Advanced Mathematical Techniques in Chemical Engineering

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Lecture No. # 19

Eigenvalue Problem in Continuous Domain

Good morning to everyone. So, we will be looking into, if you remember, whatever we have done in the last class, we have just looked into the theorems in the continuous domains, and the development will be… Basically, we are we our aim is in this portion of the syllabus is in order to solve the partial differential equations; and we will be looking into the only linear and homogenous partial differential equations.

In the last few classes, we have looked into - how we can classify and categorize the partial differential equations into parabolic, elliptical, and hyperbolic form; not only that, we have also looked into the several boundary conditions those are associated in order to solve a partial differential equations; and we categorized different types of boundary conditions as well.

Now, in this class, what we will be looking into? We will be formulating the standard eigenvalue problem in continuous domain; earlier, we have looked into - how these problems can be formulated in discrete domain, whenever we talked about the matrices; and that eigenvalue, eigenvector method was utilized in order to solve a system of algebraic equations or system of ordinary differential equation. We will be developing the theory for the continuous domain in order to solve the partial differential equation.

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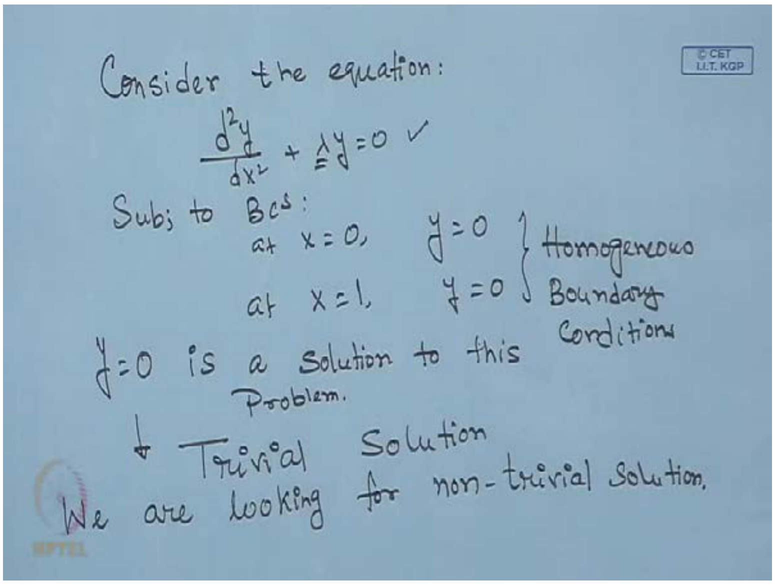

So, let us look into a particular form of equation. Consider the equation in the form, d square y dx square plus lambda y is equal to 0. Let us consider a second order ordinary differential equation of this particular form, subject to the boundary conditions, at x is equal to 0, y is equal to 0; at x is equal to 1, y is equal to 0. So, therefore, we have selected a homogeneous equation with homogeneous boundary conditions.

So, now clearly, in this equation since, the equation and the governing equation and the boundary conditions are clearly satisfied by the solution y is equal to 0. So, therefore, y is equal to 0, is a solution to this problem; but this solution is not a solution that we are looking for; this solution is known as a trivial solution. So, y is equal to 0 is of course, a solution, but it is a trivial solution and we are not looking for that; what we are looking for, is that, we are looking for non-trivial solution.

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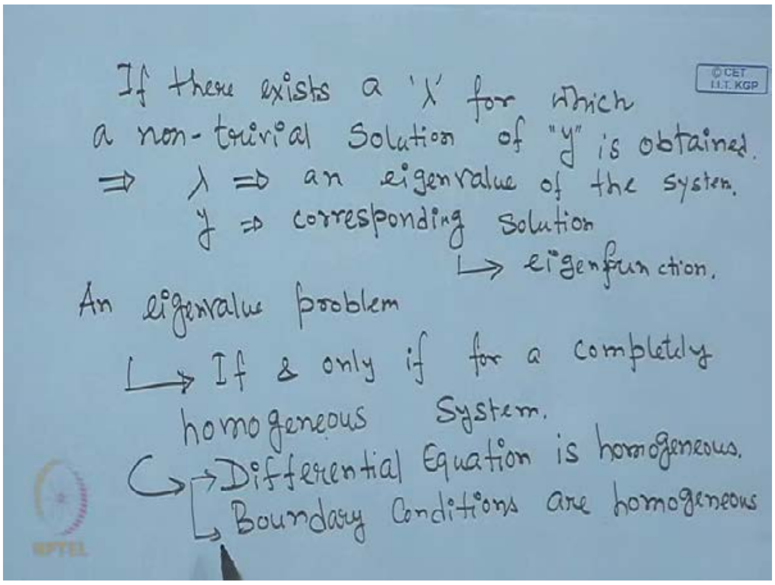

So, we are basically looking for a solution, non-zero solution, for different values of lambda, so what are the values of lambda, so that, these will be giving you a non-trivial solution. So, if that exists, if there exists a lambda that is a scalar, for which a non-trivial solution of y is obtained, then we call lambda as an eigenvalue of the system; corresponding y is or corresponding y or the solution is known as the eigen function.

So, one has to be very clear that one can formulate an eigenvalue problem, if and only if, the boundary conditions are homogeneous. We can formulate an eigenvalue problem, if and only if, for a completely homogenous system. What do you mean by the homogeneous system? By homogeneous system, we mean that the differential equation is homogeneous; the boundary conditions are also homogeneous.

If the differential equation and the boundary conditions, both are homogeneous, then we call that system as homogeneous system. Now, for a completely homogeneous system, there exists an eigenvalue problem; so, eigenvalue problem means, first we have to check whether the boundary conditions are homogeneous, as well as the governing equations are homogeneous; then only, we call that problem an eigenvalue problem. And, if we if the governing equation and the boundary conditions satisfy these two basic criteria or property, then we should go ahead for the solution for the eigenvalues and corresponding eigen functions.

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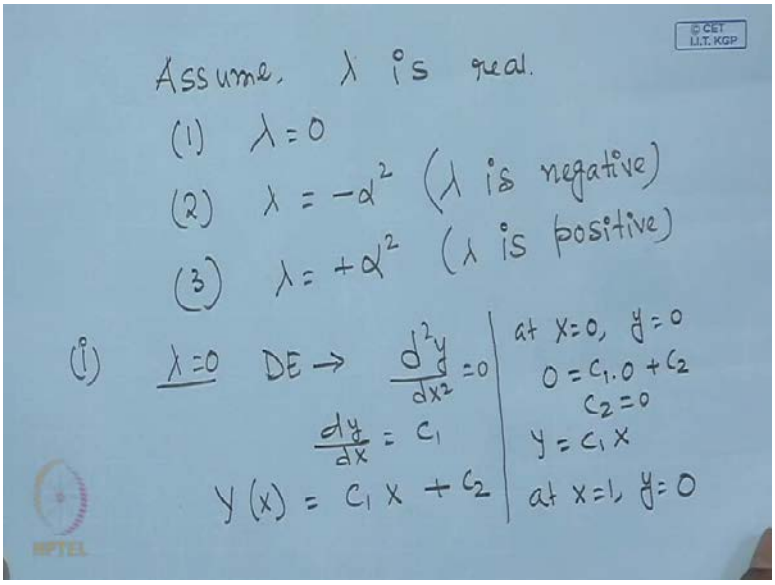

Now, for this particular problem, let us solve this equation and see what are the eigenvalues we are getting and also, the corresponding eigen functions. So, we assume lambda as the constant, lambda is real; if lambda is real, then there are three possible values lambda can assume: number 1 - lambda is equal to 0, so that is possibility one; Second part is lambda is negative; let us say, it is minus alpha square, so, this simply indicates lambda is negative; and third is lambda positive, this simply indicates lambda is positive.

So, let us consider these three cases separately and see what you get. First case will be lambda is equal to 0; if that is the case, let us see - what is the form of our differential equation. Now, the differential equation boils down to d square y dx square is equal to 0; and because lambda itself is 0, so we just integrate it, and let us look into the solution of this, we integrate this out; if we integrate this equation, the first integration results to dy dx is equal to some constant C 1 and second integration leads to y as a function of x is equal to C 1x plus C 2.

Now, these two constants - C 1and C 2 - should be evaluated from the two boundary conditions, whatever we have; let us write down the two boundary conditions: at x is equal to 0, y is equal to 0; that, if we put this boundary condition here, let us see what we get, 0 is equal to C 1times 0 plus C 2. So, therefore, C 2 is equal to 0; if C 2 is equal to 0, the form of the equation has become, now, y is equal C 1x. Now, let us put the other boundary condition that at x is equal to 1, y is equal to 0; so if we utilize this boundary condition, let us see what we get.

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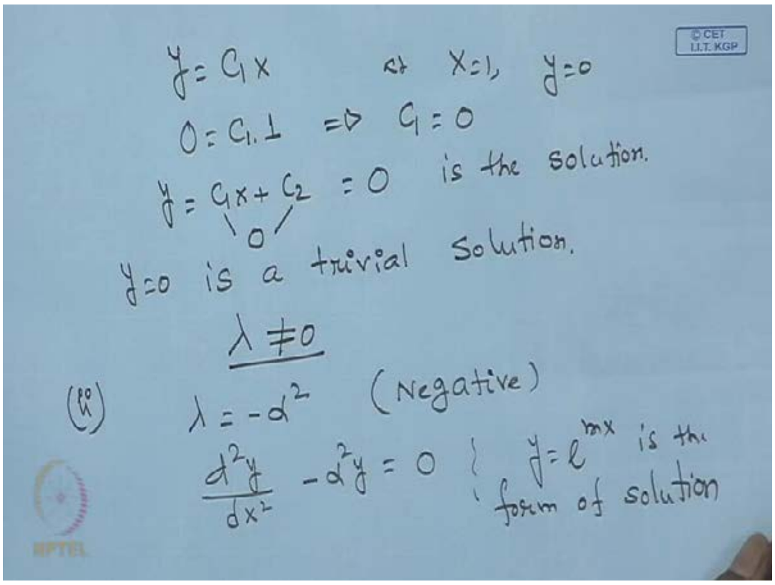

If we put x is equal to 1 into y is equal at y is equal to 0; if we look into the solution, solution is, y is equal to C 1x and at x is equal to 1, y is equal to 0 gives me an equation C 1 times 1. So that simply implies that C 1is equal to 0. So, what we get? If you look into solution, you will be getting y C 1x plus C 2, both C 1and C 2 turns out to be 0; therefore y is equal to 0 is the solution that we are getting. But this solution is a trivial solution, therefore, one should not so this trivial…

We are not looking for a trivial solution, we are looking for a non-trivial solution; so, therefore, lambda cannot be equal to 0. So, lambda is equal to 0 is ruled out, so we cannot use that anymore; so what is the let us look into the second option. Second option is lambda is negative, so, lambda is equal to minus alpha square; so, lambda is negative. Now, let us see, what is the form of our differential equation under this condition of lambda? Differential equation becomes d square y dx square minus alpha square y is equal to 0.

So, if you know the solution, form of this solution is e to the power mx. So, the form of the solution is in the form y is equal to e to the power mx is the form of the solution; so if we do that, we just put y is equal to e to the power mx and let us see what you get. You will be getting a polynomial of order 2 of m and that equation is known as the characteristic equation.

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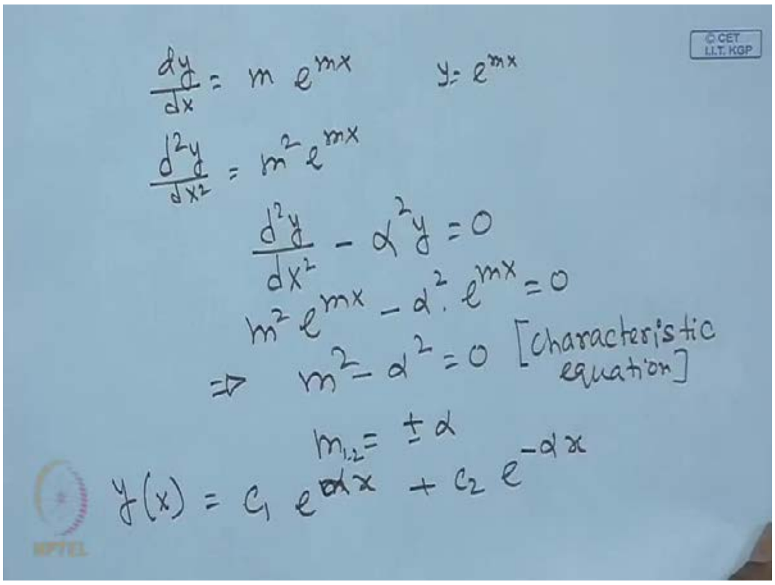

So, if we put it that way, so dy dx, if y is e to the power mx, then dy dx is nothing but m e to the power mx. We differentiate it once more, so, it becomes d square y dx square is equal to m square e to the power mx. Now, we put all these value in the differential equation, differential equation is d square y dx square minus alpha square y is equal to 0. If we put this here, m square e to the power mx minus alpha square; y is e to the power mx, so, it will be e to power mx is equal to 0.

Now, m being not equal to 0, so, e to the power mx is always positive; so, m square minus alpha square is the solution; so, m will be having a solution - plus minus alpha. So, therefore, the form of the solution will be in this form; so this is known as the characteristic equation. So, form of the equation will be solution will be C 1 e to the power alpha x plus C 2 e to the power minus alpha x. So, m will be having two roots, m 1 and m 2; so form of the solution will be C 1 e to the power m 1 x plus C 2 e to the power m 2 x; in m 1, we put plus alpha; in m 2, we put minus alpha.

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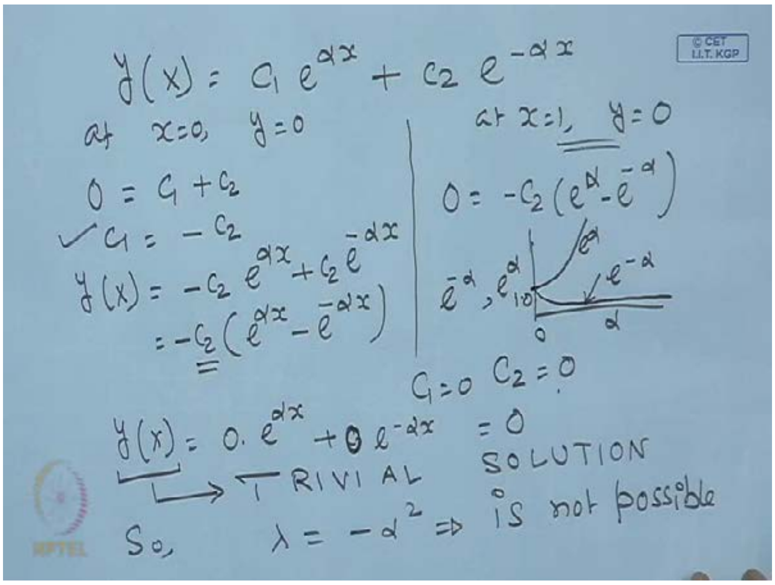

So, now, with this solution, we put the boundary conditions. So, let us write form of the solution as C 1 e to the power alpha x plus C 2 e to the power minus alpha x.

Next, we write down the boundary conditions: at x is equal to 0, y is equal to 0; at x is equal to 1, y is equal to 0, the homogeneous boundary conditions. Now, if you utilize the first boundary conditions, let us see what we get. We will be getting 0 is equal to C 1, e to the power 0 is 1, plus C 2; so, C 1 is equal to minus C 2. So, the solution is, we put, C 1 is minus C 2, so, minus C 2 e to the power alpha x plus C 2 e to the power minus alpha x; so minus C 2 we can take it common, so e to the power alpha x minus e to the power minus alpha x. Now, we put the other boundary condition and see, what is the fate of y of the constant C 2? So, one constant has to be this constant has to be evaluated and one boundary condition is left.

So, we will be having 0, minus C 2 e to the power 1 minus e to the power minus e to the power alpha minus e to the power minus alpha. Now, if you remember the variation of e to the power alpha as a function of alpha, it will be… alpha is equal to 0; we have ruled out alpha is equal to 0, because if alpha is equal to 0, we are going to get a trivial solution. So, even for alpha is equal to 0, e to the power alpha is 1, from 1 onwards, for any positive value of alpha, it will be ever increasing function; it will be always positive.

Similarly, if you look into the… So, this is the variation of e to the power alpha; I am plotting the variation of e to the power alpha and e to the power minus alpha as a function of alpha. So, e to the power alpha is always positive; the minimum value is 1 and the maximum value will be always positive, and the higher values will be always positive. If you look into the value of e to the power minus alpha, for alpha is equal to 0, e to the power minus alpha is 1, and it decreases exponentially for e to the power minus alpha, minus infinity it will be 0, but, so, it varies from 1 to 0; so, therefore e to the power minus alpha is always positive.

So, what I mean by this analysis is that, e to the power alpha is ever positive, e to the power minus alpha is always positive; so, their combination is always a positive quantity. So, in order to satisfy this equation only option is that C 2 is equal to 0; now, if C 2 is equal to zero, from this, we will be getting C 1 is equal to 0; so, what is the fate of my solution? The fate of my solution is 0 multiplied by e to the power alpha x plus, C 2 is 0, multiplied by e to the power minus alpha x, so the full solution becomes 0.

So, again we are landing up with a trivial solution, so, this is nothing but a trivial solution; and we are not looking for a trivial solution, we are looking for a non-trivial solution. So, therefore, alpha so therefore lambda cannot be a negative quantity, it is also not possible. So, lambda is minus alpha is also not possible. So, we have if we remember, we have three choices for real value of lambda; lambda is 0, that is ruled out, because you are getting a trivial solution; then lambda is negative, that is also ruled out, because you are getting a trivial solution. So, only option that we are leaving with is, only lambda is positive; and let us see what we get.

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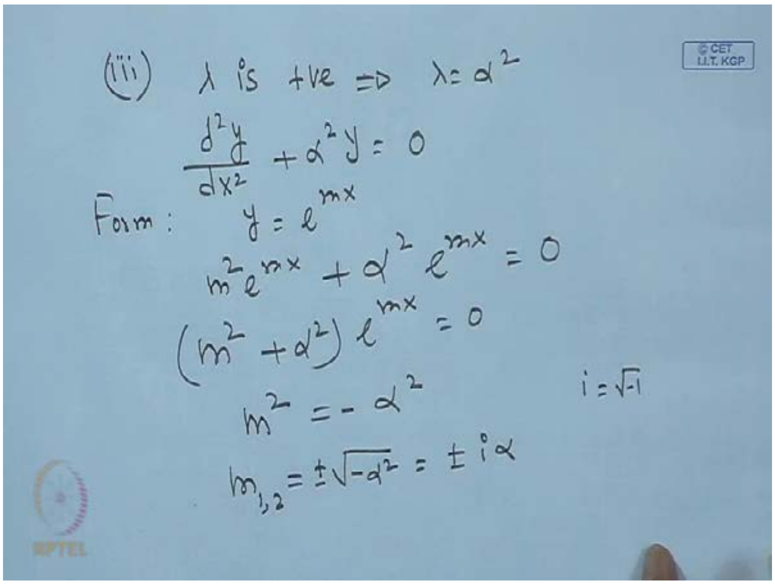

Case number 3 - lambda is positive and it is equal to plus alpha square. So, d square y dx square plus alpha square y is equal to 0. So, let us again look into the solution, the form of the solution is again y is equal to e to the power mx; so, we just put the double differentiation of this equation here, so this becomes m square e to the power mx plus alpha square e to the power mx is equal to 0.

So, what we get is, m square plus alpha square e to the power mx is equal to 0; and e to the power mx for non-zero value of m and x, it will be ever positive; so, m square is equal to minus alpha square. So, what we get is that, what is the solution? The solution is of course, minus root over plus minus root over e to the power minus alpha square; so it will be root over minus 1, it will be nothing but i, imaginary quantity; so, it will be plus minus i alpha.

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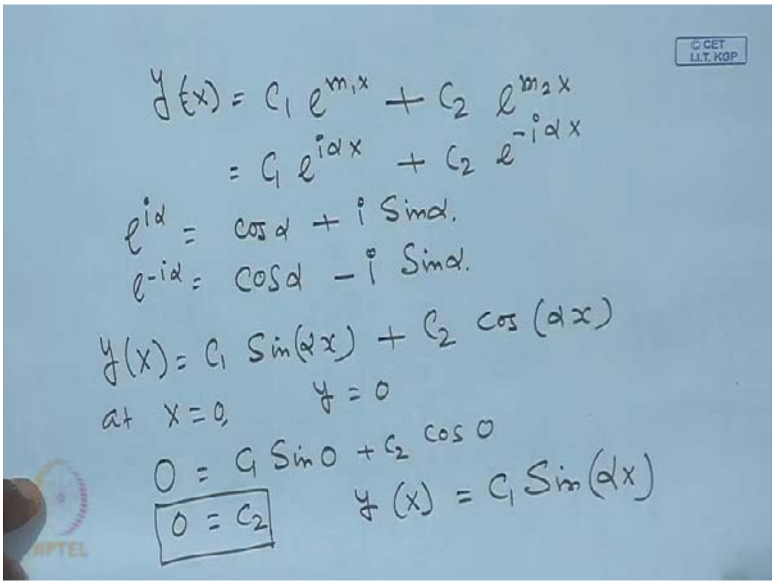

So that is the solution of this characteristic equation and the two roots are i alpha and minus i alpha. So, let us see, how the solution of the differential equation is now, takes the form. So, y is equal to a function of x; this is equal to C 1 e to the power m 1 x plus C 2 e to the power m 2 x, so, the form of the solution is C 1 e to the power i alpha x plus C 2 e to the power minus i alpha x; and if you know, you can write the then the Euler's equation; thus, e to the power i alpha is nothing but cosine alpha plus i sin alpha. Now, if you write, e to the minus i alpha is nothing but cosine alpha minus i sin alpha. If you open up these two quantities and write it down in this equation, finally, you will be getting the solution in the form of periodic functions; this cosine and sin functions are periodic functions and it will be combination of the sin function and the cosine function.

So, final form of the solution becomes C 1 sin alpha x plus C 2 cosine alpha x; these imaginary quantities i s, etcetera, will be consumed in the constants C 1 and C 2. Now, let us put the boundary conditions: at x is equal to 0, y is equal to 0. So, let us see at x is equal to 0, y equal to 0, so, put y is equal 0; then, C 1 sin 0 plus C 2 cosine 0; our sin 0 is always 0, cos 0 is 1; so, 0 is equal to… So, 0 multiply C 1 is 0, so, this be 1; so, C 2 is equal to 0; so this boundary condition gives me the solution as C 2 is equal to 0. So, what is the solution of y x? It is nothing but C 1 sin alpha x.

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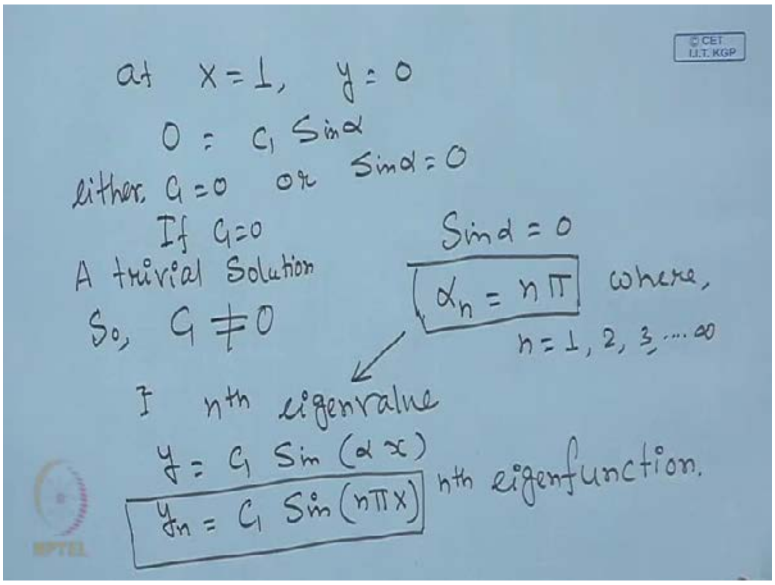

Now, let us put the other boundary condition and see what we get as the solution of this equation. Other boundary condition is: at x is equal to 1, y is equal to 0, so, if you put y is equal to 0 C 1 sin alpha. Now, there are two options of getting the solution; either C 1 is equal to 0 or sin alpha is equal to 0; if we get, if we use C 1 is equal to 0, then again, we are going to land up with a trivial solution.

So, if C 1 is equal to 0, we get a trivial solution; so, that is ruled out. So, we are not looking for a trivial solution; we are looking for a non-trivial solution. So, therefore, C 1 cannot be is equal to 0; so what is the option left? The option is left as sin alpha is equal to 0. If you look into the solution, generic solution of sin alpha is that the alpha in this solution of this equation is, alpha n is equal to n pi, where your n s, are varying from 1, 2, 3 up to infinity.

Now, n is equal to 0 is ruled out; because, if you put n is equal to 0, then alpha n is alpha will be equal to 0; if alpha is equal to 0, then again, we are going to get a trivial solution. So, for getting a non-trivial solution, the solution is alpha n is equal to n pi, where the index n runs from 1 to infinity. Now, for each alpha, for each value of n, the corresponding values of alpha are known as the eigen values. So, these are nth eigenvalue of the system and what is the corresponding eigen function? We denote the… So, what is, if you look into the solution, what is the solution?

The solution was C 1 sin alpha x; so, corresponding to nth eigen value, we will be getting the nth eigen function; we denote it as a y subscript n. So, this becomes C 1 sin alpha n; so, this becomes alpha n x; and, alpha n become n pi, so it becomes n pi x; so this is the nth eigen function. So, the corresponding eigen function is sin n pi x and corresponding eigen value is n pi.

So, next, what we will do? We just change the boundary condition from Dirichlet boundary conditions. So, what do we have? We have the boundary conditions - homogeneous boundary conditions: at x is equal to 0, y is equal to 0, and at x is equal to 1, y is equal to 0; these are the Dirichlet boundary conditions. Now, let us see, what is the form of eigen values and eigen functions in the case of a Neumann boundary condition.

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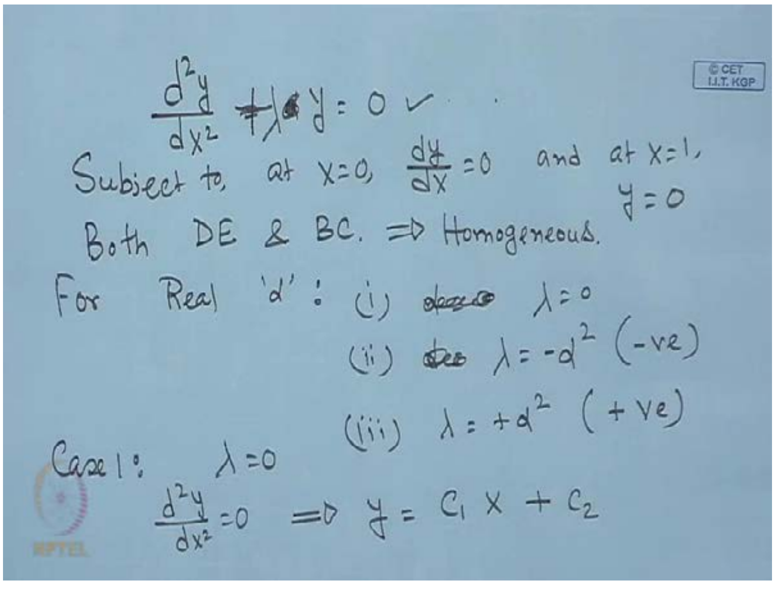

So, again, we look into the problem in a different way by changing the boundary condition. The form of the equation remains same, d square y dx square is equal to plus lambda y is equal to 0. Now, what do we do? We change the boundary condition; one boundary conditions from Dirichlet to Neumann, so, subject to, at x is equal to 0, we have a boundary condition dy dx is equal to 0; and, at x is equal to 1, we have y is equal to 0.

So, what is the difference between this problem and earlier problem? The governing equation remains the same, only the Dirichlet boundary condition at x is equal to 0 is replaced by a Neumann boundary condition dy dx is equal to 0; and at the other end, at x is equal to 1, the boundary condition remains the same; it is a Dirichlet boundary condition. So that is the difference between this problem and other problem, so it is a… But you please remember, notice that the ordinary differential equation and the boundary conditions, both are homogenous; only the nature of boundary condition has been changed at the surface or at the boundary x is equal to 0.

So, we note this down, both differential equation and boundary conditions, they are homogeneous; only the boundary condition has been changed. So, again there will be… We will solve this three cases completely, so, there will be three options for real alpha; for real values of alpha, there will be three cases; case 1: alpha is equal to 0; case 2: alpha is negative. So, let us say, that was lambda right, so, this one was lambda, so we put lambda is equal to 0; lambda is minus alpha square, so, lambda is negative; and the third option is lambda is equal to plus alpha square, so it is positive.

Now, we examine all these three cases one after another. So, case 1 will be lambda equal to 0, so, what we will be getting is that, d square y dx square is equal to 0. Now, we have already seen earlier, the solution of this equation is y is equal to C 1 x plus C 2. Next, we put the boundary conditions, utilize these two boundary conditions, and evaluate the constants C 1 and C 2 and see what we get.

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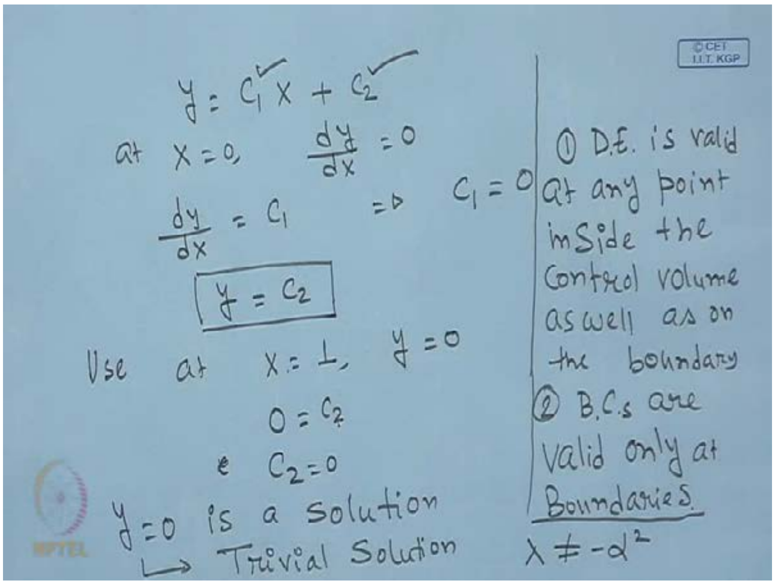

Now, if you really do that, what we will be getting is that solution is, C 1 x plus C 2 at x is equal to 0, d y dx is equal to 0. So, if we evaluate dy dx of this equation, what we get is dy dx is nothing but C 1; so, at x is equal to 0, dy dx is equal to 0; use that boundary condition, so, what you get is, C 1 is equal to 0. So, therefore, the solution is C 1 is equal to 0, so you will be getting y is equal to C 2; so, therefore, the solution is a constant solution. Now, use the other boundary condition, that is, at x is equal to 1, y is equal to 0; so, there is no variation of x with respect to y. Since y is constant, in order to satisfy this boundary condition, then you have y is equal to 0; C 2 is equal to C 2.

So, if you remember that whatever I have told in the last class, that differential equation must be valid throughout the whole boundary, as well as throughout the whole control volume, as well as it is valid only at the boundaries; but the boundary conditions are valid only at the boundaries. So, let us note down this point and use it; so, differential equation is valid at any point inside the control volume, as well as on the boundary; on the other hand, the boundary conditions are valid only at boundaries, they need not be valid within the control volume.

So, therefore, my differential equation must be satisfying every point inside the control volume, as well as, it must be satisfying the boundary condition. Now, if we… So, we have seen that solution of the differential equation is a constant and that constant is equal to C 2; but the differential equation must be equal to 0, at the boundary x is equal to 1. So, these two can go hand in hand, if and only if, we put y is equal to 0 is equal to C 2. So, therefore, C 2 becomes 0; and if you look into the solution, C 1 is 0, C 2 is 0, so you will be getting y is equal to 0 is a solution; and if you remember, this is a trivial solution.

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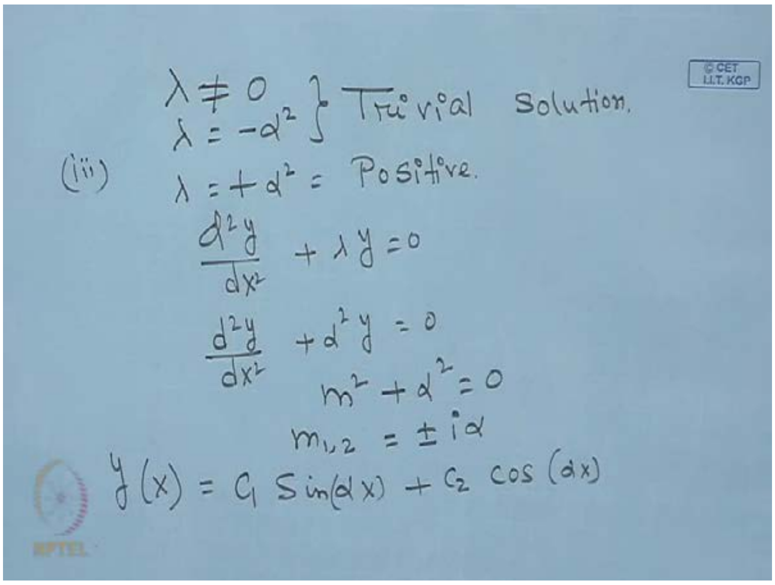

So, what is our conclusion? Our conclusion is that alpha where lambda cannot be negative; lambda cannot be minus alpha square, so that is ruled out. So, what we have done till now? We have ruled out, that lambda cannot be equal to 0; lambda cannot be is equal to minus alpha square, because both of these leads to only trivial solution; and we are looking into the non-trivial solutions. So, the third option is now left; the lambda is positive and it is equal to plus alpha square.

So, let us again look into the solution d square y dx square plus lambda y is equal to 0, so, put lambda is equal to alpha square; so, you will be getting d square y dx square plus alpha square y is equal to 0. So, again e to the power mx is the form of the solution and we have already solved this equation earlier; I am not going to solve this equation once again here. So, if you look into the characteristic equation, the characteristic equation will be in the form of m square plus alpha square is equal to 0, so you will be having two roots; m 1, 2 is nothing but plus minus i alpha.

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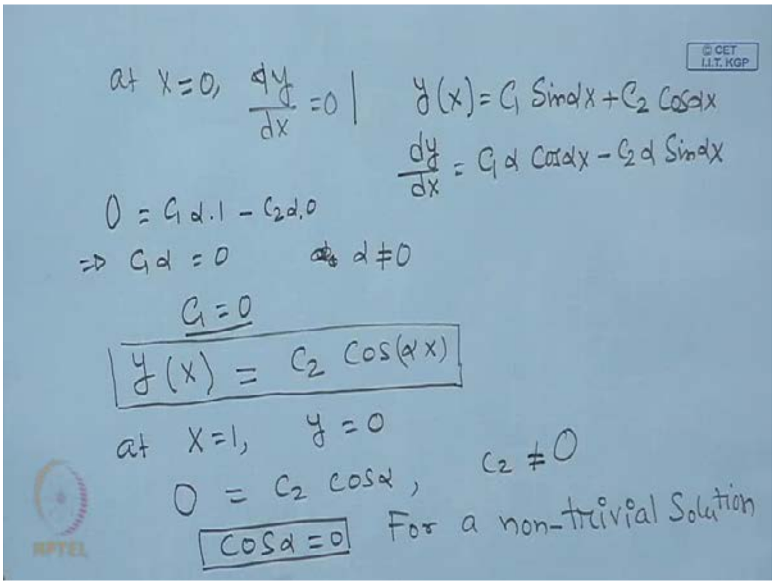

So, therefore, the solution will be composed of sine functions and cosine functions, periodic functions. So, y is equal y as a function of x becomes C 1 sin alpha x plus C 2 cosine alpha x; that is the form of the solution. And, let us put the boundary conditions and evaluate C 1 and C 2; the first boundary is: at x is equal to 0, dy dx is equal to 0. So, if we put this boundary condition and let us see what is our solution. y as a function of x is nothing but C 1 sin alpha x plus C 2 cosine alpha x. Now, we evaluate dy dx, we evaluate dy dx; dy dx becomes C 1 alpha cos alpha x minus C 2 alpha sin alpha x, remembering that differentiation of sin alpha x is nothing but alpha cosine alpha x, and differentiation of cosine alpha x nothing but minus alpha sin alpha x.

Now, putting x is equal to 0, dy dx is equal to 0, so what you will be getting is, 0 is equal to C 1 alpha cos 0 is one minus C 2 alpha 0. So, you will be getting C 1 times alpha is equal to 0; alpha is not equal to 0, because if alpha is equal to 0, you will be landing with a trivial solution again; and we are not looking for a trivial solution, that means, C 1 is equal to 0; if C 1 is equal to 0, let us see what we get as a solution. So, the solution we will be getting is, y as a function of x; since C 1 equal to 0, so you will be getting C 2 cosine alpha x. So, that is the form of the solution we are getting, and this solution is a non-trivial solution.

Now, we have to check; we have to evaluate the eigenvalues and eigen functions. So, we have one more condition that is left behind, that is, the boundary condition at x is equal to 1. So, at x is equal to 1, we have a Dirichlet boundary condition there, so we put y is equal to y x as 0, and this becomes C 2 cosine alpha; we put x is equal to alpha. Again, in this equation, C 2 cannot be equal to 0, why? If C 2 is equal to 0, then again, we are going back to a trivial solution and we are not looking for a trivial solution. So, what is the option left? The option left is cosine alpha is equal to 0 for a non-trivial solution.

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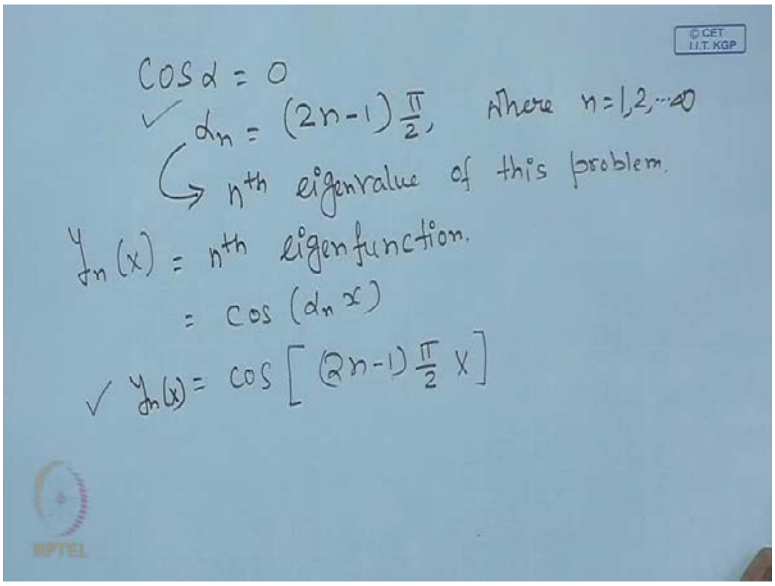

If cosine alpha is equal to 0, then let us look into what is the general solution for this equation. We have already studied the general solution of sin alpha equal to 0, cosine alpha is equal to 0, in our 10, plus 2 standard. So, the general form of this equation is alpha n is 2 n minus 1 pi by 2, where the index n runs from 1, 2 up to infinity. So, this is the general form of the solution; now, each of these values of alpha for which cosine alpha is equal to 0 is the eigenvalues for this particular problem. So, alpha n is equal to 2 n minus 1 pi by 2 is the nth eigenvalue of this problem; and the corresponding solution is the n th eigen function; so, y n as a function of x is nothing but… This is nth eigen function and this becomes cosine alpha n x. So, therefore, this becomes cosine 2 n minus 1 pi by 2 into x.

So, alpha n is equal to 2 n minus 1 pi by 2 is the nth eigenvalue and y n cosine 2 n minus 1 pi by 2 times x is the nth eigen function; so, this is the eigenvalue; this is the eigen function. So, if you look into the solution for this particular problem by changing the boundary condition from Dirichlet to Neumann, we land up with a different eigenvalue and eigen function combination. Next, what do we do? We will change the boundary condition to the third kind of boundary, which are most, which are quite common in chemical engineering applications, that is a Robin mixed boundary condition. So, we change the boundary condition, we keep one boundary condition - Dirichlet; and like this particular case, we change the boundary condition to the other boundary condition to Robin mixed boundary condition and see, what are the eigenvalues and eigen functions, we are getting in this case.

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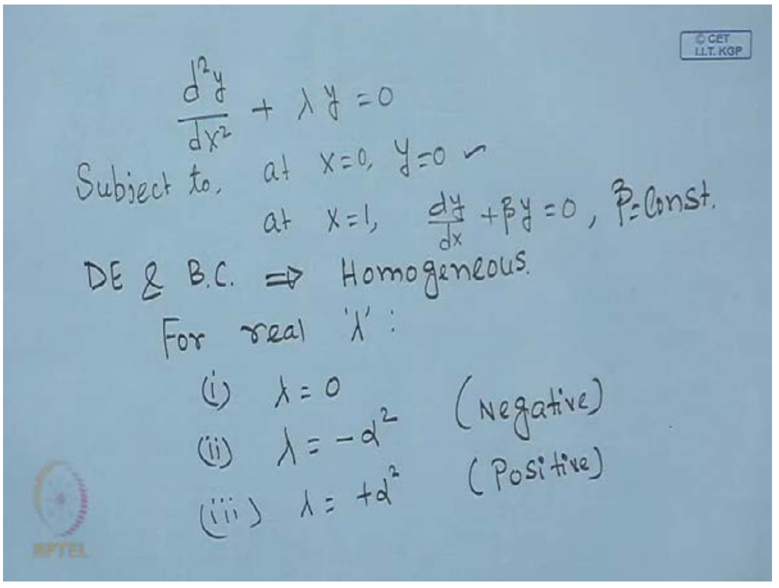

So, let us consider the same equation d square y dx square plus lambda y is equal to 0, but we change the boundary conditions in this case, subject to this is: at x is equal to 0, y is equal to 0. We keep the Dirichlet boundary condition here, as it is; we change the other boundary condition at x is equal to 1, we use a Neumann boundary we use a Robin mixed boundary condition, so, that will be dy dx plus beta y is equal to 0. Please check that, again we have a homogeneous system of equation, the differential equation and boundary conditions are all homogeneous.

What we simply did was, we keep this Dirichlet boundary condition as it is; we change the boundary condition to the other boundary condition, that is, an x equal to 1, we make it a Robin mixed boundary condition, where beta is a constant.

So, therefore, what we get is that we look into the again we this lambda can be for real values of lambda. It can be three things; lambda can be 0, lambda can be negative and third option is lambda is positive. Let us examine all the three cases in detail and see what kind of eigen values and what kind of eigen functions, we are getting in this particular case.

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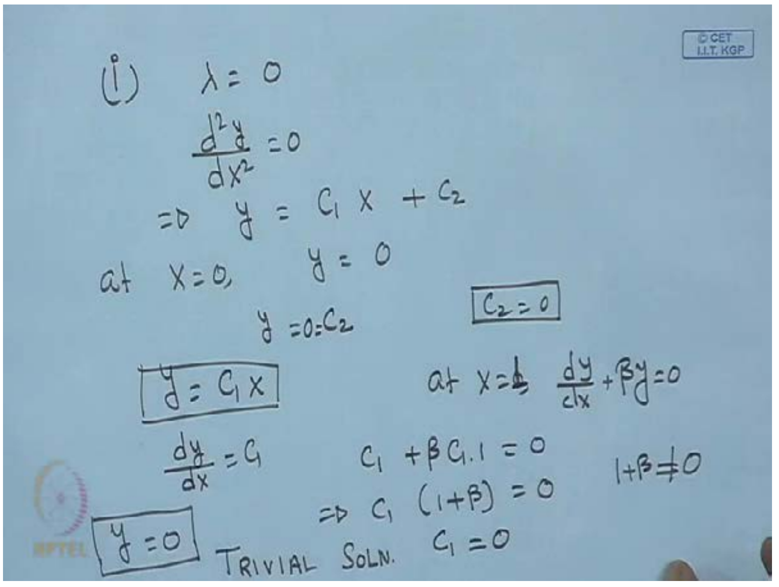

So, for lambda, case 1 is lambda is equal to 0 and again, the form of differential equation is d square y dx square is equal to 0 and we know the solution to this problem and the solution becomes y is equal to C 1 x plus C 2. Now, let us put the boundary conditions; the first boundary condition is: at x is equal to 0, y is equal to 0; if we put that you will be getting y is equal to C 1 into 0; so it will be 0. So, y is equal to 0 is equal to C 2; so C 2 will be equal to 0. So, what is the solution? The solution is, y is equal to C 1 x.

Now, if we use the other boundary condition that is at x is equal to 0, d y d x plus beta y is equal to 0. So, we put: at x is equal to 0, dy dx what? dy dx is C 1, and we put this here; so, C 1 at x is equal to 1 at x is equal to 1 dy dx plus beta y is equal to 0, that is the other boundary condition. So, boundary conditions as defined on two boundaries, at x is equal to 0 and x is equal to 1.

So, this becomes dy dx is C 1 plus beta times y; so, beta times y evaluated at x is equal to 1; so, this will be beta C 1 times 1 should be is equal to 0; so, C 1 into 1 plus beta is equal to 0. Now, beta is a constant; it is a positive constant, so therefore, 1 plus beta is always positive; so, 1 plus beta always positive, it cannot be equal to 0; so, beta can be positive or it can be negative; so, therefore, 1 plus beta is not equal to 0.

So, in order to satisfy this equation, C 1 must be equal to 0; so if C 1 is equal to 0, so what is the form of our solution? This is the solution we had; so, y is equal to C 1 is 0, so y is equal to 0. So, y is equal to 0, we are getting as a final solution for this case as well; so, this is giving a trivial solution. And, we are not looking for a trivial solution, so therefore, lambda cannot be equal to 0 in order to get a non-trivial solution. So, in order to get a non-trivial solution lambda is not equal to 0, so what is the next option? The next option is, lambda is negative and let us see what we get.

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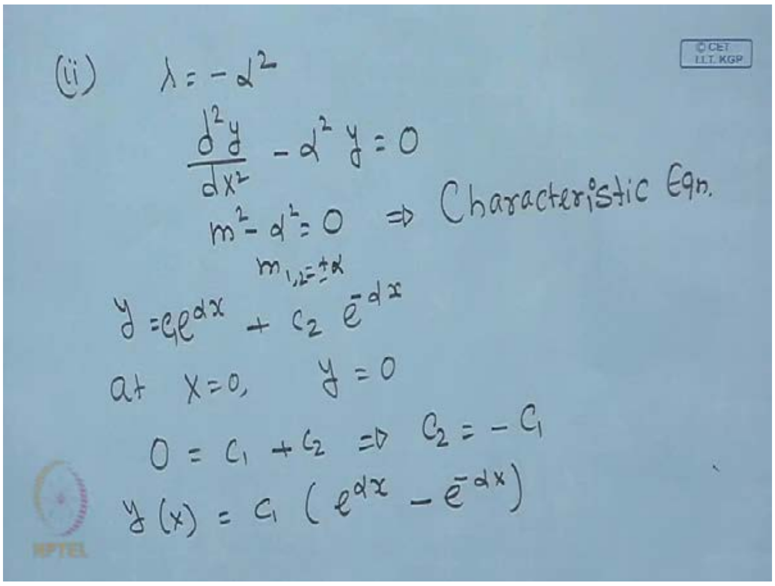

So, case 2: lambda is minus alpha square and it is negative; so, our differential equation now becomes d square y dx square minus alpha square y is equal to 0. If you remember, the form of the solution to this equation is e to the power mx, and m square is equal to minus alpha square is the characteristic equation. So, the solution will be composed of e to the power y is equal to e to the power alpha x plus C 2 e to the power minus alpha x.

Now, let us put the first boundary condition; the first boundary condition is: at x is equal to 0, you have y is equal to 0. So, therefore, this will be 0, C 1 plus C 2; so C 1 e to the power 0 is one and e to the power minus 0 is also 1, so it will be C 1 plus C 2. So, therefore, we will be getting C 2 is equal to minus C 1, so the form of the solution becomes y as a function of x, so, C 1 C 2 is equal to minus C 1; so, take C 1 common, e to the power alpha x minus e to the power minus alpha x.

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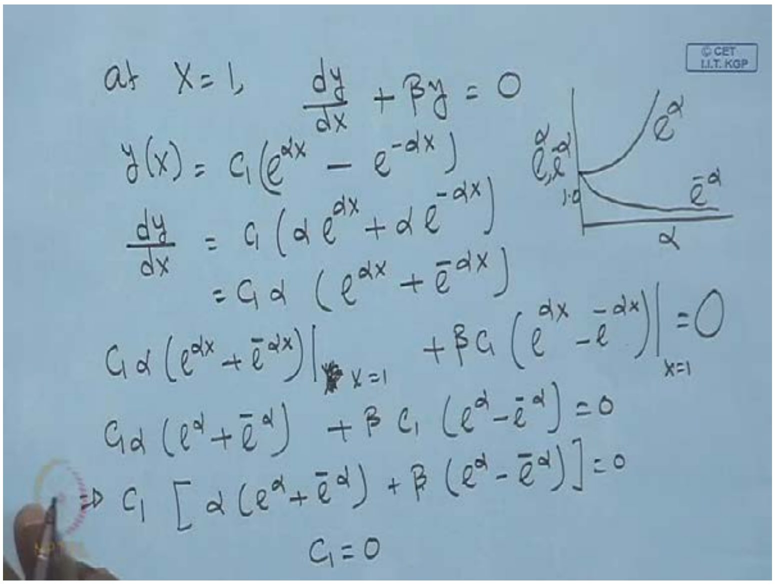

So, next what we do? We utilize the other boundary condition; the other boundary condition is at x is equal to 1, we have dy dx plus beta y is equal to 0; so let us evaluate. So, if you look into the solution, the solution is y as a function of x is C 1 e to the power alpha x minus e to the power minus alpha x; so we evaluate dy dx and see what we get. dy dx is C 1 alpha e to the power alpha x minus minus plus alpha e to the power minus alpha x, so, we can take alpha common; so C 1alpha e to the power alpha x plus e to the power minus alpha x.

Now, we put the boundary condition, at x is equal to 1; so dy dx will be evaluated, at x is equal to 1 e to the power alpha x plus e to the power minus alpha x; the whole term is evaluated at x is equal to 1 plus beta times y, y is C 1 e to the power alpha x minus e to the power minus alpha x, whole term evaluated at x is equal to 1 should be is equal to 0.

So, we evaluate that; so this becomes C 1alpha e to the power alpha plus e to the power minus alpha plus beta times C 1 e to the power alpha minus e to the power minus alpha is equal to 0. So, we can take C 1common; so this becomes alpha e to the power alpha plus e to the power minus alpha plus beta e to the power alpha minus e to the power minus alpha is equal to 0. Now, as we have argued earlier, that we have looked into the variation of e to the power alpha and e to the power minus alpha with respect to alpha; this is the variation of e to the power alpha, and that is the variation of e to the power minus alpha. So, therefore, e to the power alpha is always positive having a minimum value 1; e to the power minus alpha is always positive with a maximum value as 1.

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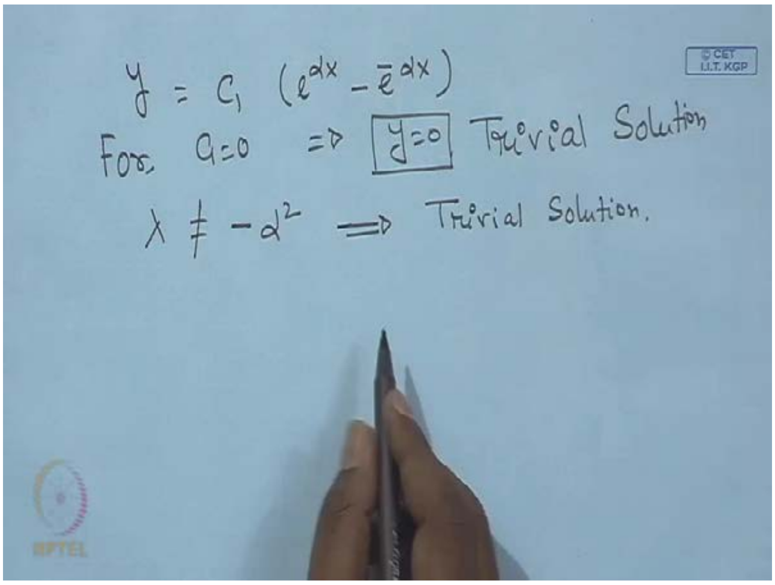

So, their combination is positive again, their combination is positive; beta is not equal to 0; the whole thing in the third bracket is a positive quantity that means so in order to satisfy this equation, we have C 1 is equal to 0. And let us see, if C 1is equal to 0 and if we look into the solution, the solution was y is equal to C 1 e to the power alpha x minus e to the power minus alpha x. For C 1is equal 0 of course, you will be getting y is equal to 0, which is nothing but a trivial solution.

So, therefore, we are going to get a trivial solution in this case as well. So, our conclusion is, our lambda cannot be a negative quantity, because it gives a trivial solution and we are looking for a non-trivial solution. So, I stop here, at the in this class at this point; I will take this point on in the next class and we will be completing this problem in the next class and then we will take a stop of what are the forms of eigenvalues and eigen functions if we change the boundary conditions slightly.

Thank you very much.