Advanced Mathematical Techniques in Chemical Engineering
Prof: S. De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur
Lecture No. # 28
Four Dimensional Parabolic PDE

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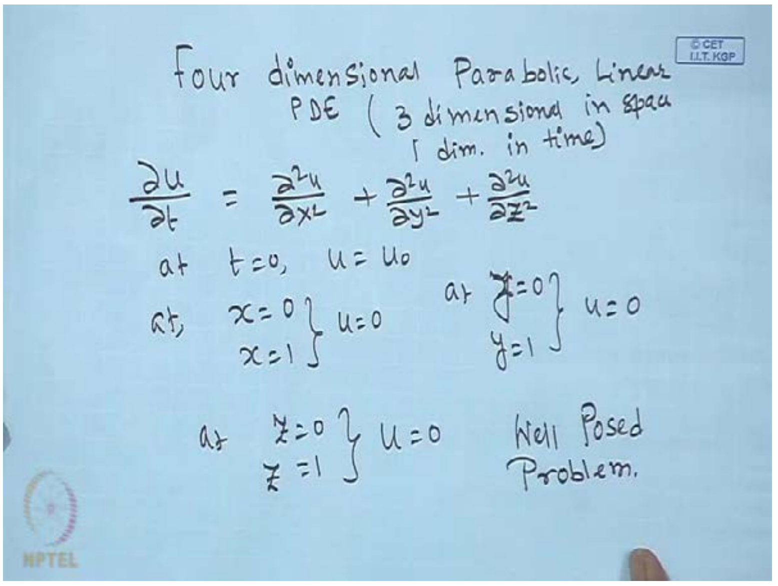

Welcome to the session of the class today's class, so we will be talking about a four dimensional parabolic and linear partial differential equation, so it is basically three dimensional in space one-dimensional in time. So, it is a three dimensional transient heat conduction problem, so the governing equation will be in general looking have to like this, del u del t is equal to del square u del x square plus del square u del y square plus del square u del z square.

Now we formulate a well posed problem and basic problem that is at time t is equal to 0, u is not equal to 0. Let us say, u is equal to u naught and the other four boundary, other six boundary conditions. We need to have two conditions specified on x two conditions on y two conditions on z.

So, all of them are dirichlet. Let us say at x is equal to 0, and at x is equal to 1. We have u is equal to 0, at x is equal at y is equal to zero and y is equal to one. We have, let say u is equal to 0 at z is equal to 0 and at z is equal to 1, we have u is equal to 0.

So, let us say all the four boundary all the six boundary conditions are homogeneous and the initial condition is non homogeneous though, so this is a well posed problem.

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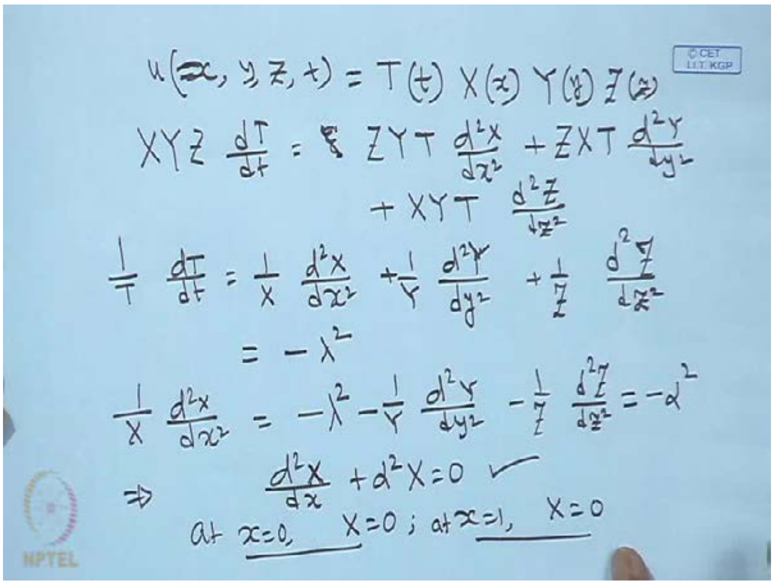

Now, in order to solve this problems since this operator is a linear operator, and all the boundary conditions are homogeneous, we can use separation or variable type of solution safely. So, if we solve, so we consider that u is constituted by product of four factors. So, this is u is a function of x y z and t so it is a product of four functions, which is a function of time alone. Another is a function of x alone, another is function of y alone, another is function of z alone.

So therefore, we substitute these equation into the governing equation. It will be getting x y z d t d t is equal to x, we will be getting z y t d square x d x square plus z x t d square y d y square plus x y t d square z d z square.

So, divide both side by x y z t, what we will be getting is that; one over t d t by d t is equal to one over x d square x d x square plus one over y d square y d y square plus one over z d square z d z square.

Now, if you look into this equation the left hand side is a function of time alone, the right hand side is a function of space and they are equal. They will be equal to some constant. If this constant is zero and positive, we will be getting a trivial solution. So this constant has to be a negative constant, so this becomes minus lambda square.

Now, we will be formulating the x containing part, so that we move y and z to the other side. So, this becomes one over x d square x d x square is equal to minus lambda square, minus one over y d square y d y square minus one over z d square z d z square.

Now again the left hand side is a function of x alone. The right hand side is a function of y and z only and they are equal again. They will be equal to some constant. Let us say this constant; if this constant is zero and positive we will be getting a trivial solution that is not allowed, therefore this constant has to be a negative constant, let us put it as minus alpha square.

So, will be getting the governing equation of x d square x d x square plus alpha square x is equal to 0. Now the x containing the x, the x containing part must satisfy the boundary condition of on the on x boundaries of the original problem. So therefore, at x is equal to 0, capital x is equal to 0 and that x is equal to one capital x is equal to 0, so that was those are the boundary condition of the original problem, so x varying part must be satisfying that.

Now, if you look into this equation, this equation is again a standard Eigen value problem and both the boundary conditions are homogeneous and they are dirichlet. So, we have already seen the solution of this problem. The solutions are the Eigen values are given by n pi and Eigen functions are given by sine n pi x.

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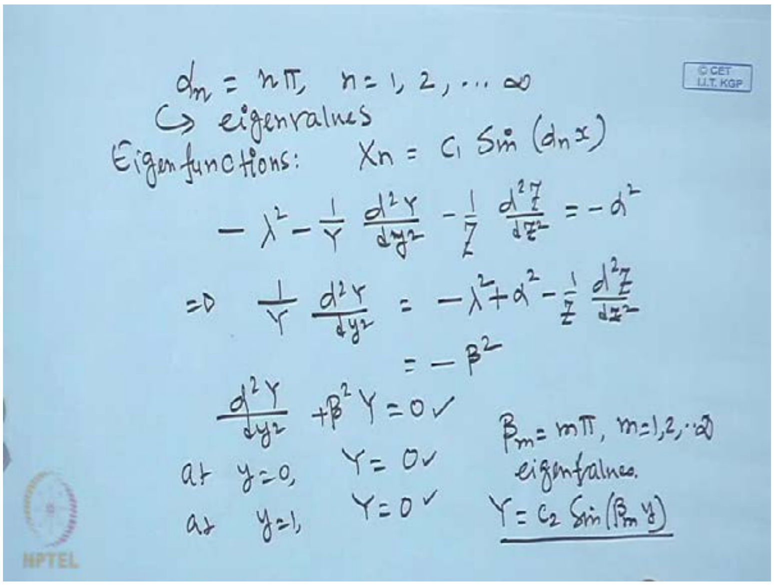

So therefore, alpha n for this particular problem is equal to n pi, where the index one n runs from one, two, three, up to infinity. These are the Eigen values, and Eigen functions are given by x n is equal to c one sine alpha n x that means c one n pi x. Now, let us look into the other two parts minus lambda square minus one over y d square y d y square minus one over z d square z d z square, is equal to minus alpha square.

So, therefore we take it as minus, we take it on the other side. So, one over y d square y d y square should be minus lambda square, plus alpha square minus one over z d square z d z square. Now this the left hand side is completely a function of y the right hand side. These two are constants, the right hand side is completely a function of z. They are equal, and they will be equal to some constant. Let us say, this is minus beta square, this constant can be positive, this constant can be negative. This constant can be zero, if it is zero and positive, this constant then, we will be getting a trivial solution. So, this constant has to be negative constant. So, you will be getting d square y d y square plus beta square, y is equal to zero subject to the boundary conditions.

Now, we formulate the boundary conditions of y. We have already seen that the boundary conditions of the original problems at y are homogeneous. Therefore the y varying part must be satisfying the boundary condition of the original problem in the at y boundaries.

So, therefore at y is equal to zero, we have capital y is equal to 0 at y is equal to 1. We have capital y is equal to zero and if you look into this problem, this is again a standard Eigen value problem and the homogeneous dirichlet boundary condition.

So, the solution to this so beta m are is equal to m pi, where m is equal to one two three up to infinity. They are the Eigen values and Eigen functions will be composed of y is equal to some constant, c two sine beta m y so m pi y, so these are the Eigen functions, ok.

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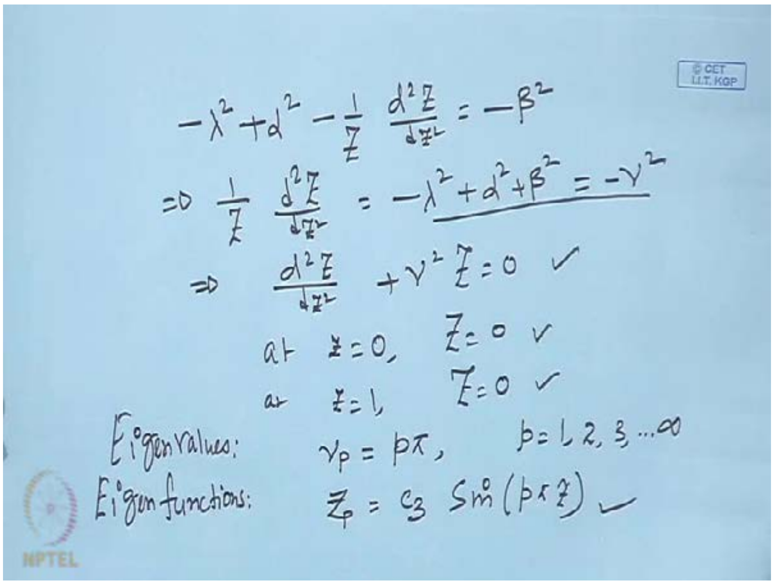

So, next we look into the z varying part. If we look into the z varying part, this becomes minus lambda square plus alpha square minus one over z d square z d z square, is equal to minus beta square, is equal to one over z d square z d z square from the other side. So bring it to the side, minus lambda square plus alpha square plus beta square.

So again, this is a combination of the constant. This constant can be, if this constant is zero and positive going to get a trivial solution. So, this constant has to be a negative constant. Let us say, it is nu square, so d square z d z square plus nu square z must be equal to 0.

Now, we get the boundary conditions on z. The boundary condition of the original problem were dirichlet homogeneous, so boundary conditions on z should be again dirichlet homogeneous on the z boundaries.

So, at z equal to small z equal to 0, capital z is equal to 0 at small z is equal to 1 capital z must be equal to 0. So again, we see this is a typical standard Eigen value problem with zero initial condition, zero homogeneous boundary conditions.

Now the Eigen values of this problem is known and the Eigen values are again given by p pi and Eigen functions are sine functions sine p pi z therefore, we will be getting the Eigen values as nu p is equal to p pi where p is equal to one, two, three up to infinity and Eigen functions or solutions are z p where this will be c three sine p pi z ok

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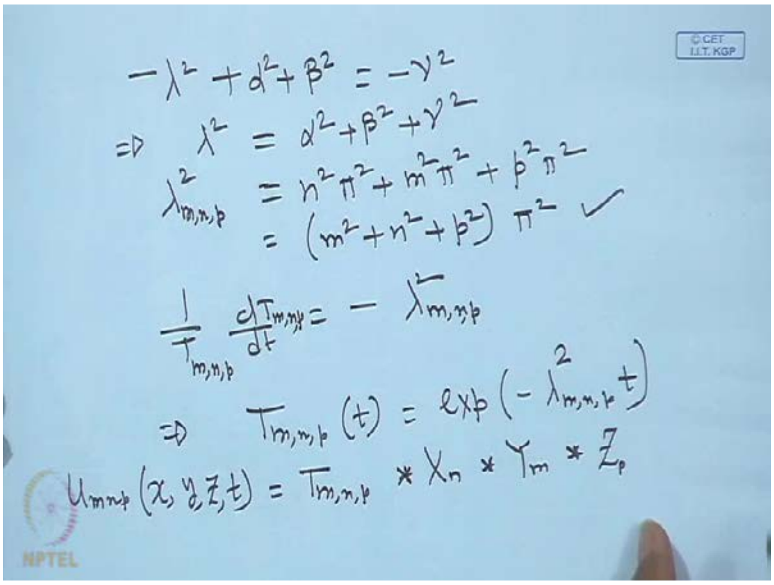

So these are the Eigen functions, and now what is left is the time varying part, but before going into the time varying part, let us examine what this, what these equation means. These equation means that minus lambda square plus, alpha square plus beta square is equal to minus nu square.

So therefore, lambda square you just take it just change over the sign. So lambda square is nothing, but alpha square plus beta square plus nu square. So, if you put values of alpha, so these are n square pi square value of beta. This is m square pi square value of gamma that is p square pi square. So, we put a subscript lambda m n p. So, this becomes m square plus n square plus p square pi square lambda square.

So, the time varying part, will look into the time varying part, one over t d t d t is equal to minus lambda m n p square. So corresponding to the m n p, the corresponding solution we will be also getting m n p. We write it m n and p, so lambda square is given by all these in in as a function of m n and p. So, therefore the solution to this time varying part is known to us, m n p function of time will be exponential minus lambda square t and where the lambda values are already known from the value of m n p and pi square.

So, we will be getting the complete solution by simply multiplying both all the four solutions of the sub-problems. So u m n p as a function of x y z and t becomes it should be multiplied by, so it should multiplied by t m n p, should multiplied by x n multiplied by y m multiplied by z p.

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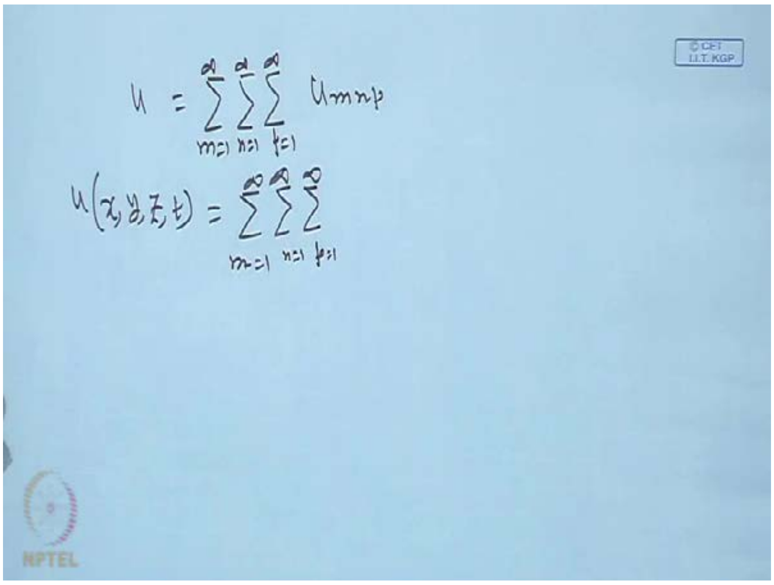

So, will be able to get the complete solution as u, u becomes summation of u m n p, so one over one summation is over m one to infinity, another summation is over n one to infinity, another summation over p one to infinity.

So, u as a function of x y z and t. This becomes triple summation m is equal to one, n is equal to one, p is equal to one to infinity and these will be, you multiply all the constants ok.

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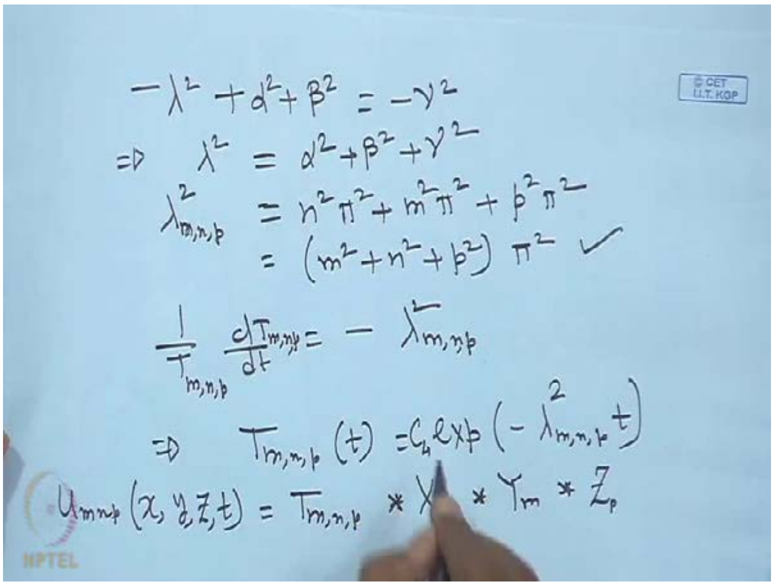

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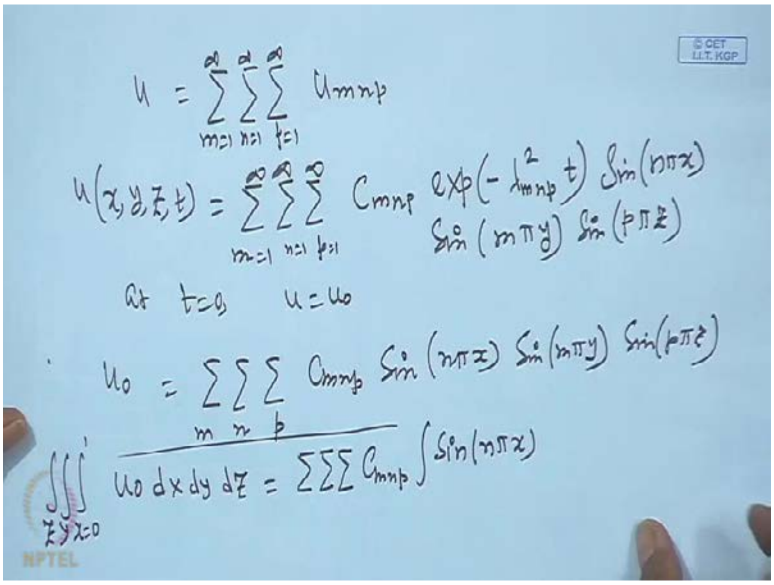

The solution of t should be associated with a constant c four,so we multiply all the constants, all the functions, so it will be having a new constant multiplication of all the constants, c, one c, two c, three and c four, so you will be having c m n p exponential minus lambda m n p square times t sine n pi x sine m pi y sine p pi z.

So, that constitutes the complete solution. Now, the constant c m n p has to be evaluated. For that, we utilize the initial condition, the initial condition was at t is equal to zero, u was equal to u naught, and we will be using the orthogonal property of the Eigen functions. At the sine function, so we multiply both side by sine m pi x sine n pi n pi y and sine let say q pi z. So and d x d y d z and integrate over the domain of x y z. So, it will be u naught. We will just write it, first three summation one is over m and another is over n another is over p c m n p. It will be one sine n pi x sine m pi y sine p pi z.

So, we multiply both side by all these. So, it will be triple integral one over from x is equal to zero to one another over y another over z. So, it will be u 0 d x d y d z. This will be triple summation c m n p integral sine n pi x, ok.

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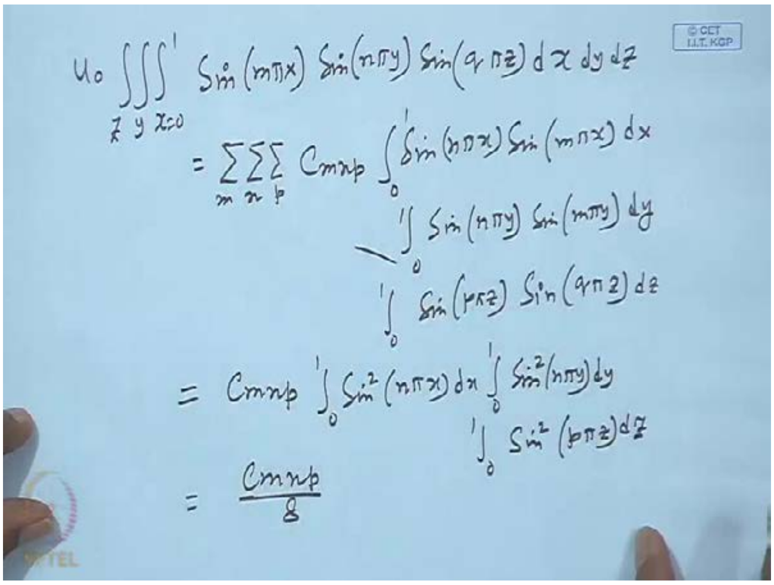

It should be multiplied by sine all these. So, I'll just try this, will not be, I will just write it on the next page, so that will be more convenient, so it will be u zero is constant it will be taken out. So, this will be one over x zero to one y z sine m pi x sine n pi y sine q pi z d x d y d z and this in the on the right hand side. It will be triple integral m n and p c m n p, integral sine n pi x sine m pi x d x from zero to one. One more integral, zero to one sine n pi y sine m pi y d y. Another will be on, sine p pi z sine q pi z d z from zero to one.

Now, we utilize the orthogonal property of the sine function and open up the summation series. If you really do that the entire all the terms except m is equal to n. They will vanish and except m is equal to n and p equal to q. They will vanish, so whatever is remaining is the one term, c m n p zero to one sine square n pi x d x zero to one sine square n pi y d y and zero to one sine square p pi z d z.

Now, value of this is half, that we have already seen half each, so it will be one by two one by two, one by two. So, it will be one by eight. So, it will be c m n p divided by eight and let us look what is the value in the of these on the left hand side.

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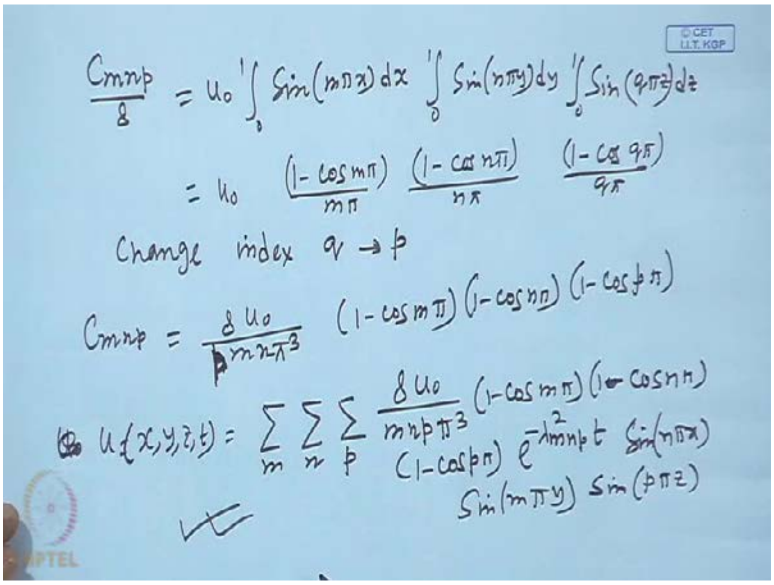

So, c m n p divided by eight should be is equal to, u naught integral sine m pi x d x zero to one, integral zero to one, sine n pi y d y zero to one sine q pi z d z from zero to one. We have already seen this one, this is nothing, but one minus cosine m pi divide by m pi, this will be one minus cosine n pi divide by n pi, and this will be one minus cosine q pi divided by q pi.

Now, we change the running index change index q to pi q to p. So, everything will be m n p. So, you will be getting c m n p is nothing, but eight u zero divide by m n pi cube m n q pi q one minus cosine m pi one minus cosine n pi one minus cosine. This should be change to q, should be change to p cosine p pi.

So, the complete solution will be evaluated as u zero, u as a function of x y z t as triple summation over m n and p. These becomes eight, u naught divide by m n p pi cube one minus cosine m pi, one minus cosine n pi, one minus cosine p pi e to the power minus lambda, m n p square times t sine n pi x sine m pi y sine p pi z. Now, this gives the complete solution of a four dimensional problem and by looking into the values of u naught, we can really compute the and compute the profile of u as a function of x y z and t, and we already know what is lambda m n p is that lambda m n p square is nothing, but pi square bracket m square plus n square plus p square.

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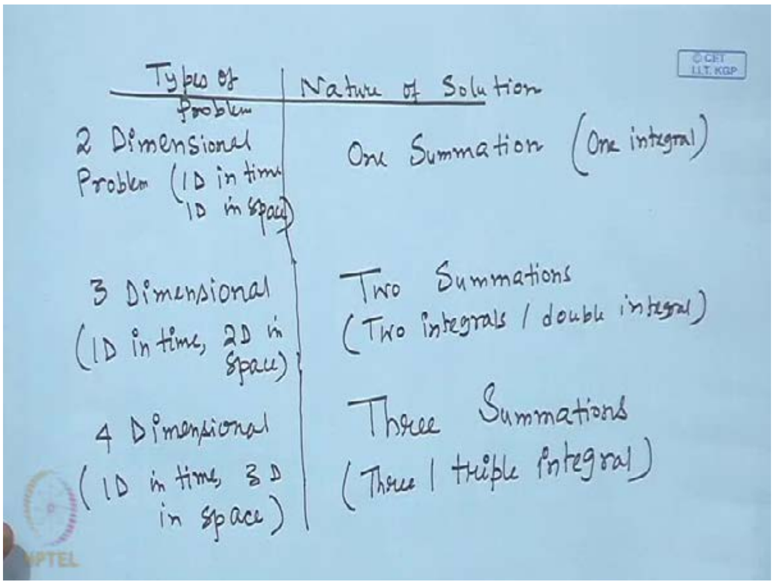

Now, if you see a typical, you know sequence of things, whatever we have done that if we have a one-dimensional, if we have a, let say types of problem and nature of solution if it is two-dimensional problem, that is one-dimensional in time, one-dimensional in space will be getting one summation in the solution.

If you have a three dimensional problem, then one dimension in time and two-dimensional in space, then we have two summations in solution. If we have a four dimensional problem, one-dimensional in time, and three dimensional in space, then you have three summations appearing in the solution.

Now, in case of one summation, it is a, it is in case two-dimensional problem, you are doing one integral is appearing on the left hand side to evaluate the final constant for two summation, two integrals, that means double integral appears to evaluate the final constant in case of three summation, we will be having three integrals or triple integral appears on to evaluate the final solution.

So, you just see the sequence of the nature of the solution changes how the you know variation of the problem, you know they differ for a two-dimensional problem will be having one summation in the final solution for a three dimensional problem. We will be having two summations for the four dimensional problem, we will be having, we will be having three summation series, and we will be having one integral two integral and three integrals double and triple integral to evaluate the final constant, ok.

So, if as we have seen in the earlier case, that depending on the solution on the nature of the boundary condition of the even for the multi dimensional problem, the Eigen values and Eigen functions will keep on changing, that means in the earlier problem, we have consider that all the boundary conditions are dirichlet boundary conditions.

If one of the boundary condition, let us say on y direction the boundary condition at let us say y is equal to the boundary condition, at y is equal to zero becomes a Neumann boundary conditions. That means it is insulated at that particular wall or it there will be constant heat flux at that particular surface or boundary, then will be having a Neumann boundary condition and all the other boundary conditions are you know dirichlet type. Then the Eigen functions for x will be sine functions Eigen functions of y will be cosine functions and Eigen functions of z will be sine functions.

If all the boundary conditions are homogeneous and dirichlet, but one of the boundary conditions at z, let us say z equal to zero it becomes a Neumann, then the Eigen functions for x will be sine functions, Eigen functions for of y will be the sine functions, but Eigen functions of z will be the cosine function.

If the Eigen where if the boundary condition on y, let say it becomes a robin mixed at x is equal to at y is equal to one, then Eigen functions will becomes sine functions, but the Eigen values will be no longer n pi. In this case, the Eigen values will be obtained by the transcendental equation, that alpha n tan alpha beta n beta m tan beta m plus some constant. Let us say biot number of some constant, those will be appearing in the governing equation should be equal to zero.

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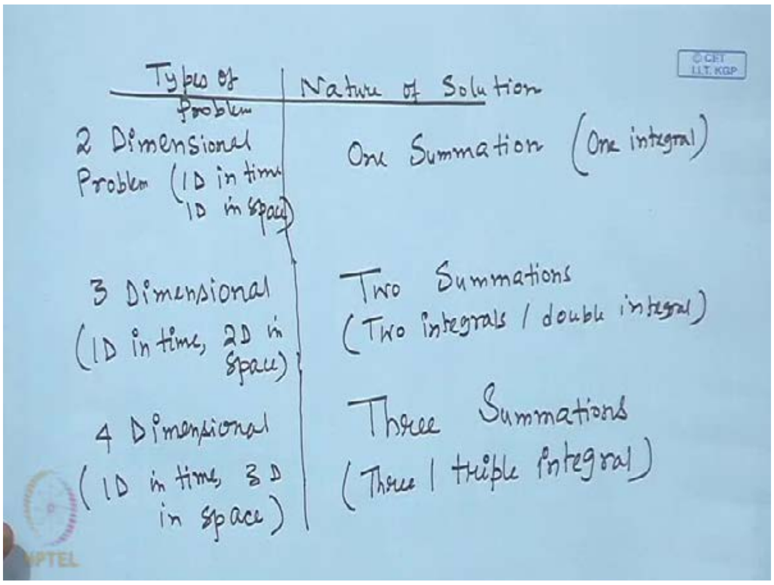

The roots of this transcendental equations will become the Eigen values therefore, depending on the nature of the boundaries, the Eigen values and Eigen functions will be done appropriately. Similarly, if we have the typical for a typical problem, where you will be having all sorts of non-homogeneities appearing in the boundary conditions. Then we have to make the problem non-dimensional such that at least one non-homogeneity will be reduced. Let us say from five to four, then this problem has to be divided into four sub-problems considering one non-homogeneity at a time, and this problem has to be solve completely by breaking down into sub-problems considering one non-homogeneity at a time.

But, out of this four sub-problems one sub-problem will be well pose because it will be containing a non-zero or non-homogeneous initial condition but, homogeneous boundary conditions, so that problem will be a well posed problem and that will be completely solved. The way you have solved this problem just earlier, but all the other problems all the other three problems will be ill posed problem, because the initial condition becomes homogeneous, but at least one boundary condition becomes non-homogeneous.

So, each such sub-problem can be will be again divided into two sub-problems. One will be time dependent part, another will be time independent part, and the non-homogeneous boundary condition will be associated with the boundary condition of the time independent part or the steady state part.

So, by doing that you force the boundary condition appearing in the time varying part to be homogeneous. So, we will be having and the initial condition of the time varying part will become the solution of the steady state part with a negative sign. So, therefore the transient problem becomes a well posed problem. It becomes a non-homogeneous initial condition, and all the four all the six boundary conditions becomes homogeneous, so that becomes a well posed problem. We have seen how to solve that problem several times in earlier classes.

So, by after solving all these four sub-problems, and then add up all the solutions, we will be getting the overall solution of this four dimensional problem.

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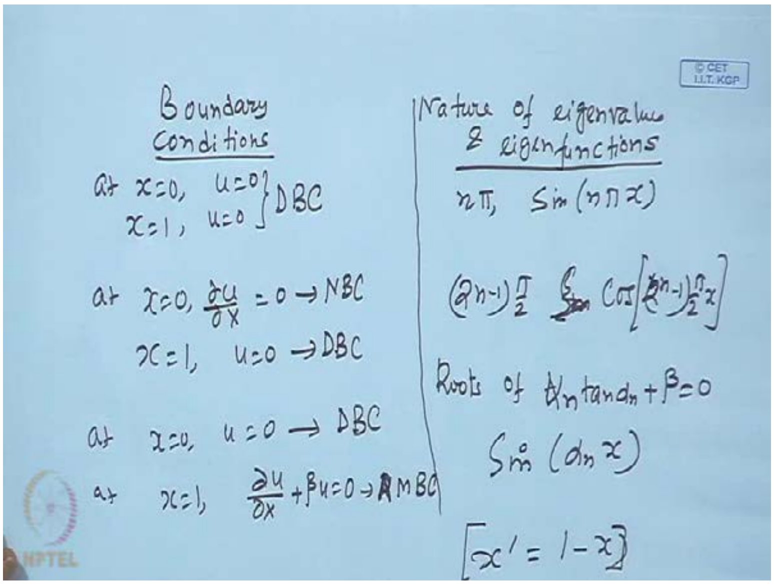

Now, one can may be wondering that if we have the boundary conditions, we have looked into all kinds of boundary conditions, but there are certain things one has to check that, we, I'll just list it down boundary conditions and nature of Eigen values and Eigen functions. This is dirichlet boundary condition at x is equal to 0, u is equal to 0 at x is equal to 1, u is equal to 0. So, these boundary condition conditions are dirichlet boundary condition, the Eigen values where n pi and Eigen functions where sine n pi x.

Now at x is equal to zero del u del x is equal to zero and at x is equal to one, u is equal to zero, so this becomes Neumann boundary condition. This becomes a dirichlet boundary condition, the Eigen values are two n minus one pi by two and sine and cosine functions are the Eigen functions, cosine two n minus one pi by two x are the Eigen functions, and at x is equal to zero, if you have u is equal to zero, and that x is equal to one, if you have del u del x plus beta u is equal to zero.

So you will be having a dirichlet boundary condition. Here, you will be having a robin mixed boundary condition at this boundary, then Eigen functions are roots of the transcendental equation, tan alpha n tan alpha n plus beta is equal to zero, and Eigen function these are the Eigen values roots of the transcendental equation, alpha n tan alpha n plus beta equal to zero and Eigen functions are sine alpha n x, ok.

Now, the point is if this boundary condition is reversed will the same thing will will will take place. Now, that means instead of dirichlet boundary condition, there is no problem, instead of Neumann boundary condition and dirichlet boundary condition, instead of x is equal to one, if we have a dirichlet boundary condition at x equal to zero, that means x equal to 0, I have u is equal to 0 and this Neumann boundary condition becomes at x is equal to one.

That means at x equal to 0, del u del x is equal to 0. Similarly, in this case at x is equal to zero, u is equal to zero, we put it the other way round, we just interchange the boundary conditions at x is equal to zero. We put del u del x plus beta u is equal to zero and at x is equal to one, we have a dirichlet boundary condition.

So, what happens when this boundary conditions are interchanged? What will happen in that case? In that case you just define a new variable, let say x prime is equal to one minus x. If we define a new variable x prime, is equal to one minus x, these boundary conditions will be reversed, and they will fall back into these fixed category, then we can directly apply the Eigen values and Eigen functions as the solution.

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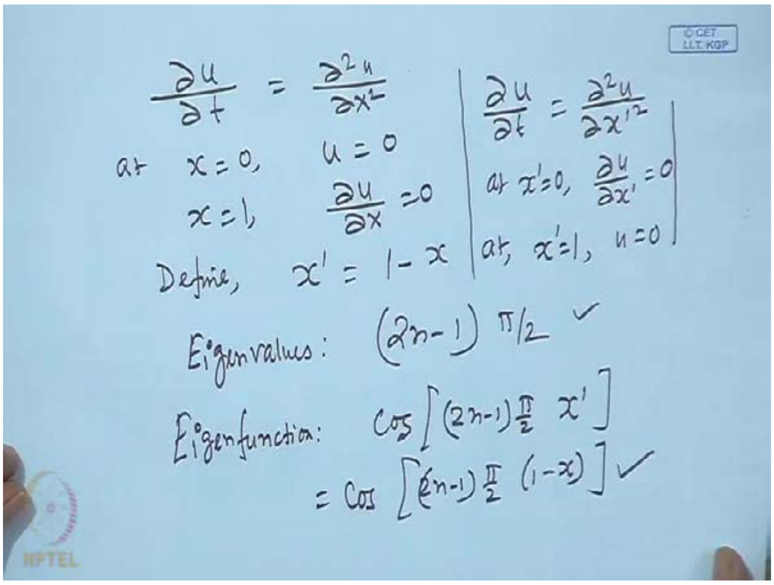

I'll just demonstrate whatever I am saying by a small example. For example, let us consider a two-dimensional parabolic partial differential equation del u del t is equal to del square u del x square, and I'll be having at x is equal to zero. We have u is equal to zero, and at x is equal to one, we have del u del x is equal to zero.

So, we has interchanged the boundaries boundary conditions. So, we define and x star such as this is one minus x, so, del u del t does not so let us look into the initial, the governing equation del u del x is equal to del u del x prime d x prime del x prime del x. So, it will be minus del u del x will be nothing but, minus del u del x prime but one more derivative, this minus minus plus so governing equation remains the same del u del t is equal to del square u del x prime square.

Now, when we define x prime is equal to one minus x, this boundary condition becomes zero at x prime is equal to 0, del u del x prime, there is a minus sign, so it will be cancelled out. So, it will be zero for the basic problem, homogeneous boundary condition, and this becomes x is equal to 0 means x is equal to x prime is equal to 1.

So, x prime is equal to 1. We have u is equal to 0. So, this problem is exactly fall under the category of this problem, so the boundary condition so this is our known form. Therefore the Eigen values becomes two n minus one pi by two and Eigen functions now become you know, Eigen functions becomes cosine two n minus one pi by two x prime.

So, they become cosine two n minus one pi by two one minus x in the original variable. So, this will be the form of final form of the Eigen values, and final form of the Eigen functions.

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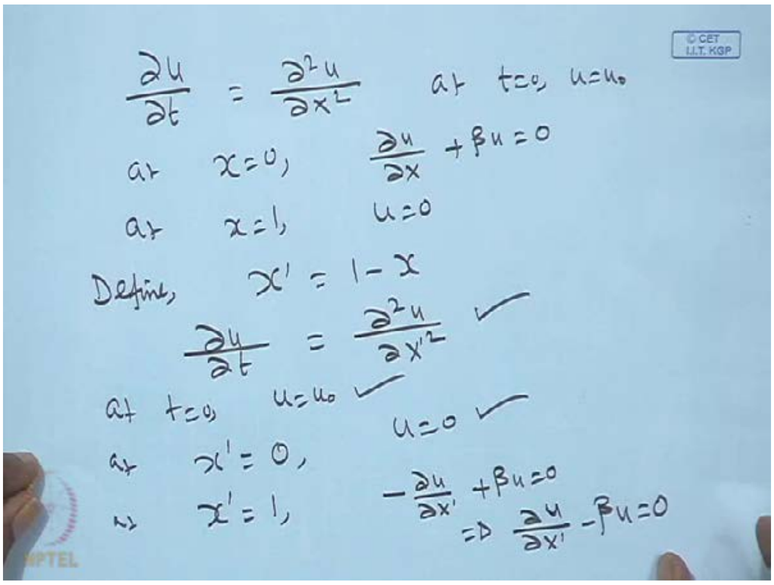

Now, if you have a mixed boundary condition, let us look into that, del u del t is equal to del square u del x square, therefore at x is equal to zero, let us say, at t is equal to u is equal to u naught at t is equal to zero u is equal to u naught, and we have at x is equal to zero. We have the mixed boundary condition, that is del u del x plus beta u is equal to zero, and at x is equal to one. We have u is equal to zero, the dirichlet boundary condition.

Now again, we define x prime as one minus x, so this becomes the governing equation remains unchanged, this becomes del square u del x prime square at t is equal to zero. u is equal to u naught at x prime is equal to one x is equal to zero, this becomes when x is equal to one, this becomes x prime becomes zero, so x prime equal to zero. u is equal to zero and at x prime is equal one, x is equal to zero, one x is equal to zero means x prime is equal to one. We have minus del u del x prime plus beta u is equal to zero, therefore del u del x prime minus beta u is equal to zero.

Now, let us look into this, examine this problem, this again, u is a u is consider to be function of now, what is the difference between this boundary, this problem, and the standard problem. We have solved, we have solved the standard problem that the governing equation remains the same, the initial condition remains the same, the boundary condition at x prime remains the same.

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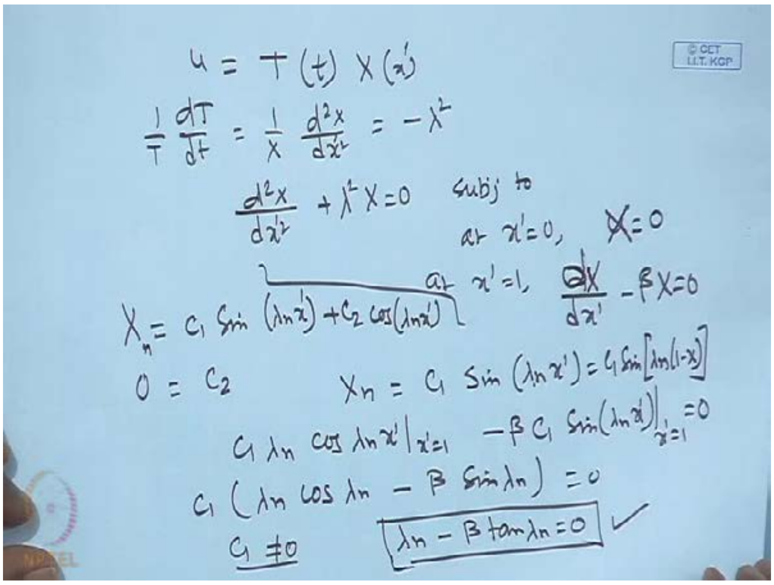

The boundary condition at one becomes the problem. We have standard problem, we have solved, we have solved del u del x prime plus beta u. In this case, it becomes minus beta u, so, let us see how these will affect our final solution, so it does not affect much only a plus sign becomes minus sign in the form of the Eigen values, the transcendental equation that governs the Eigen values.

So, we consider u is a function of t and x, so we will be having d t one over t d t d t is equal to one, over x d square x d x square is equal to minus lambda square, so we will be having d square x d x square plus lambda square x is equal to 0. This will be x will be x x prime, so will be x prime, so subject to the boundary condition at x prime is equal to 0, u capital x is equal to 0, and at x prime is equal to zero del e, del x d x d x prime minus beta x, should be is equal to 0.

Now, the solution to this problem is x n is equal to c one sine lambda n x plus x prime plus c two cosine lambda n x prime. Now put the boundary condition, that x prime is equal to zero. x is equal to 0, so this become 0 is equal to c two cos zero is one sine zero is zero, so c two equal to zero, so x n becomes c one sine lambda n x prime.

Now, we have the other boundary, now we use the other boundary condition c one lambda n cosine lambda n x prime evaluated at x prime is equal to one minus beta x n. So beta c one sine lambda n x prime evaluated at x prime is equal to one should be equal to zero. So, we take c one common so c one, and it is lambda n cosine lambda n minus beta sine lambda n is equal to zero. So, for c one not equal to zero, we have lambda n minus beta tan lambda n is equal to zero.

So, this becomes instead of minus sine, this becomes instead of plus sign, this becomes a minus sign, therefore the boundary conditions, these boundary conditions makes that the Eigen values will be roots of this transcendental equation, lambda n minus beta lambda n, and the Eigen functions remain same as sine functions, but in this case it will be sine lambda n x prime.

So, it will be in in terms of x it will be c one sine lambda n one minus x. So, only that much change will be there in the in the governing equation, in the solution therefore, if the boundary conditions are interchanged, then you have a shift of axis, and bring the boundary conditions back to the original problem, the problem that we have already defined, and you will be getting the solution straight forward with the change in the and later on in the transform variable, convert the transform variable into the original variable, by putting x prime is equal to one minus x, and or y prime is equal to one minus y something like that.

That will give you the complete control of over the all kinds of boundary conditions whether they will be appearing on this boundary, or that boundary, it absolutely does not matter. So, the method is absolutely clear to us. What to do if this, if the boundaries with the boundary conditions do not appear the same way of whatever we have done this. So, we have, we have used for solving such problems for several classes.

If the boundary conditions are interchanged, you shift the independent shift, the axis of the independent variable as x prime is equal to one minus x at read to the problem reformulate the problem, and solve the problem. As earlier, we have done so it remains the same, and the method remains the same, that gives us the ability to tackle all sorts of boundary conditions appearing in anywhere in the boundaries.

Next, so that that finishes over the three dimensional parabolic four dimensional parabolic problem, so it gives the most general time up type of parabolic problem. Now again if some of the boundaries will be having Neumann, some of the boundary conditions will be having robin mixed, the Eigen functions and Eigen values will be changed accordingly.

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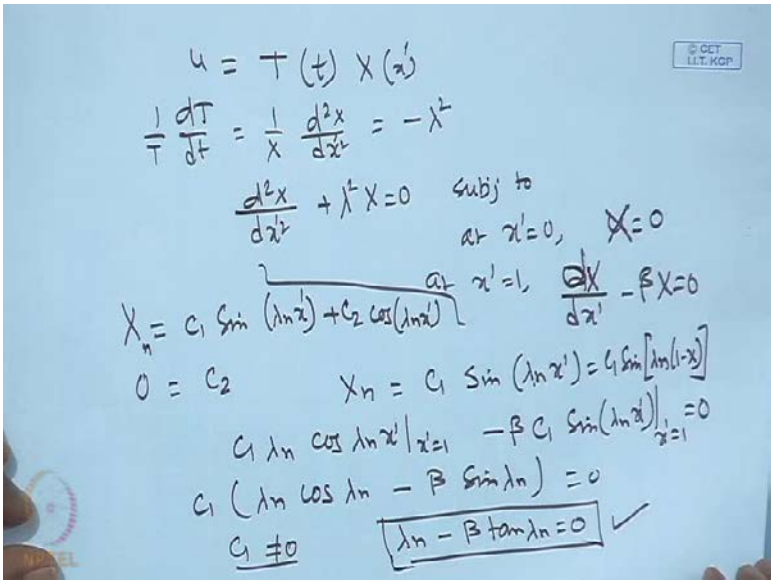

If in a in a chemical engineering problem, if there are so many, you know non-homogeneities appear, so for this particular last problem, we have solved. You know last the four dimensional problem, we have solved there were seven boundaries that we have dealt with. One boundary is the initial condition, initial condition is basically special type of boundary. One initial condition and six boundaries on six boundary conditions, on six boundaries two one x, two one y, two one z. Now out of these six boundaries if in a most general case that all of them are non-homogeneous in nature, then you have to define your temperature such that one of the boundary conditions will become homogeneous.

So, you reduce the number of non-homogeneities from seven to six, and define this problem into six sub-problems taking one non-homogeneity at a time out of this six sub-problems. One problem will be well posed, that one that one which one, will be that that will be the non-homogeneous initial condition, and all six homogeneous boundary condition, and we have already seen how to solve that problem in this class, and you will be able to that problem completely.

On the other hand the other five problems will be having the non-homogeneity in one of the boundary conditions, but homogeneous initial conditions. So each of this five problems, you have to divide into two more sub-problems, one will be the steady state part, the time varying part another is the time independent part. The non-homogeneous part of the of the problem will be associated with the steady state part or time independent part that will force the non-homo, the boundary corresponding boundary condition in the time varying part to be homogeneous, and the non-homogeneous, the homogeneous initial condition of the time varying part will becomes a solution of the steady state part with a minus sign.

So, therefore the transient part becomes absolutely well posed. It will be having a non-homogeneous initial condition, and all boundary conditions becomes homogeneous and you have already seen how to tackle that problem, how to solve that problem. Likewise you break down the other four sub-problems into two parts on his time dependent. Another is a time independent get the solution, then add all solutions up in order to get the complete over all solution.

So, that finishes the multidimensional problem in a rectangular coordinate. Next I'll be and the parabolic problem, so we have solved. We have dealt with the parabolic partial differential equations in more detail, and whatever is required for this course. So, any type of parabolic linear partial differential equations you will be getting all sorts of boundary conditions, all sorts of dimensions, two-dimensional up to four dimensional will be able to handle those problem able to solve this problem completely.

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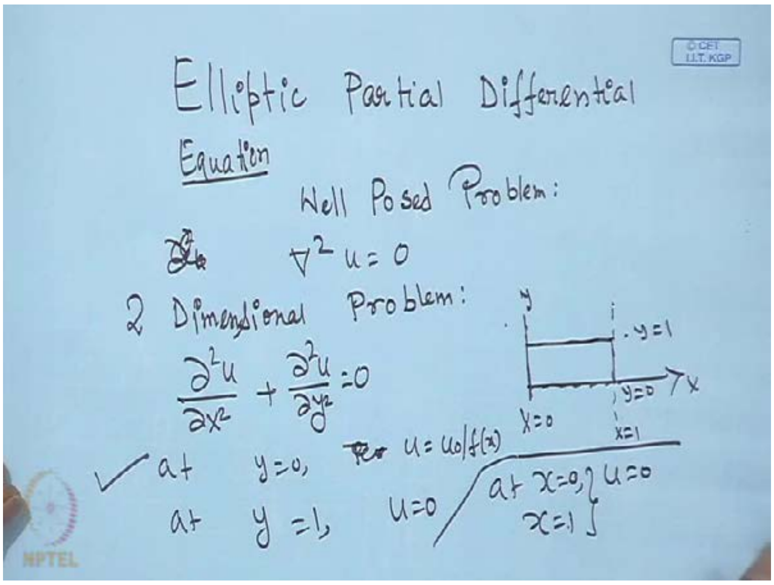

Now, let us look into the solution of elliptical partial differential equation elliptic partial differential equation will be solving the basic problem, or the well posed problem. First, the well posed problem will be something like this del square u. So, it is basically elliptical partial differential is nothing, but laplacian of u is equal to zero. First, we consider a two-dimensional problem, so this becomes del square u del x square plus del square u del y square is equal to zero.

Now, we set up the boundary conditions. So, we have two boundaries on x two boundaries on y, so you will be having the y axis. This is x axis. Let us say this is the rectangular system or the control volume. So, this boundary is located at x is equal to zero. This boundary is located at x is equal to one. This boundary is located at y is equal to zero. This boundary is located at y is equal to one.

So, let us consider we are talking about a about a square slab where the square slab is basically covered by the control volume, and it is denoted by this equation, del square u del x square plus del square u del y square. Now, let us assume that at y is equal to zero the u is a u is a constant, or in general it will be a function of x at y is equal to one. u is equal to zero. Let say at x is equal to zero, u is equal to zero at x is equal to one, u is equal to zero.

So therefore, we have out of this four boundaries, the three boundaries are boundary conditions are homogeneous. One boundary condition is non-homogeneous, if we would have obtain the four boundary conditions to be homogeneous, then would have got the got a trivial solution for this. So, one boundary condition has to be non-homogeneous in order to get a non-trivial solution, therefore we have one boundary condition to be homogeneous, non-homogeneous and all the three boundary conditions to be homogeneous.

Now, if we look into, if we analyze the boundary conditions more closely, we will see that the boundary conditions in x boundary conditions in x, they are completely homogeneous in on the both boundaries, and on the other hand, the boundary conditions on y or not homogeneous in y direction.

So, we can formulate a standard Eigen value problem in the x direction only, but not in the y direction,anyway I will stop it. I'll stop at this class, of this class at this point, I will take this problem up in the next class, and solve this problem of the elliptic partial differential equation, and then we will be looking into the various variants of elliptic p d e and will be also looking into the solution of hyperbolic partial differential equation in the next class as well. Thank you very much.