make this as 1 minus y because y is less than 1, so e to the power lambda n e to the power minus lambda n 1 minus y minus e to the power plus lambda n 1 minus y. This minus sign we take it out, so minus c 1 e to the power lambda n should be is equal to e to the power lambda n into 1 minus y minus e to the power minus lambda n 1 minus y.

Now, next what we do? We divide and multiply both side of the numerator and denominator of the third bracket by 2, so it will become minus 2 C 1 e to the power lambda n and this becomes divided by 2 e to the power lambda n 1 minus y minus e to the power minus lambda n 1 minus y. So, if you look into this term in the square bracket this becomes nothing but, a sin hyperbolic term.

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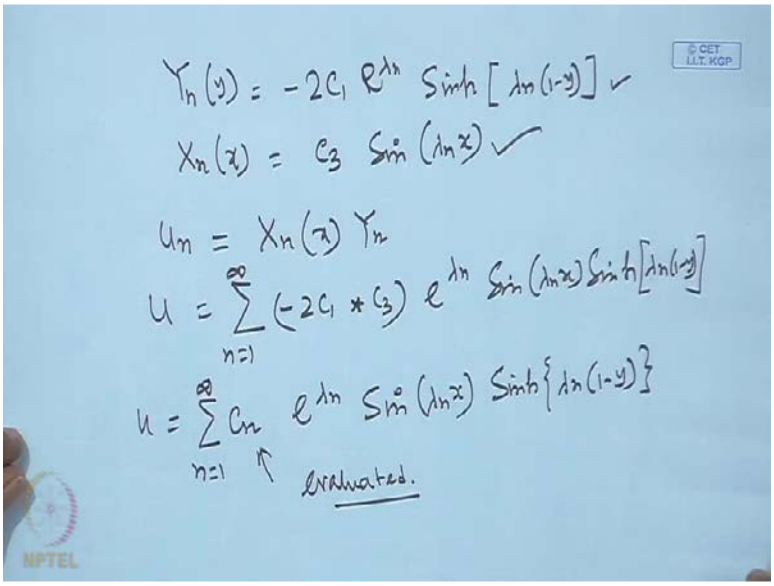

So, Y n y becomes in a compact form minus 2 C 1 e to the power lambda n sin hyperbolic lambda n 1 minus y. Now, if you look into the x varying part, the x varying part was having a solution C 3 n pi x so X varying part was some constant, C 3 multiplied by sin lambda n x. Now, we will be in a position to construct the solution of nth solution corresponding nth Eigen value, this will be simply X n and Y n and the overall solution will be obtained by a linear superposition of all the solutions. This becomes minus 2 C 1 times C 3 e to the power lambda n, sin lambda n x sin hyperbolic lambda n 1 minus y.

Now, minus 2 C 1 into C 3 it will be a new constant, let us say this new constant is C n e to the power lambda n sin lambda n x sin hyperbolic lambda n 1 minus y and this index n runs from 1 to infinity. Now we will be having the complete solution, only this constant has to be evaluated and if we remember, we had 4 boundary conditions 2 on x and 2 on y.

The homogeneous boundary conditions on x we have utilized in order to obtained the Eigen function in the x direction and we have already used the homogeneous boundary condition on y that is at y is equal to 1 and we have obtained this solution (Refer Slide Time: 17:27). Now what is left is that we have to utilize the left over boundary condition on y that is at x at y is equal to 0 u was equal to u naught.

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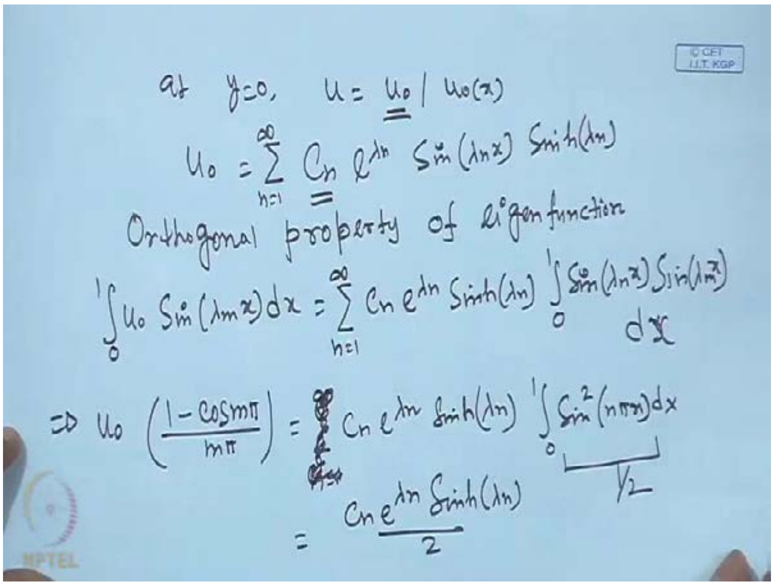

If we do that, let us see what we get at y is equal to 0, u was equal to u naught or it may be a known function of x. Let us consider it constant for the time being, so at y is equal to 0 u is equal to u naught; so u naught is nothing but, summation n is equal to 1 to infinity C n e to the power lambda n sin lambda n x and this becomes sine hyperbolic 1 minus y, so y equal to 0 this will be sin h lambda n. So only the x varying part is sin lambda n x and this lambda is nothing but, n pi so this will be sin n pi x.

In order to get rid of this summation and to evaluate C n, we utilize the orthogonal property of sine functions or the Eigen functions. That becomes, we multiply both side by sin lambda m x d x and integrate over the domain of x. So this becomes 0 to 1 u 0 sin lambda m x d x summation n is equal to 1 to infinity C n e to the power lambda n sin h lambda n integral 0 to 1 sin lambda n x sin lambda m x d x.

So this becomes u 0 lambda m is nothing but, m pi so you have already evaluated this integral several times earlier, this becomes 1 minus cosine m pi divided by m pi. This becomes summation n is equal to 1 to infinity C n e to the power lambda n sine hyperbolic lambda n and when we open up this summation series, all the terms will vanish except m is equal to n. So, this summation will be gone will be open up the summation series; this becomes 0 to 1 sin square n pi x d x and we have already seen this value of this integral is half, so this becomes C n e to the power lambda n sin hyperbolic lambda n divided by 2.

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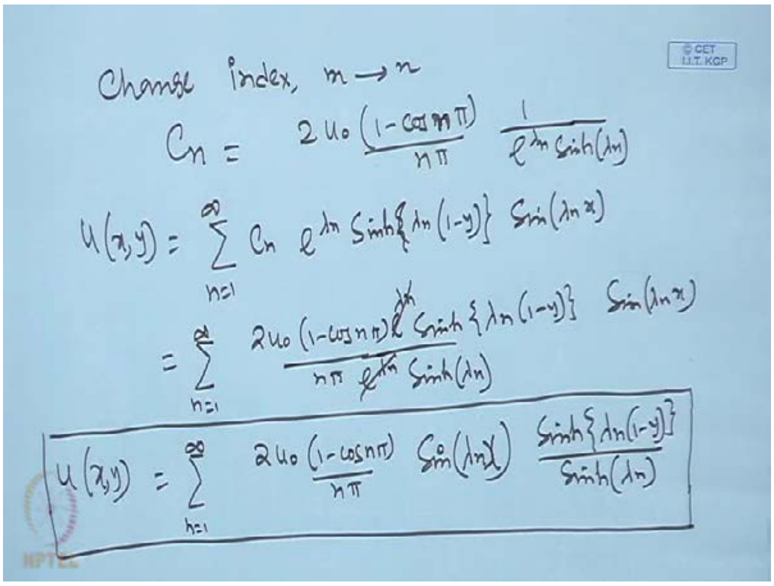

Now on the left hand side, we change the running index from m to n, change index m to n and we can get the expression of C n as 2 u naught 1 minus cosine n pi divided by n pi 1 over e to the power lambda n sin hyperbolic lambda n.

Now, we substitute this value of C n in the governing solution, the solution becomes u as a function of x and y is nothing but, n is equal to 1 to infinity C n e to the power lambda n sin hyperbolic lambda n 1 minus y sin lambda n x.

We put the value of C n there, n is equal to 1 to infinity C n is 2 u naught 1 minus cosine n pi divided by n pi e to the power lambda n sin hyperbolic lambda n sin h lambda n 1 minus y sin lambda n x. So, e to the power lambda n is there on the top so that will be cancelling out.

So, we will be getting the final solution as n is equal to 1 to infinity 2 u 0 1 minus cos n pi divided by n pi sin lambda n x; basically sin n pi x sin hyperbolic lambda n 1 minus y divided by sin hyperbolic lambda n. So, that gives the complete solution of a parabolic partial differential equation that is a parabolic partial differential equation and which is a well posed parabolic partial differential equation.

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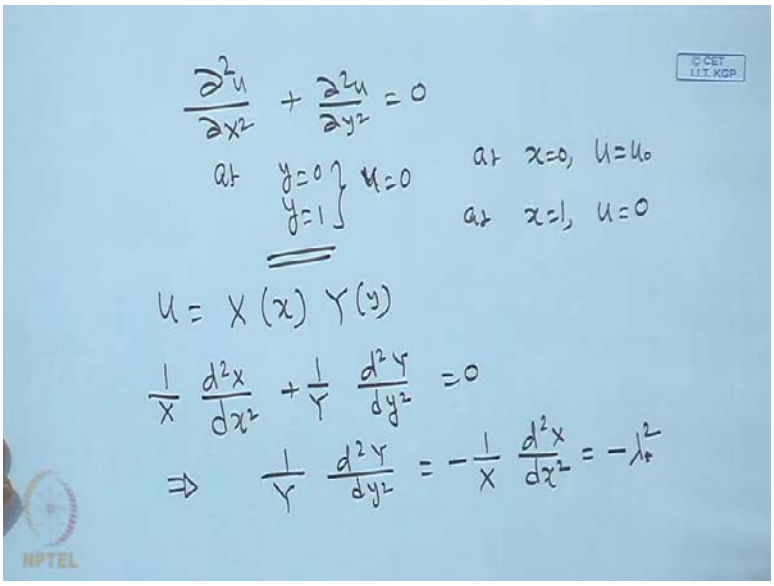

Now if in this problem, if we have a boundary condition something like this, let us repose the problem once again, del square u del x square plus del square u del y square is equal to 0. If we put the boundary conditions on y to be homogeneous, that means at y is equal to 0 and at y is equal to 1, if we have u is equal to 0 and at x is equal to 0, if we have u is equal to u naught and at x is equal to 1, you have u is equal to 0. Then we should have form a standard Eigen value problem in the y direction instead of u direction because in the x direction simply in this particular problem, both the boundary conditions in x are not homogeneous in this case, on the other hand both boundary conditions in y they are homogeneous. So, we have to formulate the standard Eigen value problem in the y direction.

If we have a solution of this type, x into y so we will be having 1 over X d square X d x square plus 1 over Y d square y d Y square is equal to 0. So, we should formulate the standard Eigen value problem not in the x direction as in the earlier case but, we have to form it from the case of y direction.

So we have to make it like this, 1 over Y d square Y d y square is equal to minus 1 over X d square X d x square; the left hand side is entirely a function of Y, the right hand side is entirely function of X. So they will be equal to some constant, this constant has to be negative constant; otherwise we will be landing up with a trivial solution. So this will be minus lambda square (Refer Slide Time: 25:50).

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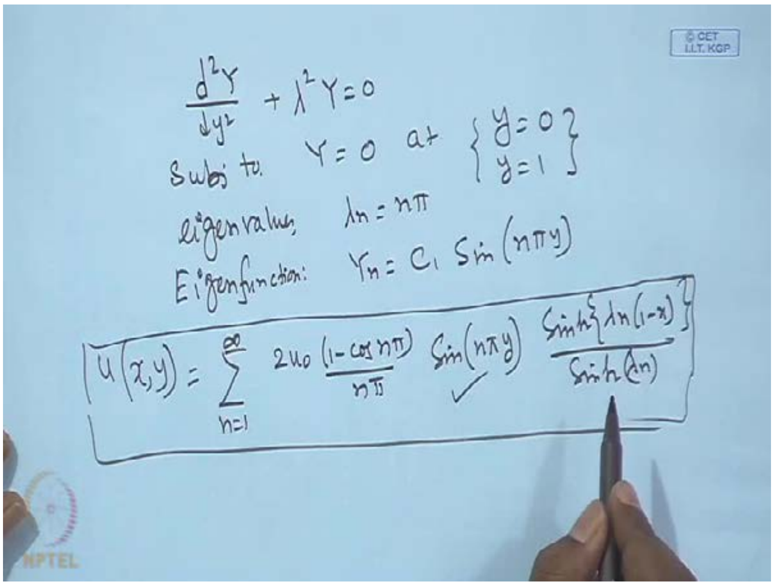

We formulate the standard Eigen value problem in the y direction, so we will be having d square Y d y square plus lambda square Y should be equal to 0, subject to Y is equal to 0 at both the boundaries, small y is equal to 0 and small y is equal to 1. Now the Eigen values lambda n will be n pi and Eigen functions will be Y n, some constant multiplied by sin n pi y.

So we will be getting the complete solution, I am just writing the complete solution. The complete solution will be summation n is equal to 1 to infinity 2 u 0 1 minus cosine n pi divided by n pi sin n pi y that is lambda n y sin hyperbolic lambda n 1 minus x sin hyperbolic lambda n.

So that gives the solution to this problem. If you look into the solution that since, we have the standard Eigen value problem in the y direction because boundary conditions in the y direction, both the boundary conditions are homogeneous. We have to formulate the Eigen value problem in the y direction, so sin n pi y will be the Eigen functions and n pi will be the Eigen values. You will be having the sine hyperbolic; the solution of hyperbolic will come in the x direction.

So that makes you understand how the Eigen value problem has to be formulated in order to for the solution of elliptic partial differential equation. Now, we have to identify the direction in which we can have the homogeneous boundary conditions; so we have to formulate the sturm liouville problem in that particular direction.

Once we do that then, we use the other boundary condition we get the solution of y the other direction and we will be using the homogeneous boundary condition to reduce two constants into one constant. Finally, we will be getting the complete solution in the form of the summation series with one unknown constant needs to be evaluated.

The unknown constant will be evaluated by using the non-homogeneous boundary condition in the other direction and from that we will be getting the complete solution using the orthogonal property of the Eigen functions.

In a general case, if you have the different boundary conditions which are not homogeneous, then we have to make the situation homogeneous first and then we have to divide the problem into sub problem considering, one non-homogeneity at a time.

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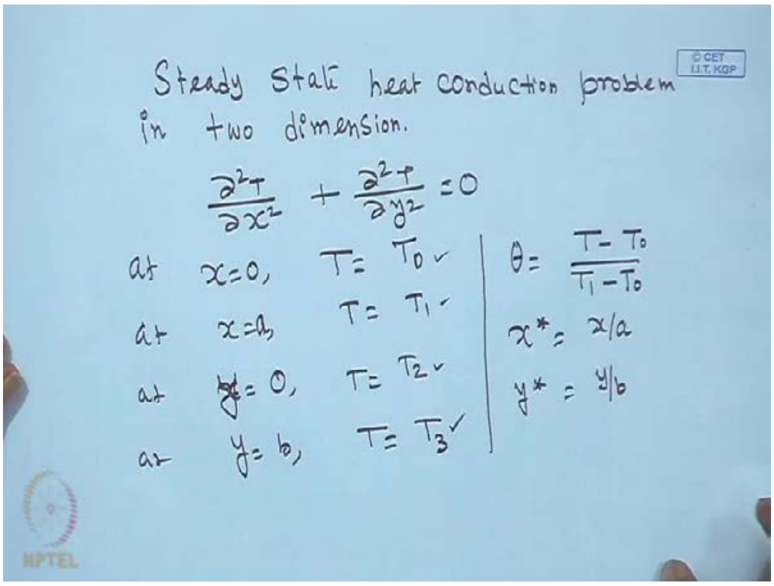

I will be again looking into solving a heat conduction problem, steady state heat conduction problem in two dimension. I will be taking up a chemical engineering application, steady state heat conduction problem in two dimension.

In one case, you will be having del square T, the governing equation will be del square T plus del square T del x square plus del square T del y square will be equal to 0. At x is equal to 0, we have T is equal to let us say T 0; at x is equal to 1, we have T is equal to T 1; at y is equal to 0, we have T is equal to T 2 let say this is x is equal to a, y is equal to b, we have T is equal to T 3.

Now there are 4 sources of non-homogeneity in this particular problem. What we do? We first make it non-dimensional and try to reduce number of non-homogeneities in this case. So we make a theta such that it becomes T minus T 0 divided by T 1 minus T 0, we make this non-dimensional temperature and write x star as x by a and y star is equal to y by b, so that x star and y star becomes 1 they vary from 0 to 1.

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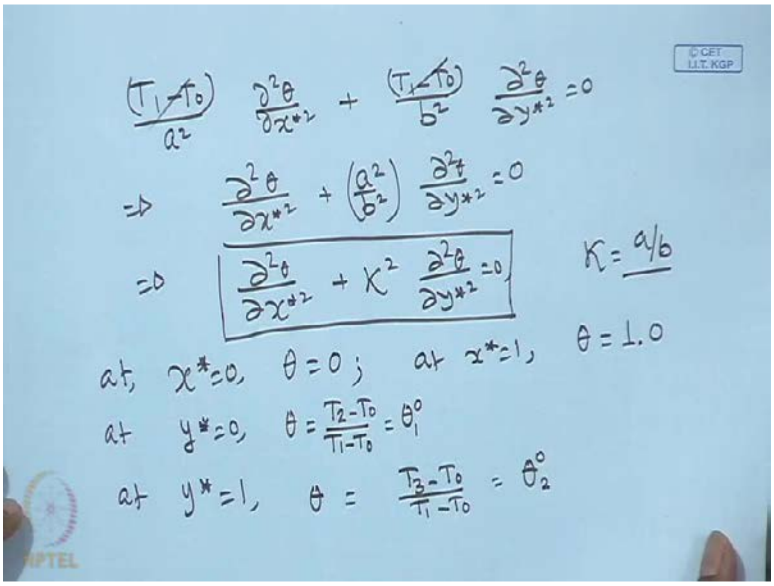

Let us try to solve this problem. So this becomes T 1 minus T 0 divided by a square del square theta del x star square plus T 1 minus T 0 divided by b square del square theta del y star square that will be equal to 0. This will be cancelled out from both the side, this becomes del square theta del x star square plus a square by b square del square theta del y star square is equal to 0.

So we are going to have del square theta del x star square plus kappa square del square theta del y star square is equal to 0 ,where kappa is nothing but the geometric factor ratio of a and b. So this is the non-dimensional form of temperature or the governing equation we are getting. Now, let us set up the non-dimensional boundary conditions; so at x is equal to 0 means at x star is equal to 0 we have t is equal to t 0 that means theta is equal to 0.

At x star is equal to 1 x equal to a means at x star is equal to 1, we have t is equal to t 1; therefore theta becomes t 1 minus 0 divided by t 1 minus 0. So this becomes 1 at y is equal to 0 means at y star is equal to 0 your theta is equal to T 2 minus T 0 divided by T 1 minus T 0. This becomes let us say, theta 1 0 and at y star is equal to 1 that is y equal to b we have t is equal to T 3 that means theta is equal to T 3 minus T 0 divided by T 1 minus T 0, so this will be theta 2 0.

Now if you look into this problem, this problem has 1 homogeneous boundary condition but, 3 non-homogeneous boundary conditions. So, this problem has to be divided into 3 sub problems considering, 1 non-homogeneity at a time.

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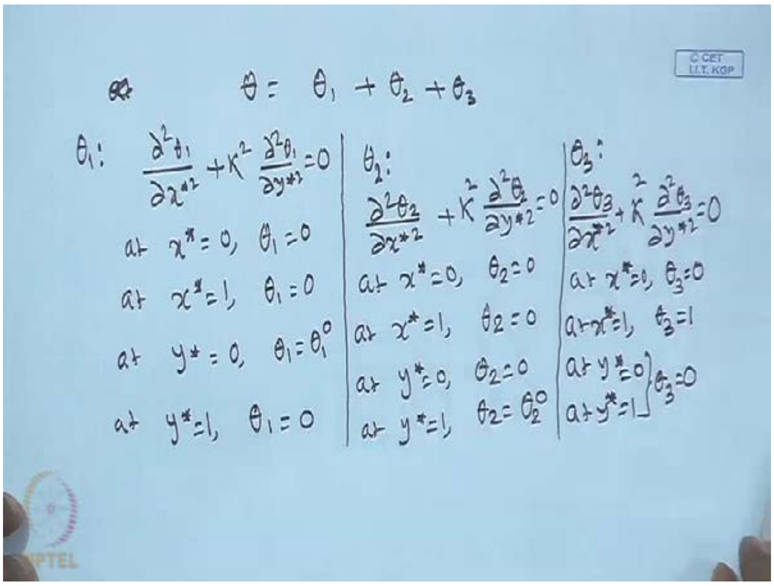

If you do that you will be getting theta will be divided into 3 sub-problems; theta 1, theta 2, theta 3. Let me solve 1 sub problem completely, and then you can take up the solution of other 2 sub problems individually. Now let us first formulate the governing equation and the boundary condition of theta 1, theta 2, and theta 3. So governing equation of theta 1 will be del square theta 1 del x star square plus kappa square del square theta 1 del y star square is equal to 0.

At x star is equal to 0 your theta 1 becomes 0, at x star is equal to 1 theta 1 is equal to 0, at y star is equal to 0 theta 1 is equal to theta 1 naught, at y star is equal to 1 theta 1 is equal to 0. So in this problem, what we have done? We have kept only one non-homogeneity at a time; we forced the other non-homogeneity to vanish. So that is the problem, theta 1 then we formulate the problem theta 2.

This will be del square theta 2 del x star square plus kappa square del square theta 2 del y star square should be is equal to 0, at x star is equal to 0, theta 2 is equal to 0, at x star is equal to 1, theta 2 is equal to 0, at y star is equal to 0, theta 2 is equal to 0, at y star is equal to 1, we kept keep that non-homogeneity intact so, theta 2 is equal to theta 2 naught.

Similarly, we formulate the third sub problem theta 3 as del square theta 3 del x star square plus kappa square del square theta t del y star square is equal to 0. Boundary conditions at x star is equal to 0, theta 3 is equal to 0, at x star is equal to 1, theta 3 is equal to 1, we keep that non-homogeneity here and we put both the non-homogeneity at y star is equal to 0 and at y star is equal to 1, we put both force the boundary conditions to be homogeneous.

So we divided the problem into 3 sub problem considering one non-homogeneity at a time; let us solve one sub problem first. Let us solve the first one theta 1, so theta 1 if you look into the boundary conditions in the x direction the theta 1 is having they are having the homogeneous boundary condition. So, we should formulate a standard Eigen value problem in x direction not in y direction for the sub problem theta 1.

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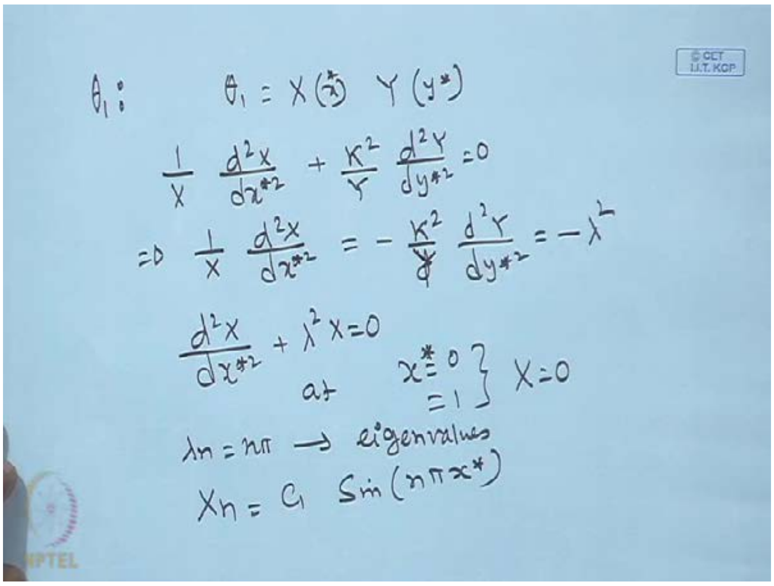

If you take up the sub problem theta 1, theta 1 should be composed of a function of X alone and a function of Y alone. So, this will be 1 over X d square X d x star square plus kappa square by Y d square Y d y star square is equal to 0.

Since the boundary conditions in the x directions are homogeneous, so you formulate the standard Eigen value problem in the x direction. So d square X d x star square is equal to minus kappa square capital Y d square Y d y star square this should be equal to minus lambda square.

So we formulate the standard Eigen value problem in the x direction plus lambda square X is equal to 0 and at x star equal to 0 and both theta 1 equal to 0, so we have at x star is equal to 0 and 1 we have both capital X is equal to 0, so lambda n are the Eigen values n pi and Eigen functions are some constant sin n pi x star.

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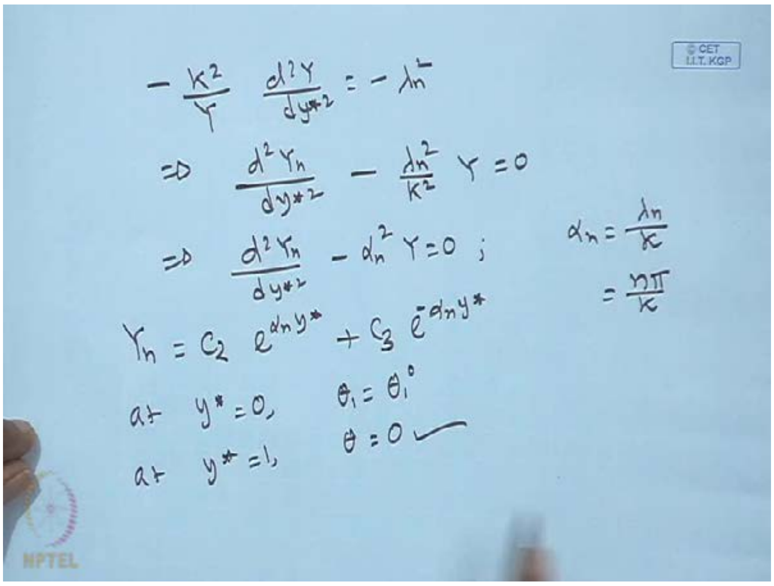

The y varying part that may be interesting, the y varying part becomes minus kappa square by Y d square Y d y star square is equal to minus lambda n square. So therefore, d square Y n d y star square plus - you bring it to the other side so minus - lambda n square by kappa square Y is equal to 0. So, this becomes d square Y n d y star square minus alpha n square times Y, where alpha n nothing but, a scaled lambda n; so it is equal to nothing but, n pi divided by kappa.

The solution of Y n is nothing but, C 2 e to the power alpha n y star plus C 3 e to the power alpha n minus alpha n y star. Now we invoke the boundary conditions, at y star is equal to 0, theta 1 is equal to theta 1 naught and at y star is equal to 1, we have theta 1 is equal to 0. We know the solution of this, so we have already solved this problem earlier the y varying part.

If you look into the solution, I am just doing it once again because I have the solution here, we have already seen it earlier and it is not a problem to solve this problem. So, we utilize this boundary condition, so that we can express C 2 and C 3 in terms of the other constant.

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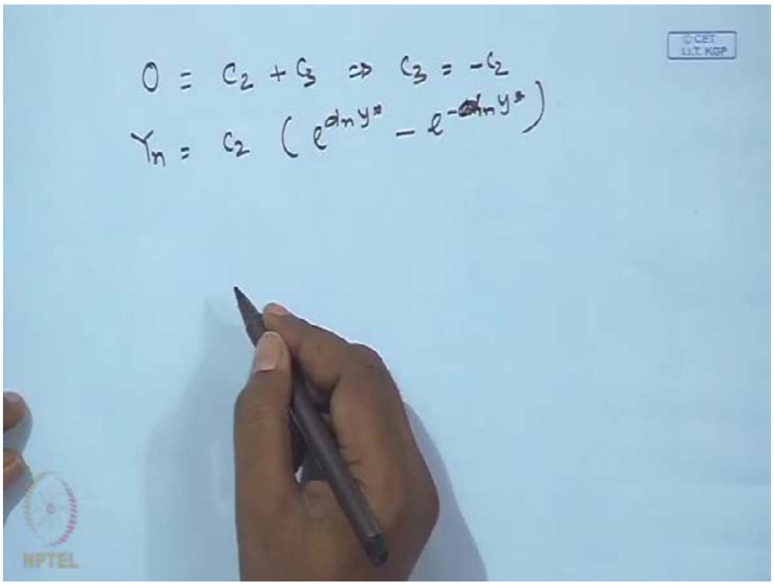

We put this is equal to 0 is equal to C 2 plus C 3; so C 3 is nothing but, minus C 2. So Y n is C 2 e to the power alpha n y star minus e to the power minus alpha n y star.

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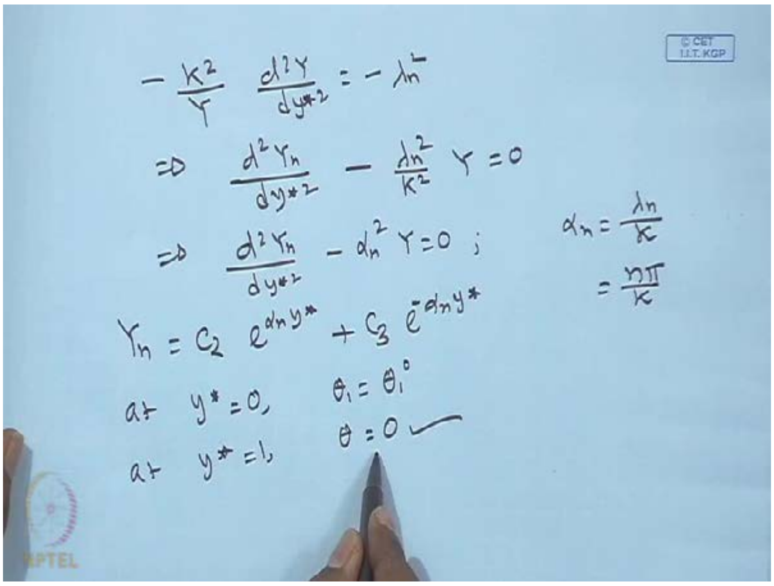

In the earlier case, we had the boundary condition that at y star is equal to 0, we had I think in the other boundary it was equal to non-homogeneous. In this case, we have y star is equal to 0, theta 1 is equal to theta 1 0 and y star is equal to 1, theta equal to 0.

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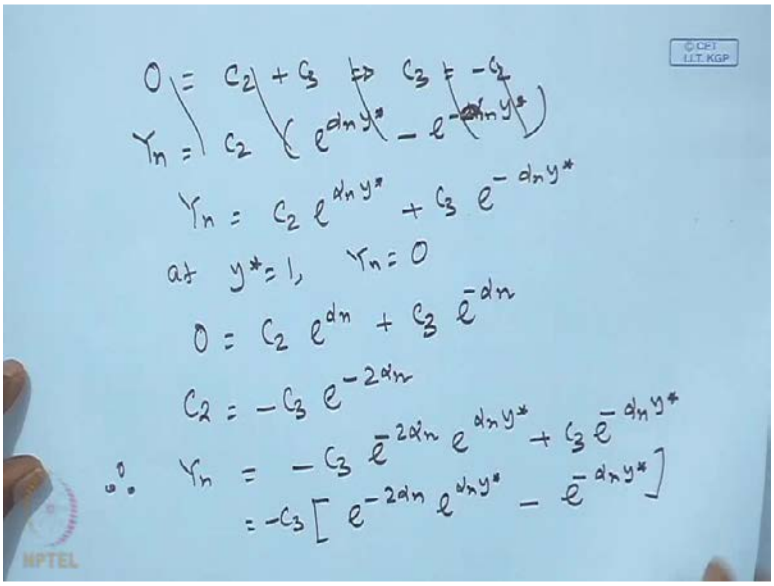

So I think, this is not correct (Refer Slide Time: 43:05). The solution is Y n is C 2 e to the power alpha n y star plus C 3 e to the power minus alpha n y star. So at y star is equal to 1, Y n theta is equal to 0 that means Y n is equal to 0. We will be having 0 C 2 e to the power alpha n plus C 3 e to the power minus alpha n so C 2 is nothing but, minus C 3 e to the power minus 2 alpha n.

So we have, Y n as minus C 3 e to the power minus 2 alpha n e to the power alpha n y star plus C 3 e to the power minus alpha n y star. We take C 3 common, so this becomes we take minus C 3 common this becomes e to the power minus 2 alpha n, e to the power alpha n y star minus so, this becomes e to the power minus alpha n y star.

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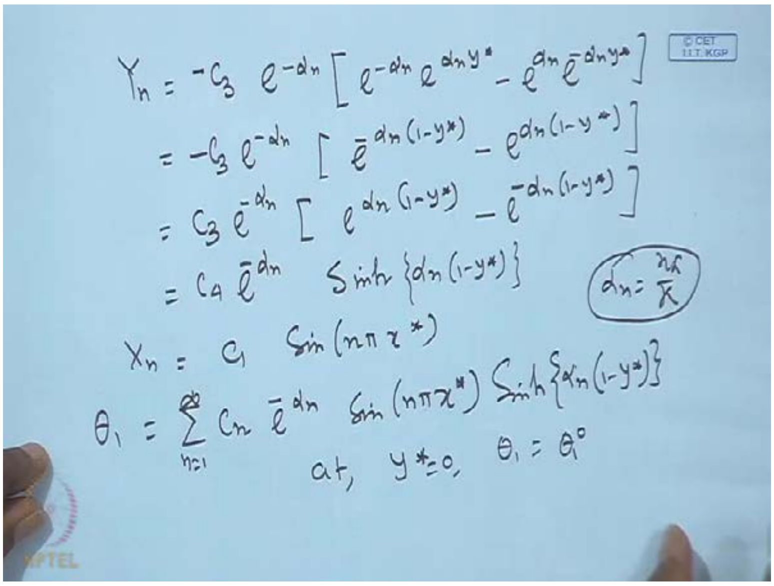

In fact, we should take e to power 1 e to the power minus alpha n common so, what will be getting is that Y n is equal to minus C 3 e to the power minus alpha n. We take it common, so this becomes e to the power minus alpha n e to the power alpha n y star minus e to the power plus alpha n e to the power minus alpha n y star.

Let us see what we get, so minus C 3 e to the power minus alpha n e to the power minus alpha n take it common, 1 minus y star minus e to the power plus alpha n 1 minus y star. So take it minus again, so minus minus this will be plus C 3 e to the power minus alpha n we will be getting e to the power alpha n 1 minus y star minus e to the power minus alpha n 1 minus y star. So divide and multiply by 2, it will be new constant let say C 4 e to the power minus alpha n and you will be getting a sin hyperbolic function sin h alpha n 1 minus y star.

Once we get the Y varying part and we have already got the X varying part and let us look into the solution. So X varying part we had the solution as C 1 sin n pi x star, so we will be getting the solution theta 1 has n is equal to 1 to infinity C 1 multiplied by C 4 it will be C n e to the power minus alpha n sin n pi x star, sin hyperbolic alpha n 1 minus y star where alpha n is nothing but, n pi by kappa; so alpha n is nothing but, n pi by kappa.

Now, we utilize the other boundary condition that is at y star is equal to 0, theta 1 is equal to theta 1 naught. At y star is equal to 0, theta 1 is equal to theta 1 naught, we utilize this boundary condition and see what we get.

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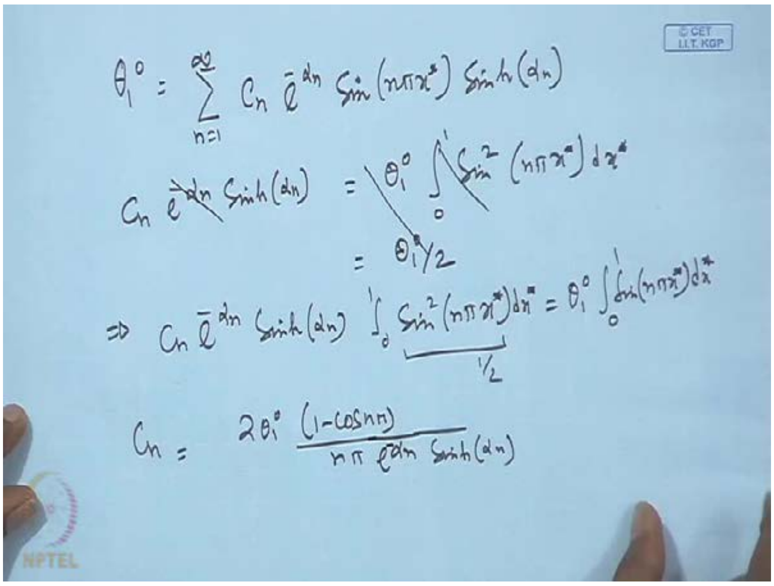

We will be getting theta 1 naught is equal to summation n is equal to 1 to infinity C n e to the power minus alpha n sin n pi x star sin h alpha n because y star equal to 0 means it will be sin h alpha n only.

So, we use the orthogonal property of the sine function, so it will be C n e to the power minus alpha n sin hyperbolic alpha n is nothing but, theta 1 naught integral of sin square n pi x star d x star. I am just omitting one step, we multiply both side by sin m pi x star d x star integrate over the domain of x star and change the index m to n only one term will survive, all the other terms will vanish because of the orthogonal property of the sine functions.

So this becomes theta 1 naught divided by 2, so C n no, it will be the other way round. So, this will be C n e to the power minus alpha n sin hyperbolic alpha n, 0 to 1 sin square n pi x star d x star is equal to theta 1 0 integral sin n pi x star d x star from 0 to 1.

This will be having a value of half, so C n will be nothing but, 2 theta 1 naught 1 minus cosine n pi divided by n pi, e to the power alpha n, sin h alpha n; C n will be e to the power of minus alpha n.

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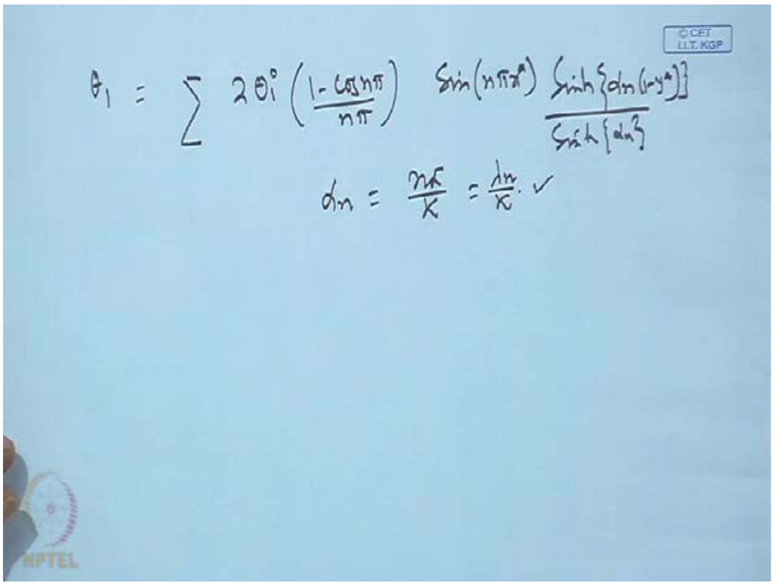

Now we can put it in the equation, so you will be getting the complete solution as theta 1 summation 2 theta 1 naught 1 minus cos n pi divided by n pi e to the power of minus alpha n and that will be cancelled out. So we will be getting sin h sin hyperbolic sin n pi x star and sin hyperbolic alpha n 1 minus y star divided by sin hyperbolic alpha n.

So that gives the solution of theta 1 so similarly, one can get the solution of the other parts as well where alpha n are nothing but, n pi divided by kappa. This will be basically in terms of original variables, this is a by b, so that is a geometric factor. So it is this nothing but, lambda n by kappa; similarly, one can obtain if you look into the theta 2, varying theta 2, so we have completely solved theta 1.

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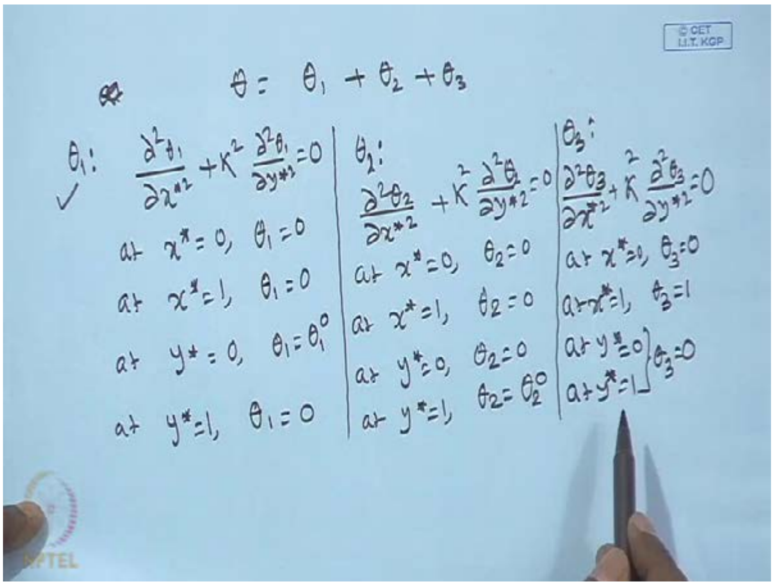

If you look into the theta 2, theta 2 is having the homogeneous boundary conditions in the x direction and non-homogeneity in the y direction, the same way we had earlier. So we will be having a since will be having a homogeneous boundary conditions in x direction, we will be having the boundary condition the Eigenvalue problem in the x direction and the boundary conditions on y star is non-homogeneous. So, we will be utilizing these boundary conditions later on to evaluate the final constant.

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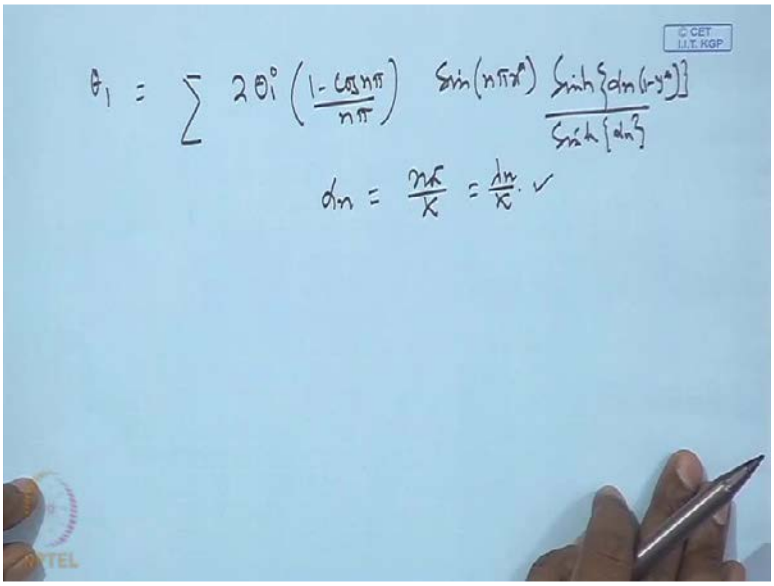

This roughly shows that i am not going to solve this problem theta 2 and theta 3 completely in this class but, the same way they can be solved and we can get the complete solution of theta 1, theta 2 and theta 3. Once we will be getting the difference solutions of theta 1 theta, 2 and theta 3 we can superpose each of them and can get the complete solution.

We have already seen how to take up the well posed basic problem for an elliptic partial differential equation. So even if the boundary conditions are not homogeneous, one can make them a homogeneous by breaking down the problem into sub problem considering one non homogeneity at a time and by that one can decompose the each sub problem as a basic problem. One can go ahead with the solution and then sum these all up; one can get a complete solution.

Only one thing has to remember in the case of elliptical partial differential equation that you have to identify the direction where the boundary conditions are homogeneous. So we have to formulate the Eigen value problem in the particular direction and then solve the problem. Ultimately you will be getting the Eigen functions in their particular direction; on the other direction, you will be getting a solution with two constants of integration using the homogeneous boundary condition you evaluate one constant in terms of the other.

So you obtained the final solution only one constant need to be determined, these constants will be determined by utilizing the non-homogeneous boundary condition of the original problem. Using the orthogonal property of the Eigen functions after that you will be getting the complete solution by superposing all such individual problem and add them up.

I will stop the lecture here, for this class. I will take up the solution of hyperbolic partial differential equation in the next class, thank you very much.