Advanced Mathematical Techniques in Chemical Engineering
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Module No. # 01
Lecture No. # 17

Partial Differential Equations

Welcome to the class. Particularly, today's class, so we are looking into the partial differential equations there and its definition. There are various types of partial differential equations – homogeneous and non-homogeneous, what is called order of a PDE? What is called a power of PDE? When we call the boundary condition, we are looking into the classification of various boundary conditions like Dirichlet, Neumann, mixed.

So, we looked into three boundary types of the boundary conditions. We will look into some more types of the boundary conditions. So, the next boundary condition that we will be looking into is that Cauchy boundary condition.

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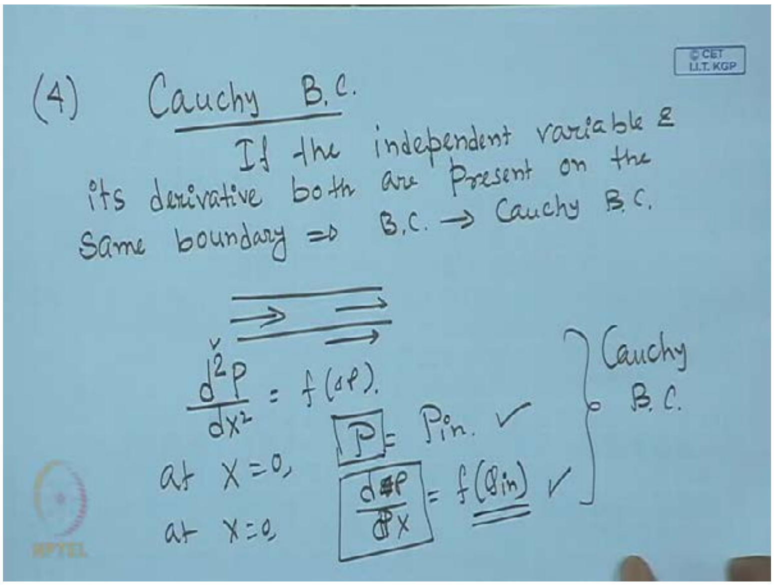

Now, we term that boundary condition to be Cauchy, if the independent variable and its derivatives, both are present on the same boundary. Then, the conditions, boundary condition is known as the Cauchy boundary condition.

Given an example in chemical engineering process, for example, if you suppose, there is a flow occurring inside a channel and we are looking into the pressure drop profile, then pressure drop profile, the equation will be looking something like this: d square p d x square is equal to some function, may be a function of delta p.

So, in order to solve this equation, we have to have about two boundary conditions. Because it is order 2, we have to have 2 boundary conditions to be specified at the boundary. has to be specified But in this case, whatever is specified is that, at inlet point at x is equal to 0; p is known; so, inlet pressure is known. And at x is equal to 0, d delta p d p by d x is known, and that is related to function of inlet flow rate because there is a measurable quantity. So, it since that is a measurable quantity, then d f d p by d x is known.

So, therefore, at x is equal to 0, the derivative is also specified; the value of the dependent variable is specified. So, this is an example of Cauchy boundary condition.

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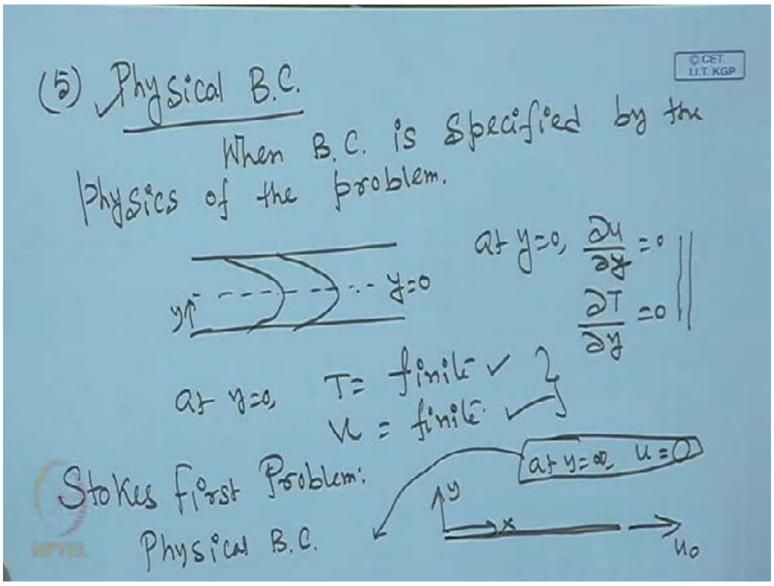

Next, we talk about something called physical boundary condition. We call that boundary condition at as a physical boundary condition where the when the boundary condition is specified by the physics of the problem. Apparently, the other boundary conditions are not specified. So, in that case, we specified that boundary condition which will be automatically evolving out of the physics of the problem.

For example, when you are talking about temperature profile or a velocity profile inside a channel, so we always define this boundary condition that the middle of the channel, that means, at y is equal to 0, del u by del x is equal to 0, or del T del u by del y equal to 0, or del T by del y is equal to 0; this comes from the physics of the problem. It is not that, that boundary is basically insulated; is basically it is symmetric symmetrical at that particular boundary.

So, or we can we can call, at y is equal to 0, T is finite or velocity is finite; that means, we understand that, at the boundary condition, boundary at y equal to 0 at the middle of the channel, that temperature must be assuming a finite value, the velocity must be assuming a finite value, but we do not know its value. So, these values, these boundary conditions are known as the physical boundary conditions.

Similarly, if we talk about talk about the Stokes first problem, what is the Stokes first problem?

In this case, suppose there is a stagnant liquid is placed in a domain, we put a we put a stationary plate there; now, at time T is equal to 0, this plate starts moving in this direction with a velocity u naught. So, therefore, the fluid elements which are which are coming in contact with this plate, they will be having a velocity u 0 in the x direction because of the no slip boundary condition and the fluid particle in the y direction. So, fluid particles adjacent to the wall, they will start moving.

Similarly, the velocity of the fluid particle will keep on diminishing because of the viscous effect, as we go along y. So, beyond a particular point, let us say, at y is equal to infinity, the fluid particle does not experience the presence of a velocity; finite velocity that is present at y is equal to 0. So, the wall moment will not be recognized or realized by a fluid particle located at y equal to infinity. So, we put the boundary condition that - at y is equal to infinity u is equal to 0. Since this boundary condition is coming from the physics of the problem, this is also categorized; it falls under the category of physical boundary condition.

There are various examples of physical boundary conditions where the boundary condition is not apparently apparent; it is not specified by the surroundings; it is not specified by the system; then, we consider that boundary condition, we specify the condition which should be which is in corroboration of the physics of the problem. So, that boundary condition is known as the physical boundary condition.

Now, I will just take up one example of a partial differential equation which will be quite common in chemical engineering system.

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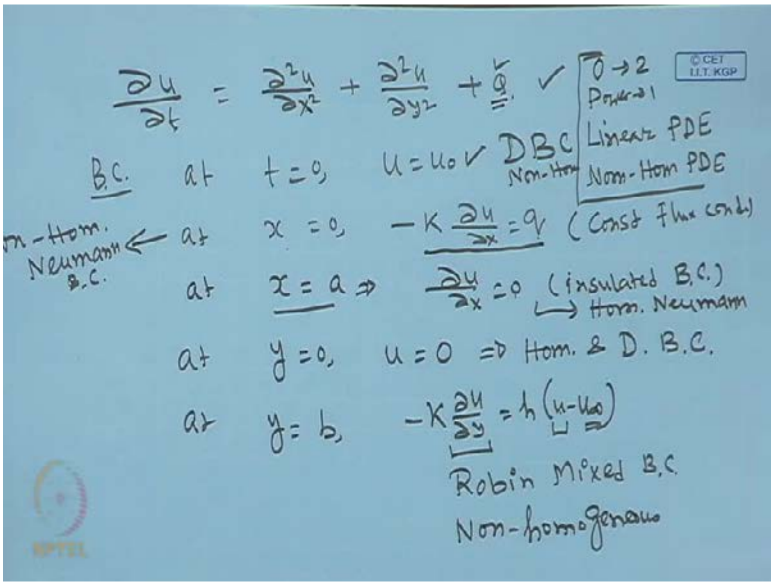

Consider this equation del u by del t is equal to del square u by del x square plus del square u by del y square plus q, and then, you are required to have one boundary condition, one initial condition, or at t, two boundary conditions on x because it is order 2 with respect to x; two conditions on y because it is order 2 with respect to y.

So, typically, we have, at t is equal to 0, u is equal to let us say u naught. At x is equal to 0, let us say, minus k del u by del x is equal to q; let us say, it is a heat conduction problem, 2-dimensional heat conduction, transient heat conduction problem; at x is equal to a del u del x is equal to 0; at y is equal to 0, u is equal to 0; at y is equal to b minus k del u by del y is equal to h u minus u infinity.

Let us explain these equations at this set of boundary condition. This equation is a second order 2-dimensional because it is 2-dimensional in x and y; 2-dimensional, transient; so, it is time dependent; so, transient heat conduction problem. And there is a source term here; that means, there is a continuously, there is a source or in the system which will be generating heat energy volumetrically. So, there is a volumetric source term present.

So, it is a there is a source present in the system which is continuously developing energy into the system (Refer Slide Time: 09:11); so, this equation is having order 2 because the highest power is 2. This equation is having a power 1; so, all the terms will be having power 1. This equation is a linear equation because all the terms, there is no terms that is existing which will be having power not equal to 1, and, or higher. And there is no term present in this equation which will be basically multiplication of the product of dependent variable and its derivative.

So, this is a linear partial differential equation. There is one term present in the equation which does not depend on the dependent variable. Therefore, this is non-homogenous partial differential equation.

So, these are the characteristic of these equations; it has order 2; it has a power 1; this is a linear partial differential equation; this is a non-homogeneous partial differential equation.

Now, let us look into the boundary conditions and initial condition at t is equal to 0, u is equal to u 0; that simply means that the initial temperature is fixed; it is given.

At x is equal to 0, minus k del u by del x is equal to q; this means, at the at x is equal to 0, we have a constant heat flux going into the system, and the constant heat flux is given by q. And, at x is equal to a, del u by del x is equal to 0; that means, the surface located, the boundary located at x is equal to 0, is insulated; so, this is a constant flux condition (Refer Slide Time: 11:24).

And this is insulated boundary condition, and u is equal to 0; that means temperature is 0 there. So, this is the value of the dependent variable is present. And, at y is equal to b, the whatever the heat that has come by conduction minus k del u by del y is equal to taken away by the air by convection h into u minus u minus u infinity.

So, therefore, this boundary condition is containing the dependent variable u and its derivative del u by del y, and they are connected by a simple algebraic equation.

Now, let us classify the boundary condition. Boundary condition is specified here. So, it is a Dirichlet boundary condition and it is non-homogeneous Dirichlet boundary condition because it is there is a term present, which does not contain the dependent variable.

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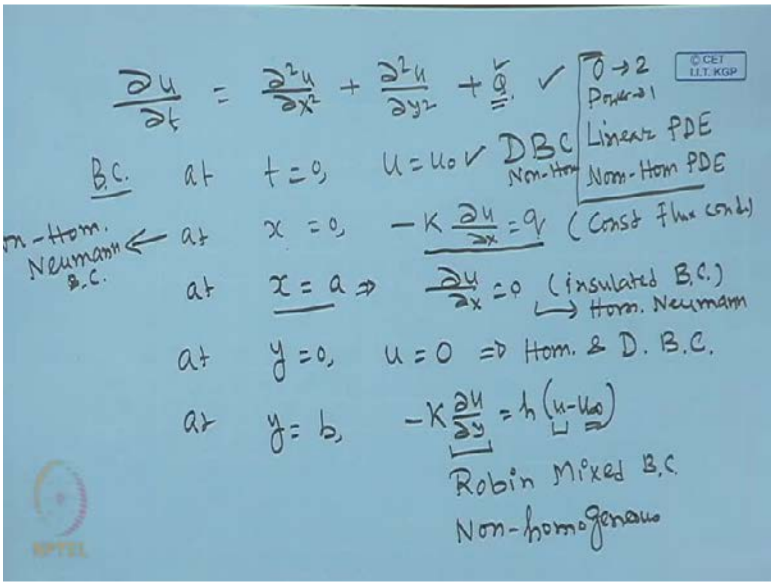

Now, let us examine this boundary condition. There is a term. contain so Since, the derivative of the dependent variable is present, this is a Neumann boundary condition, but since there is it is not 0, it is q, and that q is you know it is not a function of u. So, therefore, this equation is non-homogenous. So, this is a non-homogeneous Neumann boundary condition.

Now, let us look into this equation: x is equal to a; del u by del x is equal to 0; derivative of u is specified, dependent variable is specified, and the right hand side is 0; so, this is homogenous Neumann boundary condition, and y is equal to 0, u is equal to 0; so, value of the dependent variable is specified; so, this is homogenous and this is Dirichlet boundary condition.

On the other hand, the last boundary condition at y is equal to b, the both variable both the derivatives of the dependent variable, and the dependent variable, are connected by a simple algebraic equations. They are appearing in the same boundary condition. So, it is a Robin-Mixed boundary condition, and there is a term containing which does not contain the dependent variable or its derivative. So, this is non-homogenous Robin-Mixed boundary condition.

So, in this example, we have seen, the boundary conditions can be homogenous, can be non-homogenous, can be Dirichlet, can be Neumann, can be Robin-Mixed, at the that and all these boundary conditions can occur at the same time, for the same problem.

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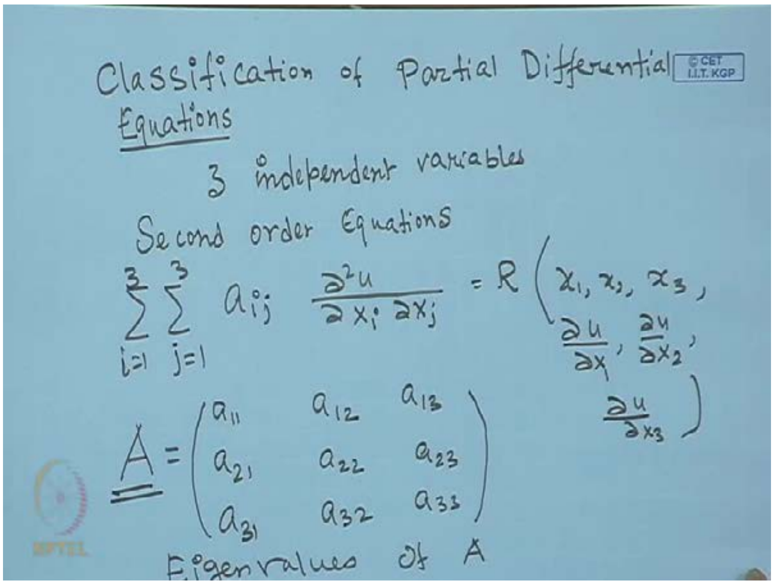

Next, we classify the partial differential equations. So, we consider three independent variables. So, one can consider the second order equation; most of the chemical engineering applications will be landing with the second order equations.

So, the general form of the partial differential equation of second order for three independent variables are can be written in this form: i is equal to 1 to j 1 to 3 j is equal to 1 to 3 a i j del square u by del x i del x j is equal to R x 1 x 2 x 3 del u by del x 1 del u by del x 2 del u by del x 3. So, these coefficients a i j can be function of axis as well. So, we can most of the equations can be cast in this form.

Now, we can have A matrix composing of the coefficients of these of the second order term. Then, this you can write it as: a 11 a 12 a 13 a 21 a 22 a 23 a 31 a 32 a 33.

Now, depending we get we formulate this matrix and look into the Eigenvalues of this matrix; Eigenvalues of coefficient matrix.

Now, depending on the sign of the Eigenvalue, the partial differential equations are classified.

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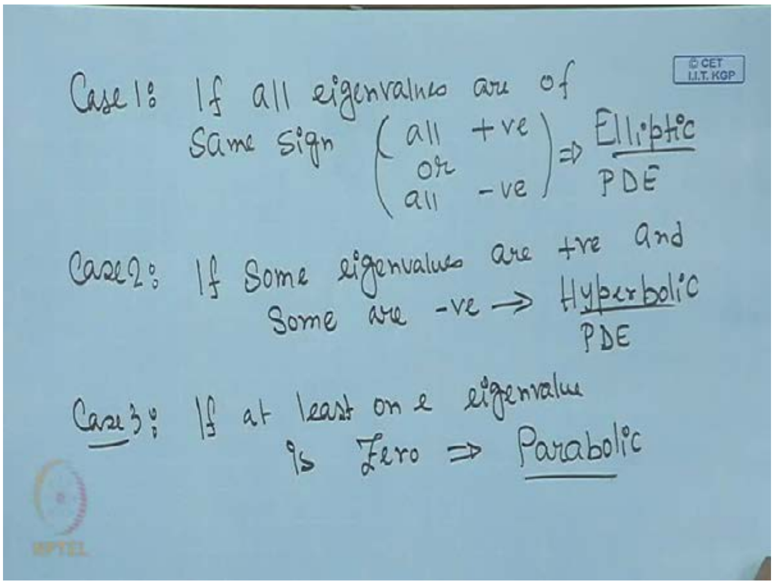

For example, case 1, the first case - if all Eigenvalues are of same sign, all may be positive, or all may be negative; then, this equation, this partial differential equation is called Elliptic PDE.

Case 2: If some Eigenvalues are positive and some are negative, then we call that equation as hyperbolic partial differential equation.

Case 3 is that, if at least one Eigenvalue is 0, then we call this as parabolic partial differential equation.

So, that way, we can classify the different partial differential equations. So, the form is the so the idea is, you formulate the matrix considering the coefficients of the second order term and evaluate the Eigenvalues of that matrix. If the Eigenvalues are of same sign, we will be landing up we will be dealing with the Elliptical partial differential equations.

If Eigenvalues are of mixed sign, that means some of them are positive, some of them are negative; then, there, it is called the hyperbolic partial differential equation. If at least one Eigenvalue is 0, then we will be dealing with a parabolic partial differential equation.

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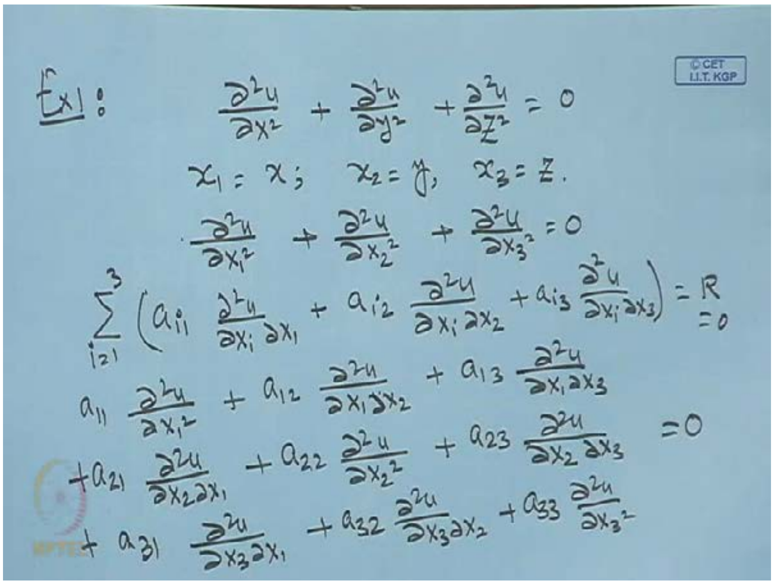

So, we will take up some of the examples. So, first example is del square u by del x square plus del square u by del y square plus del square u by del z square is equal to 0. So, in this case, x 1 is x, x 2 is y, and x 3 is z, and we will be having del square u by del x 1 square plus del square u by del x 2 square plus del square u by del x 3 square, is equal to 0.

Now, if you write it down in this form, i is equal to 1 to 3 a i 1 del square u by del x i del x 1; so, just open up the double summation; so, this will be plus a i2 del square u by del x i del x 2 plus a i3 del square u by del x i del x 3; that should be is equal to R, and R is equal to 0, in this particular case. So, you just open up this one. So, you will be having a 11 del square u by del x 1 square plus a 12 del square u by del x 1 del x 2 plus a 13 del square u by del x 1 del x 3 plus a 21 del square u by del x 2 del x 1 plus a 22 del square u del x 2 square plus a 23 del square u by del x 2 del x 3 plus a 3 1 del square u by del x 3 del x 1 plus a 32 del square u by del x 3 del x 2 plus a 33 del square u by del x 3 square is equal to 0.

Now, let us identify the coefficients. If you identify the coefficient a 11 is equal to 1, that is given here; a 11 is equal to 1; a 12 is equal to 0; a 13 is equal to 0; a 21 is equal to 0; a 22 is equal to 1; a 23 is equal to 0; a 31 is equal to 0; a 32 is equal to 0, and a 33 is equal to 1.

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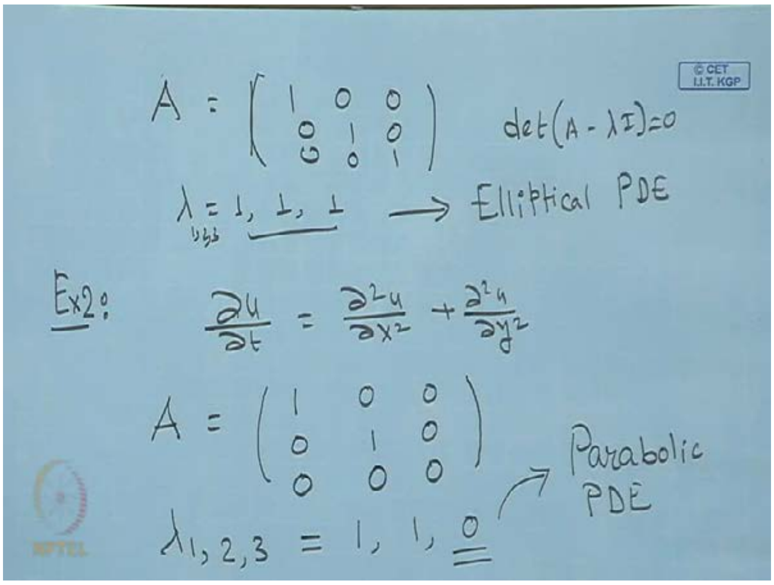

So, therefore, now, if you constitute the, formulate the coefficient matrix, let us see what is the form of the coefficient matrix; the coefficient matrix is 1 0 0 0 1 0 0 0 1.

Now, just look into the Eigenvalue of this matrix. You put determinant of A minus lambda I is equal to 0; so, you will be getting lambda is equal to 1 1 1; so, lambda 1 2 3. So, there will be three Eigenvalues of this particular problem; all are of same size. So, this is an Elliptical partial differential equation.

Next example: We take up and demonstrate how we get a parabolic partial differential equation. del u by del t is equal to del square u by del x square plus del square u by del y square.

We open up the equation as we have done earlier. So, if we really do that and look into the coefficient matrix, the coefficient matrix will look something like this - 1 0 0 because of 1; there will be the coefficient of del square u by del x square; it will be 1. Similarly, the coefficient of del square u by del y square, it is 1; so, 0 1 0, but the coefficient of del square u by del t square is equal to 0; therefore, it will be 0 0 and 0.

Now, if you look into the Eigenvalues, the Eigenvalues of this matrix are 1 1 and 0. Since, one of the eigenvalue is 0, then, this equation is a parabolic partial differential equation.

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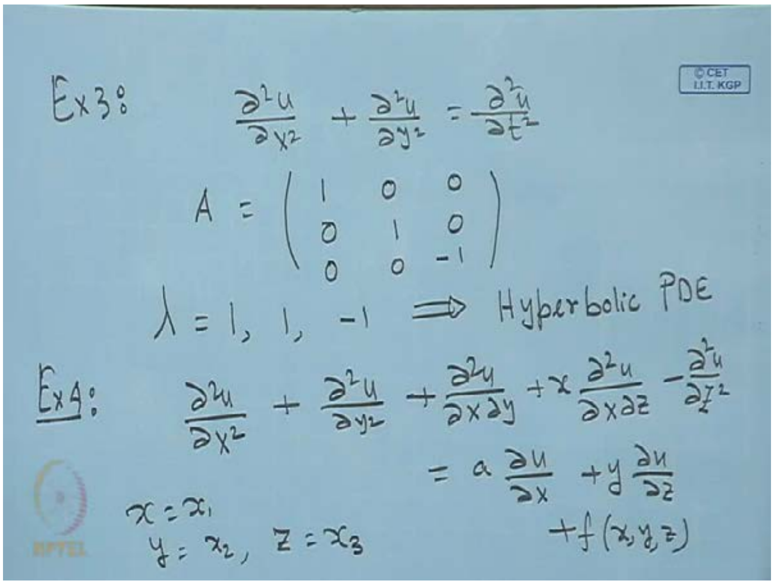

So, we go to the next example; that is a hyperbolic one. So, that is example 3. del square u by del x square plus del square u by del y square is equal to del square u by del t square.

So, suppose, we are talking about these equations, so we will be having the construct. You can construct the coefficient matrix 1 0 0 because the coefficient we del square u by del x square is 1; del square u by del y square is also 1; 0 1 0, and del square u by del t square will be minus 1 because when you bring it to other side;, so, 0 0 minus 1. So, one can get the Eigenvalues as 1 1 and minus 1. Since the Eigenvalues are of mixed signs, then this equation is a hyperbolic partial differential equation.

Now, let us look into next example; example 4. del square u, just consider this equation- del square u by del x square plus del square u by del y square plus del square u by del x del y plus x del square by u del x del z minus del square u by del z square is equal to del u by del x plus y del u by del z plus some function of x y z.

Now, let us identify x as x 1, y as x 2, and z as x 3, and recast this equation in the form of second order derivative on one side and the other derivative on the other side.

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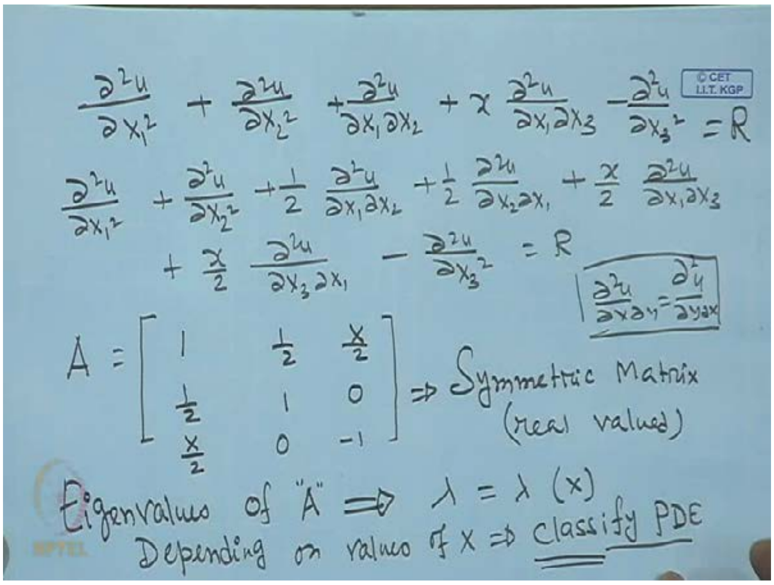

So, if you do that, what you will be getting is that del square u by del x square x 1 square plus del square u by del x 2 square plus del square u by del x 1 del x 2 plus x del square u by del x 1 del x 3 minus del square u by del x 3 square is equal to let us say R, and R contains all the first order partial derivative and the non-homogeneous term f of x, y, and z.

So, we put it in this form: del square u by del x 1 square plus del square u by del x 2 square plus half del square u by del x 1 del x 2 plus half, just bring it into two parts break it into two parts, del square u by del x 2 del x 1 plus x by 2 del square u by del x 1 del x 3 plus x by 2 del square u by del x 3 del x 1 minus del square u by del x 3 square, is equal to R.

We utilize the concept del square u by del x del y is equal to identical to del square u by del y del x.

Now, we write down the coefficient matrix A. So, if you look into the coefficient matrix, it will be 1 half x by 2 half 1 0 and x by 2 0 and minus 1. So, this is a symmetric real value matrix, and of course, it is a real value.

Now, one can identify the Eigenvalues of this matrix A. Eigenvalues of this matrix A and Eigenvalues will be depending on x; definitely, these lambdas will be function of x; so, depending on the values of x, we can we classify the partial differential equation.

So, in this case, we cannot definitely say about the nature of this PDE because we do not know the value of x. Therefore, depending on the value of x, the Eigenvalues will be determined, and that will dictate the classification of this partial differential equation.

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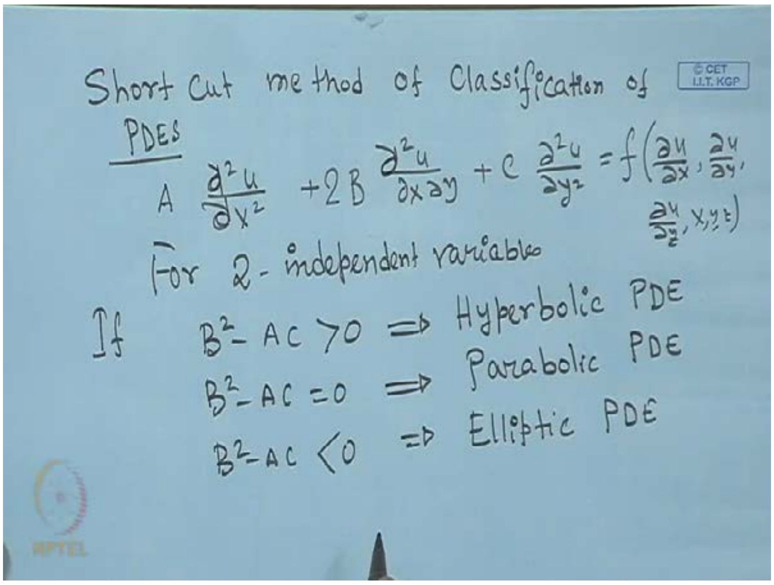

Next, we go to a shortcut method of classification of PDEs.

This is basically A d square u by d x square plus 2 B I think del square u del x square. So, it will be del square u by del x del y plus c del square u by del y square is equal to some function of del u by del x del u by del y; may be del u by del z, x, y, z, or any kind of form.

Now, if this is the case for two independent variables, if we have partial differential equation with 2 independent variables, then all these equations can be in order 2; all these equations can be cast in this particular form.

Now, by looking into the values of coefficients a, b, and c, we can classify the partial differential equations, and the classification goes like this:

If B square minus AC is greater than 0, then we are talking about a Hyperbolic partial differential equation.

If B square minus AC is equal to 0, then we are talking about a Parabolic PDE.

If B square minus AC is less than 0, then we are taking about an Elliptical PDE; Elliptic partial differential equation.

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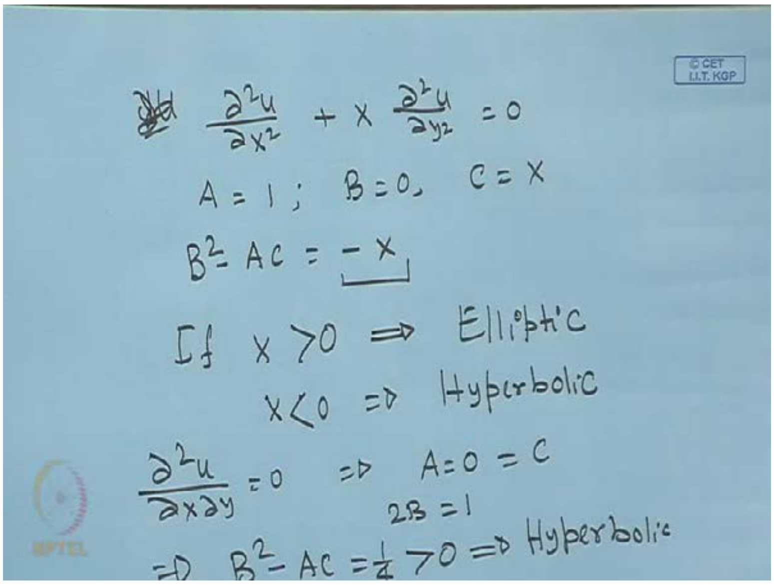

Now, that becomes very simple for a 2-dimensional problem. Then, just talk about… consider a problem like del square u by del x square plus x del square u by del y square is equal to 0; in this case, A is equal to 1, B is equal to 0, but c is equal to x.

So, if you compute B square minus AC is equal to minus x, now, depending on the value of x, the partial differential equation will be classified; that means, if x is greater than 0, then we talk about that B square minus AC is negative; then, we will be landing up with a with an Elliptic partial differential equation.

If x is negative, then B square minus AC is positive, and we are talking about a Hyperbolic equation, And if x is equal to 0, we are dealing with a Parabolic partial differential equation.

Consider the equation del square u by del x del y is equal to 0. Now, in this case, A is equal to 0 C is equal to 0, but 2 B is equal to 1.

So, therefore, you can consider, compute B square minus AC, and that turns out to be 1 by 4, and 1 by 4 is, of course, greater than 0. So, therefore, we are talking about a hyperbolic partial differential equation. So, depending on the… for a 2-dimensional case, we will be we can classify the equations in a very straight forward manner, but for a 3-

dimensional problem, we have to really compute the Eigenvalues of the of the matrix, of the coefficient matrix, where the matrix is found by the coefficients of the second order partial derivatives, and then, we evaluate the Eigenvalues of that matrix. And depending on the sign of the Eigenvalues, we can classify the partial differential equations as Parabolic, Hyperbolic, or Elliptic.

Next, we so that goes the classification of the partial differential equations, the different and how to evaluate how to evaluate the PDE's, and we defined the different values of you know boundary conditions.

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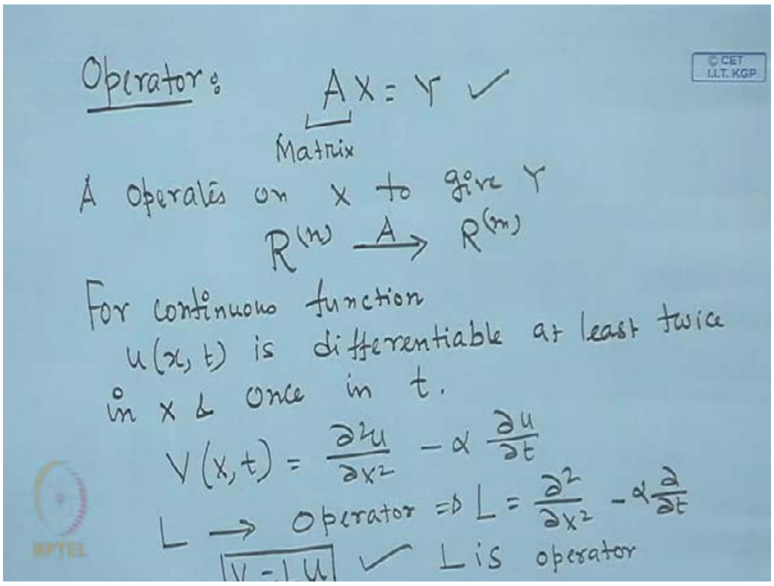

Next, we talk about an operator. What is an operator? A x Consider A x is equal to y - the matrix; A is the matrix; x is a vector; it operates on the matrix A, operates on the vector x, and it maps in y; vector y. So, A operates on x to give y, and basically, if it is n dimensional, x is x vector is n dimensional; then, y is n dimensional; so, it maps n dimensional vector to m dimensional space. So, this is the operator A. So, similarly, therefore we will be having the in case of this is for the discrete domain for continuous functions.

We consider the function u x and t is differentiable at least twice in x, and once in time t; then, we consider this function V of x t as del square u by del x square minus alpha del u by del t; then, we denote operator by L. So, the operator here, in this case, is del square by del x square minus alpha del by del t, and we write V is equal to L times u.

So, in this case, if you just look into the matrix operator and the operator in continuous function y is equal to A x; so, A is the operator; similarly, in this case, V is equal to L u where L is the operator.

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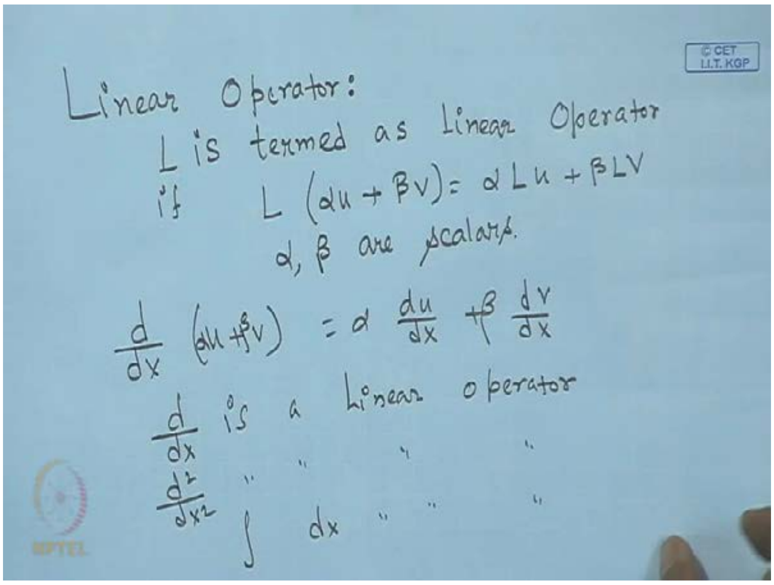

So, next, we talk about a linear operator. What is a linear operator? L is termed as linear operator, if L of alpha u plus beta V is equal to alpha L u plus beta L V. Then, this operator L is called a linear operator where, alpha and beta are scalars.

So, therefore, differential is a linear operator. So, d by d x of u plus v, let us say, alpha u plus beta v is nothing but alpha d u by d x plus beta d v by d x. So, d by d x is a linear operator; d square by d x square is a linear operator; integration is a linear operator. So, all these are basically linear operators.

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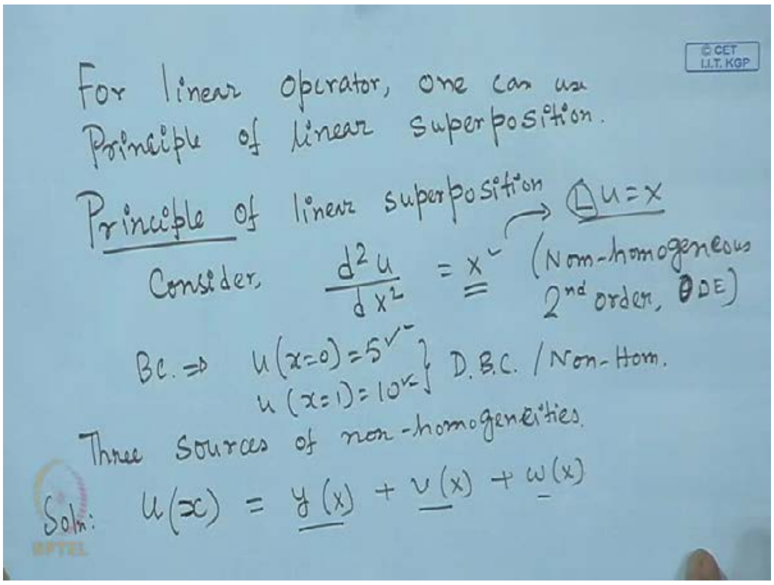

Now, the idea is, when we are having the linear operator, then we can use the Principle of superposition for linear operator; one can use Principle of linear superposition.

Now, let us look into the Principle of linear superposition. When we use the principle linear super position for construction of the solution, consider we will take up an example of ordinary differential equation first; then, we will explain the principle of linear superposition; then, we take up the partial differential equation.

Consider d square u by d x square is equal to x. This is a non-homogeneous second order ordinary differential equation. The boundary conditions are: u at x is equal to 0 is equal to 5, and u at x is equal to 1 is equal to 10; both are Dirichlet boundary conditions, but non-homogeneous.

So, now, there are three sources of non-homogeneities in this particular problem: One non-homogeneity is occurring at the differential equation one; another second non-homogeneity is occurring in the first boundary condition; the third non-homogeneity is occurring at the second boundary condition. Then, what we can do is that, we can break down this problem into three sub problems considering one non-homogeneity at a time, and then, we get the solution of each sub problem and add them up, and we will be getting the complete solution. So, there, that is possible when this so because what is the operator in this case?

The operator is Lu is equal to x; so, what is the operator? Operator L is d square by d x square, and we know that d square by d x square is a linear operator. Since the operator is a linear operator, we can break down this problem into three sub problems, considering one non-homogeneity at a time, and the complete solution can be constructed by getting the by adding them up simply. So, we break down this problem into three sub problems considering one at a time, and the whole solution can be constructed out of it.

Let us say u of x; the solution u of x can be constructed by considering y of x plus v of x plus w of x. We break down this problem into three sub parts because there are three sources of non-homogeneity 1, 2, and 3.

Next, we construct the Governing Equation and the boundary condition of each such sub problem.

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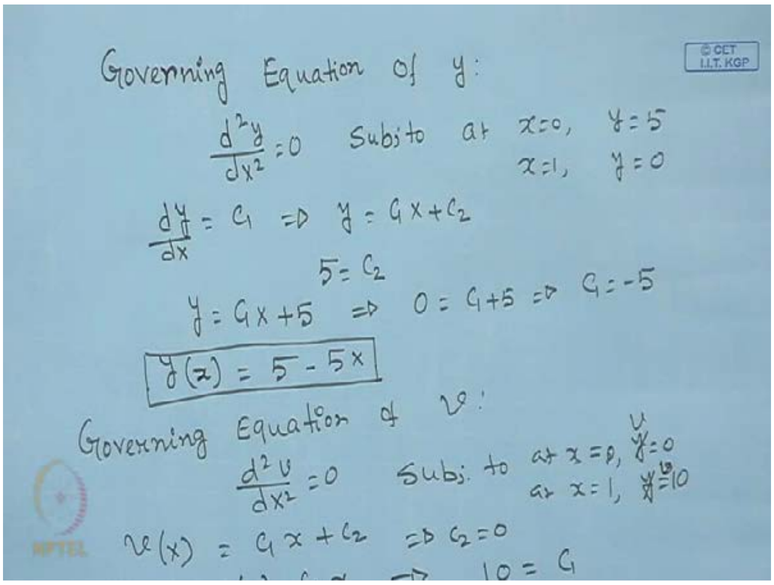

Now, let us write down the Governing Equation of y. Governing Equation of y will be d square y by d x square is equal to 0 subject to at x is equal to 0 y is equal to 5 at x is equal to 1, y is equal to 0.

So, what we have done here? We have considered the sub problem y and we have considered only one non homogeneity at a time, forcing the other two non-homogeneities to be 0.

So, there were two more non-homogeneities; one is on in the Governing Equation; another is in the boundary condition. We are forcing these two boundary, two non-homogeneities to be 0, and keeping one non-homogeneity at a time. So, that is the Governing Equation of the sub first sub problem with the boundary condition.

So, if you will have to solve this equation, the solution of this will be d y by d x will be c 1, and one more integration will give you y is equal to c 1 x plus c 2; if you put the boundary condition x is equal to 0, you will be getting y is equal to 5. So, c 2 is 5; so, the equation becomes c 1 x plus 5.

And then, we have one more boundary condition, that is x is equal to 1 y is equal to 5; so, y equal to 0. so 0 is equal to c 1 plus 5; so, c 1 is equal to minus 5. So, you can complete the solution of y; the first sub problem as 5 minus 5 x.

Now, let us look into the second sub problem v. The Governing Equation of v will be d square v by d x square is equal to 0, subject to, at x is equal to 0, y is equal to 0, and at x is equal to 1, y is equal to 10.

Now, in this sub problem, we are not including the source term in the Governing Equation, the non-homogeneous term Governing Equation. We are forcing the non-homogeneous term, the Governing Equation, and the non-homogeneous term in the first boundary condition to be 0.

We are retaining the non-homogeneous term in the boundary condition at x is equal to 1. So, we are retaining only one non-homogeneity and forcing the other two non-homogeneities to be vanished.

So, again, in this case, the solution will be constituting of c 1 x plus c 2, and using these two boundary conditions, we will be evaluating the c 1 and c 2 at x is equal to 0 y equal to at x is equal to 0, v is equal to 0. This should be v and at x is equal to 0 (Refer Slide Time: 48:08) v should be equal to 0; therefore, c 2 equal to 0.

So, therefore, v of x is nothing but, nothing but c 1 times x, and what is c 1? So, that has to be satisfied by the other boundary condition that, at x is equal to 1 v is equal to 10; so, 10 is equal to c 1. So, we can construct the solution of v.

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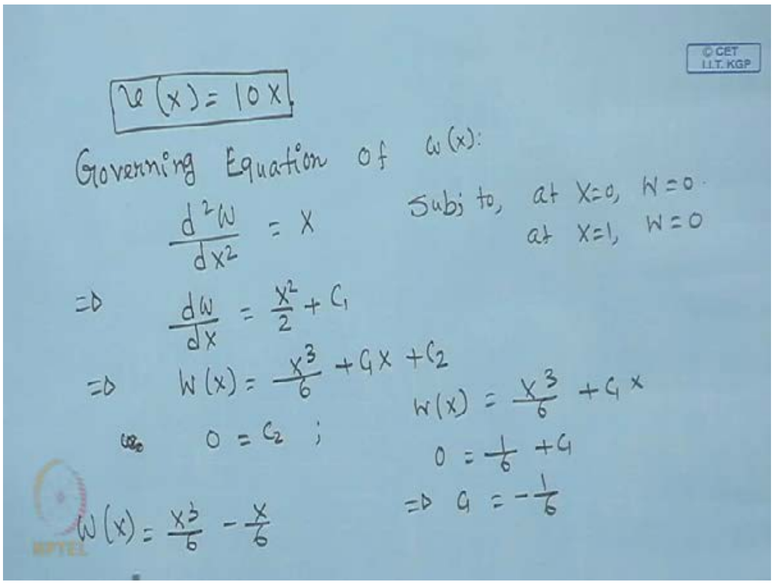

Now, we can construct the solution of v. So, v will be v of x will be nothing but 10 x. So, that is the solution of sub problem v.

Next, we talk about the sub problem of w. Write down the Governing Equation of w; it will be d square w by d x square is equal to x, subject to, at x is equal to 0, w is equal to 0; at x is equal to 1, w is equal to 0.

So, therefore, if you look into this sub problem, in this sub problem, we are keeping the non-homogeneous term in the Governing Equation intact, forcing the non-homogeneities in the boundary condition to vanish.

Now, you solve this problem, integrate it out once; so, it becomes d w by d x is equal to x square by 2 plus some constant c 1; then, do one more integration; so, this will be w, as the function of x will be x cube by 3; so, it will be 6 plus c 1 x plus c 2.

Now, utilizing these two boundary conditions at x equal 0 w equal to 0; so, 0 is equal to c 2; so, therefore, w x become becomes x cube by 6 plus c 1 x, and at x is equal to 1, w is equal to 0; so, 0 is equal to one upon 6 plus c 1 so c 1 is equal to minus 1 upon 6

So, w x becomes x cube by 6 minus x by 6. Now, if we can, if we now, we can use the principle of linear superposition and add all the solution up, and we will be getting the complete solution.

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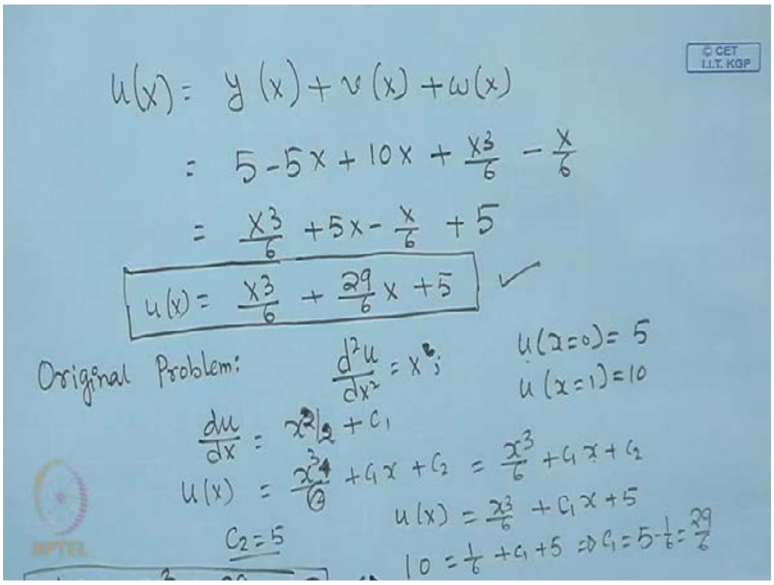

So, u of x is equal to y of x plus v of x plus w of x. We have already found out that the expressions of y and x y and v y is 5 minus 5 x plus v; v is 10 x w plus x cube by 6 minus x by 6; so, this becomes x cube by 6 plus 5 x minus x by 6 plus 5; so, x cube by 6 plus 30 minus 1. So, it will be 29 by 6 times x plus 5.

Now, if you look into the original problem, the original problem so, that is the complete solution u x. So, by using principle of linear superposition, one can construct the complete solution.

Now, if you look into the original problem, the original problem was d square u by d x square is equal to x square subject to u at x is equal to 0 is equal to 5, and u at x is equal to 1 is equal to 10. So, just do it by one integration.

So, one integration will give you d u by d x is equal x cube by 3 plus some constant c 1. One more integration will give you x to the power 4 by 12 plus c 1 x plus c 2. The original problem was d square u by d x square is equal to x. So, it will be x square by 2; then, it will be x cube by 6 plus c 1 plus c 1. So, it will be x cube by 6 plus c 1 x plus c 2. So, now, we put the boundary condition at x equal to 0, u is equal to 5.

So, c 2 equal to 5. Put x is equal to 0 and u equal to 5. So, you will be getting c 2 equal to 5. So, therefore, u of x is nothing but x cube by 6 plus c 1 x plus 5. Next, we just put x is equal to 1; u is equal to 10. So, 10 will be 1 over 6 plus c 1 plus 5.

So, if you evaluate, c 1 turns out to be 10 minus 5 is 5; 5 minus 1 upon 6; it will be 29 upon 6; so, the complete solution of this problem becomes u as a function of x cube by 6 plus 29 by 6 into x plus 5. So, that matches with our solution using the Principle of linear superposition.

So, this is quite straight forward and understandable for an ordinary differential equation. We demonstrate this particular method for ordinary differential equation.

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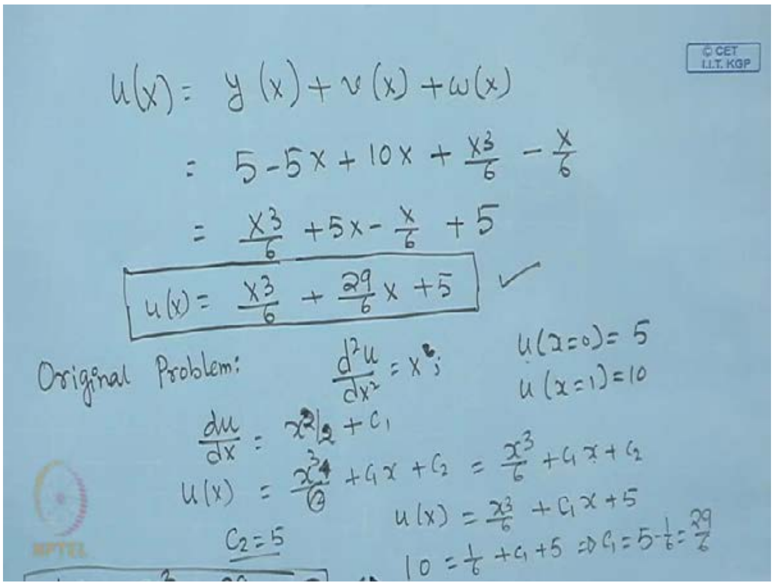

Now, nobody will solve this problem by using it, breaking into the three parts for ordinary differential equation; rather, they will be utilizing this route to solve this problem, but this problem in ordinary differential equations, we have given a demonstration to illustrate the method of application of a Principle of linear superposition. So, this method will be quite useful for solution of partial differential equations, and then, this becomes very handy because very useful method.

And in the next class, we will be outlining how these methods of Principle of linear superposition will be applicable, applied for the solution of partial differential equation.

Thank you.