Advanced Mathematical Techniques in Chemical Engineering

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Lecture No. # 18

Partial Differential Equations(Contd.)

Very good afternoon everyone. So, we were looking into the various classifications, definitions, properties, characteristics of partial differential equations, in the last class. And at the end of the last class, we defined a linear operator and we have looked into the property of the linear operator that if the operator is linear, we can use the principle of linear superposition.

We have demonstrated one example in the case of ordinary differential equation to use of to explain the use and application of principle of linear superposition. In that case, if you remember whatever we have done, we took up a problem of second order, which will be having, where they will be having to you know boundary conditions to solve the problems, and both the boundary conditions were non-homogeneous and the Governing Equation was also non-homogeneous. So, there were three sources of non-homogeneity in the problem. So, we divided the problem into three sub problems considering one non-homogeneity at a time and forcing the other two non-homogeneities to vanish.

Then, we took up each and every each and individual problem. So, we took up each of the three problems; each of these three problems will be containing only one non-homogeneity at a time now. Now, we solved these three problems separately and added the solution up in order to get the overall solution. So, that simplifies the whole problem.

So, you it may not be apparent to you, what is the benefit of this method of principle of linear superposition by solving the ordinary differential equation, but it will be quite apparent to, the advantage will be quite apparent for the in the case of solution of partial differential equation. In case of ordinary differential equation, it was taken up as for demonstration purpose; for the case of partial differential equation, the advantage will be quite apparent.

So, in this class, we will be taking up an example to demonstrate the principle of linear superposition in case of partial solution of the partial differential equation. And how that will help in overall solution? We will just look into that.

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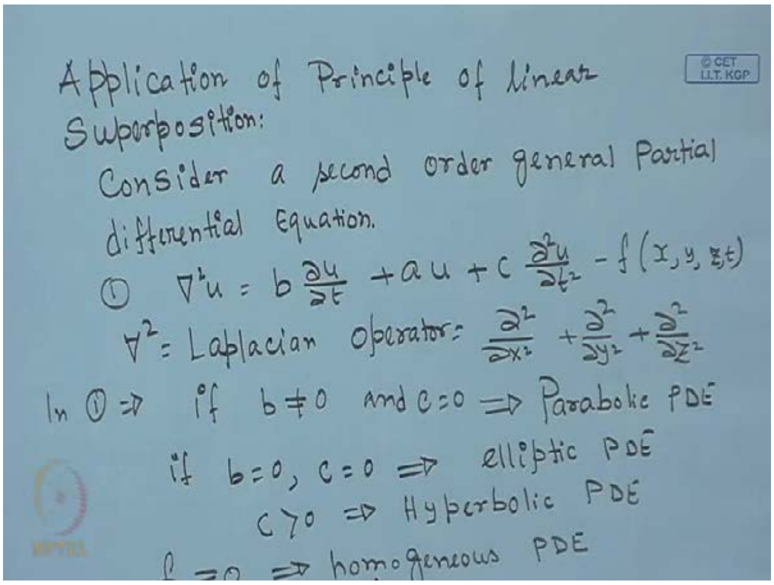

So, we look into the application of principle of linear superposition. Consider a second order general partial differential equation.

Let us say, grad square u is equal to b del u del t plus a u plus c del square u del t square minus f, say function of x, y, z, and t in general. So, we are considering a 3-dimensional, a 4-dimensional problem where, x, y, z, at the 3-dimensional mean special dimensions, and t is the time dimension; so, it is a 4-dimensional problem; grad square is a generalized Laplacian operator. So, this will be del square del x square plus del square del y square plus del square del z square.

So, in this equation, let us say this is equation number 1. In equation number 1, what we have? We have, if b is not equal to 0 and c is equal to 0, will be getting a Parabolic PDE. If b is equal to 0 and c is equal to 0, then we will be getting an Elliptic partial differential equation. For c greater than 0, we will be getting Hyperbolic partial differential equation. If f is equal to 0, we will be getting homogeneous partial differential equation.

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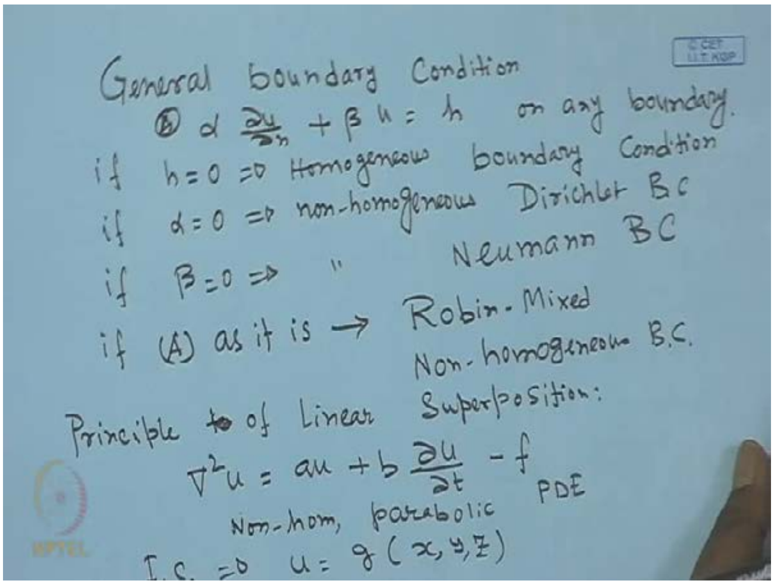

Now, general boundary condition will be, it will be alpha del u by del n; n is basically the normal derivation of any surface; beta u is equal to h. So, that is a general boundary condition on any boundary. If h is equal to 0, we will be getting a homogeneous boundary condition. If alpha is equal to 0, we will be getting a non-homogeneous Dirichlet boundary condition. If beta equal to 0, then you will be getting non-homogeneous Neumann boundary condition. If none of them will be, if the equation A as it is, then that is equation A is a Robin-Mixed, but non-homogeneous boundary condition.

Now, we like to use principle of linear superposition to simplify this partial differential equation. So, what we will do? Let us write down: grad square u is equal to au plus b del u by del t minus f is the non-homogeneous equation. Let us say this is non-homogeneous parabolic partial differential equation. So, u will be initial condition.

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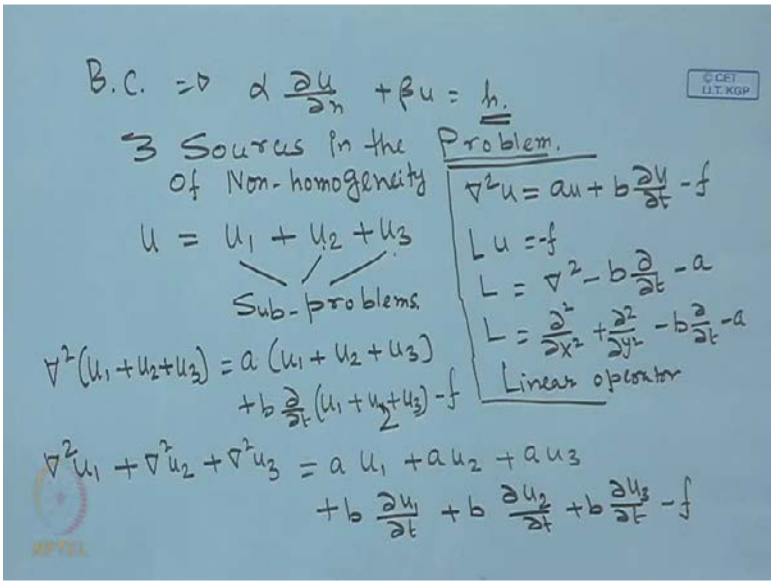

So, let us consider, this one initial condition will be let us say u is some generalized, some function of x, y, z; it is a function of space, and the boundary condition is given as alpha del u by del n plus beta u is equal to h; n is the normal derivation of the boundary, whatever we are talking about. So, it is basically normal derivative with respect to the boundary.

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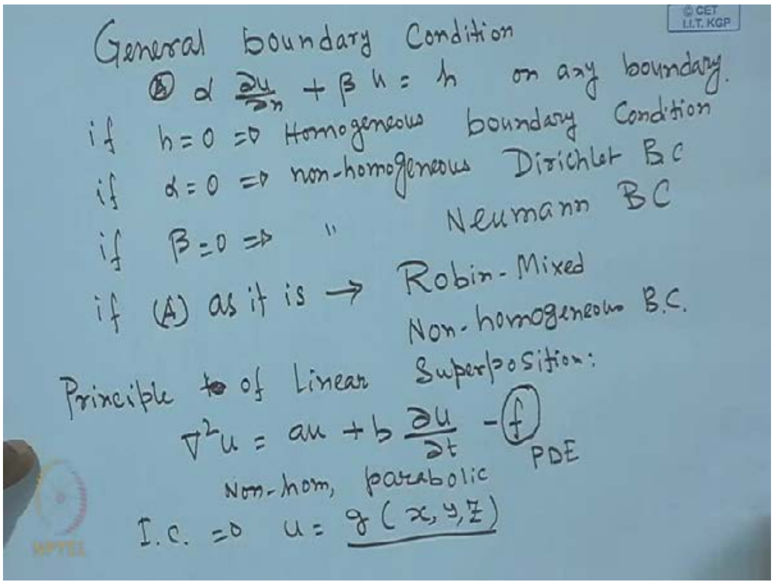

So, let us identify what are the sources of non-homogeneity to this equation. There are 3 sources of non-homogeneity to this equation. This is source number one; so, this homogeneity is appearing in the main Governing Equation. Then, in the initial condition, there is another non-homogeneity is present, and in the boundary condition there is another non-homogeneity is present. So, there are three sources of non-homogeneity in the problem.

So, whenever a differential problem is defined, including the Governing Equation and the boundary conditions, whenever we are talking about a Governing Equation, it is valid throughout the whole volume of the control volume, and whenever we are talking about the boundary conditions, they will be valid only on the boundary. Therefore, the solution must the solution of the differential equation must satisfy the Governing Equation as well as the boundary conditions. On the other hand, the boundary conditions need not to be the solutions. Boundary conditions are valid only on the boundaries, but the Governing Equation is valid throughout the whole control volume of the system.

Now, since there are three sources of non-homogeneity in this problem, we divide this problem into three parts. So, we divide this problem into 3 solutions: u is equal to u 1 plus u 2 plus u 3. Since the operator we are talking about is… what is the operator in this problem?

The operator will be so del square u. So, we will take a diversion here and talk about the operator once again, del square u is equal to a u plus b del u by del t minus f. So, the so it can be cast in the form L u is equal to minus f. So, L is the operator and this operator is grad square minus b del by del t minus a.

So, the operator is nothing but del square del x square plus del square del y square minus b del by del t minus a. If it is 3-dimensional space then one more del square by del z square will be appearing here. So, since this operator is a linear operator, we can we can simply linearly superpose all the 3 solutions and get the complete solution.

Now, what will be the solution for, what will be the governing equations for these particular sub-problems? So, divide the problems into three sub-problems; then u 1, u 2, and u 3, each are called sub-problems. So, we define each such sub-problem such that we will be considering only one non-homogeneity at a time.

Now, how this sub-problems will be form the Governing Equations of the sub-problems will be formulated since this operator is a this operator is a linear operator. What will we be doing is that we will be substituting u is equal to u 1 plus u 2 plus u 3 in the governing equation. So, if you do that we will be getting grad square u 1 plus u 2 plus u 3 is equal to a u 1 plus u 2 plus u 3 plus b del by del t of u 1 plus u 2 plus u 3 minus f.

So, if since the individual parts of this are linear; so it will be getting grad square; u 1 plus grad square u 2 plus grad square u 3 is equal to a u 1 plus a u 2 plus a u 3 plus b del u 1 by del t plus b del u 2 by del t plus b del u 3 by del t minus f.

Now, what we will be doing? We just Since this operator is a linear operator, we just consider the u 1 containing part equal, u 2 containing part equal, and u 3 containing part equal. We separate them out and while separating, we put f into one f. We associate the non-homogeneous term f along with the any of the sub-problems u 1, u 2, u 3. Let us say, we associate f with u 3; we could have associated f with u 1 as well.

So, if we compare the u 1 containing part, u 2 containing part, and u 3 containing part from this equation, and make the separate sub-problems from them, what we will be getting? We will be getting the Governing Equation of individual sub-problems.

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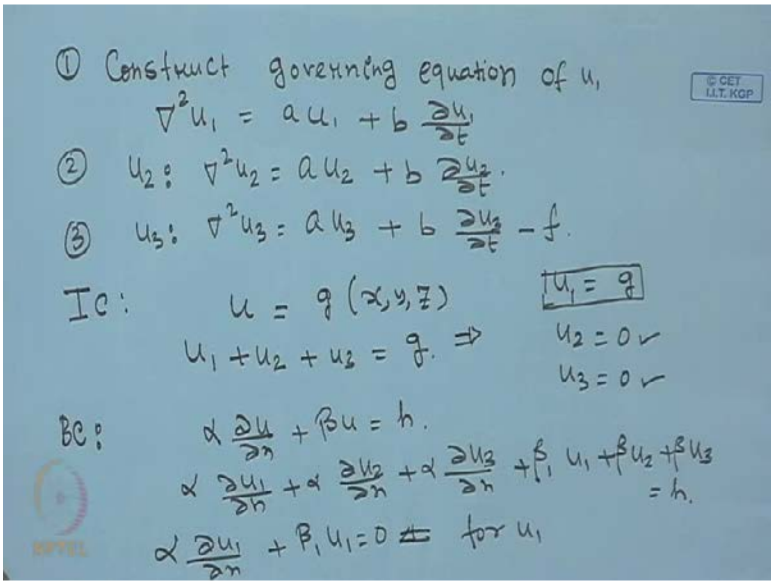

So, let us first construct the Governing Equation of u 1; take out the u 1 containing part; so, grad square u 1 is equal to a u 1 plus b del u 1 by del t.

Similarly, the Governing Equation of u 2 will be grad square u 2 is equal to a u 2 plus b del u 2 by del t, and u 3 will be nothing but, grad square u 3 is equal to a u 3 plus b del u 3 by del t minus f.

So, we associate the non-homogeneous term f with the third sub problem. If you add this 3 up linearly, then you will be you will be getting the original problem, the Governing Equation of the original problem u.

So, once we get that, now let us look into the boundary conditions as well. So, if you look into the initial condition of the original problem, the initial condition of the original problem was u is equal to g x y z. Therefore, you just write - u is equal to u 1 plus u 2 plus u 3 is equal to g. So, therefore, the initial condition can be, for each of them can be written down as u is equal to g for the u 1 is equal to g for the first sub-problem, u 2 is equal to 0, and u 3 is equal to 0.

So, this is the initial condition of the first sub-problem; this is the initial condition of the second sub-problem; this is the initial condition of the third sub-problem (Refer Slide Time: 16:50).

If you look into the boundary condition, the boundary condition of the original problem was alpha del u by del n plus beta u is equal to h.

So, if you put u 1 u 2 u 3 in in the place of u, then you will be getting del u 1by del n plus alpha del u 2 by del n plus alpha del u 3 by del n plus beta u 1 plus u 2 plus beta u 3 is equal to h. Again, we take up the u 1 containing part, u 2 containing part; collect the u 1 containing , u 2 containing, u 3 containing part separately, and associate h with the appropriate problem and we will be getting the boundary condition of each sub problem. So, if we take alpha del u 1 by del n plus is equal plus beta 1 u 1 is equal to 0. So, that is the boundary condition for u 1.

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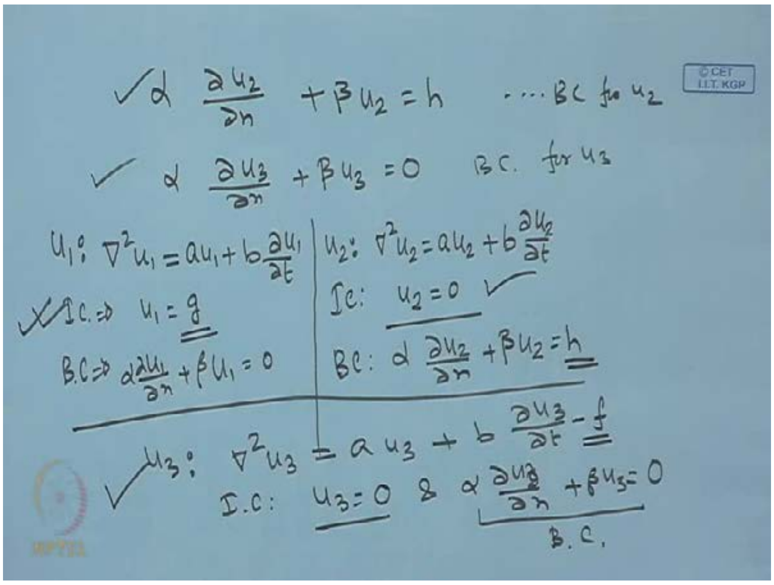

Then, similarly, what we will be getting? alpha del u 2 by del n plus beta u 2 is equal to h; that is the boundary condition for u 2. So, here, you just note, we associated the non-homogeneous part h in the boundary condition with the solution, the with the sub-problem u 2. Because we want to have, each sub problem will be containing only one homogeneity at a time.

So, we could have associated h with u 1 or with u 3, but u 1 is already having one non-homogeneity; that is in the initial condition. So, we did not associate the non-homogeneous term of h in the along with u 1. On the other hand, in case of u 3, if you remember, u 3 contains we associated the non-homogeneous term with Governing Equation in u 3. Therefore, u 3 is already containing one non-homogeneous term.

So, we intentionally did not associate the boundary condition, non-homogeneity of the boundary condition with u 3 simply because u 3 must be having only one non-homogeneous term, either in the Governing Equation or in the boundary. So, the only choice left is association of the non-homogeneous term in the boundary condition with u 2 so that u 2 will be having only one non-homogeneity in the form of this boundary.

So, alpha del u 3 by del n plus beta u 3 is equal to 0 is the boundary condition for u 3. So, if we look into the three sub-problems now, u 1 is having a Governing Equation - grad square u 1 is equal to a u 1 plus b del u 1 by del t; the initial condition is u 1 is equal to g; boundary condition is u 1 alpha del u 1 y del n plus beta u 1 is equal to 0; for this is for u

1. For u 2 - grad square u 2 is equal to a u 2 plus b del u 2 by del t and initial condition is u 2 is equal to 0, and boundary condition is alpha del u 2 by del n plus beta u 2 is equal to h. This is the one and we will be writing down for u 3. u 3's Governing Equation is grad square u 3 is equal to a u 3 plus b del u 3 by del t and the initial condition is u 3 is equal to 0 minus f was there. u 3 equal to 0 and alpha del u 3 by del n plus beta u 3 is equal to 0; that comes from this (Refer Slide Time: 21:31). So, this is the boundary condition for third sub-problem.

Now, if you closely examine each of this sub problem, that you will be having only one non-homogeneity in this formulation. You will be having only one non-homogeneity in this equation. In this problem, each of these sub problems will be having only one non-homogeneity in them.

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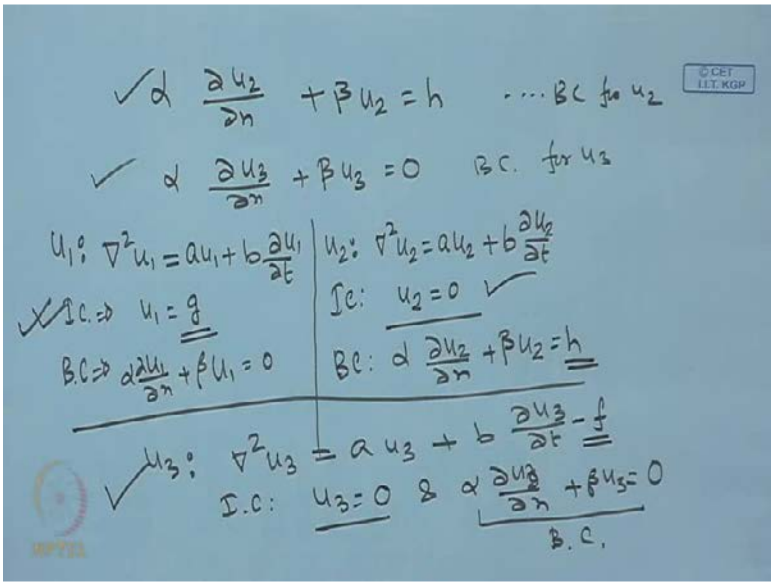

In this case first case u 1 g is the non-homogeneous term; in the second case h is the non-homogeneous term; in the third case, Governing Equation, f is the non-homogeneous term,

So, therefore, we will be dividing the problem into three sub-problems considering one non-homogeneity at a time. And probably, now it will be it must be clear to you how to reconstitute or reformulate each of the sub-problems having one non-homogeneity at a time. So, you should have a judicious selection of association of the non-homogeneous term in the boundary conditions as well as in the Governing Equation to the appropriate such sub-problem such that, that particular sub-problem under concerned will be containing only one non-homogeneity at a time.

Now, most probably, we will be the most common method of solving the second order of partial differential equations are is a using of use of separation of variables. In case of separation of variables, if you remember that we divide the solution into the two sub-parts and the entire solution is constructed by multiplication of the two parts, and we consider one part is entirely function of a particular variable, another part is a sole function of the particular variable, then, we multiply these two and construct the all the solutions and add them up simply and we will be getting the complete solution.

But in this case, if you remember that if you have the initial condition, so you will be having 3 conditions in general, 2 conditions in space ,1 condition in time; that is known as the initial condition. When you formulate the sub-problem, then these two conditions in the space will be used up to get the solution of one of the sub-problems. One of the let us say space varying time, a space varying solution, in the time varying solution, you will be getting only one condition that is the initial condition when you construct complete solution. So, one constant will be evaluated by putting the using the initial conditions. So, you will be utilizing all the 3 conditions.

Now, if the initial condition becomes 0, then the whole problem, whole purpose - the formulation of the partial differential equation becomes an ill posed problem. So, that cannot be entertained because if whenever you will be evaluating a the solution of the partial differential equation, the final constant will be evaluated from the initial condition and final constant will be multiplied to with the rest of the solution.

So, if you put the initial condition as 0, then the whole solution becomes, the whole problem becomes ill posed problem, ill-defined problem. So, we have to make the problem well posed.

Now, if you look into these three sub-problems, whatever you have constructed in 2 cases, the sub-problem u 2 and sub-problem u 3, we have the initial condition 0. So, sub-problem definition of u 2 and u 3 are not well posed; on the other hand, the sub-problem u 1 is well posed because here it has homogeneous boundary condition, and non-homogeneous or non-zero initial condition.

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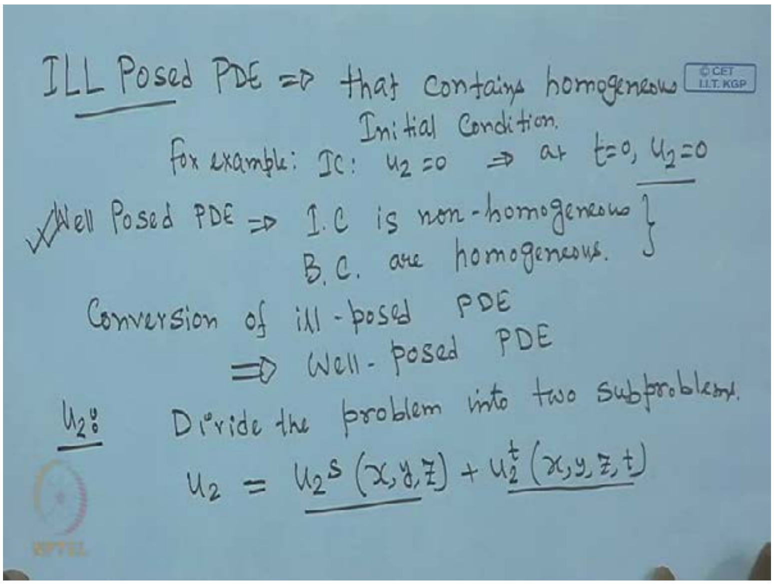

So, let us note down whatever we are trying to say, that ill posed partial differential equation is that PDE, that contains homogeneous initial condition.

For example, in the sub-problem, the initial condition u 2 is equal to 0; that means in a particular problem, at time t is equal to 0, u 2 is equal to 0. This is ill posed partial differential equation. And which one is well posed? Well posed PDE is the one where the initial condition is non-homogeneous, but boundary conditions are homogeneous.

So, in this case also, if you look into so boundary condition, so initial non-homogeneous initial condition and homogeneous boundary conditions are well posed partial differential equation. Now you cannot So, this solution is direct. On the other hand, for the ill posed problem, one has to convert the ill posed problem into well posed problem.

So, conversion of ill posed PDE to well posed PDE is very important. Now, how this conversion is done?

So, let us let us consider the problem number u 2; both problem u 2 and u 3 in the current example they are ill posed because they are homogeneous initial condition or 0 initial condition.

So, what we have to do? We have to again divide the problem into two sub-problems. So, u 2; so, one part will be time dependent and another part will be time independent. So, that will be corresponding to the steady state solution.

So, u 2 can be broken down into 2 parts; u 2 steady state which will be a function of space only and it will be having a transient part or time varying part which will be function of space as well as time.

Now, what we will be doing? Then, we will be again formulating the Governing Equation and the boundary conditions of the steady state part and the transient part. For that, we take the take recourse to the same method, as we have considered earlier.

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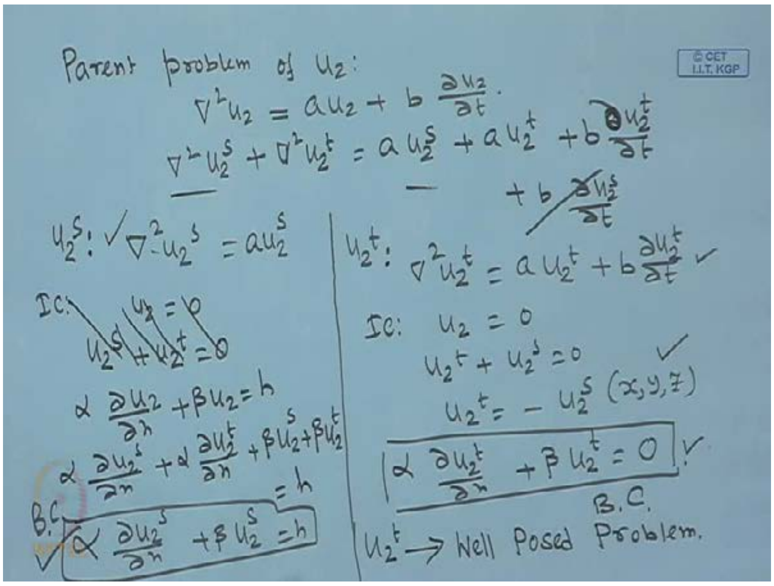

So, let us look into the what is a parent problem for u 2; the mother problem for u 2 is grad square u 2 is equal to a u 2 plus b del u 2 by del t.

So, we write down the u 2 is equal to u 2 s plus u 2 t. So, grad square u 2 s plus grad square u 2 t is equal to a u 2 s plus a u 2 t plus b del u 2 t by del t, but when you write u 2 s, you write del u 2 s by del t but u 2 s is a sole function of space. It is a function of x, y, and z. It is a steady solution. So, u 2 s is not a function of x, y, z. Therefore, the partial derivative with respect to time will be equal to 0.

So, this does not exist. So, again, we take out the we collect the similar terms and formulate the Governing Equation of u 2 s. So, what is u 2 s? The Governing Equation of u 2 s is nothing but grad square u 2 s is equal to a u 2 s. And what is the Governing Equation of u 2 t? u 2 t will be - just collect the similar terms grad square u 2 t is equal to a u 2 t plus b del u 2 t by del t.

Now, we have to formulate the initial and the boundary conditions for this problem. Let us look into the initial condition of the original problem of u 2. The original problem was u 2 is equal to 0; so, we have u 2 s plus u 2 t is equal to 0.

So, therefore, we do not need an initial condition of on u 2 s because u 2 s is a function of space only; it is not a function of time. Therefore, we need not to have initial condition on u 2 s. Therefore, the initial condition is not appropriate for u 2 s. So, we write it down it here. I think that will be better because we write need to have one initial condition on u 2 t. So, u 2 is equal to 0; so, you have u 2 t plus u 2 s is equal to 0; so, u 2 t is nothing but minus u 2 s, which will be a function of x, y, z.

So, initial condition of the second of the transient problem is nothing but the solution of the steady state part and a minus sign with it. So, we just cut it down here. Now, we get the boundary condition; the boundary condition was for u 2 was alpha del u 2 by del n plus beta u 2 is equal to h. So, alpha del u 2 s by del n plus alpha del u 2 t by del n plus beta u 2 s plus beta u 2 t is equal to h.

Again, collect the similar terms. So, u 2 containing term, you will be getting - alpha del u 2 t by del n plus beta u 2 t is equal to 0, and we associate the non-homogeneous term with the steady state solution. So, alpha del u 2 s by del n plus beta u 2 s is equal to h. So, therefore, this is the boundary condition for the steady state part and this is the boundary condition for the transient part (Refer Slide Time: 32:58).

Now, you just see the steady state part requires only the boundary conditions. Therefore, with the help of this boundary, steady state part does not require any initial condition because it is time independent. Therefore, with the help of this boundary condition, one will be able to solve the steady state part completely. So, this Governing Equation of the steady state part along with this boundary condition is a complete solution, will give will lead to a complete solution of u 2 s as a function of space x, y, and z.

Once that is that is known, then we come back to the transient part. In the transient part, this is the transient equation (Refer Slide Time: 33:36), and initial condition u 2 was u 2 t is nothing but minus of steady state solution.

We have already known the steady state solution. From here, that will be used as the initial condition of the transient problem, and if you look into the boundary condition, the boundary condition of the transient part is completely homogeneous.

So, we are having a non-zero,, non-homogeneous initial condition homogeneous boundary condition for the problem u 2, and u 2 t, is now has become a well posed problem. So, we have converted u 2 t as a well posed problem from an ill posed problem. So, u 2 t now becomes a very well posed problem. Similarly, we can look into the into the derivation for u 3 t.

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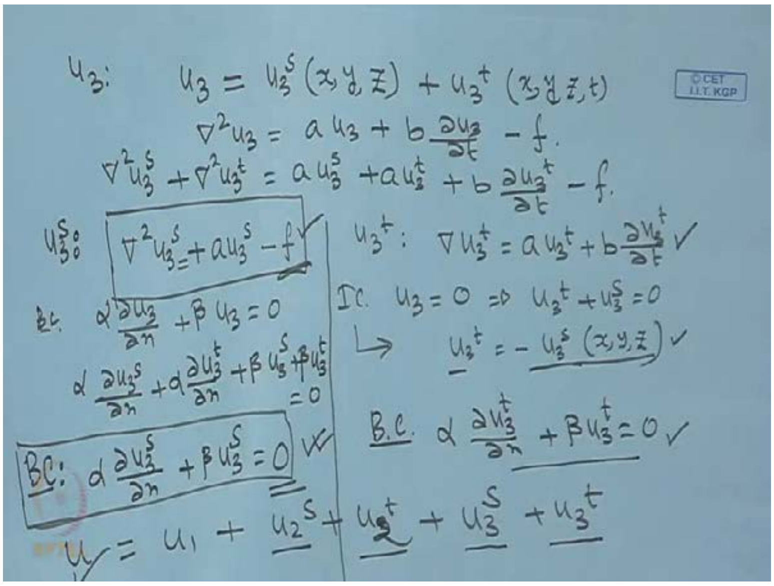

So, let us look into u 3. We have already said it is ill posed problem. So, it has any 0 initial condition or homogeneous initial condition.

So, therefore, we make it into you break into two sub-problems once again. u 3 is a function; u 3 s - that is a function of space x, y, and z; that is a steady state part plus u 3 t; that will be function of x, y, z, and t. Now, we should get the Governing Equation of u 3 s and u 3 t, and the relevant boundary conditions, and initial condition.

So, if you look into the mother problem of u 3 grad square, u 3 is equal to a u 3 plus b del u 3 by del t minus f. So, we just put u 3 is equal to u 3 s plus u 3 t. So, this becomes grad square u 3 s plus grad square u 3 t is equal to a u 3 s plus a u 3 t plus b del u 3 t by del t and del u 3 s by del t will be equal to 0 because u 3 s is a sole function of x, y, z. It is independent of t minus f.

Now, we compute the we separate the steady state part and the transient part, and formulate the Governing Equation of u 3 s and u 3 t. So, what is the Governing Equation of u 3 s? We collect the similar terms. So, grad square u 3 s plus a u 3 s minus f. So, we intentionally attach the non-homogeneous part with the space varying part or the steady state part because that will be easier to solve. That will make my the partial differential equation in terms of t and space, homogeneous.

And what is u 3 t? u 3 t will be - grad square u 3 t is equal to a u 3 t plus b del u 3 t by del t. And if you look into the initial condition, initial condition was u 3 and the original problem was u 3 equal to 0; therefore, u 3 t plus u 3 s is equal to 0; therefore, initial condition for u 3 t is nothing but minus of u 3 s which is a solution, which is a function of x, y, z, and this is the solution of the steady state part.

So, you solve the steady state part completely, and the solution of the steady state part has become the initial condition of the transient part. And now, look into the boundary condition. The boundary condition was alpha del u 3 by del n plus beta u 3 is equal to 0.

You put u 3 is equal to u 3 s plus u 3 t. So, alpha del u 3 s by del n plus alpha del u 3 t by del n plus beta u 3 s plus beta u 3 t is equal to 0. Again, we collect the similar terms. So, this boundary condition for the steady state part will be alpha del u 3 s by del n plus beta u 3 s is equal to 0, and here, the boundary condition for u 3 will be alpha del u 3 t by del n plus beta u 3 t is equal to 0.

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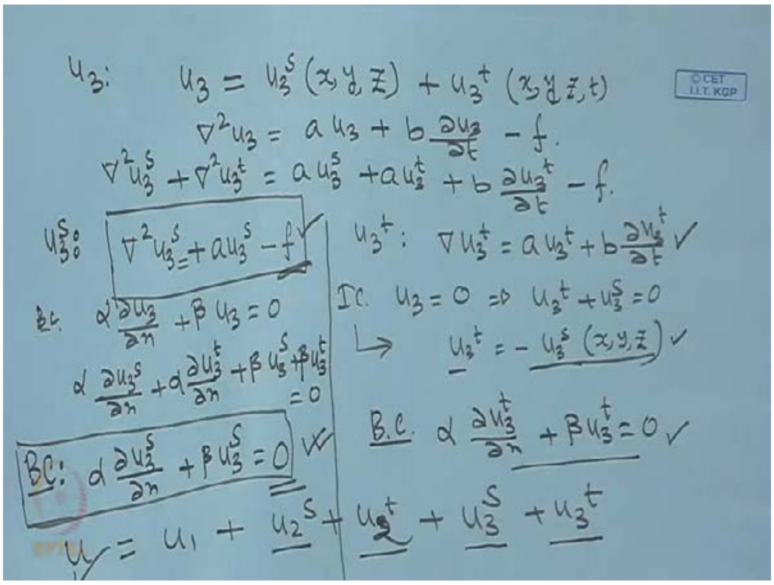

So, if you look into this problem, this is the Governing Equation (Refer Slide Time: 38:48) of the steady state problem along with this boundary condition. You do not need any initial condition to solve the steady state part because steady state part is time independent. With the help of these boundary conditions, this steady state equation can be completely solved and we will be getting u 3 s as a function of x, y, and z.

So, therefore, that solution of the steady state part will be the solution of the will be the initial condition of the transient part. So, we will be knowing it; that will be known to us with a negative sign of course, and if you look into the boundary condition, the boundary condition has become transient, but becomes homogeneous.

So, therefore, if you look into the formulation of the sub-problems, one common thing is clear to you that each of these sub-problems will be having only non-homogeneity at a time. If you look into u 3 t, the boundary Governing Equation is homogeneous; the boundary condition is homogeneous; the initial condition is non-homogeneous.

So, out of these - in the 3 sources of the equation, only one non-homogeneity is occurring here. In the case of u 3 s, the Governing Equation is non-homogeneous because of the presence of f, the boundary condition is homogeneous. So, there is only one source of non-homogeneity here.

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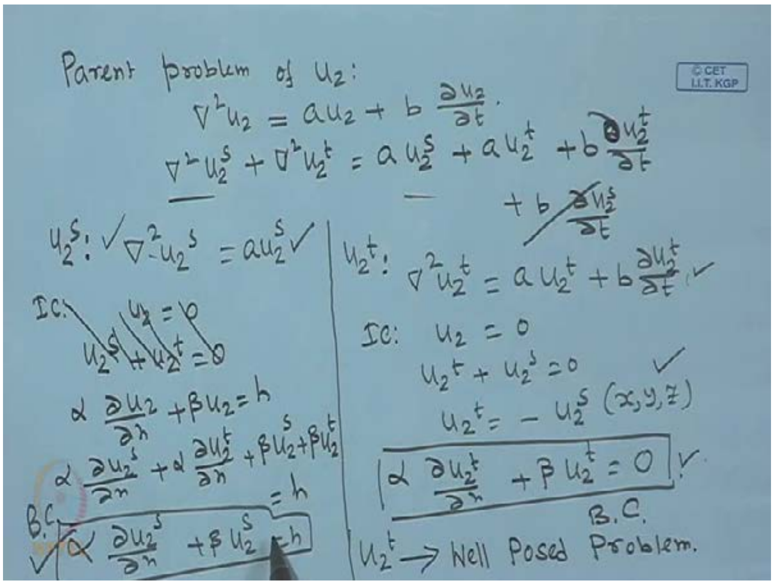

Similarly, if you look into the other problems, other sub-problems, u 2 t is having a homogeneous Governing Equation, a non-homogeneous initial condition, and a homogeneous boundary condition. In case of u 2 s, it has homogeneous Governing Equation, it has a non-homogeneous boundary condition. So, only one source of non-homogeneity is present there, and in case of u 1, it is well posed problem. Therefore, in this case, the Governing Equation was non-homogeneous; the initial condition the Governing Equation was homogeneous; the initial condition was non-homogeneous and the boundary condition was homogeneous.

So, we will be dividing the problem into sub-problems considering only one non-homogeneity at a time, and if your initial condition of a transient problem becomes homogeneous, then you are in trouble; you have to break down the problem into two more sub-problems such that each sub-problem will be containing only one non-homogeneity. So, while you are formulating the Governing Equation for the sub-problems, ensure that association of the non-homogeneous term should be done appropriately.

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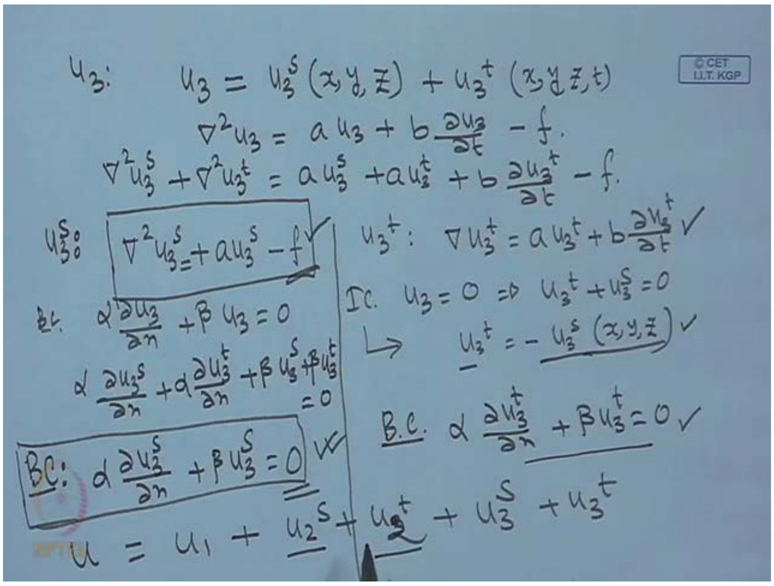

So, therefore, now you will be in a position to get that complete solution of the problem. Complete solution is given as a linear superposition of all the sub-problems. u 1 plus u 2 s plus u 3 t u 2 t plus u 3 s plus u 3 t; that gives the complete solution and u 2 s is the steady state part of the second sub-problem; u 2 t is the transient part of the second sub-problem; u 3 s is the steady state part of the first sub-problem; u 3 t is the transient part of the third sub-problem. And by using a linear superposition of all the solutions of the sub-problems, one can constitute the complete solution of the original problem.

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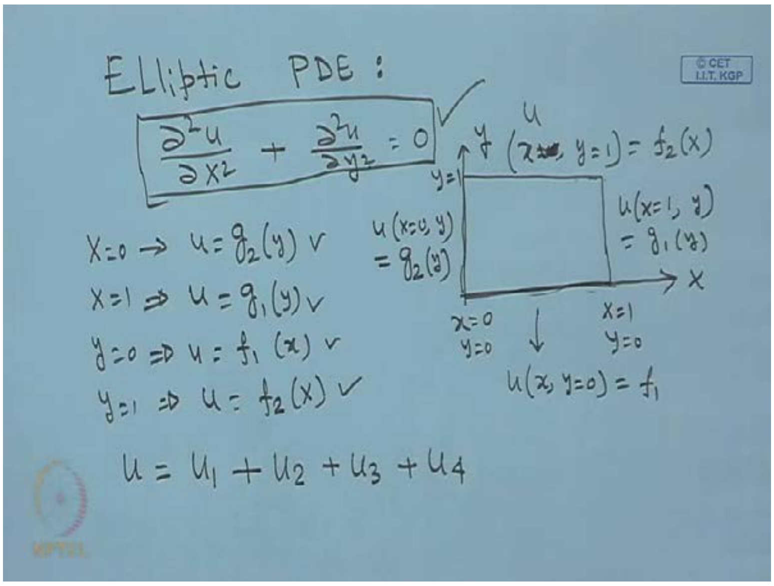

Next, we will be taking up an example of Elliptic partial differential equation, Elliptic PDE, and in this case, we are writing it the 2-dimensional Laplacian operator - del square del x square plus del square del y square del square u by del x square plus del square u by del y square is equal to 0, and in order to solve this problem, you must be having 2 boundary conditions on x because it is order 2 with respect to x; 2 boundary conditions on y because it is order to with respect to y.

Now, let us define our boundaries like this: This is x axis; this is y axis (Refer Slide Time: 43:18) this is this surface is x is equal to 0, while this boundary is located at x equal to 0, y equal to 0; this boundary is located at x equal to 1 y is equal to 0; this boundary is located at y is equal to 1 and x is equal to 1. And so, all these four boundaries can be specified. So, since and here, u is equal to at x is equal to 0, y. So, this is this boundary. Here, it is defined as the boundary condition is g 2.

In general, it will be function of y; the boundary condition on this plane, on this surface, this surface is y equal to 0, and here the boundary condition is u at any x, but at y is equal to 0, this is let us say f 1 and this boundary is located at x is equal to 1. So, u at x is equal to 1 for any y, this value is g 1 y in general and this is located at y is equal to 1 but any x.

So, x is equal to 0 x is any x, and y is equal to 1 u is equal to f 2 x; that means there are 4 boundaries: x is equal to located at x equal to 0, x equal to 1; y is equal to 0 and y is equal to 1; at x is equal to 0, you have u is equal to g 2 y; at x is equal to 1, you have u is equal to g 1 y; at y is equal to 0, your boundary condition is f 1 x, and y is equal to 1 your boundary condition is f 2 x.

So, we have if you look into the problem and its boundary conditions that, this is the Governing Equation and this Governing Equation is homogeneous, there is no non-homogeneous term present into this boundary, this partial differential equation which is elliptic in nature. On the other hand, it contains the 4 boundaries and all of these 4 boundaries are non-homogeneous in nature, and therefore, we have to break down this problem into four sub-problems, considering one non-homogeneity at a time.

So, what I will do? I will break down this problem into four sub-problems u 1 plus u 2 plus u 3 plus u 4, and we construct the solution the Governing Equation of each sub-problem.

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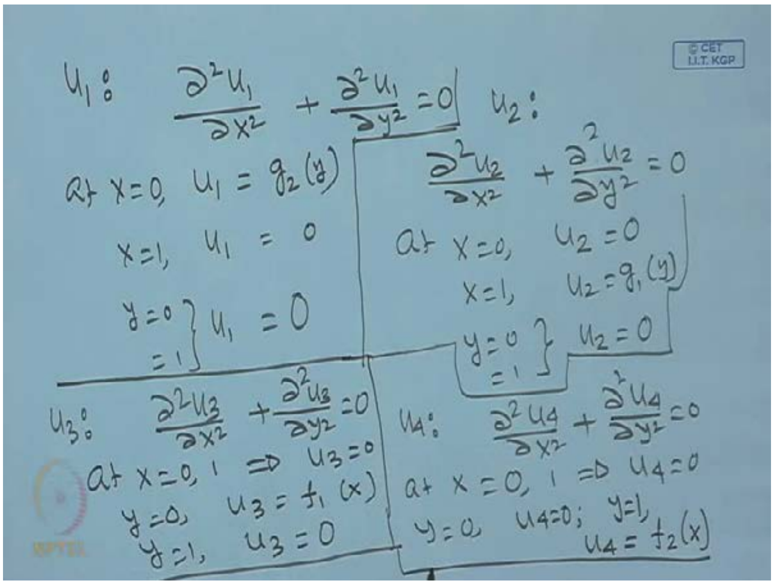

u 1 will be del square u 1 by del x square plus del square u 1 by del y square is equal to 0 and in and you just put u 1 is equal to at x equal to 0, u 1 is equal to g 2 y, and u 1 at x is equal to 1, u 1 is equal to 0 at y is equal to 0, and 1 we put u 1 is equal to 0.

So, we are keeping, only one non-homogeneity at a time forcing the other 3 non-homogeneities to be 0; that gives the construction of the Governing Equation of first sub-problem; then we look into the Governing Equation and the boundary condition of the second sub-problem. This will be del square u 2 by del x square plus del square u 2 by del y square is equal to 0. So, at x is equal to 0, we consider u 2 is equal to 0. We have the other boundary conditions, non-homogeneous boundary condition to be intact. So, u 2 is equal to g 1 y and other 2 boundary conditions at y is equal to 0 and 1; we have, let us say u 2 equal to 0.

So, therefore, we keep one non-homogeneity here, forcing the other three non-homogeneities to be 0, and we formulate the Governing Equation of u 3 del square u 3 by del x square plus del square u 3 by del y square to be equal to 0, and boundary conditions at x is equal to 0 and 1; we have u 3 equal to 0, y is equal to 0; we have we keep this non-homogeneity u 3 is equal to f 1 x and y is equal to 1. we keep the We force the non-homogeneity to vanish.

So, therefore, we keep one boundary condition at y equal to 0; that non-homogeneity, we keep intact and forcing the other three non-homogeneities to vanish. Therefore, we can formulate similarly the fourth sub-problem u 4 - del square u 4 by del x square plus del square u 4 by del y square is equal to 0, at x equal to 0 and 1, u 4 is equal to 0, and at y is equal to 0, we force the non-homogeneity to vanish, and at y is equal to 1, we keep that non- homogeneity to be intact, and that will be u 4 is equal to f 2 x. So, that completes the fourth sub-problem.

And if you look into each of this sub-problem, that only in this in problem number 1, only one non-homogeneity is kept intact; that is the boundary x is equal to 0. In problem number 2, only one non-homogeneity is kept intact; that is given in problem on the boundary condition at located at x is equal to 1. In the third sub-problem, only one non-homogeneity is kept intact; that is the boundary located at y is equal to 0, and in the fourth problem, we kept intact only one non-homogeneity that is the boundary condition located at y is equal to 1.

So, each of these sub-problems is well posed and we will be speak we will be able to solve them quite elegantly without any hassle. So, that is important. We keep only one non-homogeneity at a time; force the other all non-homogeneous to vanish so that we can make a problem well posed, and that well posed problem will be easier to solve.

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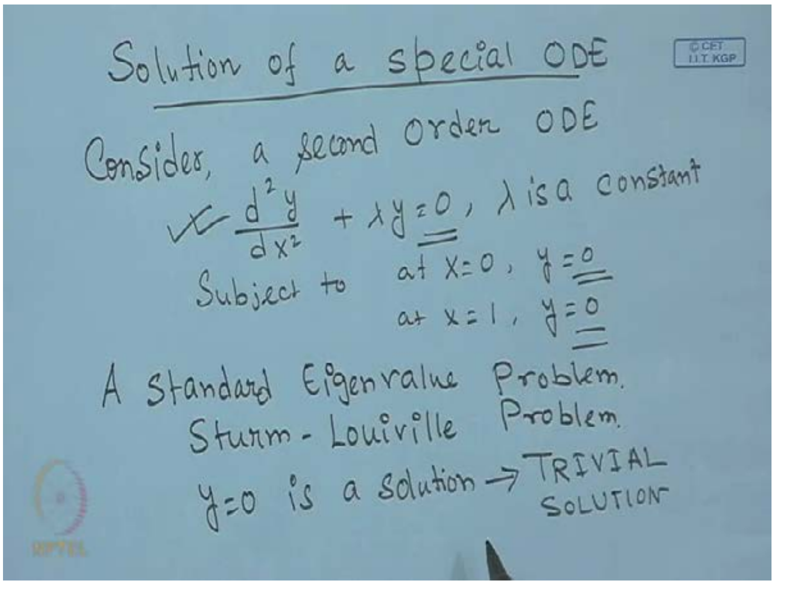

So, next we will be looking into a particular a special solution of a special type of ordinary differential equation.

So, that is how we will be using the principle of linear superposition for bringing down, for resolving their partial differential equation into sub parts so that every sub part contains only one non-homogeneity at a time and each of these sub-problems is a well-defined problem.

So, now, this point onwards, we will take a diversion and look into a special type of ordinary differential equations. And later on will connect this type of ordinary differential equations, how they will be relevant to the solution of partial differential equation.

Consider a second order ODE of the form d square y by d x square plus lambda y is equal to 0, subject to conditions at x is equal to 0 y is equal to 0, and at x is equal to 1 y is equal to 0.

Now, this type of equations, this is a special type of equation where lambda is a scalar; scalar in this means, in this case, a continuous function. Scalar indicates it is a constant. So, at x is equal to 0, both the if you look into the nature of this equation, you will be having, you can see that this equation is a homogeneous equation. So, it is not non-homogeneous; on other hand, the boundary conditions are both homogeneous. So, therefore, this is a set of homogeneous equation as well as the homogeneous boundary condition. So, this equation is known as a standard Eigenvalue problem. It has also a special name, that is called a Sturm Louiville problem, but please note, that this is not a general equation for a standard Eigenvalue problem.

This is a special case of generalized, this is a special case of Sturm Louiville problem or a standard Eigenvalue problem, and this is also a standard Eigenvalue problem.

We will be looking into more generalized version of the standard Eigenvalue problem and Sturm Louiville problem in the next class, but probably before that we should look into the solution of this equation. So, one obvious solution is that if y is equal to 0 it satisfies the Governing Equation as well as the boundary condition.

So, the solution y is equal to 0 satisfies the Governing Equation as well as the boundary conditions. So, 0 is a solution; so, y is equal to 0 is a solution, but this is an obvious solution. Therefore, this particular solution is known as the trivial solution. On the other hand, we are not looking into the trivial solution; we are looking into the specific meaningful solution.

So, therefore, in the next class, we will be looking into how the meaningful solutions or the non-trivial solutions will be obtained from the standard Eigenvalue solution, standard eigenvalue problem like this, and how one can get different kind of solutions by changing the boundary conditions.

So, we stop in this class at this point. We will look into more details of the standard Eigenvalue problem, in the next class. Thank you very much.