

Advanced Mathematical Techniques in Chemical Engineering

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Lecture No. # 13

Stability Analysis

Very good morning to everyone. So, we were looking into the applications of Eigenvalues and Eigenvectors in various chemical engineering problems. So, in the last few classes we have looked into the solution of ODE, solution of the set of algebraic equations, set of ordinary differential equations and homogenous and non-homogenous equations, those will be arising out of the chemical engineering processes and how Eigenvalues and Eigenvector method can be applied quite elegantly and replacing the numerical techniques. And, including, involving, even if you go for a higher dimensional operation the numerically calculated values will be done quite easily involving very small subroutine of elementary in nature.

Now, what we have seen in the earlier classes is that we have developed the theory based on this, to solve the set of algebraic equations and ordinary differential equations, both homogenous form and non-homogenous form, and along with that we have given we have discussed some of the problems how to apply the Eigenvalue Eigenvector method for their solution.

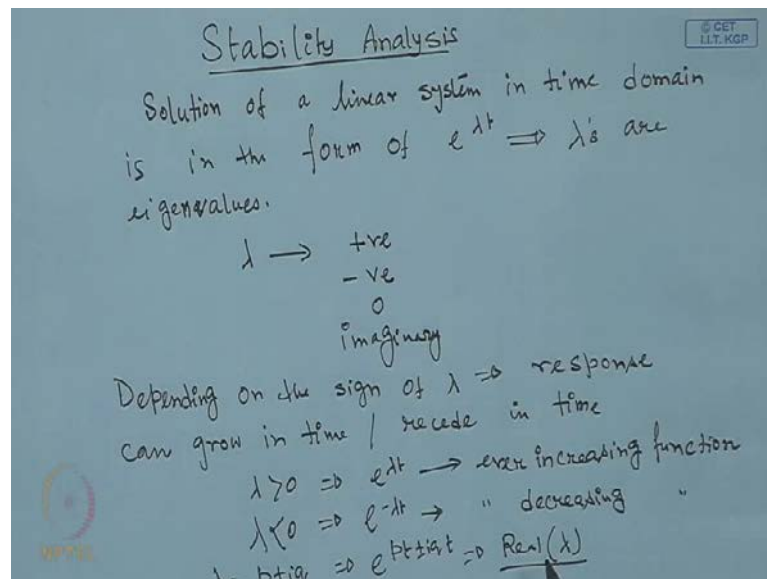
Now, in today's class, we will look into another application of Eigenvalues and Eigenvectors that will be appearing in the continuous domain. So, we will be looking into the Stability Analysis of any chemical engineering processes this stability is very important, as far as the chemical reaction engineering and chemical engineering processes are concerned, because consider any chemical reactor, if it is operating under the stable operating conditions then you can ensure the quality of the product we are going to get.

So, if the quality of the product is ensured, then it is marketability and market value will be ensured; so, we can do a very good business. If the stability of the system is not appropriate, then one cannot be very sure about the product quality; so, Stability

Analysis is very important, so we will be basically looking into the various theorems that will be applicable for analyzing the stability of any chemical engineering processes and after that we will be doing the analysis based on this theory and trying to extract the features or the set of operating conditions, under which the operation can be carried out under stable conditions.

So, stability is very important as far as the chemical engineering processes and whatever we will be doing from today's class onwards, that will be extremely important for the chemical engineers to analyze the stability of a system.

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So, let us look into the various methods involving Eigenvalues and Eigenvectors and we will develop the theory for a Stability Analysis. Now, if you look into the solution, that we have talking about that the solution **in, of a system,** of a linear system, even it is a valid for the non-linear system as well, solution of a linear system will be in time domain is in the form of e to the power λt , where typically the λ 's are Eigenvalues of the relevant problem. Now, this λ become, this λ can be positive, it can be negative, it can be when 0, it can be imaginary; if it is imaginary, then of course, it is a counter, the conjugative will be appearing; so, if there are two roots then there is a possibility of having imaginary roots.

Now, **there is,** if you can see in this problem - that **if the** depending on the sign of λ , the response can grow in time or recede in time; that means, if λ is

positive then e to the power λt is ever increasing function. On the other hand, if λ is never negative, then e to the power λt is ever decreasing function. If λ is imaginary, let say p plus minus $i q$, then e to the power, this will be entirely depending upon the real part of the λ .

Now, if real part of λ is greater than the 0, then the response will grow in time; if real part of λ is negative the response will die down in time; so, if e to the power λ is positive, if it is e to the power λt ever increasing function; that means, if you put a disturbance in a system, it will grow in time monotonically; on the other hand if we put a disturbance in a system for λ is negative the response will die down in time.

So, therefore in case of imaginary roots, it entirely depends on the real part of the root; if root is positive, then λ then the disturbance will grow in time; if this real part is negative, then the disturbance will die down or decay in time. Now, these will be extremely important as far as the stability of the system is concerned, that if you impart a disturbance in the system, if the response grow in time then we are looking, we are basically landing up into a system, which is not stable. On the other hand, if the response decays with time then will be landing up in a system, **where**, which will be stable.

So, the aim will be to have a stable region, so there will be conditions that this Eigenvalue will be negative then will be getting into the stable region and of course that will happen under a certain combination of the parameters, where the parameters are basically combination of operating variables. So, under those operating variables, so that they will satisfy those conditions, the system will be operating under stable steady state.

Now, in order to develop the theory, we will just take up a chemical engineering application and will take up whatever we have done earlier that, **is a**, in case of contraction mapping will take up an example of continuous stirred tank reactor, which was non-isothermal. So, we take up those conditions of the energy balance, those equations of energy balance and mass balance, to start up with our discussion in the context of stability of chemical engineering processes.

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Let us consider our CSTR Problem!

$$\frac{dc}{dt} = -c + Da(1-c)e^T \rightarrow \text{solute balance}$$

$$\frac{dT}{dt} = -(1+\beta)T + BDa(1-c)e^T \rightarrow \text{Heat balance.}$$

$\begin{cases} Da \rightarrow \text{Damkohler No.} \\ B, \beta \rightarrow \text{Non dimensional parameters} \end{cases}$

Steady States:

$$0 = -c + Da(1-c)e^T \dots (1a)$$

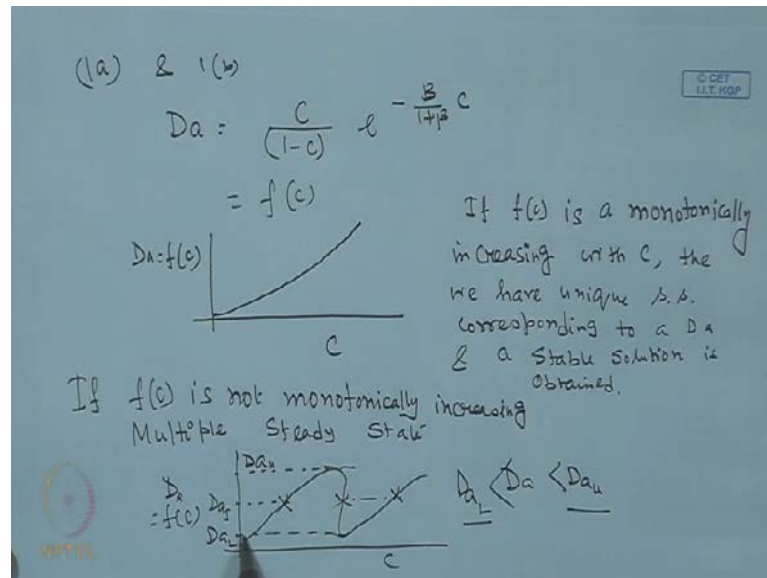
$$0 = -(1+\beta)T + BDa(1-c)e^T \dots (1b)$$

By contraction mapping if $B < 4(1+\beta) = 0$ Unique Steady state.

Now, let us consider the CSTR problem, whatever we have developed discussed in case of contraction mapping, our CSTR problem. We have the solute mass balance was given by $\frac{dc}{dt}$ is equal to minus C plus $Da(1-c)e^T$; this is nothing but solute balance. And $\frac{dT}{dt}$ is be given by minus $1 + \beta T$ plus B times $Da(1-c)e^T$; so, this will be the heat balance equation. The parameters in a system are Damkohler number, β is a non-dimensional parameter and B is another non-dimensional parameter; now, these three are the parameters of the system.

Now, let us look into the steady state of these system; the steady states will occur when this time derivate will be equal, to put, will be force to equal to 0; so, the steady state will be occurring by putting 0 is minus C plus $Da(1-c)e^T$; this is equation 1 a. And minus $1 + \beta T$ plus $B Da(1-c)e^T$; so, this is equation 1 b.

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So, we have already seen that by contraction mapping, if B is less than 4 into 1 plus β , then we will be having unique steady state for this particular problem. So, we have already seen by contraction mapping that if B is less than 4 into 1 plus β , unique steady state exists. So, we can combine equation 1 a and 1 b and we can write Da is equal to C 1 minus C e to the power minus B divided by 1 plus β times c .

So, this will be a function of concentration only. So, if we plot Da or f of C as a function of C , we can see that it will be an ever increasing function; now, if this is the case then if we have f of C is a monotonically increasing function which increasing with C . Then we have unique steady state corresponding to a Da to a Damkohler number and a stable solution is obtained; this we have proved earlier during the discussion of contraction mapping that if f of C is monotonically increasing function with concentration, then for that, for those particular Da Damkohler number will be getting the stable steady state.

On the other hand, if f of C is not monotonically increasing then will be landing up with multiple steady state; it means, if f of C is not monotonically increasing then we may land up with multiple solution or multiple steady state; convince, if we plot Da f of C Da is same as f of C verses concentration if the plot looks something like this, it is not monotonically increasing function, it is decreasing and then again it is increasing; then you will be having, they, a particular Da may be here intermediate, where there will be a

solution here, there will be a solution there, there will be a solution there; so, there will be three solutions they may exist.

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For multiple steady state, $f(C)$ must have a maxima & a minima.

$$f(C) = \frac{C}{1-C} e^{-\frac{BC}{1+\beta}}$$

$$f'(C) = \left[\frac{-(1+\beta) - BC(1-C)}{(1-C)^2 (1+\beta)} \right] e^{-\frac{BC}{1+\beta}}$$

For maxima/minima, $f'(C) = 0$

$$BC^2 - BC + (1+\beta) = 0$$

$$\Rightarrow BC^2 - BC + (1+\beta) = 0$$

$$\checkmark C_{1,2} = \frac{B \pm \sqrt{B^2 - 4B(1+\beta)}}{2B} = \frac{1}{2} \left[1 \pm \sqrt{1 - \frac{4(1+\beta)}{B}} \right]$$

Correspond to maxima & minima

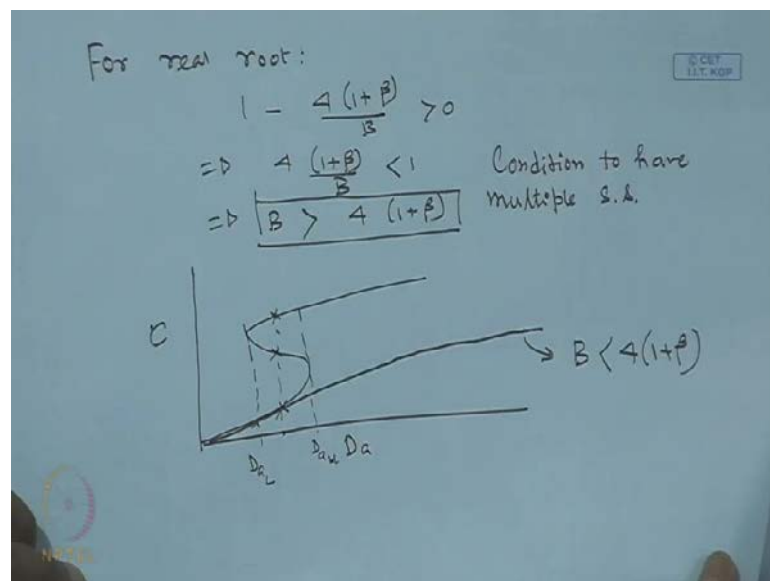
So, you can have the upper limit, so if this is the case, then you will be having the upper Damkohler number $D_{a,u}$ and this you will be getting a lower Damkohler number $D_{a,L}$; so, there will be Damkohler number in between $D_{a,L}$ and $D_{a,u}$, for which one can have a more than one solution or more than one steady state. So, between this limit $D_{a,L}$ and $D_{a,u}$, one can have multiple steady state of this particular problem. So, **of course**, what we can understand from this discussion is that in order to have existence of multiple steady state, the $f(C)$ will be having a maxima and will be having a minima, then only if the function of concentration will be having a maxima and minima, then only one can expect of occurrence of multiple steady state. Now, let us put this condition in this particular case; so, for multiple steady state $f(C)$ must have a maxima and a minima; so, $f(C)$ should be equal to C divided by $1 - C$ e to the power minus BC divided by $1 + \beta$, so for in order to have maxima or minima.

Let us get a differentiation of this, so this becomes minus, this becomes $1 + \beta$ minus BC into $1 - C$ divided by $1 - C$ square into $1 + \beta$ e to the power minus BC divided $1 + \beta$; so, this is the differentiation of $f(C)$. So, I am just writing the final expression, so for maxima or minima $f'(C)$ will be equal to 0.

So, therefore one can get this equation $B C$ square, so if basically the numerator will be equal to 0, so it will be, $1 + \beta - B C + B C^2$ is equal to 0; so, you will be having condition $B C^2 - B C + 1 + \beta = 0$. Now, this will be having two roots, this is a quadratic in C , so it will be having two roots. So, let us find out what these two roots are, they will be $\frac{-B \pm \sqrt{B^2 - 4(1+\beta)B}}{2B}$, so B times $1 + \beta$ divided by $2a$, that means, $2b$.

So, if we take B common from here, so what will be getting is that $1 \pm \sqrt{1 - 4(1+\beta)/B}$ divided by B ; so that is the condition, there are two roots, one root corresponds to maxima and other root corresponds to minima; so, these two roots correspond to maxima and one for minima. Now, we can have, in order to have a real solution, we can have the condition on this part, that this $1 - 4(1+\beta)/B$ should be greater than 0 in order to have a real root or real solution.

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So, let us do that for real root the quantity within the under root sign will be must be greater than 0; so, therefore $1 - 4(1+\beta)/B$ should be greater than 0; so, therefore $4(1+\beta)/B$ should be less than 1; so, will be having B is greater than $4(1+\beta)$. So, this is the condition for this particular problem in order to have a multiple steady state.

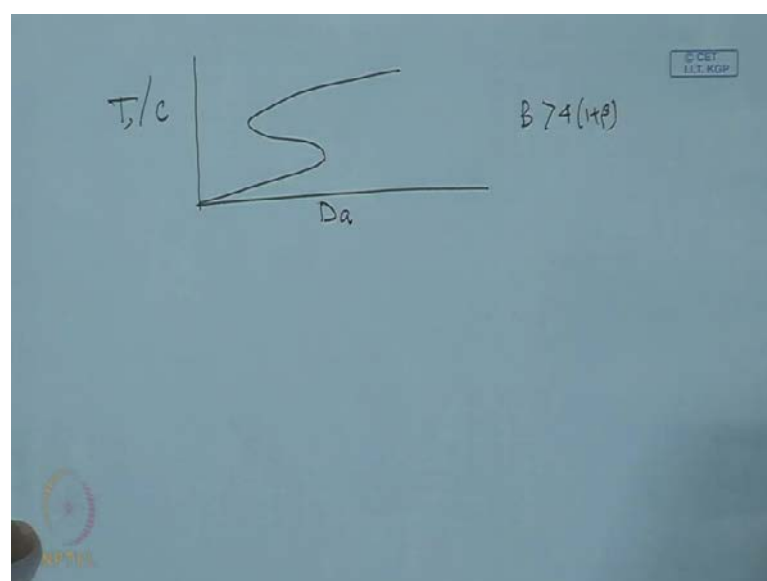
So, this becomes that condition to have multiple steady state of this problem, this is just a counter part of whatever we have obtain for the condition, of unique, having the unique

steady state of this problem. So, if you remember that the condition, we obtain from contraction mapping for existence of unique steady state is B less than $4(1 + \beta)$ solve; obviously when B is greater than $4(1 + \beta)$ that is a condition for the multiple steady state and we have derived that.

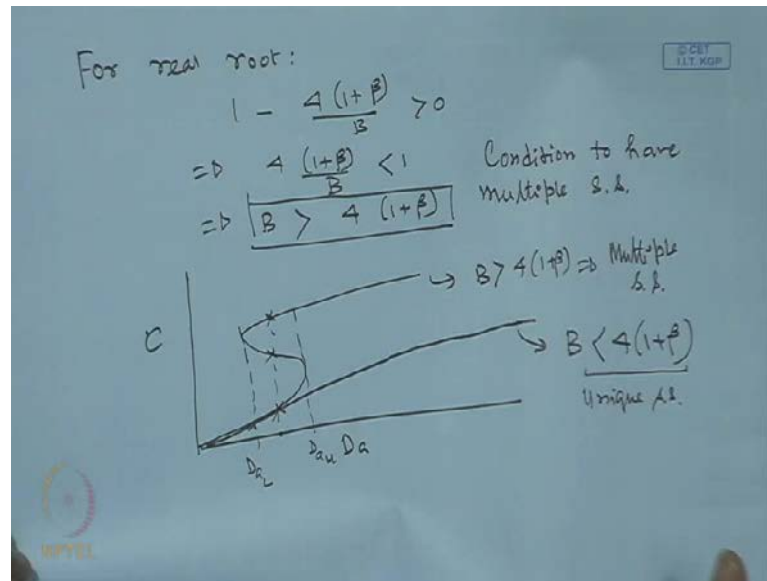
So, now, if you plot C versus Damkohler number versus C or C versus Damkohler number whatever way, so this will be, **they**, there may be one response like this, there may be a response like this. So, **this is**, in this case for every C will be having only for every Damkohler number, you will be having only one C , **for every**, for this curve, in the, for the lower curve for every existence of Damkohler number, you will be getting only one concentration, one solution steady state solution. And therefore, on this curve the unique steady state exists, so B is less than $4(1 + \beta)$.

On the other hand, if you talk about this range of Damkohler number, for this Damkohler number any Damkohler number between D_{aL} and D_{aU} ; it there will be having three steady state or multiple steady state existing for this problem. Any Damkohler number beyond D_{aU} will be leading to a steady state on the upper branch; any Damkohler number below the value of D_{aL} , one will be getting a solution at the lower branch; for any Damkohler number laying in between D_{aU} and D_{aL} , it will be having a multiple steady state in the system.

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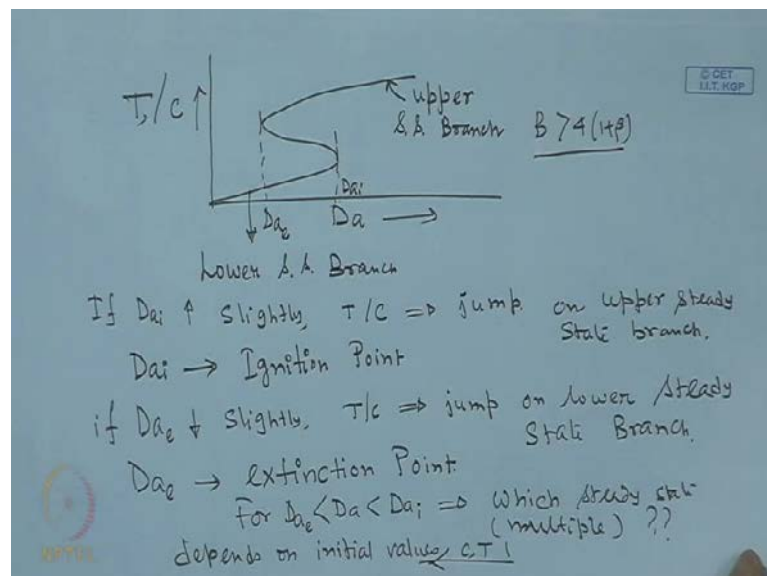


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So, if we plot now for this particular problem, if we plot now, T or C verses Damkohler number for multiple steady state part, will be getting a curve like this; this is the condition for B greater than 4 into 1 plus beta. So, if we summarize whatever we have obtained in the earlier slide, that this is for this case B is greater than 4 into 1 plus beta; that means, this is for condition for unique steady state and this is the condition for multiple steady state.

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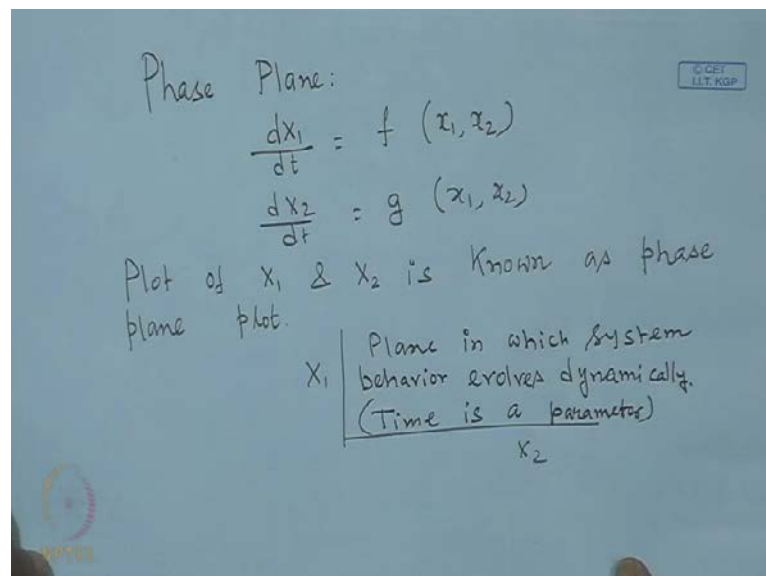


So, next, we have the T and C verses Damkohler number, for the condition B greater than 4 into 1 plus beta. This is known as upper steady state branch, and this is the lower steady state branch. Now, suppose, this is Da_i , so if Da_i increases slightly both T and C, they jump on the upper steady state branch; therefore, Da_i is called an ignition point. So, if you just slightly increase it is beyond Da_i , it will go to the upper steady state branch; so, Da_i is called the ignition point.

On the other hand, the other limit is Da_e , if we slightly decrease Da_e , if Da_e is decreased slightly, then T or C, they jump on lower steady state branch; so, Da_e is known as extinction point. So, which steady state will be, if Da is in between, the Da_e and Da_i , if we operate on any Damkohler number will be having a multiple steady state.

Now, for Da laying in between Da_e and Da_i which steady state is obtained, that will be entirely depend on the initial values of T and C, which steady state among multiple; that is the question. Then, it entirely depends on initial values of C and T; so, depending on the initial value of C and T, one can **get this, the steady state in between**. Now, next we talk about all these phenomena can be properly explained, if we talk about a phase plane plot.

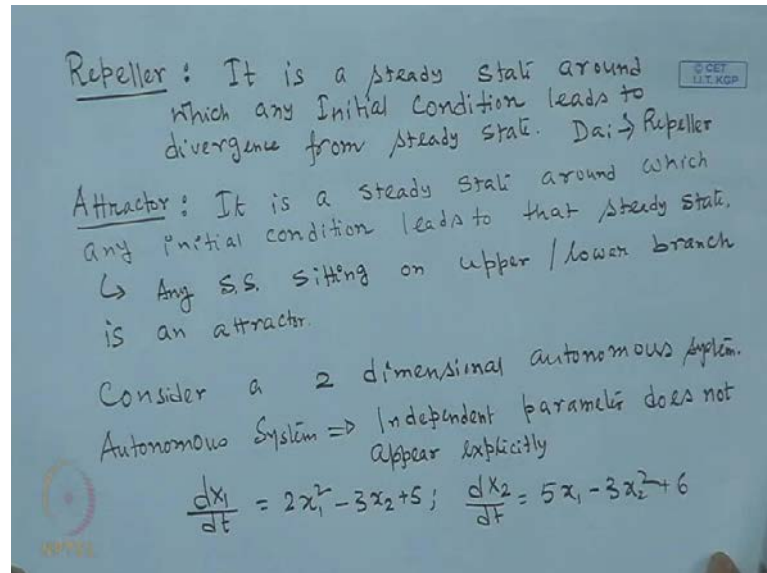
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So, let us look into the phase plane plot; so, first let us defined what is a phase plane; suppose, any system is define by two ordinary differential equations like $\frac{dx_1}{dt}$ is function of x_1 and x_2 ; x_1, x_2 are the state variable; $\frac{dx_2}{dt}$ is equal to g of x_1 and x_2

then the plane where the system behavior evolves dynamically the variation of x_1 and x_2 is called a phase plane plot; plot of x_1 and x_2 is known as phase plane plot.

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So, if we plot x_1 as a function of x_2 , it gives a plane in which system behavior evolves dynamically. Now, in this case, time is a parameter, so that gives the concepts of phase plane plot. Now, let us look into some more concepts and definitions, first one is the repeller; so, let us see what a repeller is, repeller is a steady state, it is a steady state around which, if you have any initial condition that will lead to a divergence from the steady state. So, it is a steady state around which any initial condition leads to divergence from steady state.

So, in the earlier example, D a i is a repeller; so, if we have any steady state about that point, then it will either go to the upper steady state or it will go to a lower steady state. Then attractor, this is a steady state around which any initial condition leads to that steady state; so that steady state is called an attractor.

So, **steady state**, this attractor may be, the in the earlier example, any steady state sitting on upper branch or lower branch is an attractor. And D a i or D i in the earlier example is a repeller. So, any steady state around D a I, either it will be landing on upper steady state branch or lower steady state branch, on the other hand if any steady state is sitting on the upper or lower branch, beyond the D a i and D a e region that will be acting as an attractor.

Now, let us try to develop the theory of stability for a two-dimensional system, this can be extended for a multi-dimensional system. So, for that, we consider a two-dimensional autonomous system and autonomous system, means, independent parameters do not appear explicitly; if independent parameter does not appear explicitly, then we call that system as autonomous system.

For example, if the said variables are x_1 , dependent variables are $\dot{x}_1$ and $\dot{x}_2$ and x_1 and x_2 and the independent variable is dt , if it is $2x_1^2 - 3x_2 + 5$. And if $\dot{x}_2$ is equal to $5x_1 - 3x_2^2 + 6$. Now this system is an autonomous system, because the independent variable t does not appear on the right hand side in either of the equation; so, this is an autonomous system.

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Handwritten mathematical derivation on a blue background:

$$\dot{x}_1 = \frac{dx_1}{dt} = f_1(x_1, x_2) \quad \checkmark$$

$$\dot{x}_2 = \frac{dx_2}{dt} = f_2(x_1, x_2) \quad \checkmark$$

Steady state:

$$0 = f_1(x_{1ss}, x_{2ss})$$

$$0 = f_2(x_{1ss}, x_{2ss})$$

Consider, a deviation about the steady state

$$\hat{x}_1 = x_1 - x_{1ss}; \quad \hat{x}_2 = x_2 - x_{2ss}$$

We evaluate deviation dynamics

$$\frac{d\hat{x}_1}{dt} = f_1(\hat{x}_1 + x_{1ss}, \hat{x}_2 + x_{2ss})$$

$$\frac{d\hat{x}_2}{dt} = f_2(\hat{x}_2 + x_{2ss}, \hat{x}_1 + x_{1ss})$$

Now, let us consider an autonomous system, where $\dot{x}_1$ is equal to $\frac{dx_1}{dt}$ and in general this will be a function of x_1 and x_2 . And in the second equation, $\dot{x}_2$ is $\frac{dx_2}{dt}$ is equal to another function of x_1 and x_2 ; so, it does not the right hand function f_1 and f_2 are inert functions of x_1 , x_2 and t explicitly; so that is why they are called autonomous system.

Now, most of the chemical engineering process is they fall under the category of autonomous system. Now, let us look into the steady state solution of this two equation; so, the steady states are $0 = f_1(x_{1ss}, x_{2ss})$; s stands for the steady state and 0 is equal to $f_2(x_{1ss}, x_{2ss})$. Now, let us consider a deviation about the steady state and this

deviations are $\hat{x}_1$ is nothing but x_1 minus x_{1ss} ; and $\hat{x}_2$ is x_2 minus x_{2ss} . Now, in order to evaluate, now what we are going to do, the evaluation the dynamics of the deviation variable.

So, these are called deviation variables of the two parameter, 2 dependent variables x_1 and x_2 ; this will be x_1 minus x_{1ss} , x_2 minus x_{2ss} . So, we evaluate deviation dynamics, how to evaluate that, we evaluate all the know governing, we write all the governing equations in terms of $\hat{x}_1$ and $\hat{x}_2$. So, therefore d, we can write these two equations as, $\frac{d\hat{x}_1}{dt}$, in fact, if you differentiate this equation with respect to t $\frac{dx_1}{dt}$ is identical to $\frac{d\hat{x}_1}{dt}$, so this will be nothing but function f_1 $\hat{x}_1$ plus x_{1ss} and $\hat{x}_2$ plus x_{2ss} . And similarly we have $\frac{d\hat{x}_2}{dt}$ is equal to f_2 $\hat{x}_2$ plus x_{2ss} and $\hat{x}_1$ plus x_{1ss} . Now, so, we express all the parameters, the dependent variable x_1 and x_2 , in terms of deviation variable $\hat{x}_1$ and $\hat{x}_2$.

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Linearization about the Steady State

$$\frac{d\hat{x}_1}{dt} = \hat{\dot{x}}_1 = f_1(x_{1ss}, x_{2ss}) + \left. \frac{\partial f_1}{\partial x_1} \right|_s (\hat{x}_1 - x_{1ss}) + \left. \frac{\partial f_1}{\partial x_2} \right|_s (\hat{x}_2 - x_{2ss})$$

Taylor series Expansion

$$\frac{d\hat{x}_1}{dt} = f_1(x_{1ss}, x_{2ss}) + \left. \frac{\partial f_1}{\partial x_1} \right|_s \hat{x}_1 + \left. \frac{\partial f_1}{\partial x_2} \right|_s \hat{x}_2$$

$$\frac{d\hat{x}_2}{dt} = f_2(x_{1ss}, x_{2ss}) + \left. \frac{\partial f_2}{\partial x_1} \right|_s \hat{x}_1 + \left. \frac{\partial f_2}{\partial x_2} \right|_s \hat{x}_2$$

$$\begin{cases} \frac{d\hat{x}_1}{dt} = \left. \frac{\partial f_1}{\partial x_1} \right|_s \hat{x}_1 + \left. \frac{\partial f_1}{\partial x_2} \right|_s \hat{x}_2 \\ \frac{d\hat{x}_2}{dt} = \left. \frac{\partial f_2}{\partial x_1} \right|_s \hat{x}_1 + \left. \frac{\partial f_2}{\partial x_2} \right|_s \hat{x}_2 \end{cases}$$

So, next, we linearize the problem about the steady state; next step is linearization about the steady state, if we do a linearization then $\frac{dx_1}{dt}$ or $\dot{x}_1$ becomes f_1 x_{1ss} and x_{2ss} plus $\frac{\partial f_1}{\partial x_1} (x_1 - x_{1ss})$ plus $\frac{\partial f_1}{\partial x_2} (x_2 - x_{2ss})$. It give a slight deviation about the steady state by test; so we linearize this thing by Taylor series expansion and retain only the first term of the expansion and this derivatives are about the steady state, this vary derivatives about the steady state. So, we

retain the first term of Taylor series expansion and delete the higher other terms, because the deviations are extremely small, from the deviations are extremely small compare to the steady state.

So, therefore, we can write $\frac{dx_1}{dt}$ will be nothing but $f_1(x_1, x_2)$ and x_2 plus $\frac{df_1}{dx_1}$ about the steady state, x_1 plus $\frac{df_1}{dx_2}$ at the steady state, this will be x_2 hat. So, that will be the governing equation of $\frac{dx_1}{dt}$ the deviation variable the dynamics of deviation variable by using linearization and that linearization will be restricted **a** to up to the first term and neglecting the higher terms of Taylors series expansion.

Similarly, we will be getting the next part of the deviation variable $\frac{dx_2}{dt}$ will be nothing but $f_2(x_1, x_2)$ plus $\frac{df_2}{dx_1}$ and that will be evaluated as steady state, and this will be x_1 hat plus $\frac{df_2}{dx_2}$ evaluated at the steady state, and that will be multiplied by the x_2 hat. So, therefore what we can do, we can write these two equation in a compact form and also if we remember that $f_1(x_1, x_2)$ and x_2 were equal to 0 and f_2 both f_1 and f_2 evaluated at steady state, they are equal to 0. If you look into the earlier slide that these were the governing equation of the system, we are considering and at the steady state these will be equal to 0, and $f_1(x_1, x_2)$ equal to 0, and $f_2(x_1, x_2)$ will be equal to 0.

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Handwritten mathematical derivation on a blue background:

$$\frac{d\hat{x}}{dt} = J \hat{x}$$

Where, $\hat{x} = [\hat{x}_1, \hat{x}_2]^T = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{s.s.}$$

Solution: $\hat{x}(t) = c_1 u^1 e^{\lambda_1 t} + c_2 u^2 e^{\lambda_2 t}$

Eigenvectors.

Solution / stability of perturbed variable $\Rightarrow$ depends on $\text{Re}(\lambda_i)$

Sufficient condition for instability $\text{Re}(\lambda_i) > 0$ for at least one λ_i

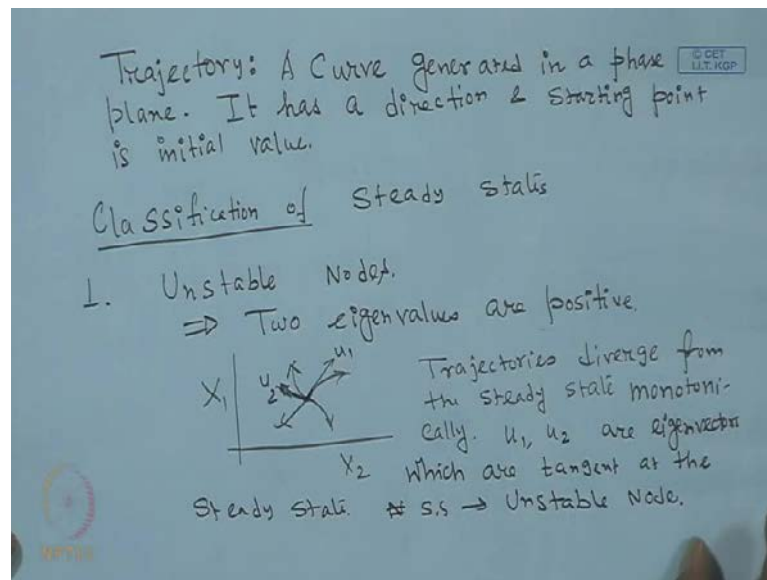
So, therefore, we have every right to make this two equal to 0, so the need at compact form will be $\frac{d\hat{x}}{dt}$ in the deviation variables will be $\frac{df_1}{dx_1}$ evaluated at steady state $\hat{x}_1$ plus $\frac{df_1}{dx_2}$ evaluated at steady state through $\hat{x}_2$. And $\frac{d\hat{x}_2}{dt}$ will be nothing but $\frac{df_2}{dx_1}$ steady state $\hat{x}_1$ plus $\frac{df_2}{dx_2}$ steady state $\hat{x}_2$. So, these two equations in the deviation variable can be written in a compact matrix form as, $\frac{d\hat{x}}{dt}$ is equal to J times $\hat{x}$; where $\hat{x}$ is nothing but a matrix, a vector, which will be having two elements $\hat{x}_1$ and $\hat{x}_2$ and transpose of that; so, it is nothing but a vector $\hat{x}_1$ and $\hat{x}_2$.

And the Jacobian matrix J becomes the elements of the derivative they will constitute the derivatives will constitute the elements of the Jacobian matrix and these will be $\frac{df_1}{dx_1}$ $\frac{df_1}{dx_2}$ $\frac{df_2}{dx_1}$ and $\frac{df_2}{dx_2}$ and all these derivatives which are the elements of the Jacobian matrix, they will be evaluated at the steady state. So, the solution of this equation will be in this form of $\hat{x}(t)$ will be is equal to $c_1 u_1 e^{\lambda_1 t}$ plus $c_2 u_2 e^{\lambda_2 t}$, because it is 2 into 2 matrix, so you will be having a two Eigenvalues λ_1 and λ_2 and u_1 and u_2 are high corresponding Eigenvectors. So, stability of the perturbation this terms value variable is nothing but the perturbation variable, through solution of solution or stability of perturbed variable or the perturbation, that we have imposed from outside, they will be entirely depend on sign of a real values of λ_i .

So, real values of eigenvalues will dictate, whether this disturb the perturbation variable will grow in time or it will die down with time. So, if real value of λ is positive then this disturb, even if one of them is positive, the whole solution the disturbance variable will grow in time; if it is negative then if all of them will be negative, then it will be the stable solution and the disturbed part out variable will die down with respect to time.

So, the sufficient condition for instability is real value of λ_i should be greater than 0. So, for at least one λ_i then, that means, if in a system there are 4 or 5 Eigenvalues, if the real part of 1 Eigenvalue is greater than 0, the system becomes instable. And that will be the sufficient condition for the instability; you do not require any more condition to check the instability. If one of the eigenvalues will be having a real part above 0 greater than positive, then will be having an instable solution.

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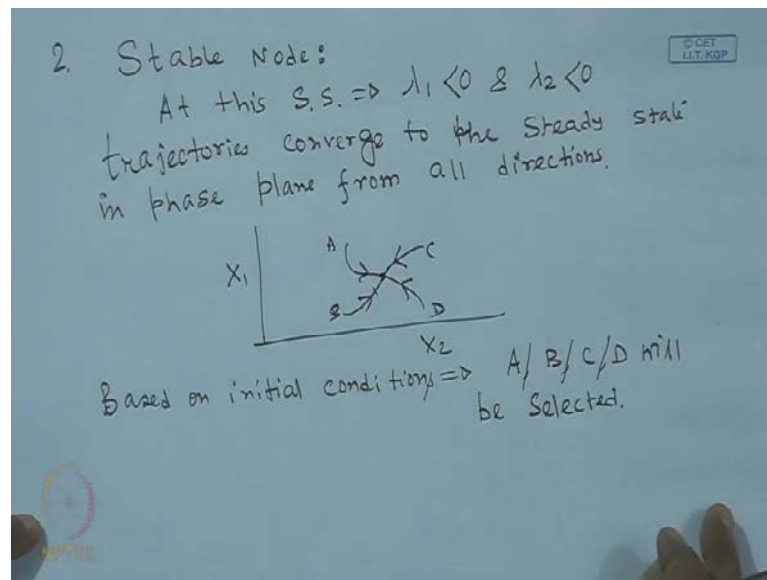


Next, we will look into some of the definitions, first one is called a trajectory; trajectory is a curve generated in a, curve generated in a phase plane that is called a trajectory. It has a direction starting with the initial value, it has a direction and the starting point is the initial value. Now, next we classify the various steady; so, classification of steady states these are quite important and you come across of several terms later on, that is the unstable saddle, unstable node, focus, stable focus, unstable focus, so on so forth.

So, let us classify the steady states, the first classification is the unstable nodes. If two Eigenvalues are positive unstable nodes occurs that when two Eigenvalues are positive. And the phase plane plot looks something like this, so you will be having an initial condition here. Then, so, any deviation in about the initial condition, it will diverge from this steady state and this node is called an unstable nodes and this will be the direction of the Eigenvectors, the tangential directions presented in the node at the Eigenvectors.

So, therefore trajectories diverge from the steady state monotonically, so that is the feature of unstable node. So, u_1, u_2 are the Eigenvectors; so, these are the direction of the Eigenvectors; u_1, u_2 are Eigenvectors which are tangent at the steady state, such kind of node or steady state is called unstable node. So, because of the presence of unstable node, any steady state around this point will diverge from this steady state. So these are called unstable nodes.

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Then we talk about a stable node that is the second type of steady. State stable node is this at this steady state, both Eigenvalues are negative; so, λ_1 is negative and as well as λ_2 is negative; so, that means, the trajectories converge to the steady state in phase plane from all direction, then, that particular node is called stable node. And if you look into the phase plane plot, so it will be x_1 versus x_2 and if this is the steady state any steady state, around this point will converge to this, but which steady state... Now, suppose A, B, C, D, now which trajectory it will follow, suppose you have a steady state here and we have a point around the steady state, from this point now it will be following this.

Now, based on the initial condition it will decide which steady state which path it will take up. So, based on initial condition, based on the initial conditions the path A, B, C or D will be selected, so that is called a stable node; that means, in case of stable node, we have the Eigenvalues less than 0; that means, both of the Eigenvalues are negative and one can have a very stable system in around that point, so any deviation about the steady state we will lead to the particular steady state, so then it is called a stable node.

In case of a system, where one of the Eigenvalues is positive, then you will be having the any deviation from the steady state will lead to divergence from that particular solution. If all steady states, if all Eigenvalues are positive, then that will be an unstable steady state, so it will be an unstable node. So, any definition from any direction of the steady

state, it will diverge from the steady state. In the next class, I will stop here at this class, in the next class, will look into some more definitions of the steady state and we will complete the classification of the steady state, then we go to the appropriate examples and further develop the theory and finally so will take up some of the examples of chemical engineering system.

Thank you very much.