Advanced Mathematical Techniques in Chemical Engineering

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Lecture No. # 15

Stability Analysis (Contd.)

Good afternoon every one. So, we in the session, we will be looking into some of the problems of chemical engineering problems, which we will be utilizing, where we will be utilizing the stability analysis for the steady state; we will be evaluating the steady state first and then we will be looking into the stability of the steady state. In some of the cases, we will be looking into the combinations of various parameters, which will be corresponding to the half bifurcation or saddle in bifurcation. So, we will be looking into some more applications of chemical engineering processes, and how this Eigenvalue or Eigenvectors method can be utilized to identify the stability of such systems.

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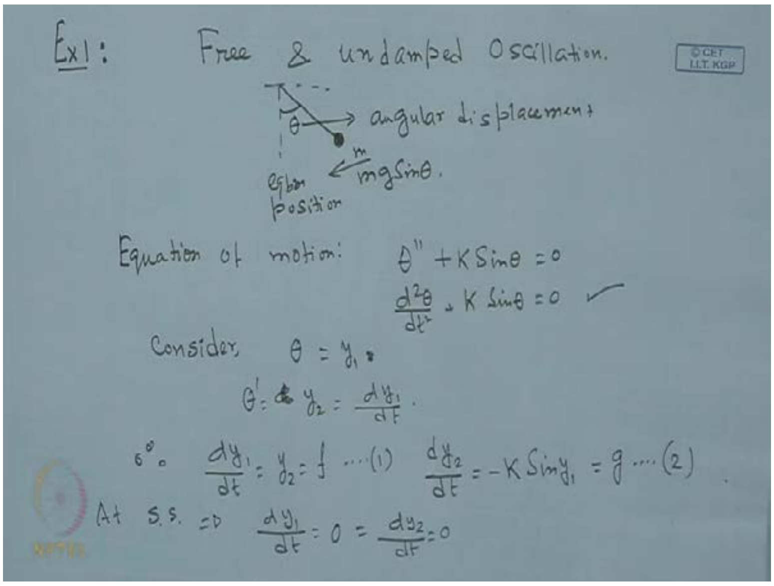

So, first example will be talking about in this session is the example of free and undamped oscillation. Just consider a system, that this the equilibrium position and we have a pendulum bomb of mass m is kept at an angular displacement theta; theta is nothing but the angular displacement. This is the equilibrium position and the restoring force in this direction will be m g sin theta.

Basically the presence, because of the presence of this m g sin theta restoring force, the pendulum will be moving towards the equilibrium position, but it cannot stop there because of the equation inertia of motion, it moves back the on other side and get, it will be acted upon by the restoring force in the opposite direction towards the equilibrium position, comes back and starts oscillating.

Now, we just look into the stability of the system; now, if you look into the equation of motion for this particle, this becomes theta double prime; theta double prime is basically d square theta d t square plus K sin theta is equal to 0; so, d square theta d t square plus K sin theta that will be equal to 0. Now, consider this transformation - thus theta is equal to y 1; that means, theta is equal to y 1; so, d theta theta prime is given as y 2, and y 2 is nothing but d y 1 d t.

So, we have by defining this, we can break down this equation motion into two equation; so, the first equation will be d y 1 d t will be y 2 that is equal to f this is equation number one; equation number two will be d y 2 d t will be equal to minus K sin y 1 and that will be equal to g this time.

So, the at the steady state will be obtained by putting d y 1 d t equal to 0 and d y 2 d t equal to 0; so that will be corresponding to the steady state of this problem.

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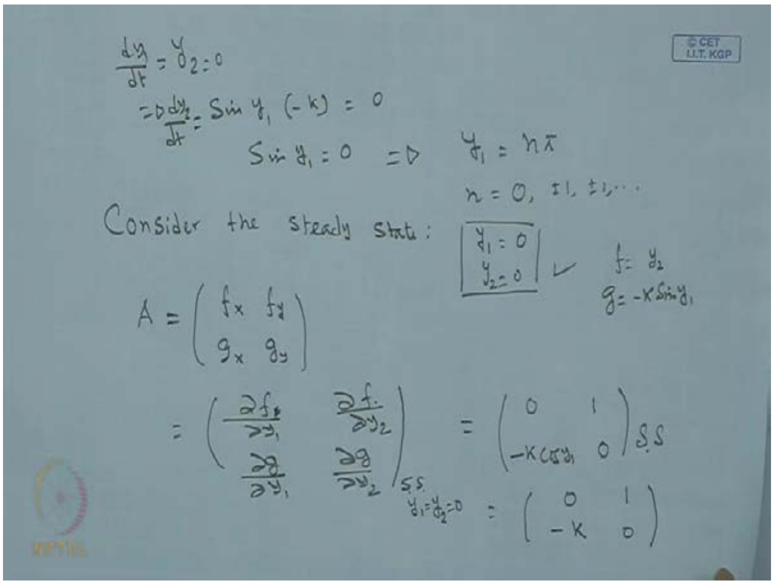

So, therefore, we will be getting as y 2 is equal to 0; so, d y 2 d d t d y 2 d y 1 d t is equal to y 2 equal to 0; that is one steady state. And at the steady state the sin y 1 d y 2 d t equal to 0; so that will give you sin y 1 with a minus k should be equal to 0. Now, k being a non zero constant; so, therefore, sin y 1 is equal to 0 and the solution is y 1 is equal to n pi, where the index n earns from 0 plus minus 1, 2 like that.

Now, consider, so the steady state is nothing but represented by this two states, y 1 equal to 0 and y 2 equal to 0; so, these are the steady state of this problem. Now, let us identify the matrix the Jacobian matrix f x f y g x g y; this simply means this is nothing but del f 1 del y 1 del f del y 1 del f del y 2 del g del y 1 and del g del y 2, evaluated at steady state, that means, y 1 is equal to y 2 is equal to 0.

Now, we are already seen that f is nothing but y 2, and g is nothing but minus K sin y 1; so, if that is the case del f del y 1 will be equal to 0, del f del y 2 will be equal to 1, del g del y 1 will be nothing but minus k cos y 1, and del g del y 2 will be equal to 0. So, this will be evaluated at the steady state, that means, at y 1 equal to 0 and y 2 equal to 0; so, this will be 0 minus K 1 0, so that is the a matrix the Jacobian matrix.

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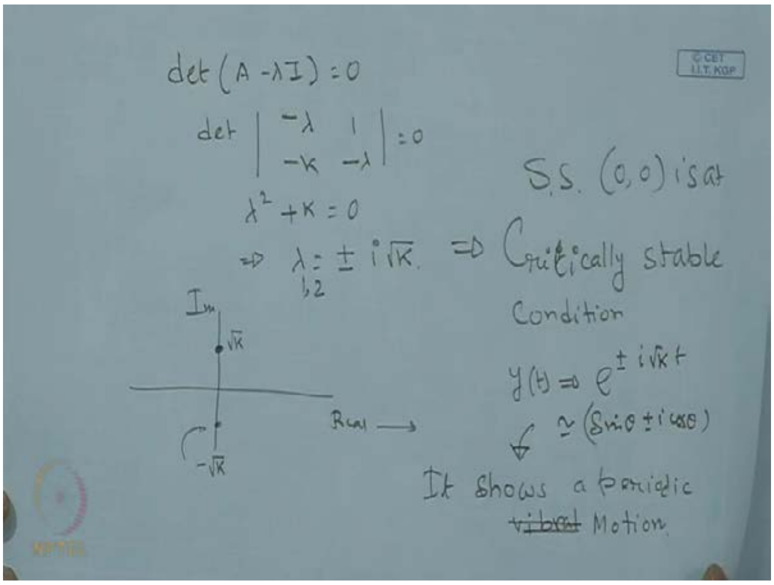

Now, if you like to find out the Eigenvalues of these determinants of this matrix A; so, let us look into the Eigenvalue of this matrix A, so that we can find out, what are the Eigenvalues of this system. So, to find out the Eigenvalue of this matrix, so determinant of A minus lambda I should be equal to 0; so, this will be, determinant of this matrix will be equal to 0 so it will be minus lambda 1 minus k minus lambda that should be equal to 0.

So, lambda square, minus into minus, plus k will be equal to 0; so, lambda becomes plus minus i root over k. So, now, this will be lambda 1, 2, if you look into the phase place plot, so this is imaginary axis this is real axis; so, this roots they are lying on this axis, so this is root over k and this will be minus root over k; so, they are on the verge of crossing over the left of plane to the right of plane.

So, we cannot say that, they are unstable or stable, so we call them as this condition is known as the steady state, is known as the critically stable. So, this is neither on the right of plane, which means, they are unstable; they are neither on the left of plane left of plane, so which is stable; so, this stability is known as the critically stable the steady state 0; 0 is at critically stable condition and it shows a periodic vibration. So, the disturbance, if you look into the solution of the disturbance, the disturbance solution will be in the form of, of e to the power lambda t; so, it will be in the form e to the power plus minus some i root over k times t; so, it will be composing of sin theta plus minus cosine i cosine theta. So, it will be in this form; so, therefore this indicates that, it shows a periodic vibration or periodic motion.

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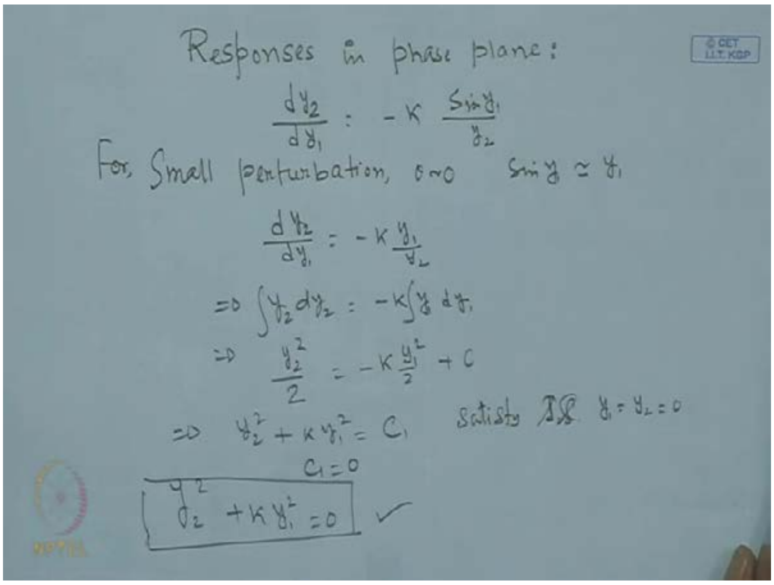

So, responses in phase plane plot we can say, we can look into responses in phase plane; we divide the governing equation into one after, one by the other; so, d y 2 d y 1 is nothing but minus k sign y 1 divided by y 2. So, for small perturbation, that means, if theta is small, the angular amplitude is small, then sin y 1 can be approximated by y 1; so, we will be having d y 2 d y 1 is equal to minus k y 1 by y 2. So, it will be y 2 d y 2 is equal to minus k y 1 d y 1. So, we can integrate it out over integration what will be getting is y 2 square by 2 is equal to minus k y 1 square by 2 plus c.

So, you will be getting y 2 square plus k y 1 square is equal to some constant c; so, it will be new constant to see that will be new constant. This should satisfy the initial condition that is y 1 is equal to y 2; this should satisfy the steady state y 1 plus y equal to y 2 equal to 2. So, if you do that, then c 1 trans out to be 0 so y 2 square plus k y 1 square will be equal to 0; that will be the governing equation in the phase plane plot.

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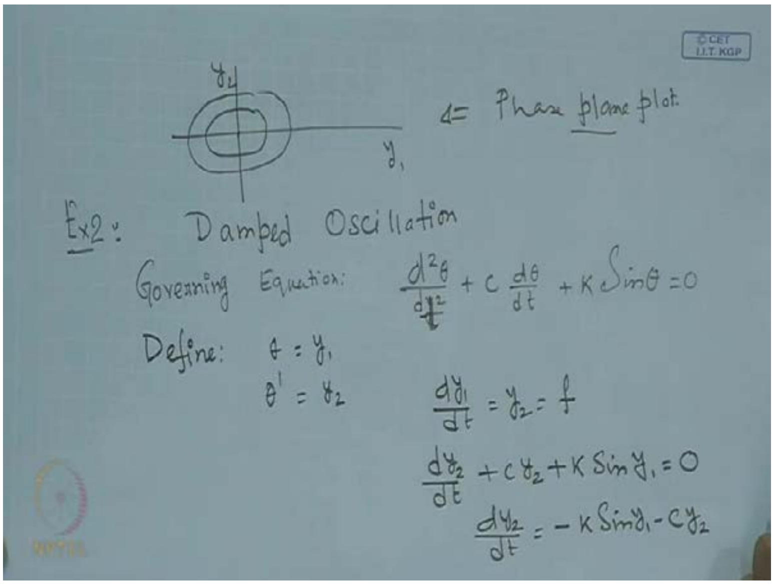

Now, if you plot this equation, so see what we will be getting. If you plot y 2 and y 1 will get will be getting the concentric, you will be getting the elliptical contuse for different values of case; so that will be the response in phase plane plot. Next, we look into the example of damped oscillation.

Now, in case of damped oscillation, the governing equation will be, d square theta d y square plus c d theta d square theta d t square plus c d theta d t plus k sin theta equal to zero. So, in that case, we define theta is equal to y 1, so, theta prime is y 2; if we define these two parameters, so d y 1 this equation govern the equation can be broken into two ordinary differential equation. So, this will be d y 1 d t is equal to y 2 is equal f d y 2 d t plus c y 2 plus k sin y one is equal to 0; so, that will be the second equation. So, you will having d y 2 d t is equal to minus k sin y 1 minus c y 2.

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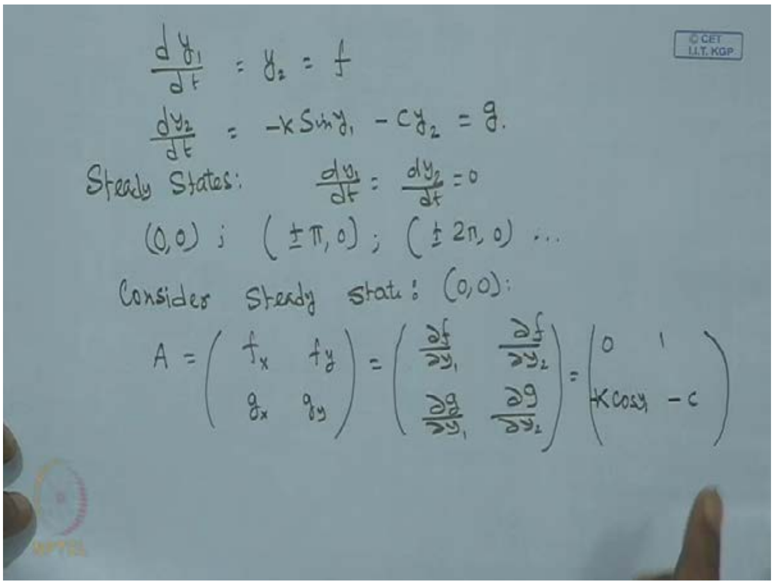

So, therefore, we get down to the governing equation of the system; so, what are the governing equation, just write it down once again, so d y 1 d t will be is equal to y 2 that is equal to f d y 2 d t is equal to minus k sin y 1 minus c y 2 is equal to g. So, let us look into the steady state of this problem; so, the existing steady states are when putting d y 1 d t is equal to d y 2 d t is equal to 0. And I am just writing the steady state, so it will be 0; 0 is 1 steady state plus minus pi, 0 is another steady state plus minus 2 pi, 0 is another steady state, likewise.

So, consider the steady state 0, 0 and see what we get out of it 0, 0; so, the matrix A is, let us write the matrix Jacobian matrix A, f x f y g x g y; so, it will be simply del f del y 1 del f del y 2 del g del y 1 del g del y 2. And this becomes del f del y 1 so it becomes 0, del f del y 2 it is 1, del g del y 1 so it will be k cosine y 1, and with a minus sin because this minus is there and del g del y 2 will be nothing but minus c.

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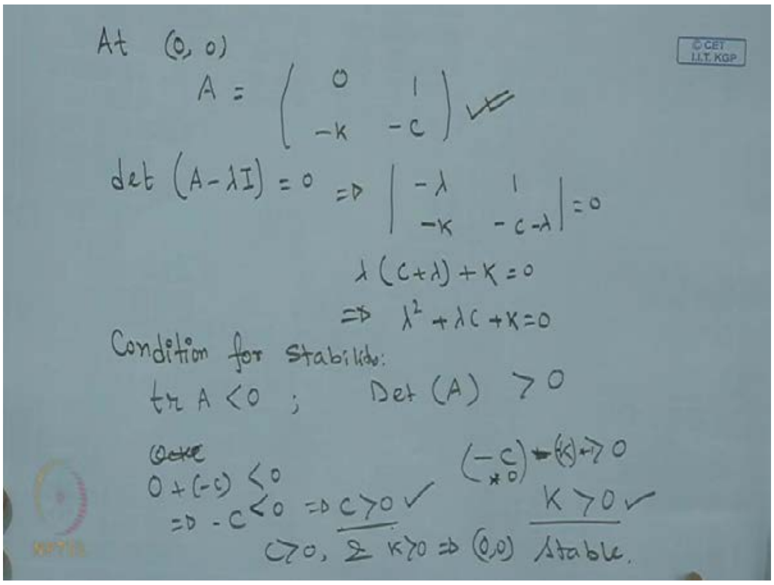

So, that is the Jacobian matrix, we evaluate this Jacobian matrix at the steady state. So, at 0, 0 we evaluate this Jacobian matrix; so, at 0, 0 we evaluate this one, A this becomes 0 1 minus k minus c.

Now, we will put the condition we evaluate the Eigenvalues; so, Eigenvalues will be evaluated by putting the determinant A is equal A minus lambda I to be equal to be 0. If that is the case, will be getting minus lambda 1 minus k minus c minus lambda is equal to 0; that means, there will be lambda into c plus lambda minus into minus plus k is equal to 0. So, will be having the condition lambda square plus lambda c plus k is equal to 0. Either you evaluate that are you can use the condition by looking into this matrix, one can find out the condition for stability. If you remember the condition for stability is that trace of the matrix A is less than 0 and determinant of matrix A should be greater than 0; so what is the trace, 0 plus c 0 plus minus c.

So, this basically addition of the diagonal element; so, 0 plus minus c should be equal to 0; that means, c is positive; minus c is less than 0, that means, c is positive, so that makes the trace of the matrix A is 0. And what is determinant of this, determinant of this will be this minus, this multiply this, minus c, minus into minus, plus k should be greater than 0; so, that means, c multiplied by 0 minus k multiplied by 1; so, minus into minus plus k, so k greater than 0. So, k greater than 0 and c greater than, c is greater than 0 because trace of c traces of a should be less than 0; so, minus c is less than 0, c greater than 0; so, c greater than 0 and k greater than 0 will lead to the steady state 0; 0 a stable steady state. So, this two condition, c greater than 0 and k greater than 0, will lead to this stable steady state.

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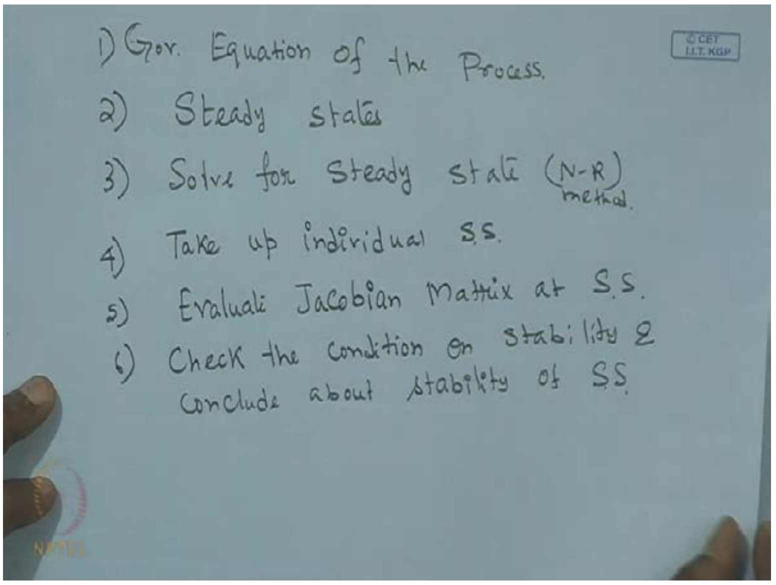

So, any such problems, the first thing, first we have to formulate the governing equation of the process, governing equation of the process. Next, we have to evaluate the steady state by putting the time variables, if the time derivatives to equal to 0. Third will be solve for the steady state, once you solve for the steady states, by you can use Newton Raphson, may be one of the method for a complicated problem. Then you take up individual steady state, evaluate Jacobian matrix at that steady state. And next is that you check the condition on stability and conclude about stability of steady state.

So, next we go head, so this will be the steps. So, once we get the stability of the steady state, then you take up the another steady state and repeat this process evaluate the Jacobian matrix, at the particular steady state; check the condition of stability and conclude about the stability of the steady state or you find out the conditions of the parameters, so that the steady stable or unstable.

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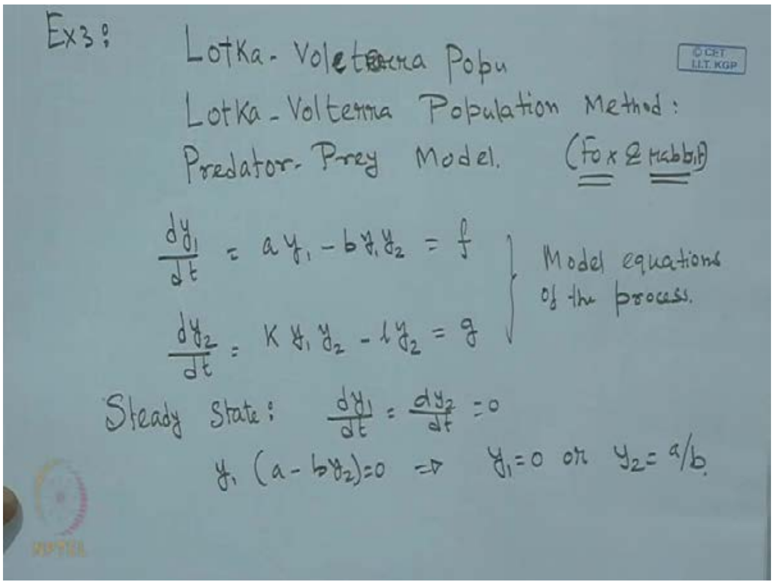

Next, we look into the third example; example 3, this is about the Lotka-Volterra population method and there is spelling mistake here and it will be Lotka-Volterra population method and in this method the model is known as the Predator-Prey Model. So, it is like a fox and rabbit population in a forest; if number of foxes, that if the population of the fox increases when the population of the rabbit decreases; once the population of fox population of rabbit decreases, then because of dearth of scarcity of food, the population of fox decreases and population of rabbit increases and it keeps on going like that. So, the governing equation for this problem is given by d y 1 d t is equal to a y 1 minus b y 1 y 2 that is equal to function f; d y 2 d t is equal to k y 1y 2 minus l y 2 is equal to g.

Now, these are the two models, model equation of the process. So, now, let us look into the steady state; the steady states are obtained by putting d y 1 d t is equal to d y 2 d t is equal to zero. So, if you put the first equation to be 0, it becomes y 1 a minus b y 2 equal to 0; so, it has a two solution y 1 equal to 0 or y 2 is equal to a by b.

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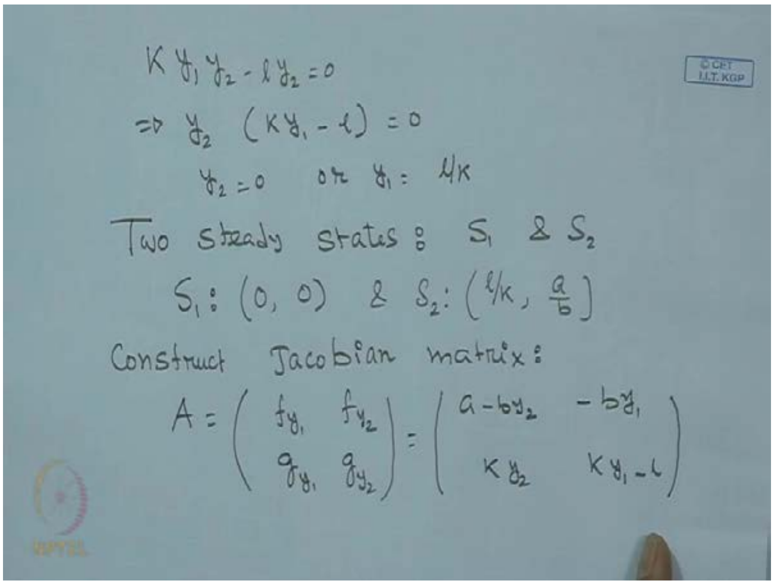

Now,, if you put the second equation to be 0 second transient equation to be 0, will be getting k y 1 y 2 minus l y 2 equal to 0. So, you will be getting y 2 into k y 1 minus l equal to 0; so, therefore, either y 2 equal to 0 or y 1 is equal to l by k. So, there exists two steady states in this particular problem; these two steady states are denoted by S 1 and S 2, S 1 and S 2 and S 1 is 0, 0 and S 2 is l by k and a by b; so, these are the two steady state of this particular process and we will check the stability of this two steady states one after another, but before that we construct the Jacobian matrix.

So, we construct Jacobian matrix and if you look into the Jacobian matrix, this will be f y 1 f y 2 g y 1 g y 2; so, this will be del f del y 1 del f del y 2 del g del y 1 del g del y 2; so, this will be a minus by 2 minus by 1 k y 2 k y 1 minus l. So, this is the construction of Jacobian matrix. Now, we take up the steady state S 1 and evaluate the Jacobian matrix A and look into the stability by looking into the trace and the determinant conditions.

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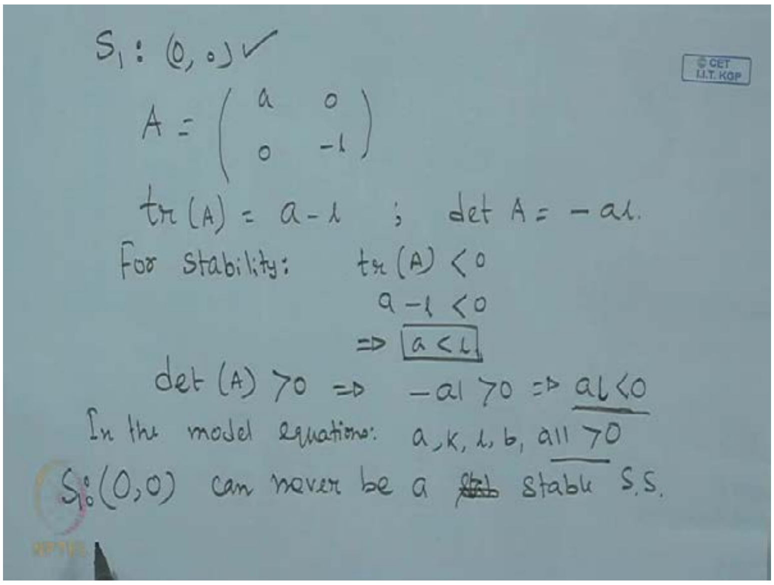

So, consider steady state 1 that is nothing but 0, 0. So, evaluate the Jacobian matrix at 0, 0, so this will be a 0 0 minus l; so, this will be the construction of the evaluation of Jacobian matrix at the steady state 0, 0. So, you find out trace of a will be nothing but this plus, this so a minus l; and determinant of A will be nothing but minus a l.

Now, for stability of the, of this particular steady state, trace of A has to be equal to minus 1; so, a minus l should be less than 0; so, a must be less than l that is required condition number one. And next condition is determinant of A should be greater than 0, that means, minus a l should be greater than 0 and a l has to be less than 0.

Now, in the model equation, if you look the model equation, the constants those appear a, k, l and b, all are positive constants so multiplication of two positive constants cannot be 0; therefore, this steady state cannot be a stable steady state. So, 0, 0 that is S 1 can never be a stable steady state, because the constants those are appearing the governing equations, they all positive and multiplication of two positive numbers cannot be negative.

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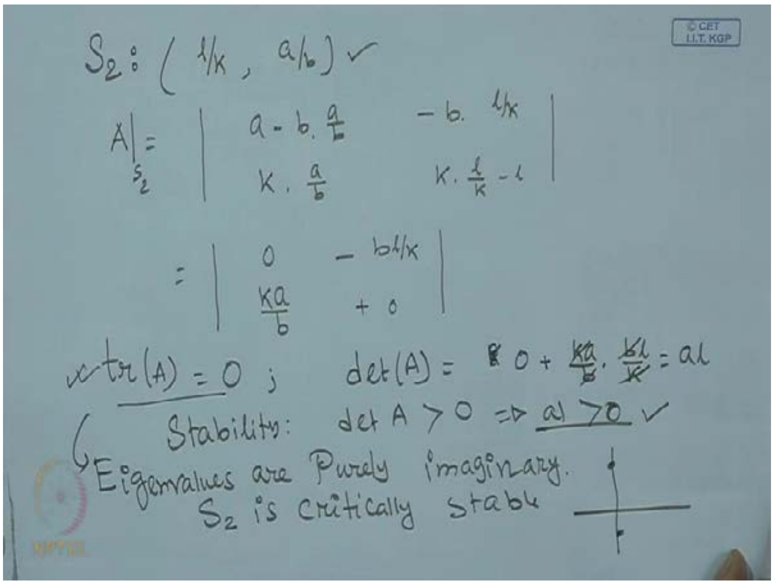

So, therefore, this particular steady state can never be a stable steady state. Then, let us look into the steady state number 2 that is S 2; we examine the stability of this particular steady state S 2 is l by k and a by b, evaluate the Jacobian matrix as S 2, so this will be a minus evaluated at a by b, k into a by b minus b l by k and k multiplied by l by k minus l.

So, therefore, this becomes zero minus b l by k ka by b plus 0; so, this will be the Jacobian matrix evaluated at the steady state l by k and a by b. Again, we look into the trace, trace of A is basically some of the, of this two values. So, in the corner elements, so it will be 0, and determinant of A will be nothing but this minus, this, so determinant of a will be k by b, so there we will be 0 into 0, 0 minus into minus, plus ka by b into b l by k, this becomes b b will be cancelling out, and k k will be cancelling out; so, this becomes a times l.

Now, for the stability determinant of A has to be greater than 0; therefore a l has to be greater than 0; there is no problem in that because both a and l are positive constants. So, multiplication of them only greater than 0, but trace of A makes them equal to 0; that means, the Eigenvalues, if you remember if trace becomes 0, the Eigenvalues are purely imaginary; this simply means the Eigenvalues are purely imaginary. Therefore steady state 2 is critically stable, because the both the steady state are lying on the imaginary axis, both the steady states are lie on the imaginary axis and they are critically stable slight deviation, they may be unstable.

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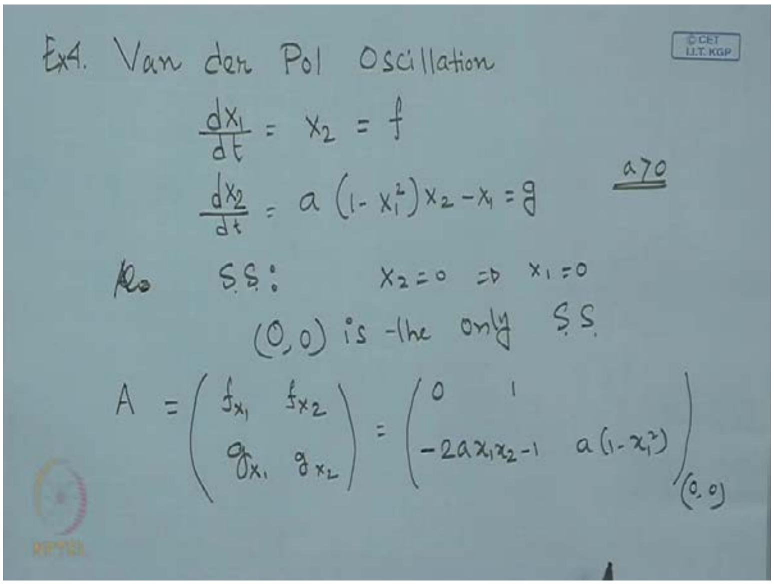

The next example, we will be talking about the something called Van der pol oscillation; that is example number 4. This is Van der pol oscillation, the first governing equation of this process is given as d x 1 d t is equal to x 2 that is equal to f; the second equation is d x 2 d t is equal to a 1 minus x 1 square multiplied by x 2 minus x 1 this is equal to g and this constant a is a positive constant.

Now, we evaluate the Jacobian matrix, so we find out the steady state; the steady state will be obtained by putting x 2 d x 1 d t equal to 0; that means x 2 equal to 0. And if you put x 2 is equal to 0 then from the other equation will be getting x 1 is equal to also 0; so, 0, 0 is the only steady state of this particular problem. So, next we evaluate the Jacobian matrix A is f of x 1 f of x 2 g of x 1 g of x 2. And this will be equal to 0 1, and g of x 1 is nothing but 2 a 2 2 x 1, so 2 a x 1 x 2 minus 1, and g of x 2 will be simply, a times 1 minus x 1 square.

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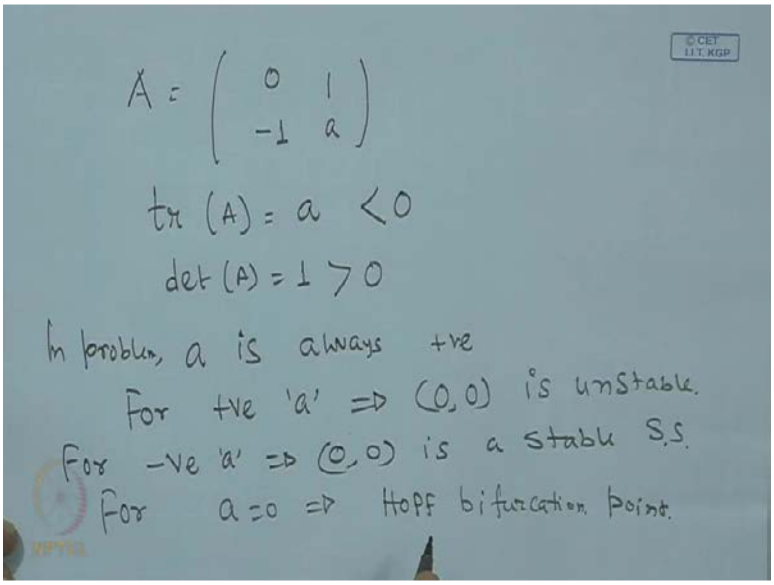

So, these are the foue elements of the Jacobian matrix and we evaluate this Jacobian matrix at the steady state 0, 0, if we do so, then after evaluation the Jacobian matrix becomes 0 1 minus 1 and a, these are the elements of the Jacobian matrix. So, evaluate trace of A the trace of A is a summation of the diagonal elements so it becomes a. And on the other hand determinant of A becomes 0 into a, that is 0, minus into minus, plus so it will be 1.

Now, determinant a, of a, is always 0, but trace of a should be negative. But as the problem is given that in the problem, it was given that a is always positive. So, therefore, for positive a, the steady state 0, 0 is unstable. On the other hand, if in the problem it was given that a is negative, for negative a trace negative a, that is a is less than 0, that is satisfied, then this 0, 0 is a stable steady state. For a equal to 0, then we have the HoPF bifurcation because trace becomes equal to 0 that presents a HoPF bifurcation point.

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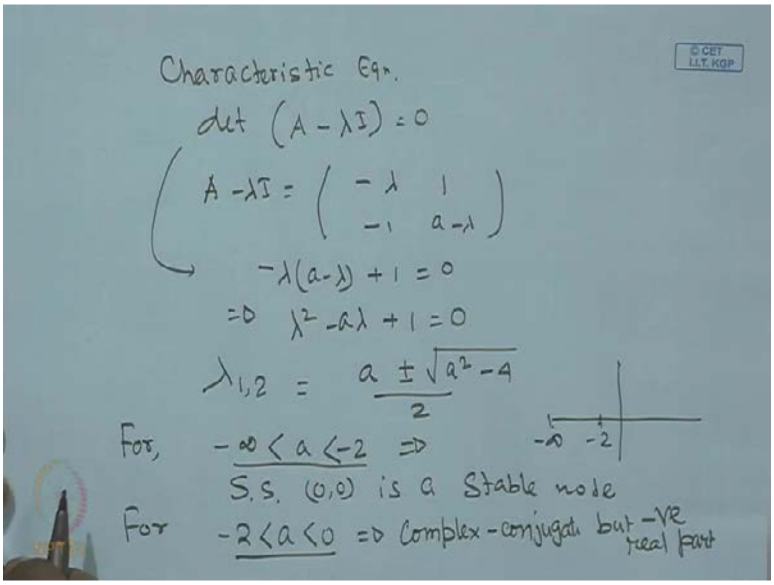

Now, we can further analyze the system, if we get into the characteristic equation of the Eigenvalues. So, if we look into the characteristic equation of the Eigenvalues will be given by, determinant of by setting, determinant of A minus lambda I is equal to 0. So, you will be having A minus just construct the A minus lambda I, matrix minus lambda 1 minus 1 a minus lambda; so determinant of that will be equal to 0; so, minus lambda a minus lambda, minus into minus, plus 1 will be equal to 0. So, this will be lambda square minus a lambda plus 1 is equal to 0. Now, if you look into the roots of this characteristic equation which will be corresponding to the Eigenvalues, two Eigenvalues this will be minus b plus minus under root b square minus 4 divided by 2.

Now, if we see that, for if you try to fix up a domain of the parameter a, then for minus infinity lie, a lying in between minus infinity to minus 2; that means a is in the negative domain. So, if it is minus infinity to minus 2, then if a is lying between minus infinity 2 minus 2, then stable then the steady state 0, 0 is a stable node.

So, for minus 2 a lying in between minus 2 to 0, if a is lying in between minus 2 to 0, then you will be having a negative real part, but at the same time, this will be the material with the argument, with in the square root becomes less than 1; so, if it is lying between minus 2 and 0 the argument with in the under root becomes less than 1. So, it becomes, the whole thing becomes imaginary, at the same time the real part of this will be negative.

So, this is condition, then we will be having imaginary roots, may know complex conjugate roots with negative real part. So, Eigenvalues becomes complex conjugate but negative real part, so for this case we will be having a stable focus.

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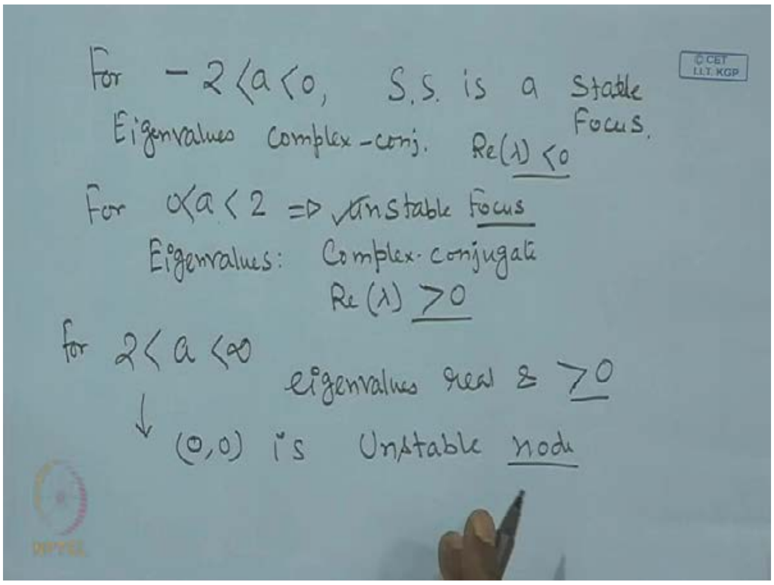

So, for a lying between minus 2 and 0, the steady state is a stable focus, because roots the Eigenvalues are imaginary and Egenvalues are complex conjugate and real part of Eigenvalues are negative. For a lying in between 0 and plus 2, what we will be having is that, wiil be having a unstable focus, because in this case Eigenvalues will be again complex conjugate, but real part of the Eigenvalues will be 0 greater than 0.

So, therefore, you will be having a focus, but that focus will be unstable focus because the real part of the Eigenvalues are greater than 0. Now for a lying in between 2 to infinity, we will be having, if this is the case, then if you look into the solution a plus minus under root a square minus 4. So, therefore, we will be having the Eigenvalues real and greater than 0, always positive; so, we will be getting the unstable node. So, for this condition, the steady state 0, 0 is nothing but an unstable node.

So, for different domains of a, one will be getting the different times, types of steady state. In one case, we will be getting the stable focus; in another case, will be getting the unstable focus; in two domains of, for a will be getting the stable node and unstable node always.

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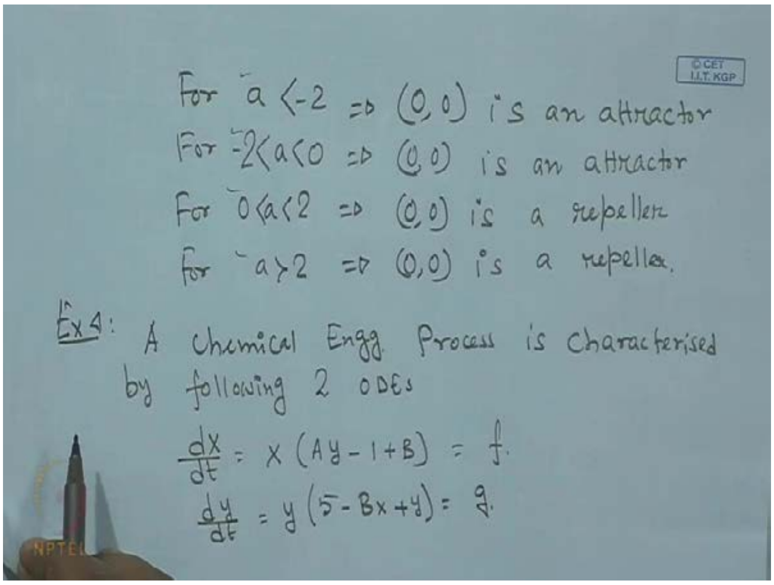

So, you can make this phase plane plot, that for a is less than minus 2, you can come to conclusion that steady state 0, 0 is an attractor, because it will be a stable node. For a lying in between minus 2 and 0, the steady state 0, 0 is an attractor, because that will give you a stable focus. For a lying between 0 and 2, the steady state 0, 0 is a repeller. And for a greater than 2 the steady state 0, 0 is again a repeller.

So, therefore, this gives a particular demonstration of example, where different values of the parameter. You will be landing up the different steady states which can be an attractor or can be repeller, can be a node, can be a focus, can be a stable node, can be unstable node, can be stable focus, can be unstable focus.

Then, will move on to another example and that will be quite interesting example, because in this case, we will be not only checking the steady state, we will be putting up conditions in the parameters, such that, stability will be implemented or it will be imposed.

So, in this particular problem, what we are doing, a chemical engineering process is characterized by the following 2 ODEs. And this ODEs are given by d x d t is equal to x A y minus 1 plus B is equal to we called that as f and d y d t is equal to y times 5 minus B x plus y and we called that as g.

Now, let us find out the steady; so, the question is find out the steady state of this process and evaluate the stability of the steady state. And find out the conditions in the parameter, so that the steady state is stable or unstable.

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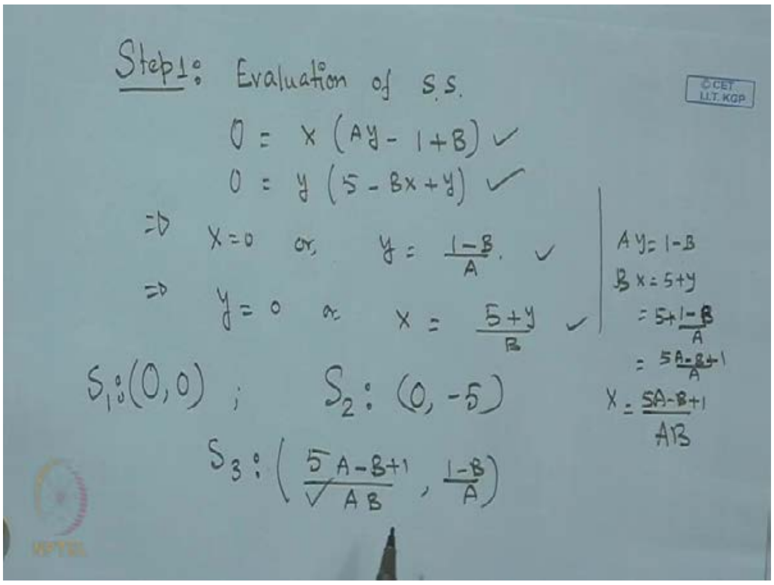

So, first step that we are going to do is evaluation of steady state. To evaluate the steady state - so, it becomes 0 x A y minus 1 plus B, and 0 y into 5 minus B x plus y; so, the first equation will give you the solution x is equal to 0 or y is equal to 1 minus B divided by A. From the second one you will be getting y is equal to 0 or x is equal 5 plus y divided by B. So, 0, 0 is one steady state, so you are having two steady states in this problem and the second steady state 0 minus 5 is another steady state.

If x is equal to 0, you can put, you can get y is equal to minus 5; so, 0 minus 5 is another steady state and by solving these two equations, by solving these two equations, one can get the third steady state. And the third steady state is 5 A minus B plus 1 divided by A B and 1 minus B by A. How to get this steady state? If you simply solve these two equations A y is equal to 1 minus B and B x is equal to 5 plus y.

So, if you 1 minus, so if you put the value of y there, so it becomes 5 minus 1 minus A by 1 minus B by A; so, 5 A minus B, 5 plus y, so B x will be 5 plus y, y is 1 plus 1 minus B, so it becomes 5 A minus B plus 1 divided by A.

So, x becomes 5 A minus B plus 1 divided by A B. So, you will get the first value of the steady state and corresponding to that you, put into here y becomes 1 minus B by A. So, you will be having three steady state into this problem; by you can get these three steady states by solving these 2 equations. One will be getting the steady state number 1 that is 0, 0; steady state number 2 that is 0 minus 5; this is a steady state number 3 5 A minus B plus 1 divided by AB and 1 minus B over A.

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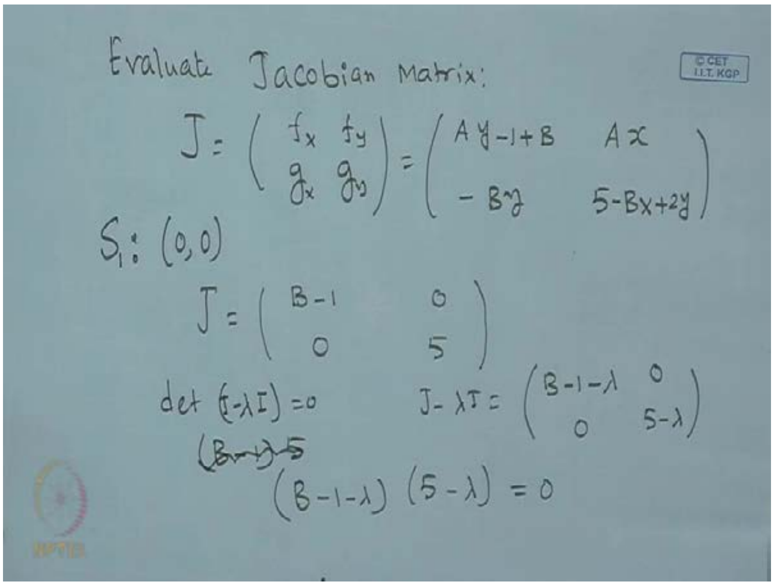

Now, we evaluate the each and every steady state and check the condition for their stability, but before that what we should do, we should evaluate the Jacobian matrix. If we evaluate the Jacobian matrix, J is nothing but f x f y g x g y; and this becomes A y minus 1 plus B A x minus B y 5 minus B x plus 2 y.

Now, let us, so this is the Jacobian matrix; now, we take up one steady state after another and check the stability of the steady state. So 0, 0 is the, for steady state, we check the stability of this one; so, evaluate the Jacobian matrix at 0, 0, so this becomes B minus 1 then it becomes 0 0 and 5. So, determinant, if you evaluate the Eigenvalues, so the characteristic equations becomes determinant of j minus lambda I should be equal to 0; so B minus 1 multiplied by 5, so this becomes J minus lambda I becomes B minus 1 minus lambda 0 0 5 minus lambda, so determinant of this will be equal to be 0. So, B minus 1 minus lambda multiplied by 5 minus lambda that should be is equal to 0.

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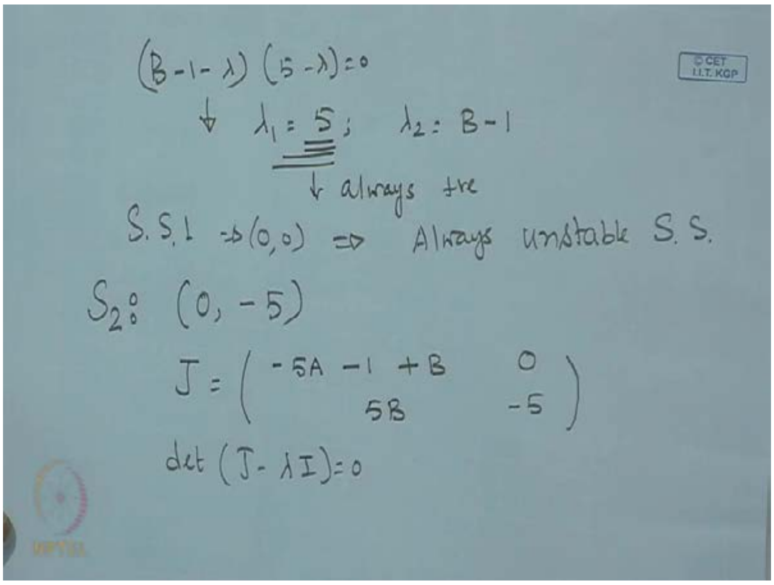

Now, if we land up with a quadratic and evaluate the roots of the quadratic. So this quadratic equation has to be evaluated, B minus 1 minus lambda multiplied by 5 minus lambda equal to 0. So, one root will be getting as 5, another root will be getting as B minus 1. Now, since, one root is always positive, now depending on the value of the lambda 2 can be positive and negative; for B less than 1 lambda 2 is negative for B greater than 1 lambda 2 is positive, so it does not matter because the other root is always positive.

So, this is always positive, that means, the steady state 1 that is 0, 0 is always an unstable steady state; so, it does not matter to the values of a and b and the parameters 0, 0 steady is always unstable.

Now, next, we examine the stability of the steady state 2, steady state 2 is 0 and minus 5. So, evaluate the Jacobian matrix at these steady state values, so it becomes minus 5 A minus 1 plus B 0 and this will be 5 B and minus 5.

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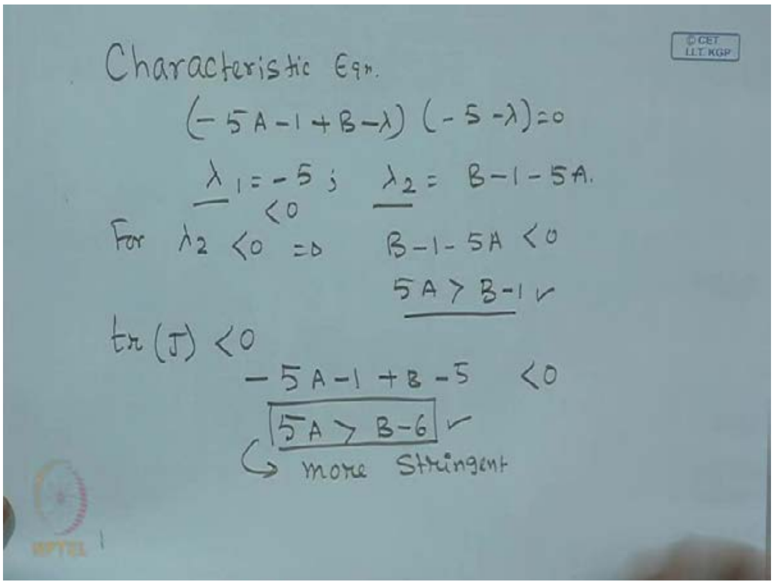

So, we again we evaluate the Eigenvalues by putting determinant of J minus lambda I is equal to 0. So, if we do that the characteristic equation becomes, so the characteristic equation, we write down the characteristic equation that becomes, minus 5 A minus 1 plus B minus lambda multiplied by minus 5 minus lambda is equal to 0. So, therefore first equation, so first root lambda 1 is equal to minus 5; and second lambda 2 is equal to B minus 1 minus 5 A. Now, lambda 2, so lambda 1 is minus 5, it is less than 0 and for lambda 2 is less than 0, we have to have, B minus 1 minus 5 A should be less than 0.

So, therefore, 5 A should be greater than B minus 1, so that is the condition so that both roots are negative and we will be having this steady state stable. Now, if you put into the other condition that is the trace of determinant of the Jacobian matrix is less than 0 will be landing up with minus 5 A minus 1 plus B minus 5, that is the summation of the diagonal elements of the Jacobian matrix should be less than 0.

So, this will be getting 5 A should be greater than B minus 6; in fact, between these two conditions 5 A is greater than B minus 6 is more stringent condition between these two; therefore, we can consider this thing, so that trace becomes less than 0.

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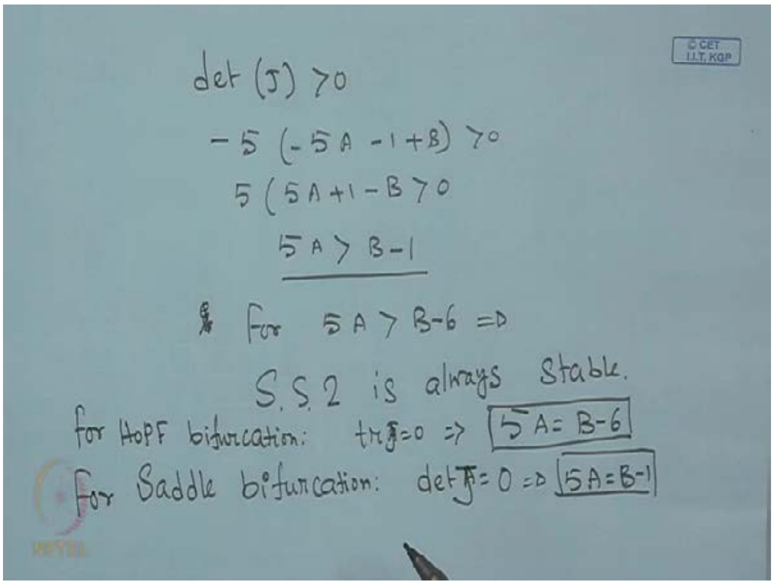

Then, we will also check the determinant of a should be, determinant of J the Jacobian matrix should be greater than 0. If you look into the determinant, this becomes minus 5 minus 5 A minus 1 plus B should be greater than 0.

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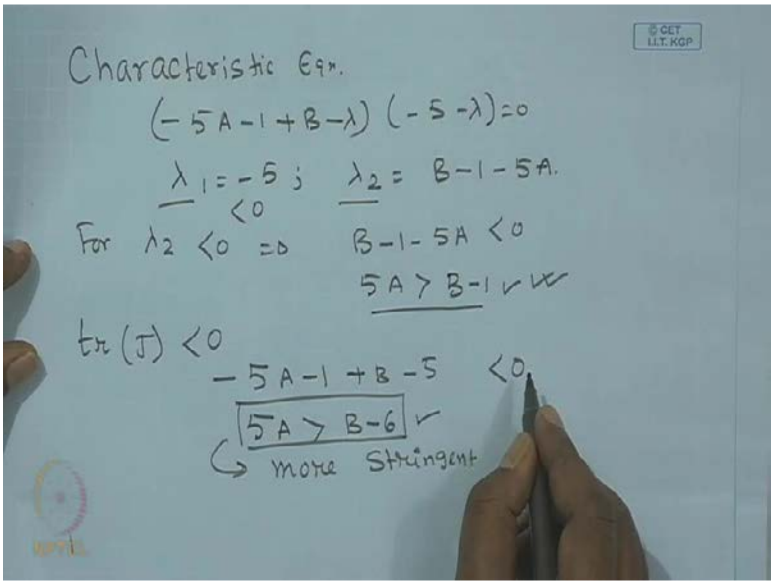

So, minus can be consumed here, so this will be 5 A plus 1 minus B should be greater than 0. So, one can have 5 A is greater than B minus 1; so, if you remember that we already got this equation earlier, so this equation we got from earlier; so, 5 A is always greater than B minus 1.

So, therefore between these two conditions is more stringent condition; therefore, we can say that lambda 1 that for 5 A is greater than B minus 6. The steady state S 2 is always stable, we can talk about the HoPF bifurcation and Saddle point bifurcation. For HoPF bifurcation trace of a should be, trace of Jacobian matrix should be equal to 0; so, the condition for that is 5 A is equal B minus 6. So, this is the condition on the parameters where the HoPF bifurcation occurs. Similarly, one can get the Saddle node bifurcation point; for Saddle node bifurcation point, you will be getting determinant of A should be, determinant of Jacobian matrix should be equal to 0 and you will be having 5 A is equal to B minus 1; so that is the condition for Saddle node bifurcation. And what is left now is that will be doing, probably the third steady state is left and again we will be evaluating the Jacobian matrix for the third steady state and see the stability values where in a stability criteria.

The trace of and evaluate the Jacobian matrix at steady state number 3, we evaluate the trace of the Jacobian matrix and determinant of the Jacobian matrix by putting the condition trace is less than 0 and determinant of is greater than 0 will be getting the conditions on a and b for the stability of the third steady state.

So, we stop here, we will take up the examination of third steady state in the next class and start from that point onwards.

Thank you.