Advanced Mathematical Techniques in Chemical Engineering
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Module No. # 01
Lecture No. # 25
Solution of Parabolic PDE: Separation of Variables Method.
(Contd.)

Welcome to the session. So, in this particular session, we will be taking up one actual example in a chemical engineering application and we will forward the governing equation boundary condition. We will go step by step by making it non-dimensional and then, we will be looking into the separation of variable type of solution and finally, we will be constructing the complete solution.

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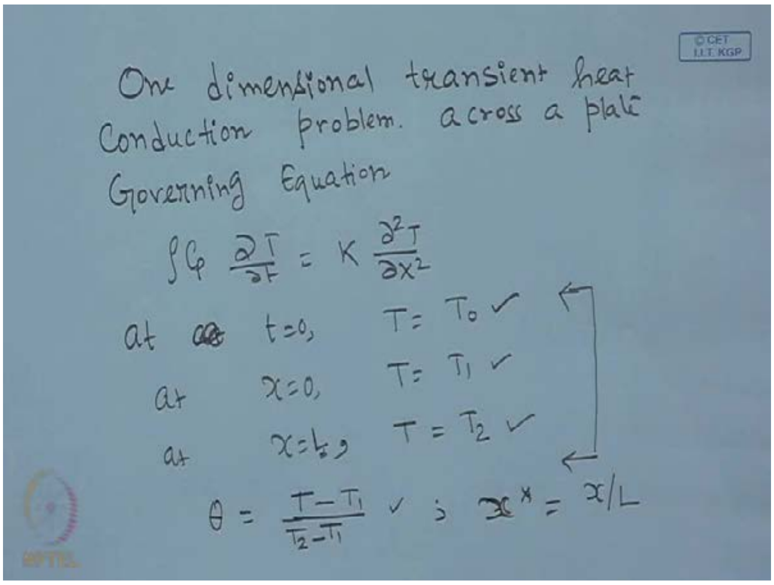

Now, let us take up one example of chemical engineering application, that is, one dimensional transient heat conduction problem. Now, if you look into the across may be a plate so that we can take recourse to the cartesian coordinate system.

So, if you write down the governing equation, the governing equation to this heat conduction problem becomes rho c p del T del t is equal to k del square T del x square. So, we are considering the heat conduction takes place across the, in the x direction and the boundary conditions were at t is equal to 0, temperature was T naught, at x is equal to

0, temperature was T 1 and at x is equal to 1, temperature was… at x is equal to L, temperature was T 2.

So, we are maintaining Dirichlet boundary conditions on both the boundaries, at the boundary located at x is equal to 0, we maintain a temperature T 1, boundary located at x is equal to L, we are maintaining a temperature T 2. Now, we make this system, first you make this system non-dimensional. Now, you if you if you look into the, let us analyze this problem in a more detail, the analysis go like this. Now, you can identify the number of homogeneities number of non-homogeneities in this particular problem. This particular problem consists of three sources of non-homogeneities, one in the initial condition and both the boundary conditions.

So, if you consider the separation of variable, if you remember the principle of linear superposition, this problem has to be divided into three sub-problems considering one non-homogeneity at a time. So, in order to reduce the labor of doing the solution, what do we do, we will define the define a non-dimensional temperature such that one of the boundary condition becomes homogeneous automatically.

So, therefore, what we will do, we will define a non-dimensional temperature as T minus T 1 divided by T 1. What that will do, we can make it T minus T 1 divided by T 2 minus T 1, what that will do, that will simply make this boundary condition to be homogeneous, that is at x is equal to 0, T is equal to T 1.

So, we will be getting, we will be reducing the number of non-homogeneities from 3 to 3 to 2. So, please try to understand this point, if we do not do anything, this problem will be having three sources of non-homogeneities, by defining this non-dimensional temperature, we can force the boundary condition at x is equal to 0 to be homogeneous and number of non-homogeneities will be brought down from 3 to 2.

So, we define a non-dimensional temperature theta is equal to T minus T 1 divided by T 2 minus T 1, we define a non-dimensional distance as x over L. So, therefore, we put this non-dimensional quantity into governing equation and see what we get.

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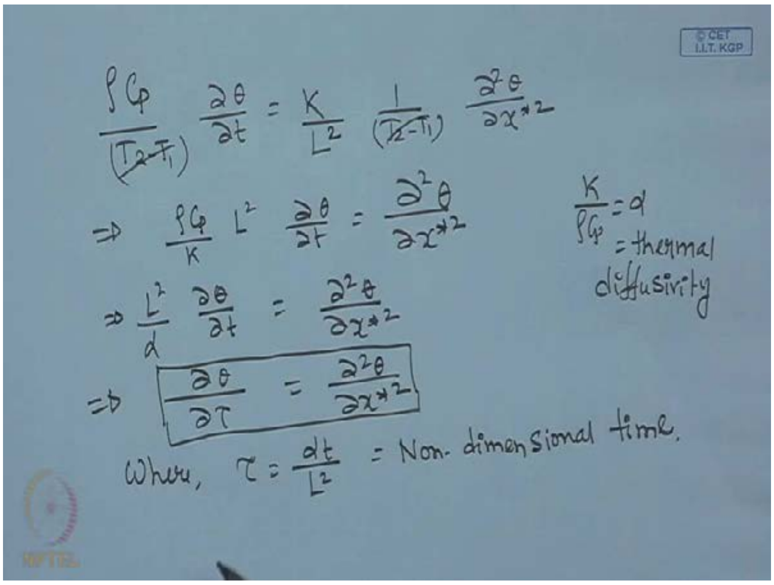

Rho c p T 2 minus T 1 del theta d t is equal to k divided by L square del square T, del square T will be 1 over T 2 minus T 1 del square theta divided by del x star square. So, we have, this will be cancelling out, so rho c p over k times l square del theta del t is equal to del square theta del x star square, therefore we define k rho c k by rho c p as alpha, alpha is nothing but the thermal diffusivity.

So, therefore, we write it down as del theta del T is L square by alpha is equal to del square theta del x star square. So, therefore, we will be getting del theta and if you look into alpha by L square, it will be having a unit of second inverse, so this becomes del tau is equal to del square theta del x star square. So, where tau is equal to alpha t over L square, this is a non-dimensional time. So, we get the non-dimensional form of the equation, in fact this is a technique how to make the things non-dimensional.

If some of the if the parameter, some parameter is not apparent, how to make it non-dimensional, it is basically, the rule is that you make the those parameters to be non-dimensional with the known values of, with the known parameters and then substitute in the governing equation, so automatically the non-dimensional parameter will be evolving out.

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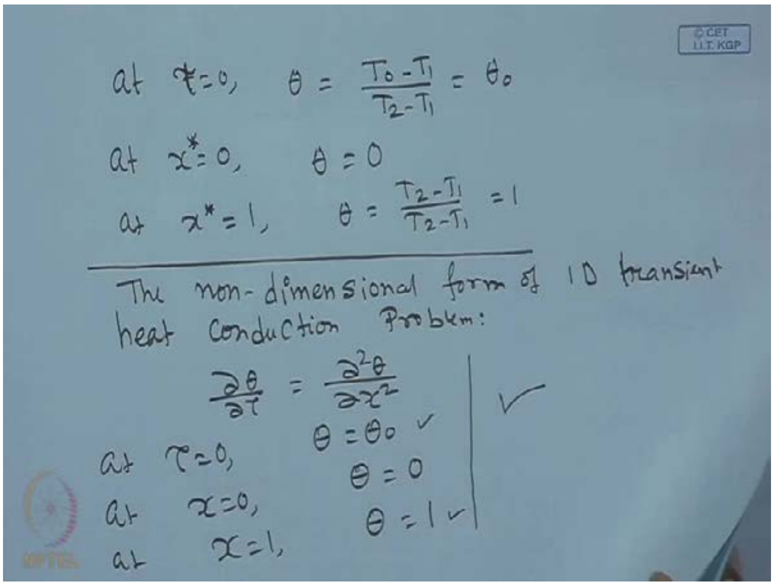

So, let us put the get the boundary conditions in it is non-dimensional form, at t is equal to 0, means at t tau is equal to 0, your T was equal to T naught, so theta was equal to T naught minus T 1 divided by T 2 minus T 1, so this becomes theta naught. At x is equal to 0, T is equal to T 1, so your theta becomes is equal to 0. At x star x is equal to L means x star is equal to 1, theta becomes T 2, so it is it will be T 2 minus T 1 divided by T 2 minus T 1, so it will be having 1.

So, therefore, let us write down the so we so, these are the boundary conditions in their non-dimensional version, so we (( )) with x star and all these, so you put them as x x. So, the non-dimensional form of one dimensional transient heat conduction problem becomes del theta del tau is equal to del square theta del x square at tau is equal to 0, theta was equal to theta naught, at x is equal to 0, theta was equal to 0, at x is equal to 1, theta was equal to 1.

So, we solve this problem, we just omitted the x star part, we make it x 2 to make our calculations convenient. So, we are going to solve this equation, so if you now check this equation has now become in the original problem, we had three sources of non-homogeneity and but in this problem, we have only two sources of non-homogeneity.

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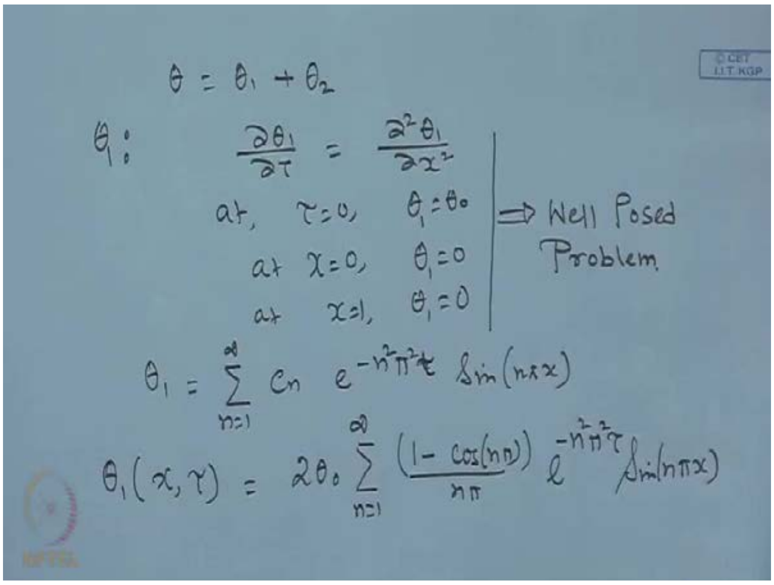

Now, therefore, this problem has to be divided into two sub-problems, because there are two sources of non-homogeneity. So, we will be having theta is equal to theta 1 plus theta 2, where each such sub-problem will be having only one non-homogeneity at a time. So, therefore, we have the governing equation of theta 1 as del theta 1 del tau is equal to del square theta 1 del x square, at and at tau is equal to 0, theta is equal to theta naught, at x is equal to 0, we had theta is equal to 0, at x is equal to 1, we had theta equal to 0. So, we consider theta 1 as this and considering one non-homogeneity at a time, so this if you remember, this is a well posed problem.

Now, if you remember, the solution of this well posed problem is nothing but this will be theta 1, so theta 1 if I remember, this becomes n is equal to 1 to infinity C n e to the power minus n square pi square t sine n pi x. Since these are the Dirichlet boundary conditions, the eigenvalues are n pi and eigenfunctions are sine n pi x. So, therefore, if you can evaluate C n from the initial conditions theta 1 is equal to theta 1 is equal to theta naught and probably we have already solve this problem earlier, I am just putting the complete solution. So, theta 1 as a function of x and tau, will be nothing but 2 theta naught, summation n is equal to 1 infinity one minus cosine n pi divided by n pi e to the power minus n square pi square tau sine n pi x. So, that gives the complete solution of theta 1.

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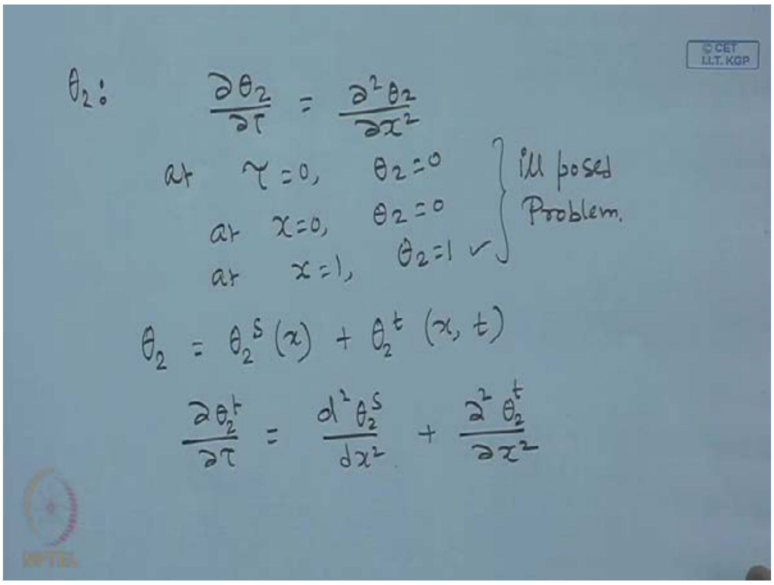

Next, we will be doing for the theta 2. So, theta 2 will be the governing equation theta 2 will be del theta 2 del tau is equal to del square theta 2 del x square. At tau is equal to 0, we made theta 2 is equal to 0, at x is equal to 0, we had theta 2 is equal to 0 and at x is equal to 1, we had theta 2 is equal to 1. So, if you in this sub-problem, we are considering one non-homogeneity, the non-homogeneity at the boundary condition we are considering, forcing the non-homogeneity in the initial condition to be 0.

So, we are considering one non-homogeneity at a time, therefore if you look into the solution of this, this solution we have already looked, so this is an ill posed problem, because we have a 0 initial condition. This problem should be again, we have to make this problem well posed problem, for that this problem will be again broken down into two parts, one is the time varying part or the transient part and another is the steady state part at the time independent part.

So, theta 2 has to be again broken down into two part, one will be theta 2 s, which will be function of x alone, it is a steady state part plus theta 2 t, which is a function of x and t both. Now, exactly the same way you have proceeded earlier, we will be solving this problem del theta 2 del tau, we just put these equations there. So, this becomes del theta 2 t del tau is equal to d square theta 2 s d x square plus del square theta 2 t del x square, we collect the similar order terms and can get the solution of theta 2 and theta s.

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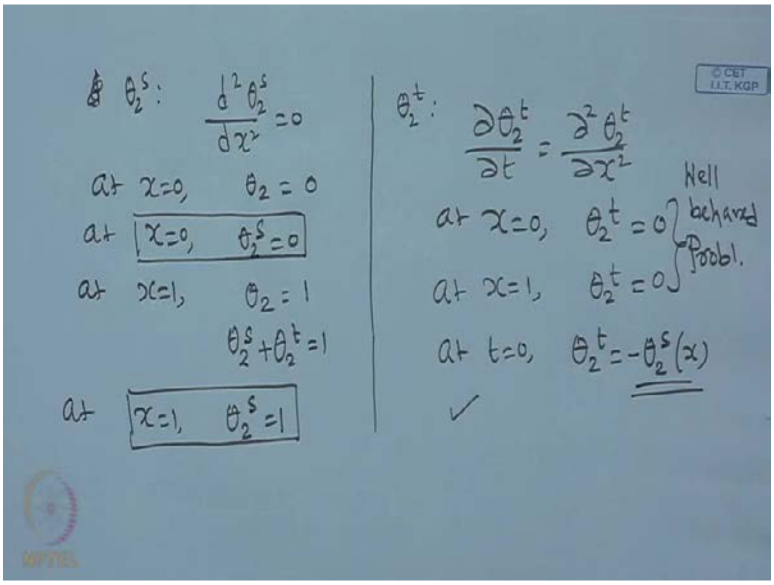

So, we formulate theta 2 s as d square theta 2 s d x square is equal to 0 and at the same time, we will be getting the governing equation theta 2 as del theta 2 t del t is equal to del square theta 2 t del x square. So, d square theta 2 s d x square will be, have let us put it into set the boundary conditions, at x is equal to 0, the original problem is theta 2, in this problem, the original problem, the mother problem becomes theta 2 and theta 2 equal to 0.

So, we have at x is equal to 0, theta 2 s is equal to 0 and at x is equal to 0, theta 2 t is equal to 0, at x is equal to 1, the mother problem theta 2 was having a boundary condition was 1. So, this will be putting theta 2 s plus theta 2 t is equal to 1, so we associate the non-homogeneous term one with theta 2 s, the steady state part. So, at x is equal to 1, my theta 2 s becomes one that forces the boundary condition of the transient part to be homogenous at x is equal to 1.

So, therefore, at x is equal 1, the theta 2 t will be equal to 0. And now we are in a position to get the initial condition of theta 2, at t is equal to 0, theta 2 was equal to 0 and therefore theta 2 t will be nothing but the solution of the steady state part, which is a function of x. So, this is the boundary condition of the x varying part at x is equal to 0 and this is the complete formulation of theta 2 t, which has a non-homogenous initial condition and homogenous boundary condition, so this is a well behaved problem.

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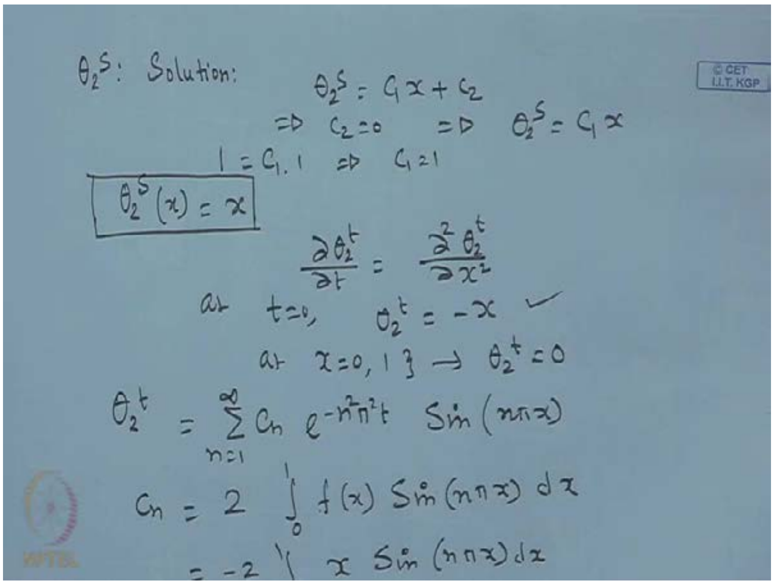

Now, we solve this problem completely, I think that will be important, we get first the theta the space varying part theta 2 s solution of that. The solution of theta 2 s is nothing but C 1 x plus C 2. At x is equal 0, theta 2 s is equal to 0, that means C 2 is equal to 0. So, that gives theta 2 s is equal to C 1 x and at x is equal to 1, theta 2 was, theta 2 s was 1.

So, therefore, 1 is equal C 1 times 1, therefore will be having C 1 is equal to 1. So, the steady state solution is given as x, so this is the steady state solution. Now, the let us look into the transient part, del theta 2 t del t is equal to del square theta 2 t del x square and we have at t is equal to 0, theta 2 t was equal to minus x at x is equal to 0 and 1, both will be having theta 2 t is equal to 0.

So, we have a non-zero initial conditions or non-homogeneous initial condition and homogenous boundary conditions, both the boundaries are Dirichlet in nature. So, n pi are the eigenvalues and sine functions of the eigenfunction, so theta 2 t will be comprised of summation of C n, n is equal to 1 to infinity e to the power minus n square pi square t sin n pi x. So, that gives the theta 2 t and what is the value of C n? C n will be twice integral 0 to 1 f of x sin n pi x d x. So, the initial condition will be, the this f x is nothing but minus x.

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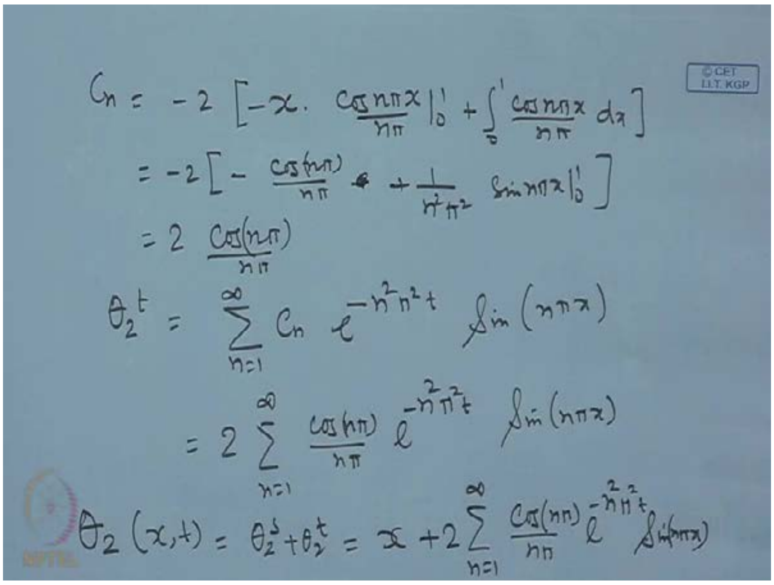

So, this will be minus 2 0 to 1 x sin n pi x d x, so if you get that, so we obtain the solution of C n by carrying out this integration by parts. So, minus 2 first function x, integration of second functions will be minus cosine n pi x divided by n pi from 0 to 1 minus differential of fist function is 1 minus minus plus cosine n pi x divided by n pi d x from 0 to 1.

So, this becomes minus 2 minus 1 cos n pi divided by n pi minus minus plus, 0 multiplied these things, whole thing becomes 0, this becomes 1 over n square pi square sin n pi x, from 0 to 1 sin n pi is 0 sin 0 is 0. So, no contribution from there, you will be getting minus minus plus 2 cosine n pi over n pi.

So, therefore, C n becomes 2 cosine n pi by n pi and theta 2 t becomes summation of n is equal to 1 to infinity C n e to the power minus n square pi square t sin n pi x. And this C n is 2 summation cosine n pi divided by n pi e to the power minus n square pi square t sine n pi x, n is equal to 1 infinity. So, we complete this problem, so we complete the second problem theta 2 as a function of x and t, theta 2 s plus theta 2 t and theta 2 s was x, so x plus 2 summation n is equal to 1 to infinity cosine n pi divided by n pi e to the power minus n square pi square t sin n pi x.

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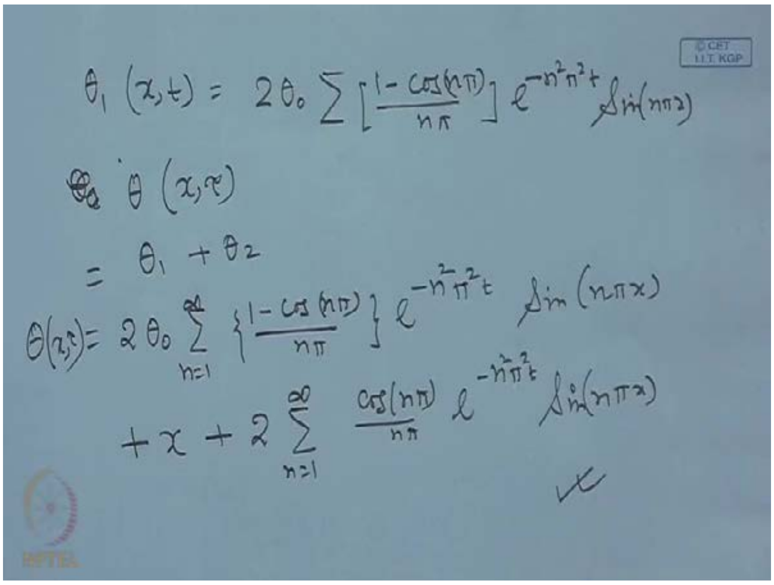

And we have already got the solution of theta 1, so if you remember the solution of part theta 1 as a function of x and t should be is equal to 2 theta naught summation of 1 minus cosine n pi divided by n pi e to the power minus n square pi square t sin n pi x, this t is basically tau. So, theta 1, so the complete solution theta as a function of x and tau should be theta 1 plus theta 2.

So, theta 1 is 2 theta naught summation n is equal to 1 to infinity 1 minus cosine n pi divided by n pi e to the power minus n square pi square t sin n pi x plus x plus 2 summation n is equal to 1 to infinity cosine n pi divided by n pi e to the power minus n square pi square t sin n pi x. So, that gives the complete solution of a transient heat conduction problem in its non-dimensional form.

So, this gives a demonstration how a chemical engineering problem can be solved in it is in the form of by using the separation of variable type of solution. Next, we take up a chemical engineering problem, a one dimensional transient heat conduction problem, but at one of the boundaries, the boundary is insulated, if the boundary is not insulated, there is a constant heat flux that is going putting into the system and we will be having a Neumann boundary condition there.

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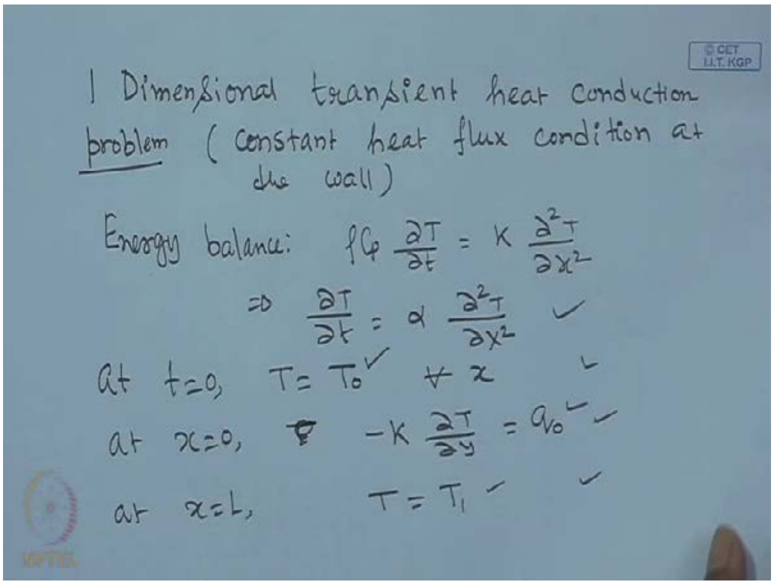

So, it will be again an example of one dimensional transient heat conduction problem, again we will be solving it completely; constant heat flux condition at the wall. So, the energy balance equation is given by rho c p del T del t is equal to k del square T del x square. So, we make del T del t as k by rho c p, that is alpha del square T del x square, alpha is known as the thermal diffusivity, that will be equal to k by rho c p.

Now, let us put the other boundary conditions, an initial condition, at t equal to 0, temperature was equal to t naught at for any x, for every x, this is true. At x is equal to 0, the boundary at x is equal 0, t was minus k del t del y is equal to q naught. So, this is the, q naught is the constant heat flux that is going into system and at x is equal to L, we had, we are maintaining a constant temperature at the wall T is equal to T 1.

Now, this there are if you look into the system, this in this system of equations, there are three sources of non-homogeneity, both at the initial condition is non-homogeneous, both the boundary conditions are non-homogeneous. Therefore, what we will do, we make this equation non-dimensional, by making the equation non-dimensional we are getting two benefits at a time. The first one is we make this problem well behaved; that means, the boundaries are made from 0 to 1 and we normalize all the variables, that is number 1; number 2 is that the we can judiciously selects the our non-dimensional variable, so that we can reduce the non-number of non-homogeneities from 3 to 2.

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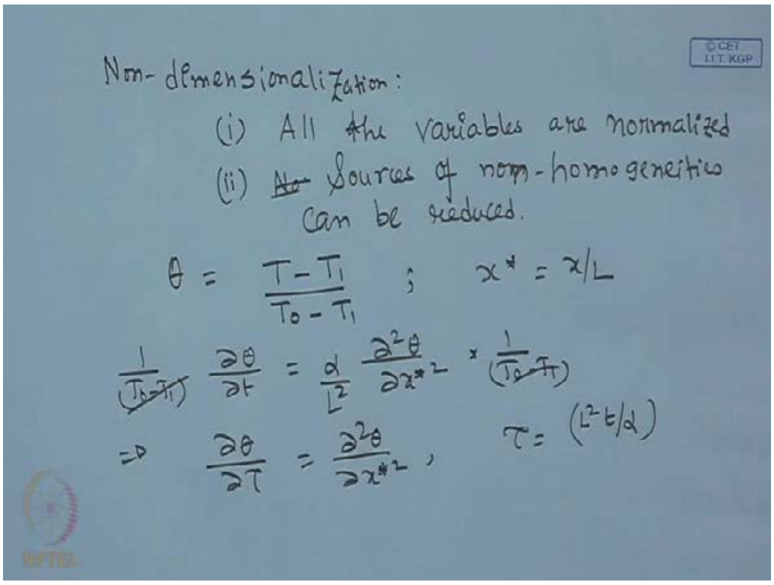

So, therefore, the non-dimensional is necessary, so if you remember, the non-dimensionalization is done because of two things, as I mentioned, first one, it has to be, the all the variables are normalized, so, therefore, they are in a domain that can be handled quite easily. And secondly, the number of sources of non-homogeneities can be reduced so that the rigger of calculations will be decreased.

Now, we define a non-dimensional temperature as theta equal to T minus T 1 divided by T 0 minus T 1. If we define this non-dimensional theta, let us see an x star is equal to x by L, let us see how the non-dimensional equation looks like. So, this becomes del 1 over T 0 minus T 1 del theta del t is equal alpha del square theta L square del x star square, so into 1 over T 0 minus T 1 is there, so that will be cancelling out. So, this becomes del theta del tau is equal to del square theta del x star square, where tau equal to L square t over alpha.

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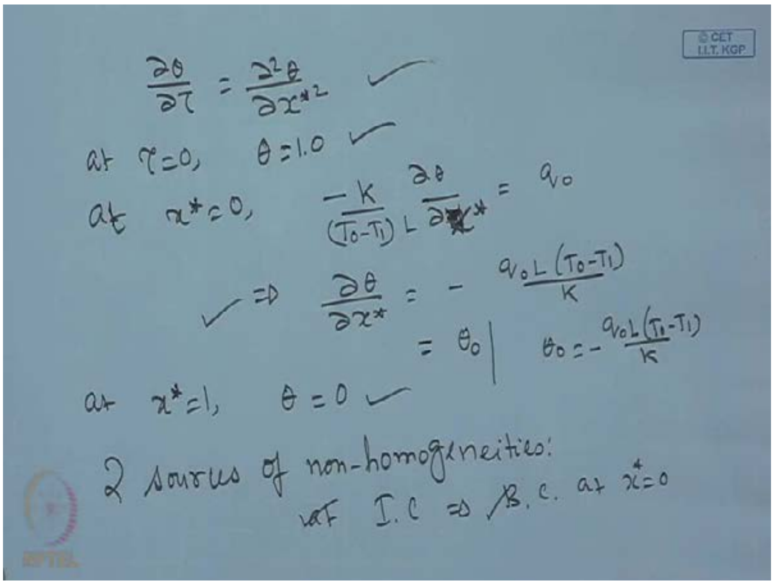

Now, let us use the boundary conditions and make the boundary conditions non-dimensional. So, you just write the equation del theta del tau is equal to del square theta del x star square. The non-dimensional temperature at t is equal to 0, that means at tau is equal to 0, temperature t was equal to t naught, so theta was equal to 1.

At x star is equal to 0, minus k del t del y is equal to q naught, so minus k del t del naught. So, it will be del theta divided by T naught minus T 1 del y, so it multiplied by L, del x star del y star, yeah it will be del x star. The boundary condition at x is equal to 0 minus k del T del x equal to q naught, so this will be del theta del x star is equal to q naught. So, we will be getting del theta del x star is equal to minus q naught L T 0 minus T 1 divided by k. So, we call this as some quantity, let us say this is as theta naught.

So, theta naught is nothing but minus q 0 L T 0 minus T 1 divided by k and at x star is equal to 1, we have t is equal to T 1, therefore theta becomes equal to 0. So, now, if you see the boundary conditions and initial condition of this problem, the problem itself is homogenous, the initial condition is non-homogenous, the boundary condition is non-homogenous at x star is equal to 0, but the boundary condition at x star equal to 1 is homogenous.

So, therefore, two sources of non-homogeneity in this equation, one is at initial condition and second is at boundary condition at x is equal to x star is equal to 0. So, we have these two sources of non-homogeneity in this system.

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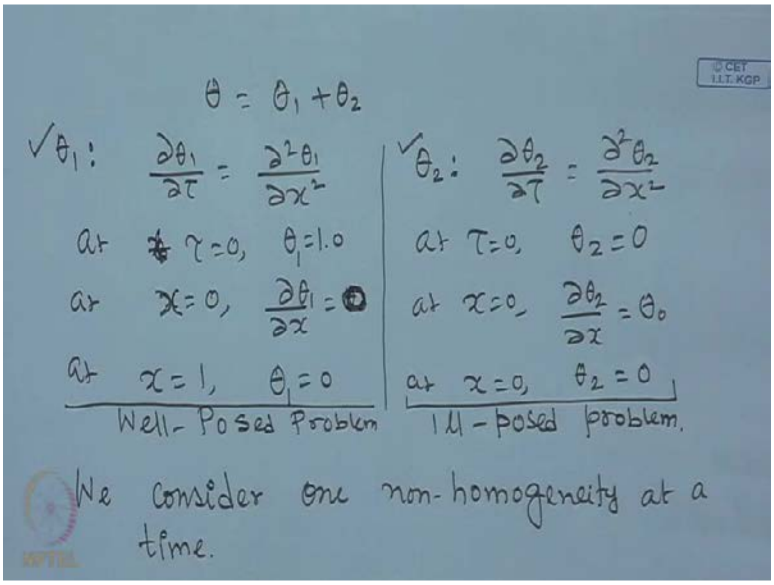

Now, therefore, this problem is again divided into 2 sub-problems and if you do that, so this sub-problem will be divided into 2 theta 1 plus theta 2, will be formulating both these sub-problem considering one non-homogeneity at a time. So, del theta

So, let us look into definition of theta 1, it will be del theta 1 del tau is equal to del square theta 1 del x star square. So, we make it, we substitute x star by x, so that the we need not to put star everywhere. So, at x is equal at t is equal to, at tau is equal to 0, we had theta is equal to 1, at x is equal to 0, we had del theta del x is equal to theta naught.

At x is equal to 1, we had theta is equal to 0, we make this boundary condition to be 0, we force this boundary condition to be 0. So, for theta 1, we make this boundary conditions, at tau is equal to 0, theta 1 equal to 0, we keep this non-homogeneity with theta 1. And at x is equal to 0, the non-homogeneous boundary condition del theta 1 del x is equal del theta, del x is equal to theta naught, we force it to be 0, so at x is equal to 1, theta 1 is equal to 0.

So, this non-homogeneity, let us associate with the next problem, so theta 2 will be defined as del theta 2 del tau is equal to del square theta 2 del x square, so at tau is equal to 0, we have theta 2 equal to 0. So, if force this non-homogeneity is 0, in this case and we will keep the non-homogeneity in the boundary condition intact in the second problem.

At x is equal to 0, del theta 2 del x is equal to theta naught and at x is equal to 0, theta 2 was equal to 0. So, if you as you have done earlier, will be divided this problem into 2 sub-problem, considering one non-homogeneity at a time. So, the crux of this problem is that, we consider one non-homogeneity at a time, so in order to do that we have defined these two sub-problems theta 1 and theta 2.

And if you look into this problem, theta 1 is a well posed problem. Why it is well posed problem, simply because it has a non-homogenous initial conditions condition and homogenous boundary conditions, on the other hand theta 2 is an ill posed problem. We already know the well posed problem like this and we have already solved it earlier. If you remember the boundary conditions at x is equal to 0 is having a Neumann boundary condition, so the eigenvalues to this problem are 2 n minus 1 pi by 2 and cosine functions are the eigenfunctions.

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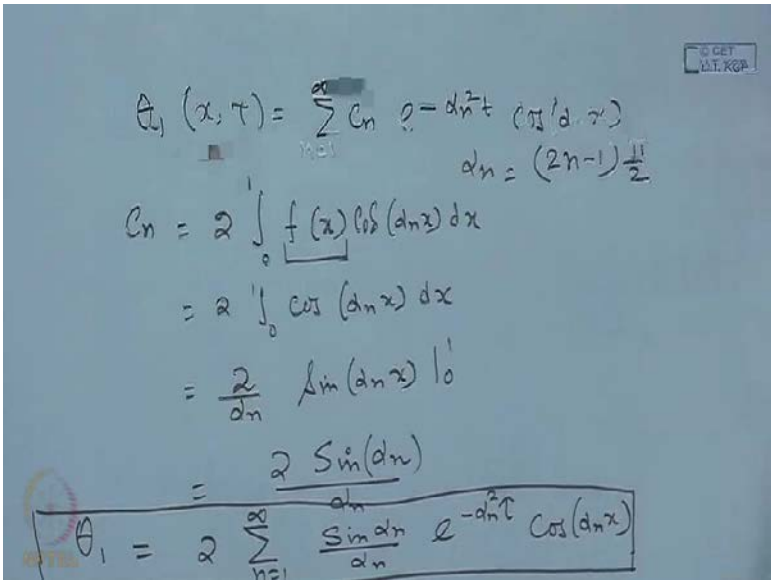

So, therefore, we write the solution, we can write the complete solution of theta 1 as a function of x and tau, that will be simply summation of C n e to the power minus alpha n square t cosine alpha n x, where n runs from 1 to infinity, where alpha n is equal to 2 n minus 1 pi by 2. And let us look into what C n's are, the C n's was 2 summation of f of x cosine alpha n x d x from 0 to 1 and what is this f x, this f x was the initial condition at t is equal to 0.

Now, for this particular problem, f x is equal to 1, so the initial condition is equal to 1, therefore we put 2 integral 0 to 1 cosine alpha n x d x, therefore integration of this will be nothing but 2 by alpha n sin alpha n x 0 to 1. So, sin 0 is 0, sin alpha n x will be, it will be 1 if you put the upper limit.

This becomes 2 sin alpha n divided by alpha n, so the complete solution of first part theta 1 is nothing but 2 summation n is equal to 1 to infinity sine alpha n divided by alpha n e to the power minus alpha n square tau cosine alpha n x, so that is the complete solution of theta 1 part.

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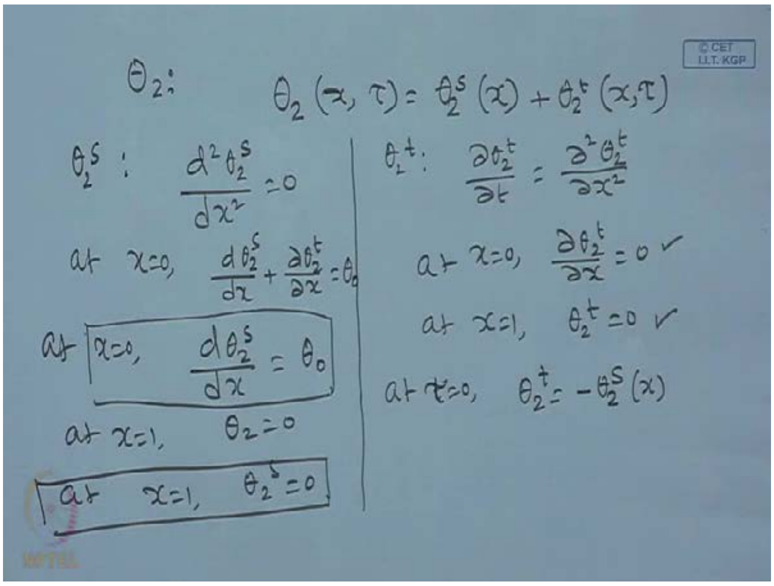

Now, let us look into the theta 2 part. The theta 2 part is having an ill posed problem, so it has a homogenous initial condition and a non-homogenous boundary condition. So, this problem will be divided into 2 sub parts, one is the time dependent part, another is the time independent part. So, theta 2 x and tau should be divided into 2 sub-problem, theta 2 s which is a time varying part and theta 2 t which is a function of both x and time.

So, we defined the like exactly the same way we have done earlier, theta 2 s is given as governing equation, so the parent problem for theta 2 s and theta 2 t at theta, so d square theta 2 s d x square is the governing equation of theta 2 s. And theta 2 t will be having del theta 2 t del t is equal to del square theta 2 t del x square.

So, boundary conditions of this problem of the steady state part are at x is equal to 0, we have del theta 2 del x is equal to theta 0. So, you put d theta 2 s d x plus del theta 2 t del x is equal to theta naught. And we intentionally associate the non-homogenous term with the steady state solution, steady state boundary condition, so that boundary condition of the transient problem will be homogenous.

So, at x is equal to 0, we should have d theta 2 s d x should be is equal to theta naught, and at x is equal to 0, we should have del theta 2 t del x equal to 0. At x is equal to 1, we have theta 2 is equal to 0, so therefore, at x is equal to 1, we should have theta 2 s is equal to 0 and at x is equal to 1, we should have theta 2 t is equal to 0.

So, the boundary condition at x is equal to 1 becomes homogenous, the boundary condition at x is equal to 0 for the steady state part is non-homogenous. On the other hand, the boundary condition of the time varying part at x is equal to 0 becomes homogenous, the boundary condition of the time varying part of at x is equal to 1 becomes homogenous.

So, what is the initial condition of the time varying part? At t is equal to 0 or tau is equal to 0, theta 2 t will be nothing minus nothing but theta 2 s, which is a minus of theta 2 s, which is a solution of the steady state part.

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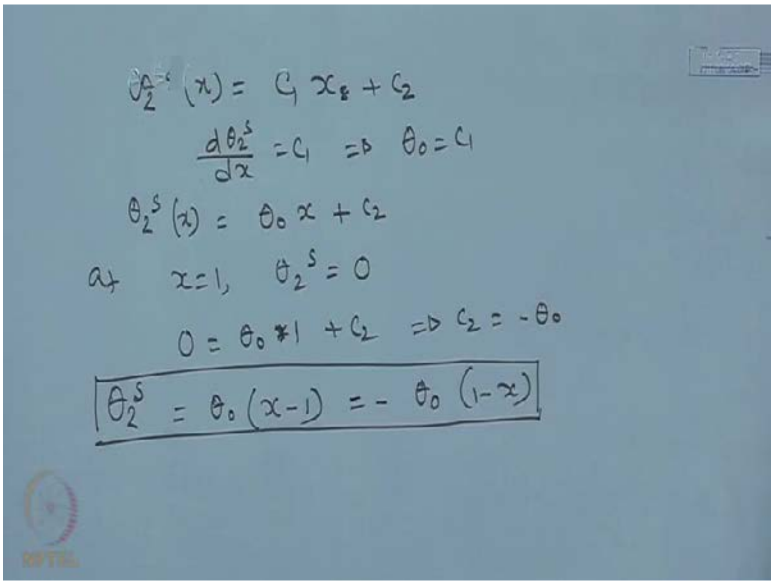

Now, let us look into the solution of the steady state part. The solution of the steady state part theta 2 s will be obtained, since d square theta 2 s d x square is equal to 0, so it is a linear profile C 1 x plus C 2 at x is equal to 0. d theta 2 s d x is equal to C 1, therefore C 1 must be is equal to theta naught, so theta naught is equal to C 1, so therefore theta 2 s is nothing but theta naught x plus C 2. The other boundary is at x is equal to 1, theta 2 s is equal to 0, so, therefore, 0 is equal to theta naught into 1 plus C 2, x is equal to 1, so C 2 will be minus theta naught. So, theta 2 s will be nothing but theta 0 into x minus 1 or minus theta 0 into 1 minus x, so that is the steady state solution.

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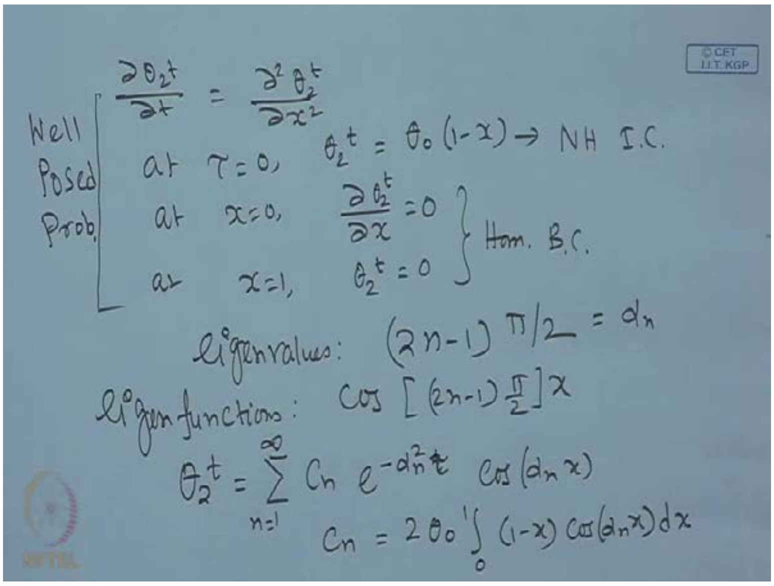

Now, let us look into the transient solution. If you remember the theta 2 part, it will be del theta 2 t del t is equal to del square theta 2 t del x square. At tau is equal to 0, we have theta 2 t is minus of theta 2 s, so this will be theta naught into 1 minus x and at x is equal to 0, we have del theta 2 t del x will be equal to 0. And at x is equal to 1, we have theta 2 t will be equal to 0.

We have already looked into the solution of this problem earlier, so this problem will be having homogenous boundary conditions and a non-homogenous initial condition, so this is a well posed problem. Now, since the boundary condition at x is equal to 0 is Neumann and boundary condition at x is equal to 1 is Dirichlet, the eigenvalues to this problem where, so it is the basic problem of Neumann boundary condition, the eigenvalues are 2 n minus 1 pi by 2 and cosine 2 n minus 1 pi by 2 x are the eigenfunctions.

So, I am not going into detail solution, we have solved this problem several times earlier, the solution will be summation of C n one to infinity, n is equal to 1 to infinity C n e to the power minus alpha n square t tau cosine alpha n x, where alpha n are eigenvalues 2 n minus 1 pi by 2 and C n is nothing but 2 theta 0 integral 0 to 1 1 minus x cosine alpha n x d x. So, I am not going to evaluate this integral, you can evaluate this integral for the time being.

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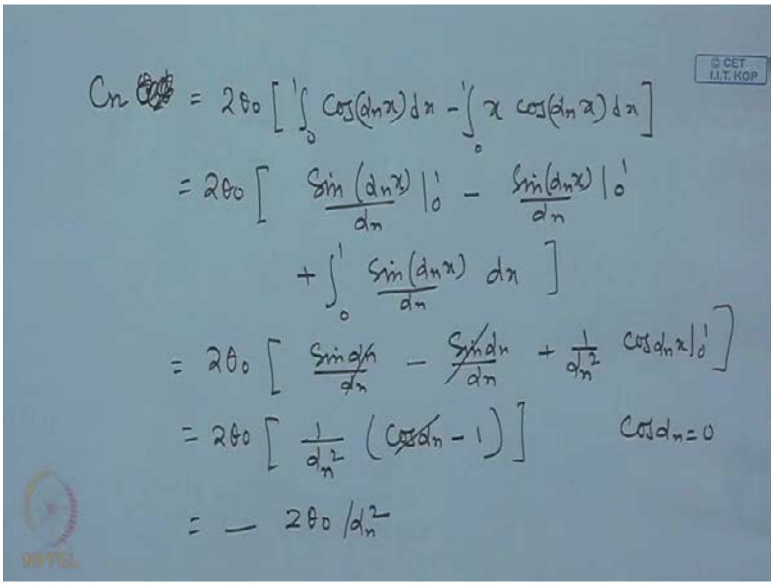

So, theta 2 t is equal to 2 theta naught, so 0 to 1 cosine alpha n x d x minus integral x cosine alpha n x d x 0 to 1. So, this becomes 2 theta 0, cosine alpha n x becomes sine alpha n x divided by alpha n, so sin alpha n x divided by alpha n from 0 to 1 minus first function integral of the second function, so sine alpha n x divided by alpha n from 0 to 1 minus minus plus differential first function 1 integral of the second function, so sin alpha n x divided by alpha n d x from 0 to 1.

So, we will be getting 2 theta naught, this will be sin alpha n divided by alpha n minus sin 0 is 0 minus sin alpha n divided by alpha n, sin 0 0 plus this will be cosine alpha n x, so 1 by alpha n square. So, this will be cosine alpha n x from 0 to 1, so this becomes 2 theta 0, these two will be cancelling out, 1 by alpha n square, this becomes cosine alpha n minus cos 0 is 1.

Now, cosine alpha n is 0, those are the eigenvalues, so this will be 1, so 0, this will be minus 2 theta 0 by alpha n square as we have already seen earlier. So, this is not theta 2, this is basically constant C n.

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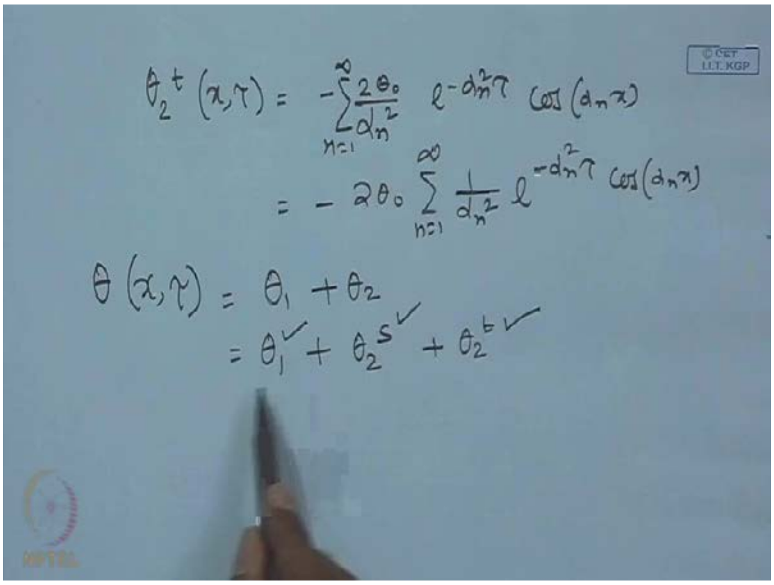

So, theta 2 t, now we are in a position to write the complete solution of theta 2 t. So, theta 2 t as a function of x and tau, should be minus 2 theta 0 by alpha n square, so that is a summation there, e to the power minus alpha n square tau cosine alpha n x, n is equal 1 to infinity. So, you can take minus 2 theta 0 common, so it will be 1 by alpha n square e to the power minus alpha n square tau cosine alpha n x, so n is equal to 1 to infinity.

So, that gives the complete solution of theta 2 t, similarly therefore, you can construct the overall solution theta of our original problem, it was equal divided into two parts theta 1 as a theta 1 plus theta 2 and we completely solved the theta 1 part and theta 2 part. We had two parts, theta 2 steady state plus theta 2 transient and we have we have completely solved the theta 2 s, we have completely solved theta 2 t, we have completely solved theta 1, so summation of all this three parts will give rise to the complete solution of theta.

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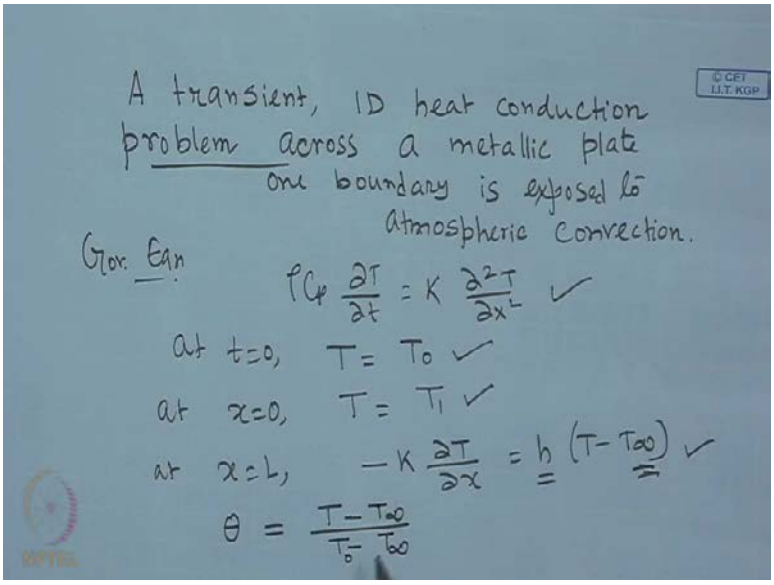

Now, I would like to take up a complete chemical engineering problem, where one of the boundary condition is a robin mixed boundary condition, so it is again a transient heat conduction problem in one dimensional. Transient one dimensional heat conduction problem across a plate, across a metallic plate such that the boundary at x is equal to 1 is opened to atmosphere.

So, if we write down the energy balance, one boundary is exposed to atmospheric convection, so this is the one dimensional heat conduction problem with a mixed boundary condition. We will be just looking into that, let us write down the governing equation, governing equation is nothing but the energy balance equation, rho c p del T del t is equal to k del square T del x square, that is the governing equation at time t is equal to 0.

We have an initial temperature, let us say T naught at x is equal to 0, let us say, we have a temperature, we maintain a temperature T 1 and at x is equal to L, the end of the thickness L, the boundary is opened atmosphere. So, whatever the energy that comes by conduction is taken away by convection of, by taken away convection of the ambient environment, h is the heat transfer coefficient and T infinity is the temperature of the surroundings.

So, h into T minus T infinity is the amount of heat flux that is taken away from the boundary and that should be equal to amount of heat flux that is arriving at that boundary by conduction by minus k del T by del x. Now, if you look into this problem, the governing equation is homogenous, the initial condition is non-homogenous, the boundary condition at x is equal to 0 is non-homogenous and boundary condition at x is equal to L is also non-homogenous because of the presence of term T infinity there.

So, we define this problem, we make this problem non-dimensional, so that the number of non-homogeneities can be reduced significantly and we are aiming to reduce the number of non-homogeneity at least 1. So, if we define a non-dimensional temperature as T minus T infinity divided by T minus T 0 divided by T 0 minus T infinity and then probably we can get down the number of non-homogeneities to 2 from 3.

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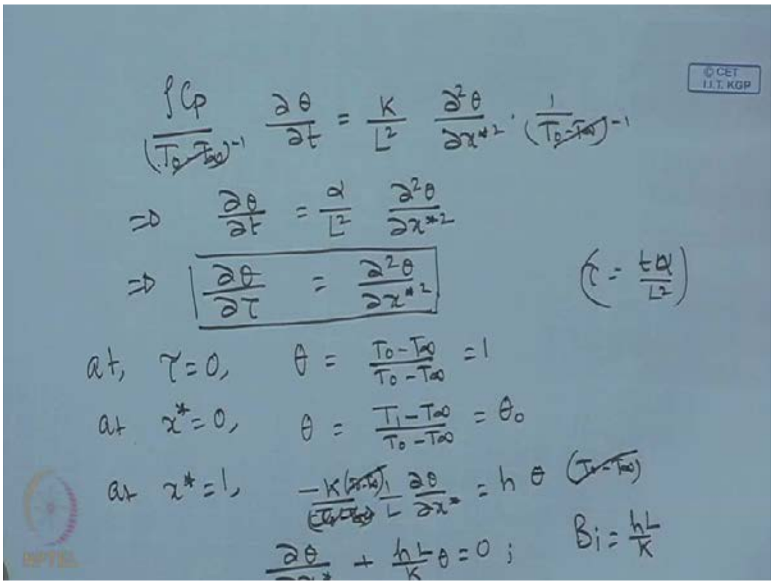

So, if you really do that, let us see what the fate of the governing equation is. The governing equation of non-dimensional form will be rho C p 1 over T 0 minus T infinity del theta del t is equal to k over L square del square theta del x star square. There will be 1 T 0 minus T infinity in the denominator, this 2 will be cancelled out, so you will be getting del theta del t is equal to alpha over L square del square theta del x star square. So, alpha is the thermal diffusivity, so you can define del theta del tau is equal to del square theta del x star square.

But, tau is equal to t alpha over L square, this is the non-dimensional type. So, this is the governing equation of non-dimensional governing equation heat conduction problem.

Now, let us make the boundary condition non-dimensional, at t is equal 0, means at tau is equal to 0, t is equal to t naught, therefore theta becomes T naught minus T infinity divided by T naught minus T infinity is equal to 1. At x is equal to 0, that means at x star is equal to 0, t is equal to T 1, therefore theta is equal to T 1 minus T infinity divided by T 0 minus T infinity, let us put it as theta 0.

And at x star is equal to 1, so we will be having minus k del t, del t means 1 over T 0 minus T infinity, del theta del x that means L 1 over L del x star is equal to h t minus t infinity, we are writing t minus t infinity as theta into T 0 minus T infinity. In fact, T 0 T infinity will be coming in the numerator, so I think this will be minus 1, this will be in inverse, so they will be on the numerator, they will be cancelling out, does not matter. So, these two also will be cancelled out, so we will be having del theta del x star plus h L over k times theta is equal to 0.

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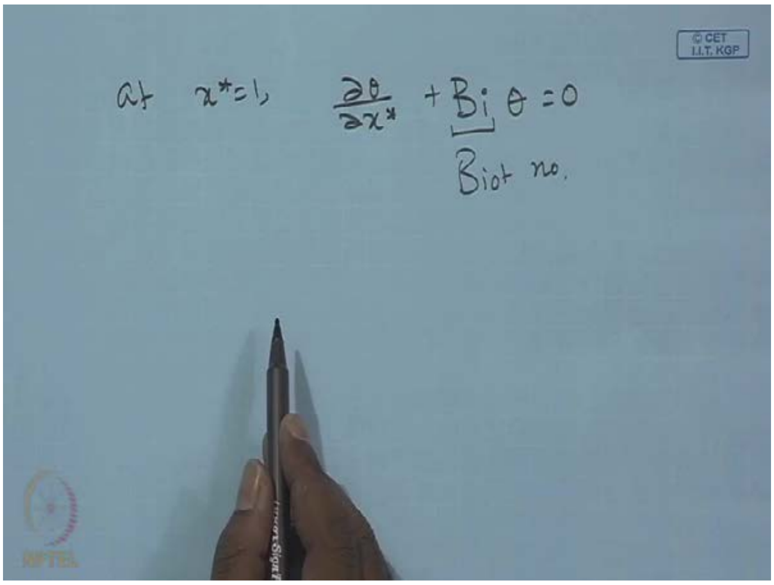

So, if you remember that what is h L over k, h L over k is nothing but the Biot number. So, we write Biot number is equal to h L over k, so the non-dimensional form of temperature at this boundary becomes at x star is equal to 1, is del theta del x star plus Bi times theta, Bi is the Biot number. So, we made this governing equation to be non-dimensional and we will just see there how many number of you know… we have reduce the number of non-homogeneities from 3 to 2.

And I will stop here in this class; in the next class, will just see how this problem can be handled and divide into sub-problems and how the complete solution will be evolved out of this problem. And will be getting a complete solution of one dimensional transient heat conduction with one boundary is expose to the atmosphere, so that we will be having a, dealing with a mixed boundary condition; thank you very much.