Advanced Mathematical Techniques in Chemical Engineering

Prof. S. De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 27
Solution of Higher Dimensional PDEs (Contd.)

Good morning everyone. So, we are looking into the higher order differential equations for multidimensional partial differential equations and parabolic equations to you and use the separation of variable to solve these equations.

Now, in today's class, in the last class whatever we have done is that, we have taken up a 3-dimensional problem in the and with all Dirichlet boundary conditions, 2 dimension in space and 1 dimension in time and we solve the problem completely by using separation of variable technique.

Now, we have also discussed in the last class, what are the conditions where one will be using a lumped systems analysis in heat conduction problem in chemical engineering applications, where the governing equation is basically an ordinary differential equation. And under what conditions, the special variations of the temperature within the body of the material become very important and the problem becomes a multi-dimensional problem and you will be landing with a partial differential equation.

Now, in today's class, we will be looking into the 3-dimensional problem and 4-dimensional problem in more detail and in the last class, we have done the we have taken up an example, where we have solved the problem using all Dirichlet boundary condition un embassy basic problem. Now in today's class, what you discuss, I will discuss out of the 4 boundary conditions, at least some of the boundary conditions will be containing are Dirichlet, Neumann and Robin mixed boundary condition for a basic problem.

Then we will take up one more example for an actual problem, how to reduce the actual problem in the form of the basic problem and one can get the complete solution. And then, we will move into the 4-dimensional parabolic partial differential equation.

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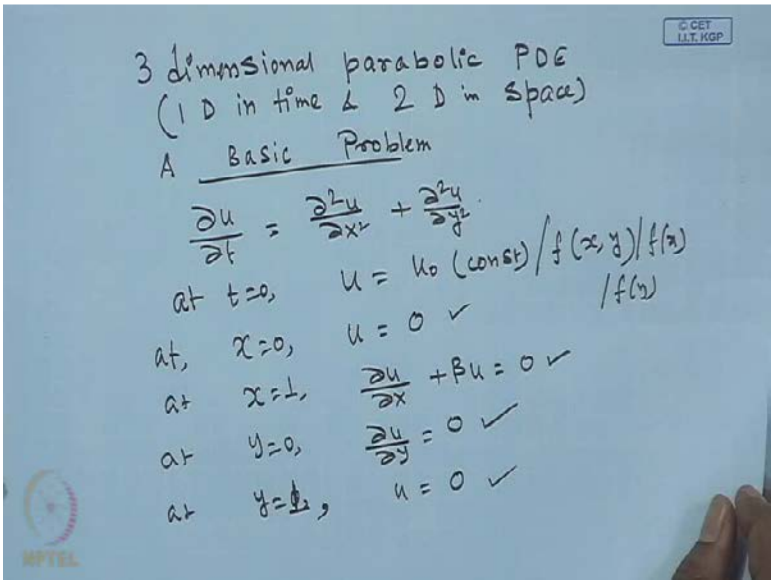

So, we will be considering again a 3-dimensional parabolic PDE, 1 dimension in space, 1 dimension in time and 2-dimensional in space and we consider a basic problem. So, the governing equation of this problem will be del u del t is equal to del square u del x square plus del square u del y square is equal to ok so, that remains the same.

Now, we have the 4 boundary conditions, we fix up some of the boundary conditions as robin mixed and Neumann and Dirichlet are mixture of the of all, at t is equal to 0. Since it is a basic problem, we must be dealing with a non-homogeneous initial condition. So, at t is equal to 0, u is equal to let say u naught or that means, it is a constant or in more general, it may be a function of x and y, where or it may be a function of x alone or it may be a function of y alone.

So, all of these 4 combination can be specified can be this functions can be specified as the initial condition. And the boundary conditions, where let us say at x is equal to 0, u is equal to 0, at x is equal to 1 we have u del u del x plus Beta u is equal to 0, at y is equal to 0, we have let say del u del y is equal to 0 and at y is equal to 0, y is equal to 1 we have let say u is equal to 0.

So, if you look into this problem and compare with the earlier problem solved in the last class, the difference is that, both the problems are 3-dimensional parabolic partial differential equations, both the problems of a homogeneous boundary condition and non-homogeneous initial condition.

In the earlier problem the difference is that, in the earlier problem all the boundary conditions were Dirichlet boundary condition, but in this problem, we have a Dirichlet boundary condition present at x is equal to 0; we have a mixed boundary condition present at x is equal to 1, we have a Neumann boundary condition present at y is equal to 0 and we have a Dirichlet boundary condition present at y is equal to 1.

So with this, let us move forward to solve this problem almost completely, since this is a linear and homogeneous governing equation and linear boundary linear boundary conditions and homogeneous boundary condition, we can go ahead with the separation of variable type of solution. That means, the solution u is composed of a product of 3 quantities, 3 functions, which will be a function of time alone, another function will be function of x alone, another function is function of y alone.

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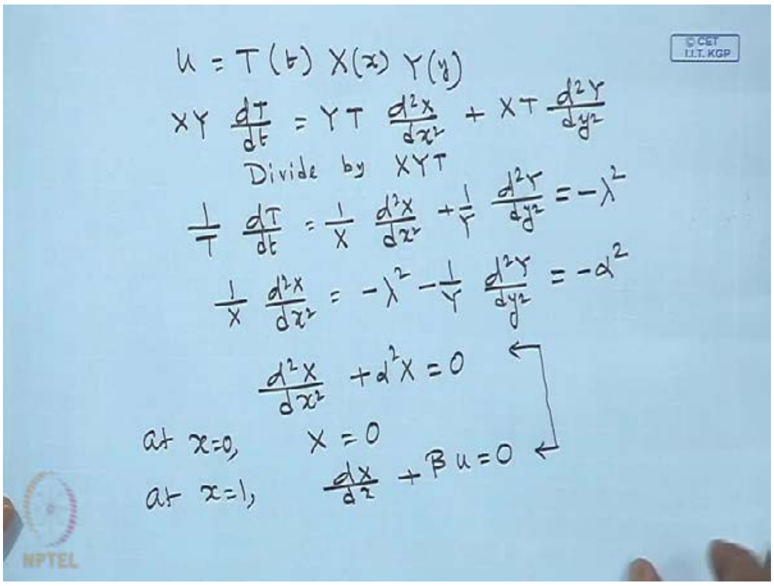

So, therefore, we can write u as a product of T of t X of small x and capital Y for small y, then we substitute this in the governing equation. If we do that, then you will be getting X Y d T d t is equal to Y T d square x d x square plus X T d square Y d y square, divide by X Y and T. What will be getting is 1 over T d T d t is equal to 1 over X d square x d x square plus 1 over Y d square Y d y square.

Now, the left hand side is a function of time alone, the right hand side is a function of space alone and they are equal, they must be equal to some constant. And this constant is minus lambda square, if this constant can be 0 or positive then, will be landing up with a trivial solution, we have already proved it earlier. Now, let us compose decompose the spatial part, 1 over X d square x d x square is equal to minus lambda square minus 1 over Y d square Y d y square.

So, again the left hand side is completely a function of x, the right hand side is entirely a function of y and that. So, they will be equal to some constant, again if this constant is 0 or positive, we will be landing up with a trivial solution. So, this constant must be a negative constant that will be minus alpha square. Next, we put it we formulate the governing equation of x.

So, we equate 1 over X d square X d x square with minus alpha square. So, you will be getting d square X d x square plus alpha square x is equal to 0. Now, let us put down the boundary conditions on capital X, it must be satisfying the x varying part must be satisfying the boundary conditions on x of the original problem. So, at x is equal to 0 your capital X is equal to 0, because u was equal to 0 at that boundary and at x is equal to 1, you have d X d x plus Beta u is equal to 0 per Beta is scalar parameter.

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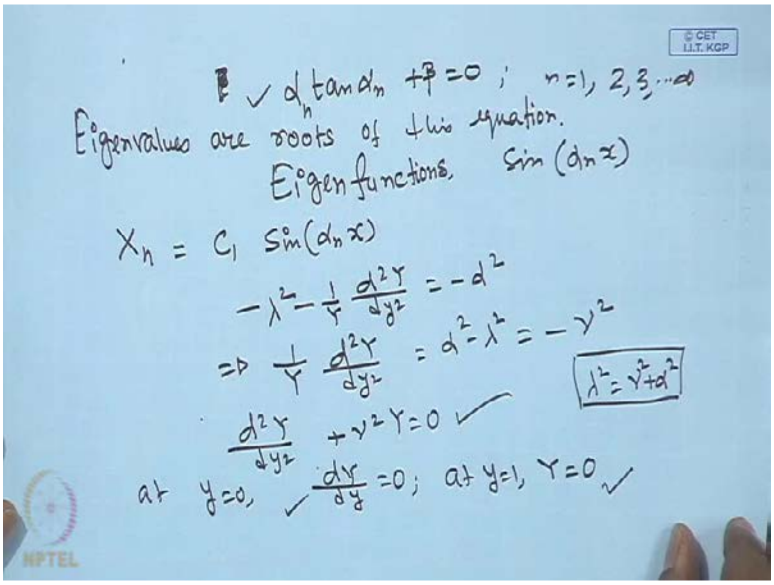

Now, if you look into this problem and examine this problem, this problem is a special form of a standard Eigen value problem or Sturm Liouville Problem with the boundary conditions as homogeneous boundary conditions. We have already solve this sub-problem earlier, if you remember that the Eigen values of this equation will be obtained from the transcendental equation, b Eigen values of this equations will be obtained from the equation write alpha n tan alpha n plus Beta is equal to 0. So, Eigen values are roots of this equation where n goes from 1, 2, 3 up to infinity.

So, Eigen values are the roots of this equation and Eigen functions are sine functions are sine functions, they are sine alpha n x. Next, we have also solve this problem and got the complete solution, let us look into the complete solution. X n is the Eigen function of corresponding to n th Eigen value, so this will be C 1 sine alpha n x.

So, these are the Eigen functions and then, let us look into the other problem, that is the y varying part. If we look into the y varying part, the governing equation becomes lambda square minus with 1 by Y d square Y d y square is equal to minus alpha square.

So, this will be 1 over Y d square Y d y square is equal to alpha square minus lambda square, again if you look into the right hand side, alpha is a constant, lambda is a constant this constant can be positive. So, the whole thing becomes a constant, again this constant is positive, if this constant is 0, we are going to get a trivial solution. So, this constant has to be a negative constant. So, let us say this negative constant is nu square. So, we can get an expression of lambda square will be nothing but, nu square plus alpha square.

So, d square Y d y square plus nu square Y becomes 0 and if you look the into the boundary conditions on y, the boundary conditions on y of y must be satisfying the boundary conditions of the original problem on y. So, therefore, at y is equal to 0, d Y d y must be equal to 0 and at y is equal to 1, capital Y must be equal to 0.

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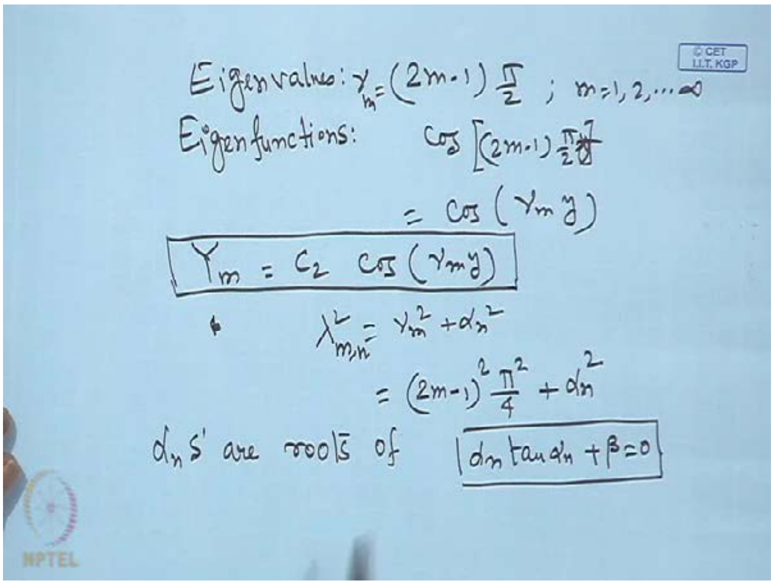

So, if you look into this problem, this problem is again a standard Eigen value problem with the homogeneous boundary condition a Neumann at the center at y equal to 0 and at y is equal to 1 a Dirichlet boundary condition. And we have already solved this problem earlier and we if you remember that for this particular problem, the Eigen values will be 2 m minus 1 pi by 2 and Eigen functions are cosine function. So, this is a again a standard Eigen value problem. So, therefore, the Eigen functions, the Eigen values will be 2 m minus 1 pi by 2, where the index m runs from 1, 2, 3 up to infinity and Eigen functions are cosine functions cosine 2 m minus 1 pi by 2.

So, this we write as nu m and y. So, this will be cosine nu m y, where nu m is nothing but, 2 m minus 1 pi by 2. So, let us write down the y m as c 2 cosine nu m y, next we look into the time varying part. If you look into the time varying part, this time varying part will be nothing but, one over ok. So, if you look into the lambda square now, lambda square we have already obtained is equal to nu m square plus alpha n square and if you really look into the value, you know value of lambda, lamda is a function of nu m and alpha n.

So, we write lambda subscript m comma n. So, nu m nu m is basically 2 m minus 1 whole square pi square by 4 plus alpha n square, but in this case, we do not have the explicit expression of alpha n, because alpha n’s are root is of the transcendental equation that we have already written earlier, that is alpha n tan alpha n plus beta is equal to 0.

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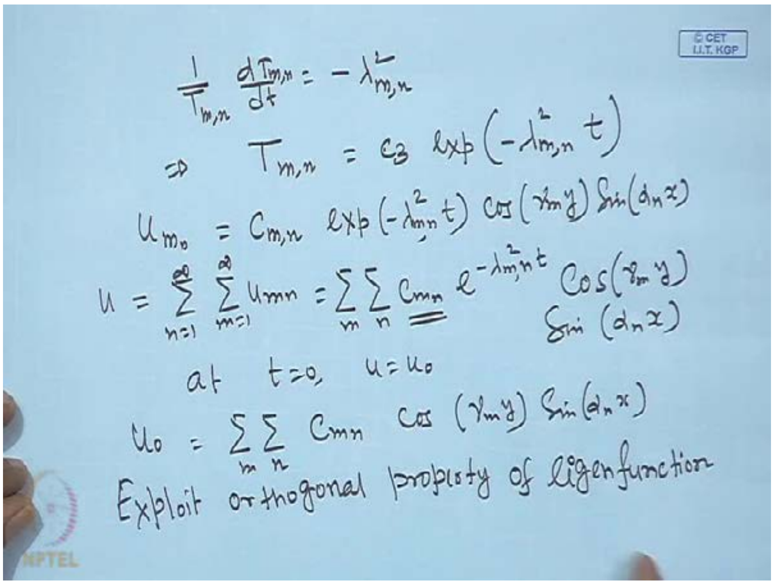

So, therefore, what is left for this problem is the time varying part, the time varying part if you look into the time varying part, this becomes 1 over T d T d t is equal to minus lambda square. So, since lambda is a function of m and n, so we put a subscript m n corresponding to lambda and corresponding solution on T we put m and n. So, therefore, we integrate this out, T m n can be expressed as x, some constants c 3 exponential minus lambda m n square times t.

So, now, we have solved each of the segments, each of individual segments separately and we can construct the complete solution. The complete solution first will be having let say U m n, U m n will be nothing but, 3 constants multiplication of this three functions and the 3 constants will be multiplied and giving rise to a new constant, let us have it C m n. So, C m n exponential minus lambda m n square times t cosine nu m y into sine alpha n x.

So, the overall solution will be obtained as u is nothing but, double summation over u m n, one is over m another is over n. So, therefore, this will be double summation C m n e to the power minus lambda m n square t cosine nu m y sine alpha n x. Now, this problem is solved and what is left behind is the solution of the determination of the constant C m n. So, we get this constant from the initial condition at t is equal to 0, u was equal to u naught. So, this will be u naught summation, double summation one over m another over n. So, this becomes c m n cosine nu m y sine alpha n x.

So, what we do next? We utilize the orthogonal property of the Eigen functions, cosine functions and sine function. We Exploit orthogonal property of Eigen functions cosine function and sine function. So, therefore, what we are going to do? We are going to multiply both sides of this equation by sine nu n y and cosine alpha n x d x d y.

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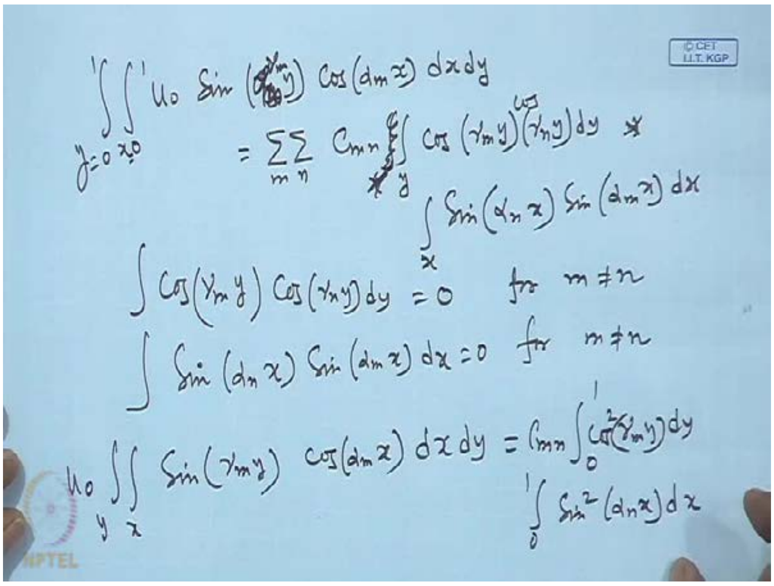

So, if you do that, we will be getting u naught double integral u naught sine nu n y cosine alpha m x d x d y, this will be from 0 to 1, this over x, this over y from 0 to 1 that will be is equal to summation 1 over m, another over n C m n integral double integral 1 over X, another over y cosine nu m y nu n y d y. So, that will be integration over y, then this should be multiplied by we another integration over x, that will be sine alpha n x sine alpha m x d x.

This will be sine alpha m and this will be cosine nu, we multiply both side by sine nu m y and sine alpha n sine alpha m x (Refer Slide Time: 19:49). So, this is fine, this is sine nu m y cosine alpha m x. So, what we are going to get here that, once open up this summation series what will be getting is that, cosine nu m y, this will be cosine here cosine nu n y d y will be is equal to 0, for m not is equal to n and integral sine alpha n x sine alpha m x d x will be is equal to 0, for m not is equal to n.

So, all if you open up this summation series, all the terms will vanish, only one term will remain that will be 1 m is equal to m. So, what we will be getting is integral y integral x, u 0 will be taken out because that is a constant, integral sine nu m y and cosine alpha m x d x d y is equal to C m n integral cos square nu m y d y from 0 to 1 and integral sine square alpha n x d x.

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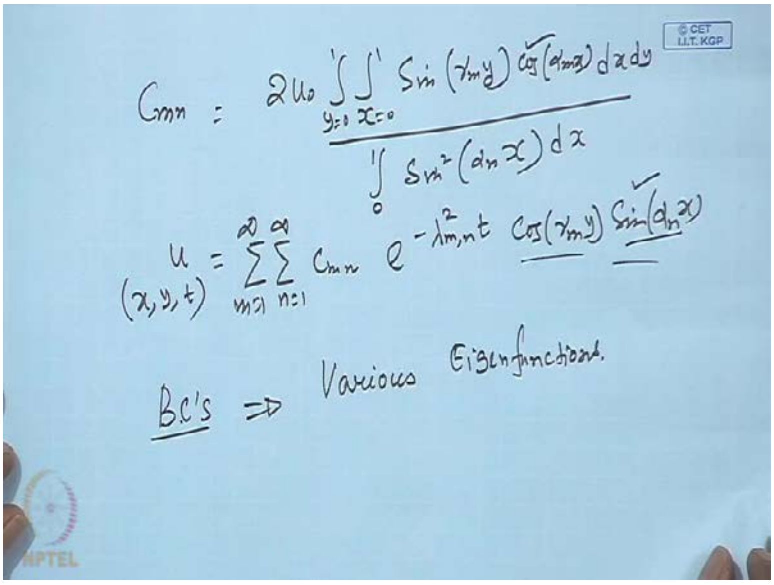

So, we know the value of cos square nu m y will be 0 to 1 will be half, but this value is not half because that will be not the you know, so will be getting this equation by solving whatever we have done earlier. So, we already know how to handle this integral. So, I will just write down the value of C m n, C m n will be 2 u naught integral over y integral over x, sine nu m y cosine alpha m x d x d y divided by integral from 0 to 1 sine square alpha n x d x.

So, we can evaluate this integral analytically and we will be able to solve this problem completely. So, that gives you the solution of the equation, u is equal to summation double summation C m n e to the power minus lambda m n square t plus e multiplied by cosine nu m y sine alpha n x, this is over m 1 to infinity, this is over n 1 to infinity, this integral from 0 to 1, this integral from 0 to 1 (Refer Slide Time: 23:13).

So, we can analytically evaluate this constant and one will be able to obtain the complete solution of the function of the variable u as a function of x y t. So, this gives a presentation that when the boundary conditions are not of same time, if they are mixed with the Dirichlet boundary condition, Neumann boundary condition and mixed boundary condition, what will be the nature of the solution one will get? Since you will be having the Neumann boundary conditions, the Eigen functions will be cosine functions and Eigen values will be 2 n minus 1 pi by 2, since the one of the boundary conditions in a y direction is basically they are in the Eigen functions, they are in the mixed boundary condition.

So, you will be getting the sine functions as the Eigen functions and sine and the Eigen values will be obtained from a transcendental equation. So, depending on the boundary conditions, one will get the various Eigen functions and can get either sine function or cosine function in your solution. Now, I will be taking up one complete example in a 3-dimensional problem in chemical engineering application and see, how that will be reduced to the solution. So, we have already known, how to solve a basic problem in 3-dimensional basic problem in 3-dimensional analysis.

And I will be taking up just 1 chemical engineering application, where we will be getting the boundary conditions all non-homogeneous sine generalized and then, we can break down the problem in the form of basic problem; then one can construct the complete solution by superposing a by using principle of linear superposition of all the solution to add them up and you will be getting the complete solution.

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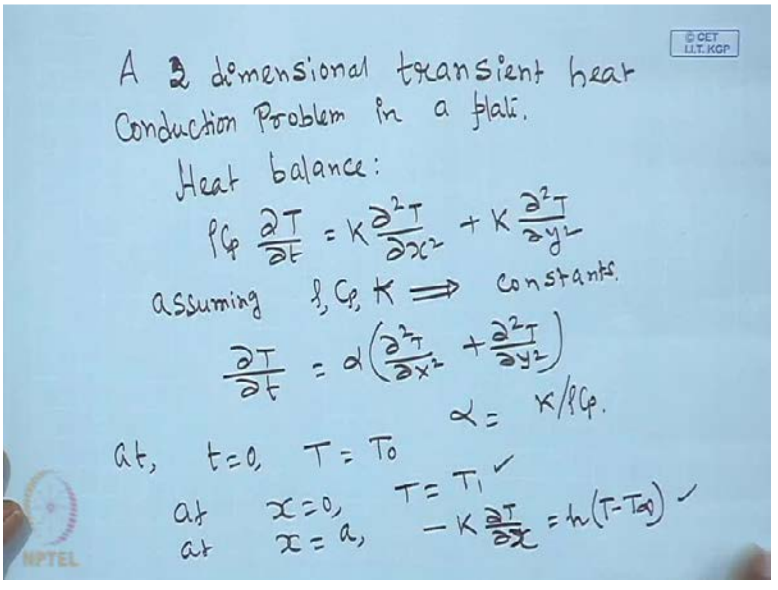

So, the next practical example I will be taking again a 3-dimensional a 2-dimensional transient heat conduction problem, in a plate rectangular plate and this is 2-dimensional in space transient in time. So, therefore, it is a ultimately a 3-dimensional problem. If you write down the energy balance equation, what you will be getting is that, you will be getting del u rho C p del T del t is equal to K del square T del x square plus K del square T del y square assuming rho C p K thermal conductivity all are constants.

So, if you divide both side by rho C p, what you will be getting is del T del t is equal to alpha del square T del x square plus alpha del square T del y square a bracket. Now, alpha is the thermal diffusivity and this will be K divided by rho C p, now we set up the boundary conditions at t is equal to 0, we have T is equal to T naught. Let say T naught is the temperature that is existing for T is equal to at initially at time T equal to 0 at x is equal to 0, let us say we have a Dirichlet boundary condition, let say x is equal to 0, T is equal to T 1 and at x is equal to 1, at x is equal to a.

So, this will be let say minus K del T del y del T del x is equal to h times T minus T infinity; that means, the boundary at located at x is equal to a is expose to the environment. That means, how much whatever the heat that is coming by conduction at the boundary is taken up by the convection.

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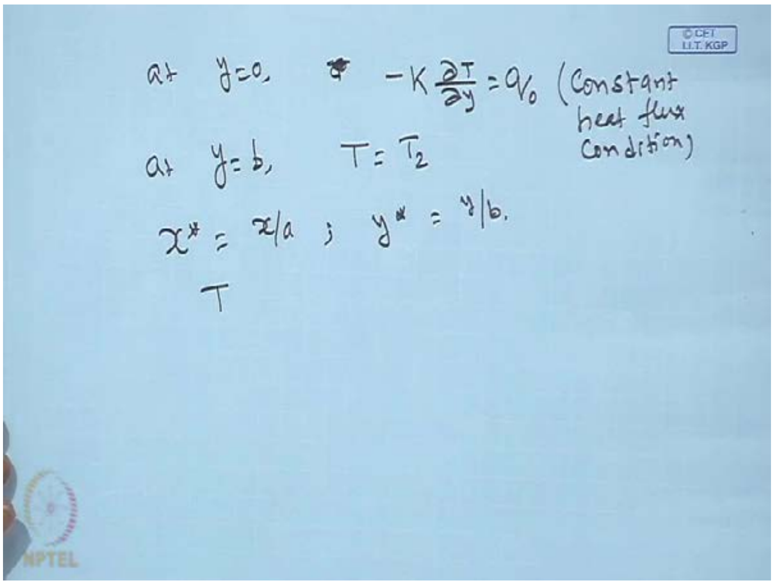

So, you have a Dirichlet boundary condition at x is equal to 0, you have a robin mixed boundary condition at x is equal to a, next what we do? We set up the boundary conditions for y, at y is equal to 0, we have T is equal to let say, it is a we are supplying let say constant heat flux to the system. So, minus K del t del y will be is equal to q naught, q naught is the constant heat flux that is going into the system.

So, this is known as the constant heat flux condition, then let us put the other boundary that is at y is equal to b, T is equal to T 2, we maintain a Dirichlet boundary condition or a constant temperature at the boundary located at y is equal to b. Now, let us first non-dimensional this equation as we have done earlier, if you really set it up what will be getting is that, at we define x star as x by a and y star as y by b.

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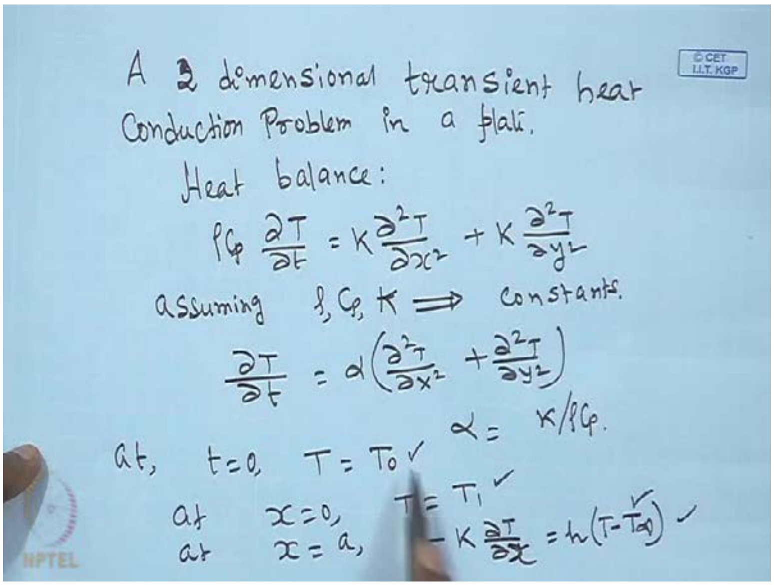

And we defined, so there are how let us look into the how many sources of non-homogeneity are present in this problem? In this problem, initial condition is a non-homogeneous initial condition, the boundary condition at x is equal to 0 is non-homogeneous, the boundary condition at x is equal to 0 is non-homogeneous because of this term.

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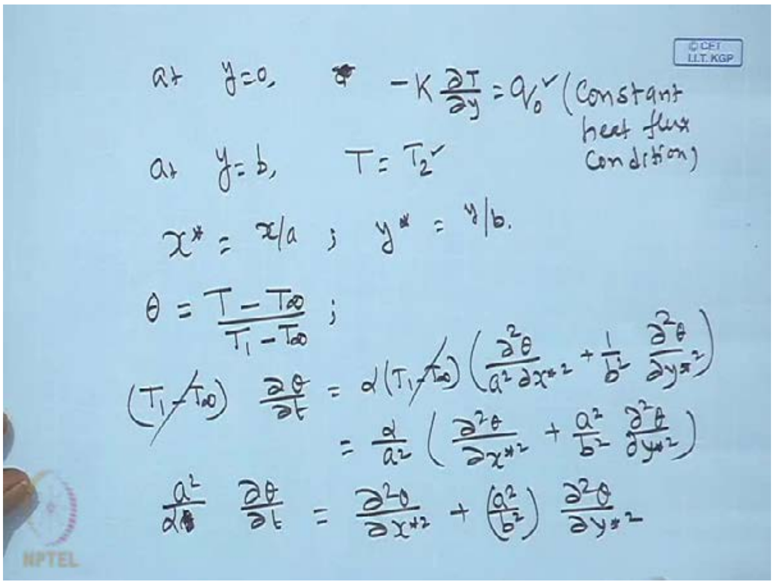

So, we have 1, 2, 3 non-homogeneities here and both the boundaries on y, that is minus k del T del y is equal to q naught and T is equal to t 2, both are the both of this term they contribute as the non-homogeneous term in the governing equation. So, there are 5 sources of non-homogeneity in this particular problem. So, if we define a temperature such that, we can reduce at least 1 non-homogeneity and we can reduce them from 5 to 4.

Let us define a non-homogeneity a dimensional temperature theta is equal to T minus T infinity divided by T 1 minus T infinity and x star is x by a, y star is equal to y b y by b now let us put all this equations, all this non-dimensional containing the governing equation. So, this becomes T 1 minus T infinity del theta del t and this will be alpha T 1 minus T infinity del square theta, this will be a square del x star square plus 1 over b square del square theta del y star square.

So, this T 1 minus infinity, T 1 minus infinity will be cancelled, what we can do, we multiply both side by a square. So, this becomes alpha by a square you take a square common. So, this becomes d square theta d x star square plus a square by b square del square theta del y star square. So, we take it on the other side, so it becomes a square by alpha t del theta del t is equal to del square theta del x star square plus a square by b square del square theta del y star square.

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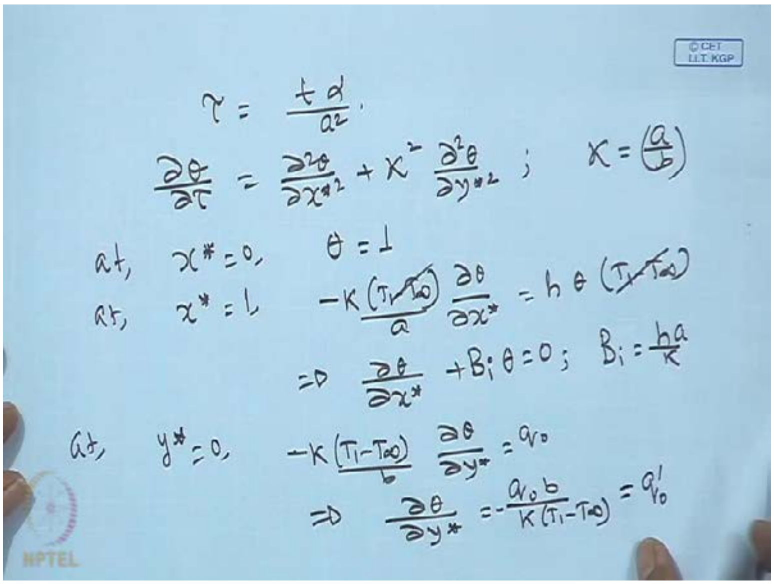

Now, if you remember that, a square divided by a square divided by alpha. So, this not T here a square divided by alpha as a unit of time. So, we can define a non-dimensional time. So, the right hand side is entirely non-dimensional. So, left hand side has to be non-dimensional. So, we define a time as tau t alpha divided by a square as a non-dimensional time.

So, our governing equation now becomes del theta del tau is equal to del square theta del x star square plus let us say, kappa square del square theta del y star square, where kappa is equal to a by b, the geometric factor. Now, let us set up the, make the boundary conditions non-homogeneous the dimensionless. So, at x is equal to 0 means, at x star is equal to 0, t is equal to t 1, so therefore, theta is equal to t 1 minus t infinity divided by t 1 minus t infinity. So, theta becomes 1.

So, that is the boundary condition at x is equal to x star is equal to 0, at x star is equal to 1 that means, that at x equal to a, this becomes minus K theta t becomes T 1 minus T infinity del theta del x. So, divided by a del x star is equal to h theta, into T 1 minus T infinity. So, this will be cancelling out. So, what will be getting is del theta del x star plus Biot number times theta is equal to 0, where Biot number is equal to h a over k.

So, that becomes the boundary condition at x star is equal to one, now if you see this boundary condition as become homogeneous boundary condition. So, therefore, let us make the other boundary conditions non-dimensional at y is equal to 0; that means, at y star is equal to 0 minus k del t del y is equal to q naught. So, minus K T 1 minus T infinity del theta del y star is equal to q naught. So, if we take it other side, del theta del y star becomes q naught b divided by K 1 T 1 minus T infinity with a negative sign.

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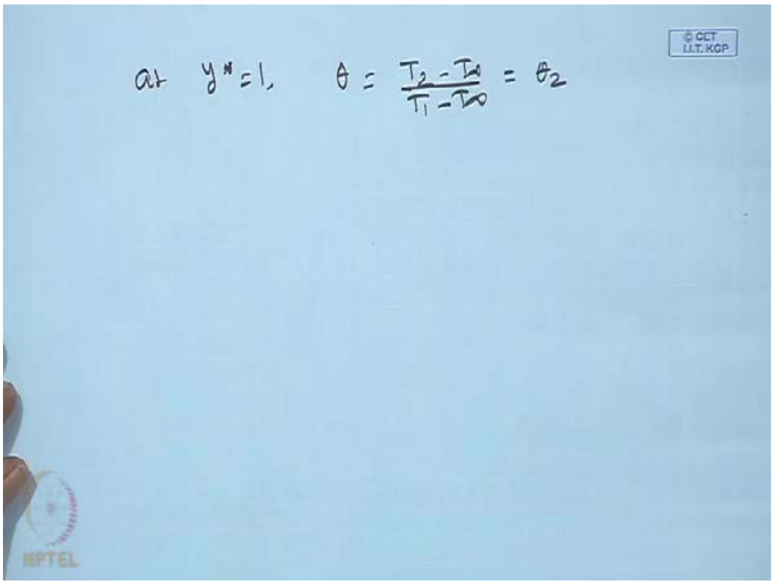

So, the left hand side is completely non-dimensional. So, right hand side has to be a non-dimensional quantity, let us put this as q 0 prime, which is a non-dimensional quantity. Next, we put the other boundary condition, so that is at y is equal to b, t is equal to t 2. So, at y star is equal to 1, we have theta is equal to T 2 minus T infinity divided by T 1 minus T infinity is equal to let say theta 2. Now, let us see in this problem, how many sources of non-homogeneities are there.

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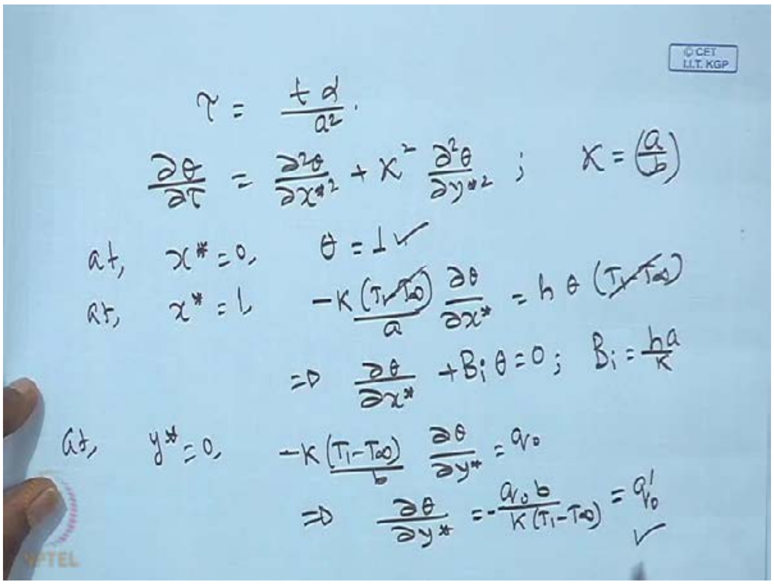

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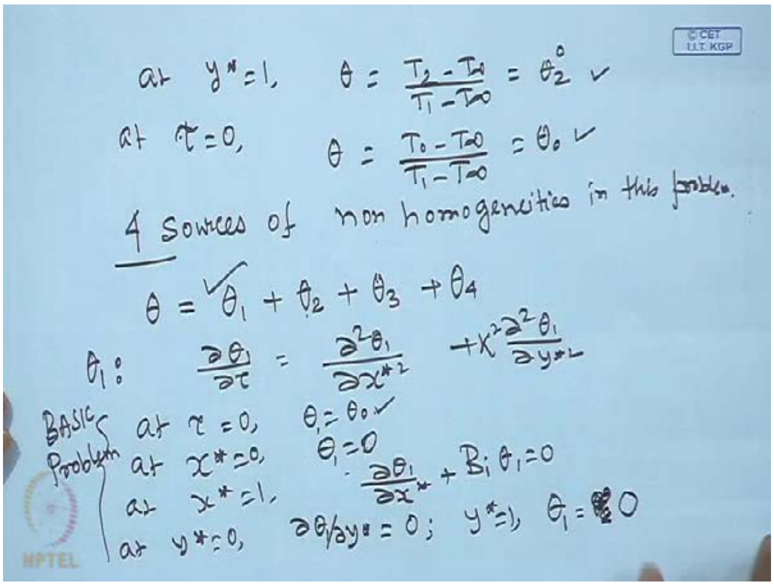

There is one sources of non-homogeneity, the initial condition is non-homogeneous, the boundary condition at x is equal x star is equal to 0 that is non-homogeneous, this becomes homogeneous, the boundary condition at y star is equal to 0 that becomes non-homogeneous and the boundary condition at y star is equal to 1, this becomes non-homogeneous (Refer Slide Time: 36:00).

So, we put the initial condition, we make the initial condition non-dimensional as well. So, this becomes T was is equal to t naught. So, therefore, theta was is equal to T naught minus T infinity divided by T 1 minus T infinity. So, that becomes theta naught. So, if you look 1, 2 and 3 and 4, so there are 4 sources of non-homogeneities in this problem ties in this problem and this if you remember in the original problem, we had 5 sources of non-homogeneities; so, therefore, we have reduced at least one non-homogeneity in this by doing this non-dimensionalization.

Now, since there are 4 sources of non-homogeneity, so therefore, the original theta the will be divided into 4 sub-problem considering, 1 non-homogeneity at a time. Let us say, theta is equal to theta 1 plus theta 2 plus theta 3 plus theta 4. So, what I will be doing? I will be formulating all these sub-problems one after another, but I would not be solving it completely, because you have already looked into the solution of the basic problem in the earlier two examples.

So, one can get these sub-problems and break down these sub-problems up to the form of the basic problem then, one can go ahead with the solution whatever we have done in the earlier class. So, let us look into the formulation of the governing equation of theta one, considering one non-homogeneity at a time, del theta 1 del tau is equal to del square theta 1 del x star square plus del square theta 1 del y star square with a kappa square here.

So, this with the now we consider, one non-homogeneity at a time therefore, we keep the non-homogeneity of the initial condition with this forcing all the other non-homogeneities on the boundaries to be vanish. So, at tau is equal to 0, we write theta is equal to theta naught at x star is equal to 0, we had theta is equal to 1, theta is equal to 0, we force the non-homogeneity to vanish, at x star is equal to 1, this is already non-homogeneous.

So, this becomes del theta 1 del x star plus Biot times theta 1 is equal to 0, at y star is equal to 0, we have we make this forces boundary condition to be homogeneous, del theta del y del theta 1 del y star is equal to 0 and y star is equal to 1, we have the boundary condition as theta is equal to theta 2.

So, we make this as homogeneous. So, we force it to be homogeneous. So, theta 1 becomes theta 2 naught, we put it as theta 2 naught in order to avoid the mixing up of the nomenclature later on, because this is also theta 2. So, you make it as theta 2 naught. So, what we did out of the 4 sources of non-homogeneities in this particular problem of definition of theta 1, we kept one non-homogeneity intact and we forced all the other non -homogeneities to vanish, we force it to 0.

So, we force all the other 3 non-homogeneities to vanish. So, we have 1 non-homogeneity here this is force to be homogeneous, this is already homogeneous, the boundary condition at y star is equal to 0 is force to be homogeneous, the boundary condition at y star equal to 1 is forced to be homogeneous. So, this is a basic problem basic problem or a well-posed problem. So, we have already seen the solution of this. So, we know the solution of theta 1.

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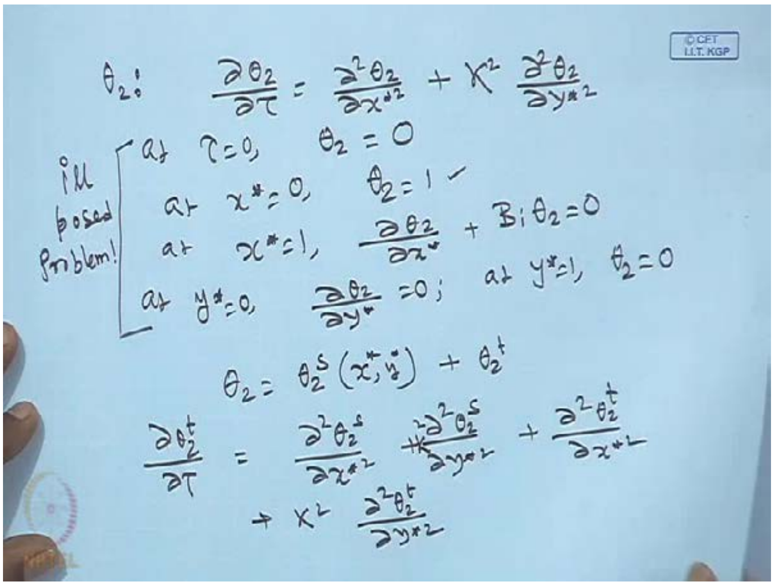

Now, let us look into the other parts, the governing equation of theta 2 will be del theta 2 del tau is equal to del square theta 2 del x star square plus kappa square del square theta 2 del y star square. Now at tau is equal to 0, theta 2 we make it homogeneous, the initial condition and one boundary condition, we keep as non-homogeneous that is at x star is equal to 0, we keep this non-homogeneity intact. So, theta 2 is equal to 1 and at x star is equal to 1, we have del theta 2 del x star plus Biot times theta 2 is equal to 0.

So, it is already there, it is already homogeneous and at y star is equal to 0, del theta 2 del y star is equal to 0 and at y star is equal to 1, theta 2 is equal to 0. So, we keep this non-homogeneity intact forcing all the other non-homogeneities to vanish, but this is an ill posed problem; simply because this becomes a ill posed problem, the initial condition is 0 and one of the boundary condition becomes homogeneous.

So, this problem has to be divided into two sub-problems theta 2 becomes f a theta 2 s which will be function of x and y alone, and there will be other part that is theta 2 t.

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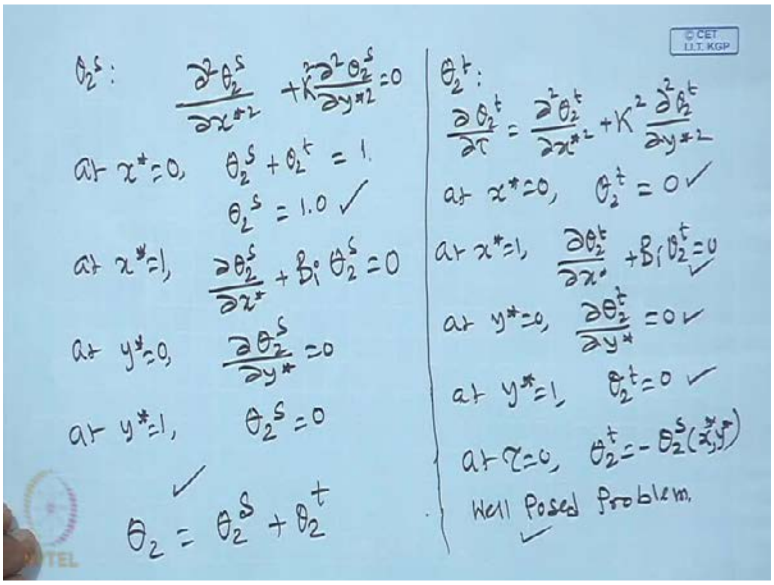

Now, what we do? We put this value there. So, this becomes del theta 2 t del tau is equal to del square theta 2 s, del x star square plus del square theta 2 s, del y star square plus del square theta 2 t, del x star square plus there will be kappa square here, plus kappa square del square theta 2 t del y star square. So, we collect the similar terms and formulate the governing equation, first we solve the steady state part, we take the steady state part the steady state part is del square theta 2 s. So, it becomes del square theta 2 s del x star square plus del square theta 2 s del y star square is equal to 0 and next we formulate the theta 2 t.

So, that will be del theta 2 t, del tau is equal to del square theta 2 t del x square plus kappa square del square theta 2 t del y star square. Now, let us set up the boundary condition of the steady state part, the boundary condition should satisfy the boundary condition of the original problem. The original problem in this case is theta 2, that is the parent problem for this, at x star is equal to 0, we have theta 2 is equal to 1.

So, therefore, we put theta 2 s plus theta 2 t is equal to 1. So, we associate the non-homogeneous part with the steady state solution and we associate the homogenous, we force the boundary condition of theta 2 t the time varying part, we force it to be homogeneous. Therefore, at x star equal to 0, we put theta 2 s is equal to 1 and at x star is equal to 0, we put theta 2 t is equal to 0.

So, we force it to be homogenous and this at x star is equal to 1, if you look into the original problem it was del theta 2 del x star plus B i theta 2. So, this will be del theta 2 s del x star plus B i theta 2 s is equal to 0 and for this, at x star is equal to 1 del theta 2 t del x star plus B i theta 2 t is equal to 0 and at y star is equal to 0 we had del theta 2, del y star is equal to 0. So, it is homogeneous, so no problem.

So, this becomes del theta 2 s del y star is equal to 0 and at y star is equal to 0, we have del theta 2 t del y star is equal to 0. And similarly, at y star is equal to 1, there will be kappa square here, at y star is equal to 1 we had theta 2 is equal to 0 so, therefore, theta 2 s is equal to 0 and at y star is equal to 1 theta 2 t is equal to 0.

And let us put into the initial condition at tau is equal to 0, theta was theta 2 was theta 2 s plus theta 2 t, that will be equal to 0. So, it will be theta 2 t is nothing but, theta 2 s which will be a function of x star and y star. Now, if you look into this particular problem, this will be the initial condition is non-homogeneous and this is not equal to 0 and this is nothing but, the solution of the steady state part that is number 1.

Then boundary condition at x star equal to 0 this is homogeneous, boundary condition at x star is equal to 1 this is homogeneous, the boundary condition at y star equal to 0 this is homogeneous, the boundary condition at y star equal to 1 this is homogeneous. So, this is a well posed problem and we know the solution of this provided, the solution of this steady state function, the steady states solution is known that becomes negative of that becomes an initial condition.

Now, let us come back to the steady state problem. In this steady state problem, this is an elliptical partial differential equation, we have not seen till now, how to solve the elliptical partial differential equation. We will see shortly after couple of classes may be then this boundary condition, all these 3 boundary conditions are homogeneous, but 1 boundary condition is non-homogeneous will be able to solve this problem completely. So, that problem is solvable, this is a well posed problem.

So, we have already seen the solution of this, provided we can supply the initial condition as the solution from this problem and we have homogeneous boundary conditions. So, absolutely no problem, we can get the solution of theta 2 which will be nothing but, a linear superposition of theta 2 s plus theta 2 t.

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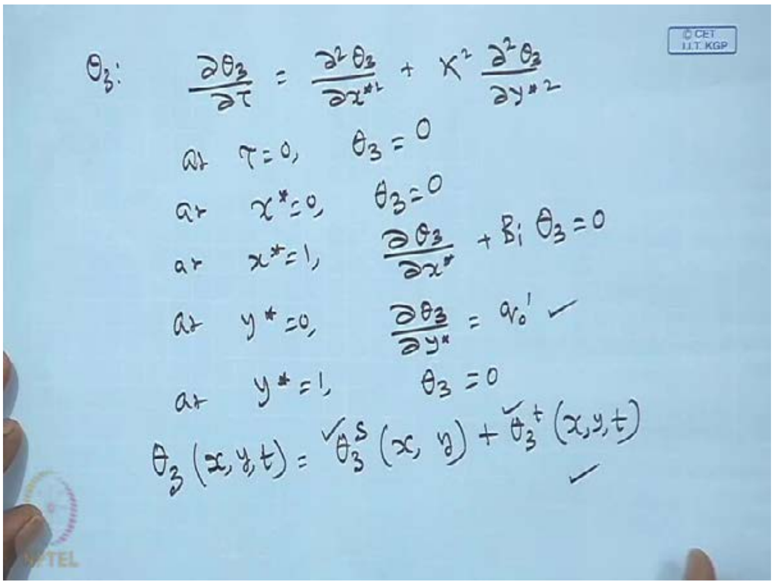

Similarly, if you look into the third problem and we can solve the third problem as a we and fourth problem like that. So, if we formulate the third problem, keeping 1 non-homogeneity at a time. So, third problem will be del theta 3 del tau is equal to del square theta 3 del x star square plus kappa square del square theta 3 del y star square at tau is equal to 0, theta 3 is equal to is equal to 0 at x star is equal to 0, we have theta 3 is equal to 0 at x star is equal to 1. We had del theta 3 del x star plus B i Biot theta 3 is equal to 0 that is already homogeneous from the parent problem, at y star is equal to 0, we have del theta 3 del y star is equal to is equal to q 0 prime, this non-homogeneity we keep intact.

And at y star is equal to 1, we force the non-homogeneity to vanish. So, again if you examine in this problem, in this problem only 1 non-homogeneity we have kept in the formulation and this non-homogeneity is appearing in the boundary condition with a 0 initial condition. So, again this problem has to be divided into two sub-problem, one is the function of space there is the steady state part, another is the function of time and space, both that is the transient part.

So, theta 3 is nothing but, theta 3 s which is a function of x and y plus theta 3 t which is a function of x y t both. We have already seen in the earlier example that, how to get the governing equation of theta 3 s and theta 3 t and we will by selecting we will be associating the boundary condition, non-homogenous boundary condition with the steady state part forcing this boundary condition for that transient part to be homogeneous.

So, that way and the initial condition of the transient part will be nothing but, the minus of the solution of the steady state part. So, we can completely solve the steady state part which turns out to be a parabolic lead and elliptical partial differential equation in this case as well and you know transient part will be having non-homogeneous initial condition and all the boundary conditions becomes homogeneous. So, this problem becomes a well-defined or well posed problem and we have already seen the solution to that and we can get the solution of this problem as well.

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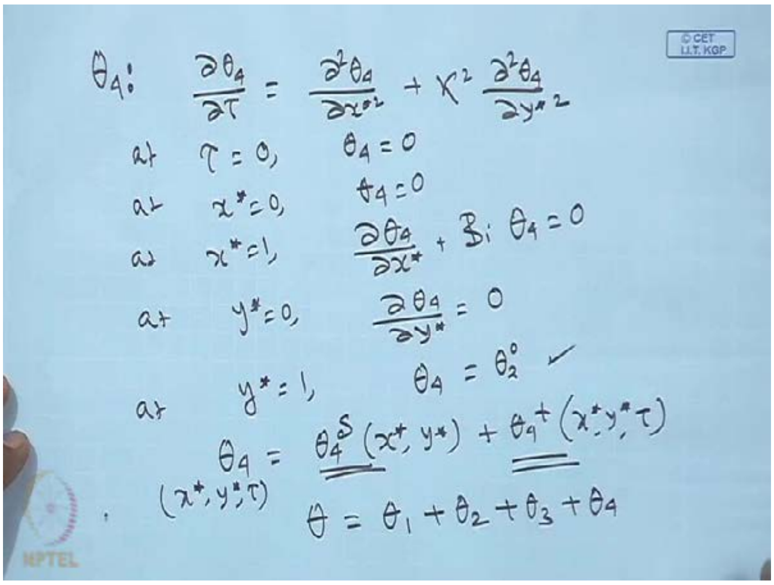

Next, we look into the formulation of the sub-problem theta 4, that will be del theta 4 del tau is equal to del square theta 4 del x star square plus kappa square del square theta 4 del y star square. Now, we keep one non-homogeneity here and forcing the others to vanish. So, this will be tau is equal to 0, theta 4 is equal to 0, at x star is equal to 0 theta 4 is equal to 0, at x star is equal to 1 del theta 4 del x star plus Biot times theta 4 is equal to 0, at y star is equal to 0 del theta 4 del y star is equal to 0.

And at y star is equal to 1, we keep the non-homogeneous term theta 2 naught. Now, in this problem, we have kept all we have forced all the non-homogeneous term to vanish and kept the boundary condition non-homogeneity of at the boundary at y star is equal to

1. Again, this problem has 0 initial condition, non homogeneous initial condition, non-homogeneous boundary, 1 non-homogeneous boundary condition.

Then again it is an ill posed problem, we have to convert this problem into well posed problem. So, therefore, theta 4 is equal to theta 4 as a function of theta 4 s, which is a function of x star y star plus theta 4 t, which is a function of x star y star and tau both all. So, this will be function of x star, y star and tau; now like the earlier problem, we formulate differently the governing equation of steady state part, the transient governing equation of the transient part and we can we associate the non-homogeneous term to with the steady state solution, force forcing that the transient part should be assured have a homogeneous boundary condition at y star is equal to 1.

So, therefore, will be having and the initial condition of the transient part theta 4 2 t will be nothing but, at time t is equal to 0, it will be nothing but, the minus negative of the steady state solution, which will be completely solvable. Therefore, again the transient part becomes a well posed problem and will be getting complete solution from theta 4, we have we already seen the solution of theta 4 s and theta 4 t.

We have not seen the solution of the of the partial of a differential equations in the as the elliptical characteristic, we will be looking the solution of that shortly. So, will be getting the complete overall solution as theta is equal to theta 1 plus theta 2 plus theta 3 plus theta 4. And thus an appropriate that actual chemical engineering 3-dimensional problem can be reduced into 4 sub-problems and we can do the solution by doing a linear superposition of all the individual solutions.

So, I stop it here at this particular class and then, I will move into the next class, I will move into the next topic, that is a formulation of a 4-dimensional problem or a highest possible dimensional in our realty. So, we will be talking about a 4-dimensional problem, 3 dimension in space at 1 dimension in time. Thank you for your kind attention.