Advanced Mathematical Techniques in Chemical Engineering

Prof. S. De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

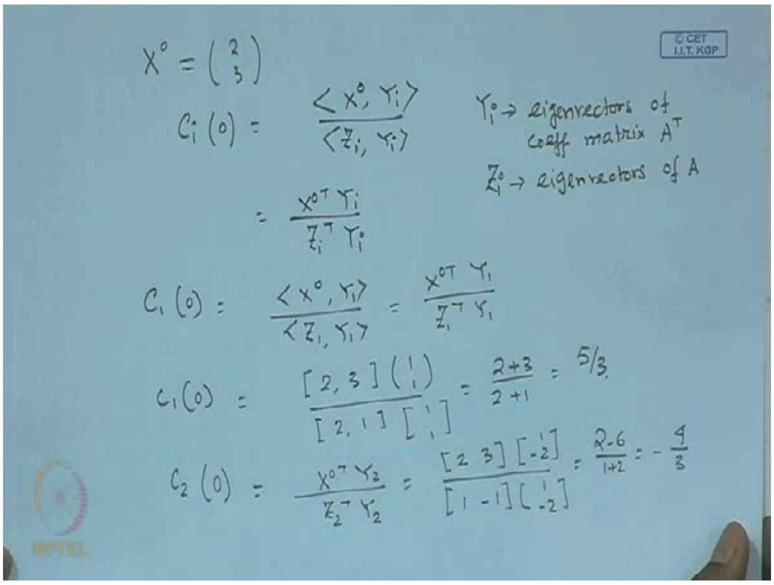
Lecture No. # 12
Eigenvalue Problem
(Contd.)

Good afternoon every one. So, we are in this class, we will be looking into the specific examples of Eigenvalue problems and how this Eigenvalue problems will be used for solving the chemical engineering application oriented process equations.

So, we have already seen one example in the last class - how to solve a non-homogeneous set of non-homogenous algebraic equations. We have taken up two examples: in the first example the Eigenvalues were real and the solutions an eigen and all the elements of the Eigenvectors are real and you are getting a real solution of the system. In the next example, we have considered a problem where the Eigenvalues were imaginary; so, therefore, the elements of the Eigenvectors are also imaginary, but ultimately you will be getting a real valued solution, as a, as a final answer, because the chemical engineering problem that was - that we have considered, they are basically real of a real valued.

Next, we have taken up the theory for solution of a set of homogeneous or in differential equations. And we have taken up one example, we have developed the theory for it, and we have taken up an example how to solve them, and we are almost at the last step of the solution.

(Refer Slide Time: 02:00)



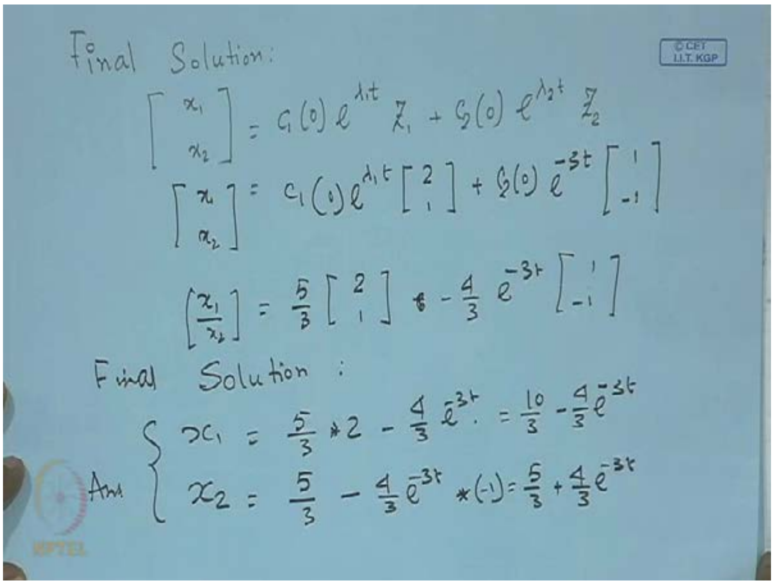
So, if you remember that we have the initial value set of the conditions where 2 and 3; these are the initial value problem, at time t is equal to 0; these are there values of the, of variable. And we are looking into the solution and that for the solution you are going to get the coefficients C i naught and these coefficients are defined as the inner product of X naught and Y i and this is divided by inner product of Z i and Y i. If you remember that Y is are the Eigenvectors of coefficient matrix transpose of coefficient matrix A transpose and Z i where the Eigenvectors of A.

So, therefore, this will be nothing but X 0 transpose Y i Z i transpose Y i. And C 1 0 becomes X 0 inner product of X 0 Y 1divided by Z 1inner product of Z 1 and Y 1; so, this will be X 0 transpose Y 1 divided by Z 1 transpose Y 1. So, you have the coefficient C 1 0, if you remember X 0 transpose, where it was 2 3, it is the transpose of the vector 2 3 and Y 1 is nothing but eigenvector 1 1 divided by Z 1was the vector 2 1 and this will be Y 1 will be 1 1. And this will be 2 plus, 2 into 1, 2 plus, 3 into 1, 3 divided by, 2 into 1, plus 1 into 1; it will be 5 by 3.

Similarly, we have, we had seen, we have already seen that C 2 naught will be the inner product of X 0 T Y 2 divided by Z 2 T Y 2. And this will be 2 3 and it will be Y 2 will be 1 minus 2 divided by 1 minus 1 and 1minus 2; and ultimately it will be 2 minus 6 divided by 1 plus 2; so, it will be minus 4 by 3. So we are able to estimate the coefficient C 1 and

C 2 and then we have solved almost everything and the, all the quantities are known to you.

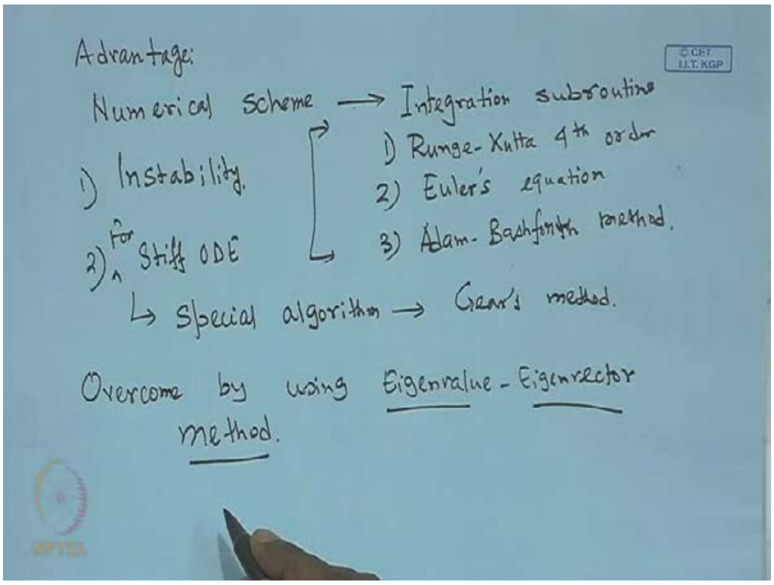
(Refer Slide Time: 05:16)



Now, we will write down the final solution, the final solution will be, if you remember the final solution is in this form, final solution becomes x 1 x 2 is equal to C 1 0 e to the power lambda 1 t Z 1 plus C 2 0 e to the power lambda 2 t Z 2. And if we remember the thing becomes the lambda 1 trans out to be, we just put the values of lambda 1 and lambda 2, so this becomes x 1 x 2 is equal to C 1 0 e to the power lambda 1 t and this becomes Z 1 was 2 and 1 the vector of 2 and 1 and this becomes C 2 0 e to the power minus 3 t, and Z 2 was 1 minus 1.

So, therefore, the x 1 and x 2 becomes put the value of C .1 C 1 is 5 by 3, lambda 1 was 0, so it will be one and this will be 2 1 plus C 2, C 2 will be minus 4 by 3 e to the power minus3 t 1 and minus 1. So, therefore we can construct the final solution as x 1 is equal to 5 by 3 into 2 minus 4 by 3 e to the power minus 3 t times 1; so, this becomes 10 by 3 minus 4 by 3 e to the power minus 3 t. And x 2 becomes 5 by 3 into 1 minus 4 by 3 e to the power minus 3 t multiplied by minus 1; so, it will be 5 by 3minus plus 5 by 3 e to the power minus 3 t; so that is the final solution of, what are the problem we are looking into. So, this gives a direct application of Eigenvalue, eigenvector problem, to solve the set of ordinary differential equations which are homogenous.

(Refer Slide Time: 08:06)



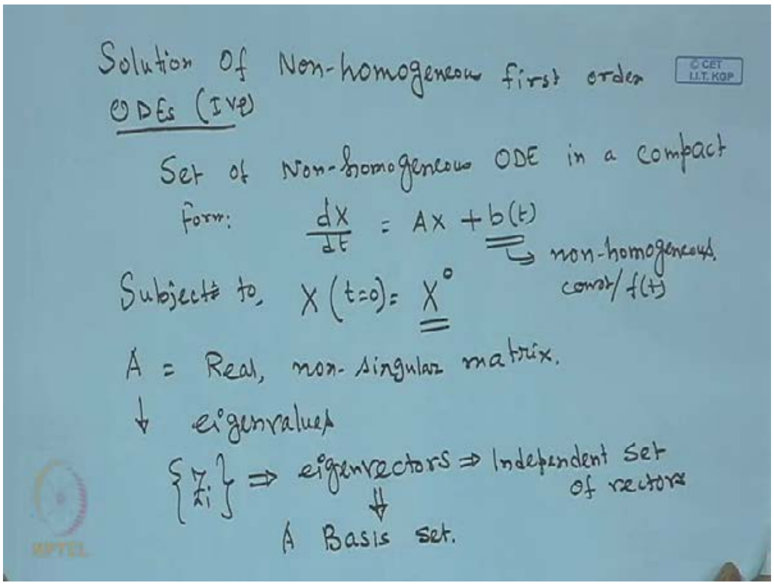
Now, let us talk about the advantage of this method, compare to any numerical scheme. If you look into the numerical scheme for the numerical scheme, then you could have preferred any integration subroutine to solve this type of problem; these subroutines include Runge Kutta 4, forth order or Euler's equation, Euler's method or Adam-Bashforth method or predictor corrected method.

So, therefore, but all these numerical schemes will be having the disadvantage of instability, there may be numeric, if the, where the step size of numerical integration is not properly selected that instability may be a problem. If the equations are stiff equations, for stiff ordinary differential equations, none of this method will be, can be utilized; then you have to use the special algorithm like Gear's, Gear's method may be used. The stiff ODE, means, if the variation of dependent variable, a small variation of independent variable gives a large defect variation of the dependent variable, so that gives a, that is called whether stiff ordinary differential equation.

So, you require special algorithm for them; so, you could have of all these short coming can be overcome, by using this Eigenvalue method. Eigenvalue - eigenvector method that gives us more accurate and elegant method of solution trouble free; so, what are the various subroutines, one has to write, one has to have a subroutine of evaluation of the Eigenvalues that is not a problem; one can use the Honor's method, they, one can use the eigen the subroutines for evaluation of Eigenvectors. So, Eigenvalues - Eigenvector then matrix transpose of, you have to write a couple of line routines for matrix transpose and then one has to go for the simple calculations like n matrix multiplication.

So, using four such basics subroutines, one can utilize the Eigenvalue - Eigenvector method, in order to solve the complicated problems of linear ordinary differential equation, in chemical engineering. And can do away with the a numerical, the complicacies or you know short comings of numerical integration.

(Refer Slide Time: 11:28)

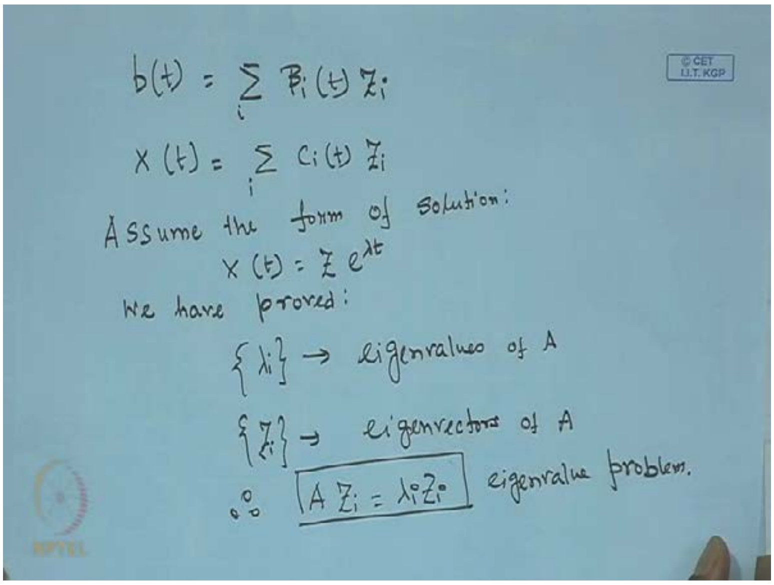


Next, we talk about the non-homogenous solution of, non-homogenous first or first order ordinary differential equation, but it is, they are non-homogenous; in earlier case we have solve the homogenous. And in this case we will be taking about the solution of non-homogenous first order ODEs; of course they are IVPs. In fact, even boundary value problem can be broken down into series of ordinary differential equations, by assuming the unknown, the initial condition and then matching with the corresponding boundary condition anyway.

So, let us write down the set of non-homogenous ODEs in a compact form, in a compact form and that will be d x d t is equal to A x plus b of t. In general this term corresponds to non-homogenous term. So, this can be the non-homogenous term, can be constant term or it can be function of time, but this cannot be function of X; then, this term, the extra term in the additional term on the right hand side is called a non-homogenous term, then this set of equation has to be solved by subject to the boundary condition, the initial conditions that, at X vector at time t is equal to 0 is given by this represented by this vector x naught.

Now, A is a real non-singular matrix and we know how to evaluate the Eigenvalues and Eigenvectors of this matrix; and for this matrix, you can evaluate the Eigenvalues. And let us say Z I, forms a set of vectors which are nothing but the Eigenvectors. And since Eigenvectors are, they form the independent set of vectors, we have already proved earlier these are independent set of vectors and they constitute a basis set. Therefore any other vector in the domain can be expressed as a linear combination of the basis set vectors or the Eigenvectors of the matrix A.

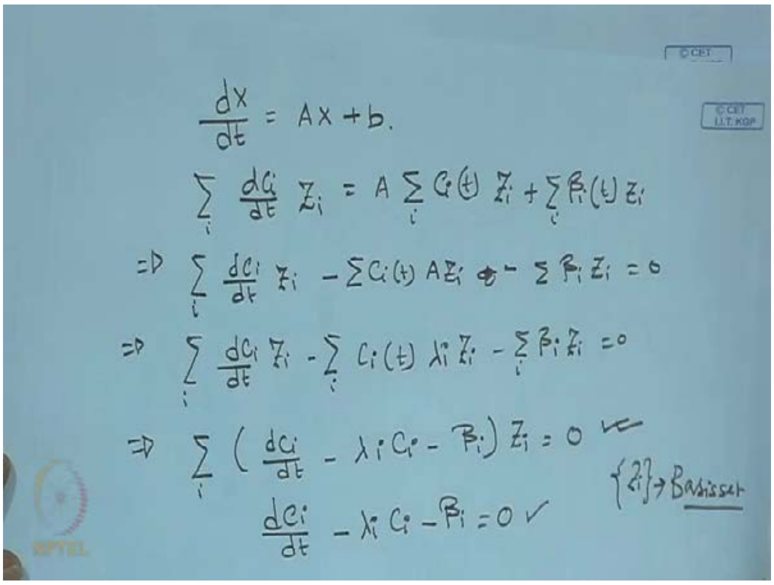
(Refer Slide Time: 14:47)



Now, we assume the vector b t, b which is a function of t is expressed as a summation of beta I, as a summation of, as a linear combination of this vector the i Eigenvectors Z i. And the solution X t is expressed as combination of these Eigenvectors Z I. So, we have to assume the form of the solution as X t is equal to Z e to the power lambda t would have proved that, if this is the form of the equation, a form the solution, we have proved earlier, that lambda i corresponds to Eigenvalues of A and Z i correspond to the Eigenvectors of A.

So, therefore, we have the Eigenvalue problem A Z i is equal to lambda i Z I; so, lambda i is i th Eigenvalue and Z i is the corresponding eigenvector and this Eigenvalue problem is defined.

(Refer Slide Time: 16:40)

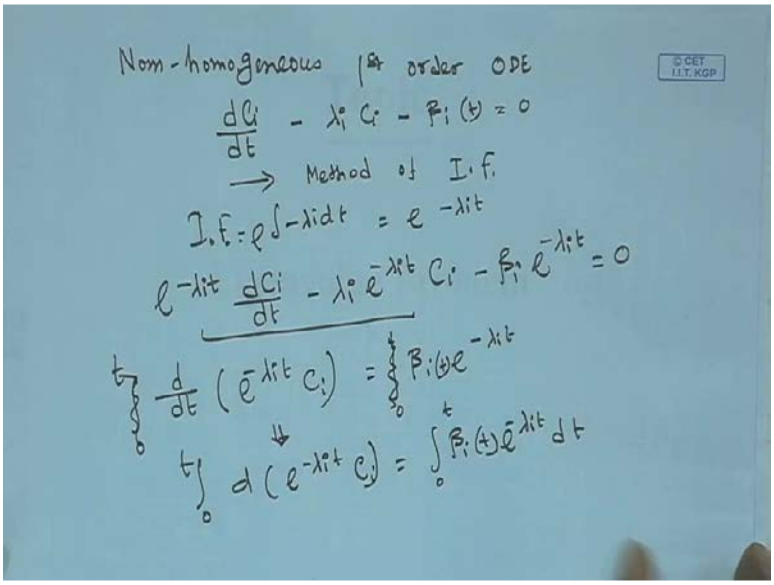


So, next we write down the equation d x d t is equal to A x plus b. And write down the solution, write down the expression of X, X is summation of C i Z I; so, therefore this will be summation d C i d t, if you remember C i was a sole function of time, so it becomes d c e i d t Z i is equal to A summation of C i which is a function of t Z i plus summation of beta i times Z i. So, therefore one can bring this up on the left hand side and put everything under the summation, so these becomes d C i d t Z i minus summation of C i t A Z I, so matrix is an operator, so operate on Z plus minus summation of beta i Z i will be equal to 0. So, we have summation of d c d t Z i and we identify that this A Z i is nothing but lambda i Z I, so therefore this becomes C i t lambda i Z i minus summation beta i Z i is equal to 0 and all these summations are over the index i runs from 1 to n for n-dimensional problem.

Next, we bring everything under the summation, so it becomes so d C i d t minus lambda i c i minus beta i times Z i equal to 0 and if we remember that what this Z i's are this Z i's are Eigenvectors and they are independent vectors and they form a basis set. So, therefore, if these vectors are independent vectors and they are in this form the coefficient of all these vectors has to be independently equal to 0, in order to satisfy this equation.

So, therefore, we have the condition d C i d t minus lambda i C i minus beta i should be equal to 0; in order to have, in order to satisfy this equation because the Eigenvectors are set of independent vectors, to satisfy this equation all the corresponding individual coefficient must be equal to 0.

(Refer Slide Time: 19:48)

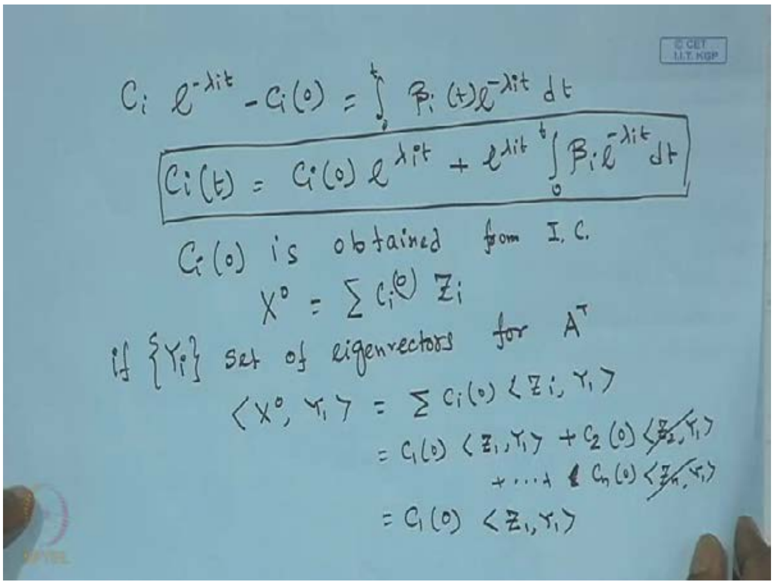


So, we get an ordinary non-homogeneous first order equation, non-homogeneous first order ODE as d C i d t as a condition d C i d t minus lambda i C i minus beta i, which is in general function of time.

Now, this is a non-homogeneous PDE, this method this ODE can be non-homogenous ODE ordinary differential equation, this ODE can be solved by the method of integrating factor, integrating factor. So, we multiply both the sides by integral e to the power e to the power minus lambda i d t, this is the integrating factor, so this becomes e to the power minus lambda i t; so, you multiply both side by e to the power minus lambda i t, so this becomes e to the power minus lambda i t d C i d t minus lambda i e to the power minus lambda i t C i minus beta i e to the power minus lambda i t is equal to 0.

So, we can combine these two parts, so it will be, d d t of e to the power minus lambda i t times c is equal to beta i e to the power minus lambda i t. So, we can integrate this out, so it will be 0 to t beta e is in general function of time, so 0 to t d t, so we can we should write it in the next step. So, it will be in the next step, we will be getting, 0 to t d of e to the power minus lambda i t times C is equal to integral beta i t e to the power minus lambda i t d t 0 to t.

(Refer Slide Time: 22:10)



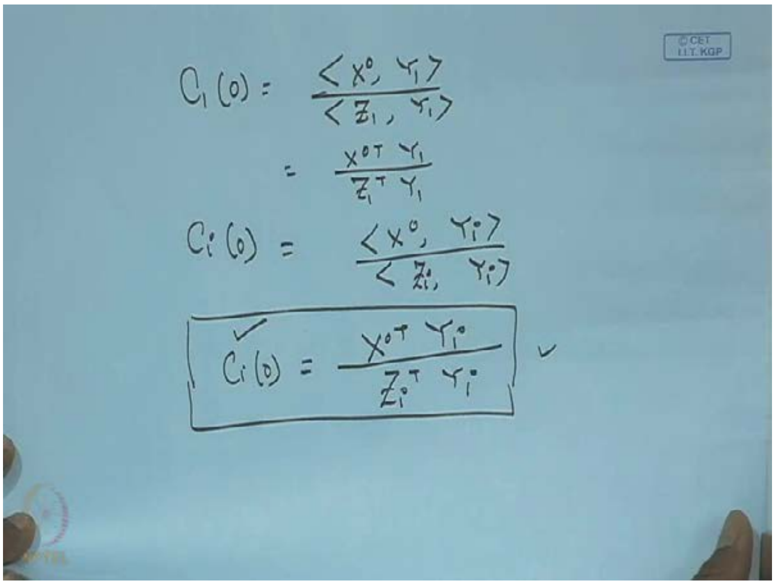
So, we can get this integral evaluated, so this will be giving you that, after evaluation this integral will be getting C I, we will be having the sub script i in all the cases of C i of the c’s; so, therefore, after integration we will be getting C i e to the power minus lambda i t minus C i at time t is equal to 0, at time t is equal to 0 e to the power 0 becomes one is equal to 0 to t beta i t e to the power minus lambda i t d t. And you will be getting C i as a function of time as C i naught e to the power lambda i t plus e to the power lambda i t 0 to t beta i e to the power minus lambda i t d t.

So, we will be getting these expressions as the final solution. And let us see how the C i naughts are obtained C i naught, the vector C i naught is obtained from the initial condition, obtained from initial condition; and what is initial condition, we know the vector X naught, that means, all the solutions were known as the initial condition at initial time t is equal to 0. So, these vector can be expressed as a linear combination of independent vectors Z I; so, therefore, one can obtain this by combining this with the Eigenvectors of A transpose.

So, now, if Y i is a set of Eigenvectors for A transpose, then inner product of X naught and Y 1 should be given as you take the inner product of this equation with respect to Y 1, so this becomes C i naught inner product of Z i and Y 1; in fact if you open up this series, then we will be getting C 1 0 inner product of Z 1 Y 1 plus C 2 0 inner product of Z 2 Y 1 plus like that and C n 0 inner product of Z n Y 1.

Now, Z 1 and Y 1, Z we have already proved that Eigenvectors of A and A transpose will form an a bi orthogonal set, so inner product of Z 2 and Y 1 will be 0, inner product of Z and Y 1 will be Y 1 equal to 0; so, only one term will survive in this summation that will be C 10 inner product of Z 1 and Y 1.

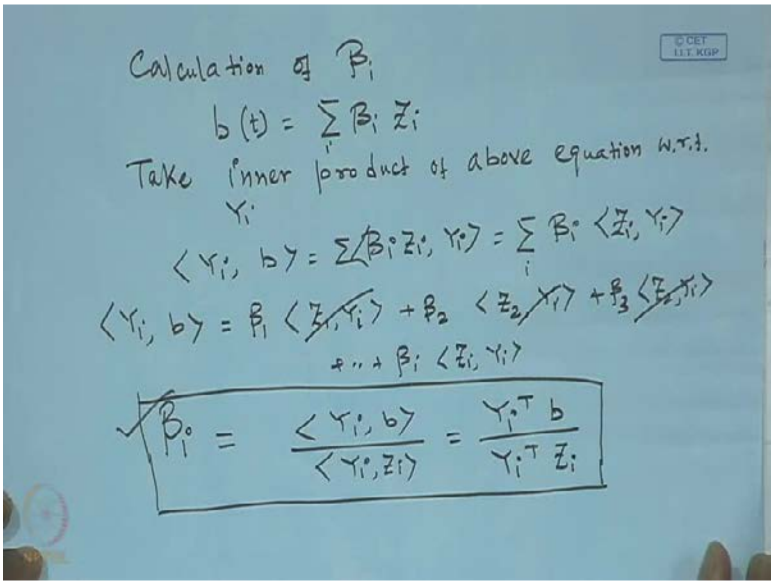
(Refer Slide Time: 25:42)



So, in that case what we will be getting as C 1 0 is that, so you will be getting C 1 0 is equal to inner product of X naught Y1 divided by inner product of Z 1 and Y 1. And we can get as X 0 transpose Y 1 divided by Z 1 transpose Y 1. And ultimately you can get the generic form of C i naught as inner product of X naught and Y i divided by inner product of Z i and Y i. So, you can put into the matrix in vector multiplication form, matrix multiplication X 0 t Y i Z i transpose Y I; so that will be the form of coefficient, since C is in the above solution.

And next we will be talking about the other parts of the solution. So, the C i naught vector will be determined by this, because we can evaluate the Eigenvectors of A transpose, we can evaluate the Eigenvectors of A, we know the initial condition vector, so all of these coefficients C i naught will be calculated.

(Refer Slide Time: 27:08)

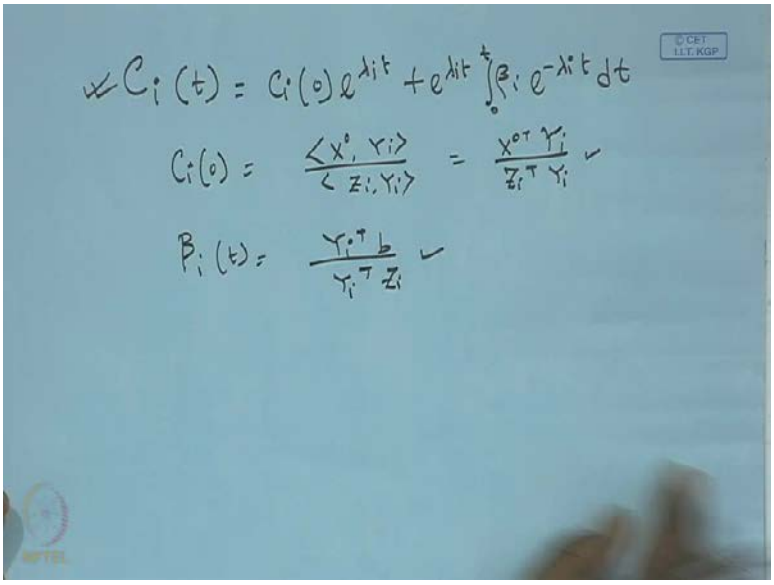


Next is the calculation of beta I, next step is calculation of beta i, so if you remember the vector b as a function of time can be expressed as linear combination of the Eigenvectors of a as beta i Z i.

Then we can take the inner product of this equation with respect to Y i of above equation with respect to Y I; so, you will be getting Y i inner product of Y and b is summation beta i Z i and Y i beta i being a constant, so it will be summation beta i inner product of Z i and Y i. Again, one can open up this summation series and see what you get so it will be, let us considered for one, so beta t Y i inner product of Y and b, you will be getting beta 1 inner product of Z 1 Y 1 plus beta 2 Z 1 Y i, so beta 2 inner product of Z 2 Y i plus beta 3 inner product of Z 2 and Y i, like that you will be getting beta i inner product of Z i and y i. So, since Z i's are the Eigenvectors of A and Y i are the Eigenvectors of A transpose, they form the biorthogonal set and we have already proved that except i is equal to j, they will be for the inner product of the individual Eigenvectors will be equal to 0; this will vanish, this will vanish, this will vanish, only this term will survive out of this summation.

So, therefore, in general one can evaluate beta i is summation of Y i b divided by inner product of y i Z I, inner product of Z i Y i is identical to inner product of Y i and Z I, so these will be nothing but inner product the multiplication of Y i transpose and the vector b and this will be multiplication of Y i transpose and vector Z i.

(Refer Slide Time: 30:16)

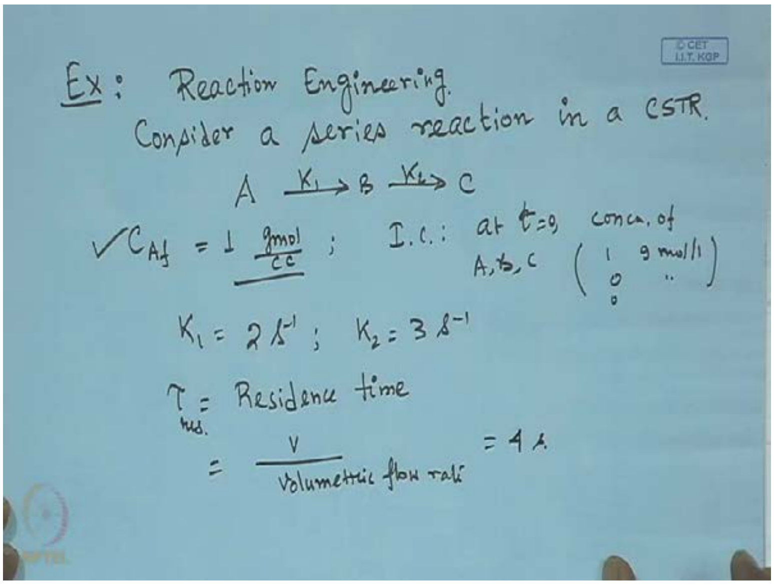


So, that is how the coefficients beta i can be evaluated. So, once you obtain the expression of beta i, once you obtain the expression of C i, then you can construct the complete solution as. So, the complete solution, if you look into the complete solution, we wrote C i t is equal to C i naught e to the power lambda i t plus e to the power lambda i t 0 to t beta i e to the power minus lambda i t d t.

So, therefore, where the coefficient C i naught can be expressed as inner product of X naught and Y i divided by inner product of Z i and Y I; so, this will be X naught t Y i we have already written this, so this will be Z i transpose Y i. And beta i we have already obtained as, Y i transpose b divided by Y i transpose Z i. So, once we get this, we can get the complete the solution of C i t and will be getting the complete solution.

Next, we will be with this theoretical formulation of solution of non-homogenous ordinary differential equation by Eigenvalue - Eigenvector method, we will be taking up one particular example a chemical engineering application example and see how this problem will be solved by using this Eigenvalue - Eigenvector method

(Refer Slide Time: 31:53)

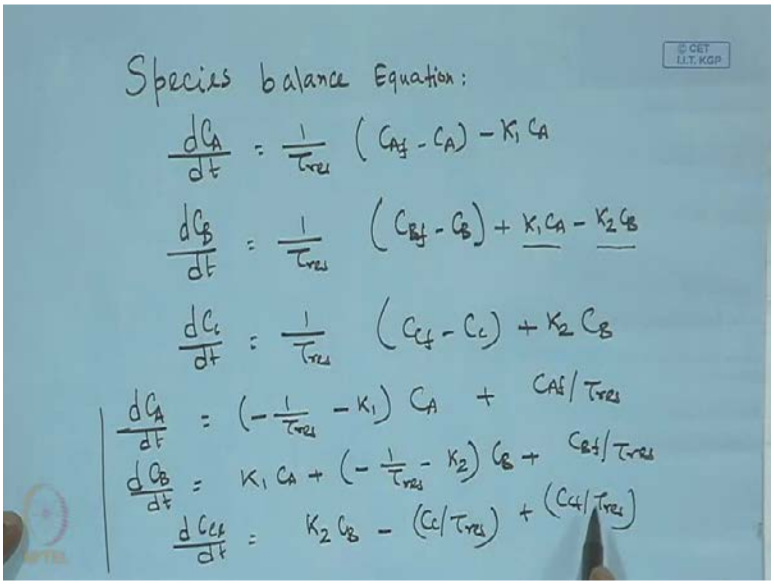


So, the example that we will be talking about is highly relevant for chemical engineers and this is the problem of reaction engineering. And we will be taking up a series reaction, considered a series reaction in a CSTR and we have already, I have already discussed what a continuous start tank reactor is in earlier classes.

So, considered a series reaction in a CSTR, so you will be having a reaction like this A going to B going to C and they are elemental reaction let us say and the rate expressions are given as first order rate expression. The feed concentrations was given C A F as 1gram mol per c c and initial conditions where at time t is equal to 0 the concentration of A, B, C where given by this vector 1 gram mol per liter 0 gram mol per liter and 0 gram mol per liter.

So, at time t is equal to 0, there was nothing in the system except C, A feed that was one gram mol per cc and the K 1 and K 2 values are given. K 1 is given by 2 second inverse K 2 is given as 3 second inverse. And tau that is residence time and it is defined as volume divided by the volumetric flow rate. So, volume will be in meter, volumetric flow rate will be in meter cube per second, so it will be second and this tau will be 4 second.

(Refer Slide Time: 34:32)

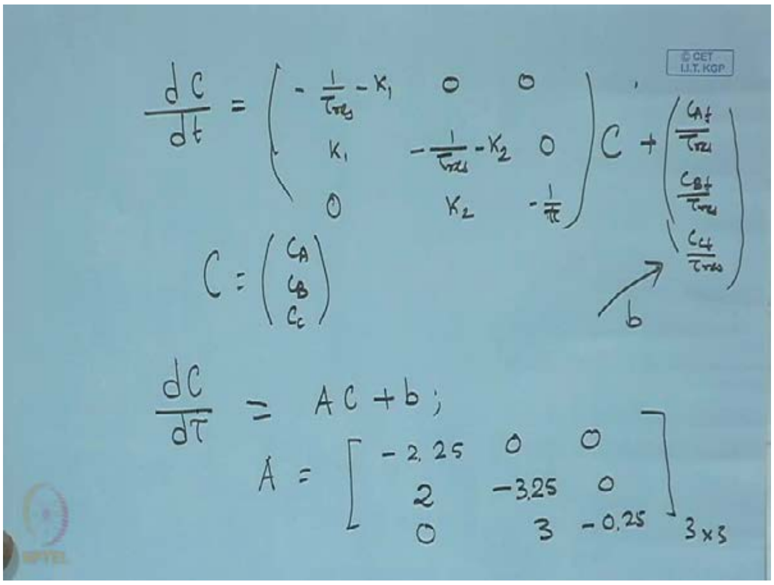


So, we have now in a position to write down the species balance equation, we write it as tau residence, the time. So, write down the species balance equation, this will be d C A d t is equal to one by tau C A F minus C A minus K 1 times C A; that is the A balance. B balance will be one by tau C B F feed minus C B plus k 1 C A minus K 2 C B; so, this is the accumulation term, this is net material in minus, net material out, minus, plus rate of generation here. And in the case of A, it will be consumption; in fact this is a combination of rate of generation and rate of consumption, because B is produced at the rate K 1 C A and it will be consumed at the rate K 2 times C B.

So, d C C d t can be written as one by tau C C final, if C C in the feed minus C C plus rate of generation K 2 C B. So, we can write it more amenable form d C A d t will be minus 1 by tau res minus K 1 times C A plus C A f divided by tau; d C B d t can be written as K 1 C A plus minus 1 by tau minus K 2 times C B plus C B feed divided by tau resistance. And d C C d t can be written as K 2 C B minus C C by tau plus C C f by tau; so, you will be having the form of set of non-homogeneous ordinary differential equation.

So, this will be 3 the dx dt part or the component were C A C B and C C. These are the coefficient matrix and these terms correspond to the non-homogeneous part.

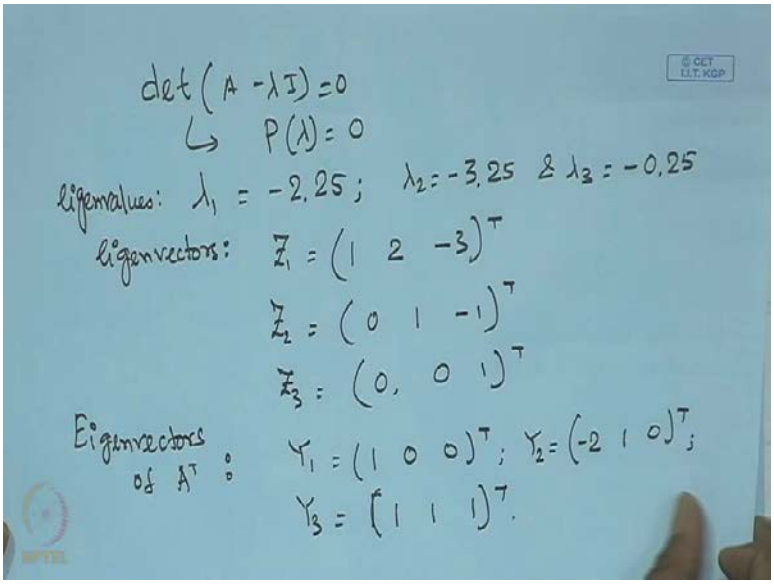
(Refer Slide Time: 37:40)



So, therefore one can get these equations, so we can write it as d C d t in a more compact form matrix notation minus 1 by tau minus K 1 0 0 K 1 minus 1 by tau minus K 2 0 0 K 2 minus 1 by tau times C plus the vector C A f divided by tau C Bf divided by tau and C C C f divided by tau; where the matrix C is that the vector C is basically given by C A C B C C; so, it has three elements.

Now, we can write down this in a compact form d C d tau is equal to A C plus b; where the b vector is given by this, so this is a b vector. And the coefficient matrix A can be given as just put all the values, so if you put all the values, this becomes minus 2.25 0 0 2 minus 3.25 0 0 3 minus 0.25, put all the values there; so, you will be landing up with the coefficient matrix like this. Now this is a 3 by 3 matrix, so we can use, we can evaluate the Eigenvalues of this matrix and we can, we can obtain the Eigenvectors.

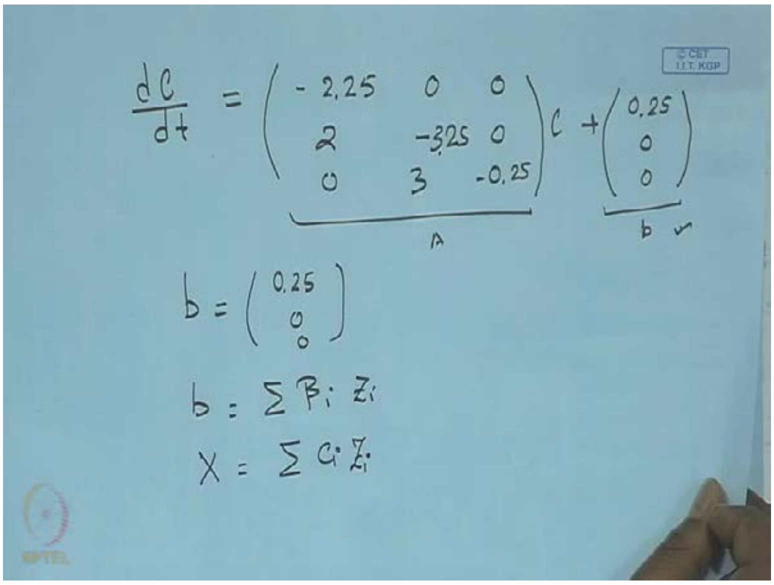
(Refer Slide Time: 39:39)



So, for the Eigenvalues what we do is we put determinant of A minus lambda i is equal to 0; so, you will be getting a polynomial of lambda and this polynomial will be a third order polynomial and you will be getting 3 roots, those will be corresponding to the Eigenvalues. So, lambda 1 is equal to minus 2.25; lambda 2 is equal to 2 minus 3.25 and lambda 3 is equal to minus 0.25 and the corresponding, so these are the Eigenvalues. Corresponding Eigenvectors will be Z 1 and it will be 1 2 minus 3 transpose and at again deriving it. We have already seen in the earlier examples, how to evaluate the Eigenvalues and corresponding Eigenvectors of a square matrix Z 2 is Z 1 minus 1 transpose and Z 3 is 0 0 1transpose.

We can get the Eigenvalues of the transpose matrix will be identical that we have already proved earlier. And we can get the Eigenvectors of A transpose and this will be Y 1 is 1 0 0 transpose; Y 2 is minus 2 1 0 transpose; and Y 3 will be 1 1 1 transpose; so, this will be the Eigenvectors of A transpose.

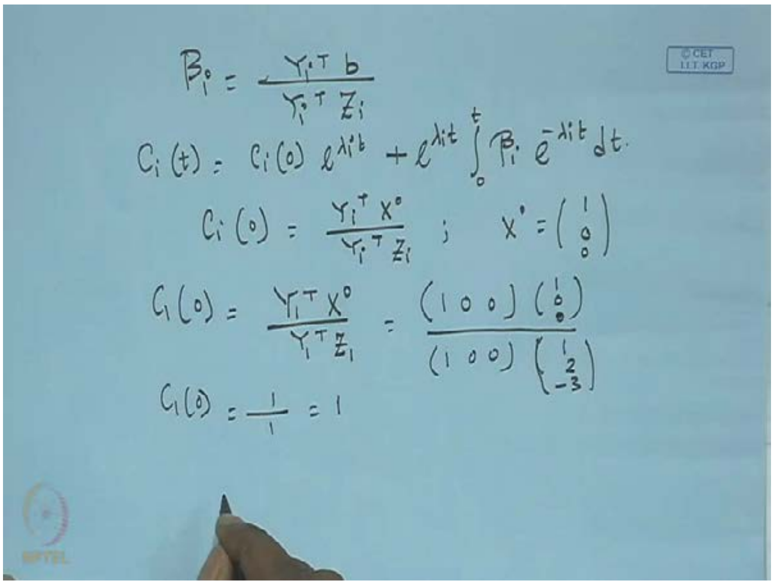
(Refer Slide Time: 41:40)



So, we can write the compact notation that d C d t is equal to minus 2.25 0 0 2 minus three 3.25 0 0 3 minus 0.25 times the vector C plus the vector 0.25 0 0. So, therefore, this is the set, this is the non-homogeneous term, this is the A, this is b; so, d C d t equal d x d t equal to in the form of A X plus b, this is the non-homogeneous term.

Now, we are going to solve this equation, so our non-homogeneous vector becomes 1 by 4 0.25 0 0. And we write beta b is summation of beta i Z i. We write X as summation of C i Z I, where X is the solution is, X is the solution matrix solution vector. So, therefore you can evaluate since all these Eigenvalues and Eigenvectors of A and A transpose have already been develop and derived, we need not to, you just go ahead with the solution procedure.

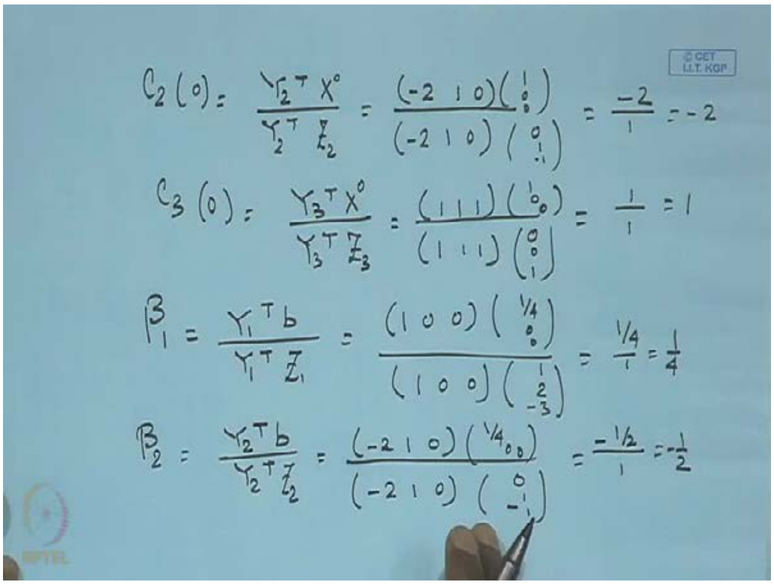
(Refer Slide Time: 43:16)



So, if you write down the expression of beta i this becomes Y i transpose times b Y i transpose times Z i. And the expression of C i as a function of t, if you remember this becomes C i 0 e to the power lambda i t plus e to the power lambda i t 0 to t beta i e to the power minus lambda i t d t.

So, therefore, we can, if you remember the expression of the vector C i naught the element of the vector C i, it will be Y i transpose X naught divided by Y i transpose Z I; where X naught where nothing but the vector which C A f C B f and C C f the initial values. So, therefore C 1 0 will be nothing but Y 1 transpose X naught divided by Y 1 transpose Z 1, so this becomes 1 0 0, this becomes 1 0 0 that is the initial value vector divided by 1 0 0 that is Y 1 transpose and Z 1 will be 1 2 minus 3 that is the first eigenvector of the matrix A. And this becomes C 1 0 becomes 1 plus 1 divided by 1, so it will become 1; so, all the other terms will be 0.

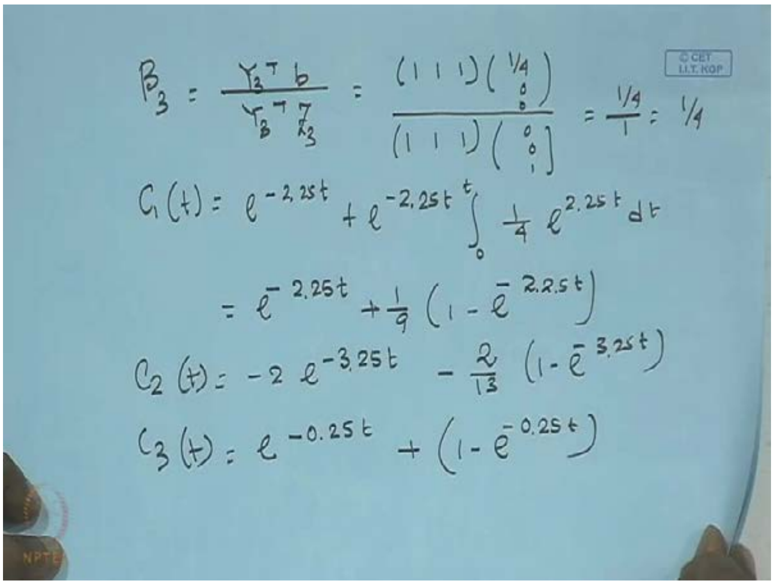
(Refer Slide Time: 45:11)



Similarly, we can evaluate that C 2 0 and C 3 0, if you do that C 2 0 trans out to be Y 2 transpose X naught divided by Y 2 transpose Z 2; and this becomes minus 21010 0 and this will be minus 2 1 0 0 1 minus 1. So, this becomes minus 2, this becomes 1 divided by 1, so it becomes minus 2. And C 3 0 becomes Y 3 transpose X 0 divided by Y 3 transpose Z 3. And this becomes 1 1 1 1 0 0 divided by 1 1 1 0 0 1 that is the third Eigenvector; so this becomes 1 and this also 1, so it becomes also 1.

So, now you are in a position to evaluate the coefficient beta 1, so beta 1 becomes Y 1 transpose times b divided by Y 1 transpose times Z 1; so, this will be 1 0 0 that is Y 1 transpose and b will be 1 by 4 0 0 divided by 1 0 0 1 2 and minus 3; so, this becomes 1 by 4 divided by 1; so, this becomes 1 by 4. Similarly, one can get beta 2 as Y 2 transpose b divided by Y 2 transpose Z 2; and this becomes minus 2 1 0 will be 1 by 4 0 0 divided by Y 2 transpose and Z 2, so minus 2 1 0 and Z 2 will be 0 1 minus 1that is the second Eigenvector; this will be equal to minus half divided by 1, so it will be minus 1 by 2.

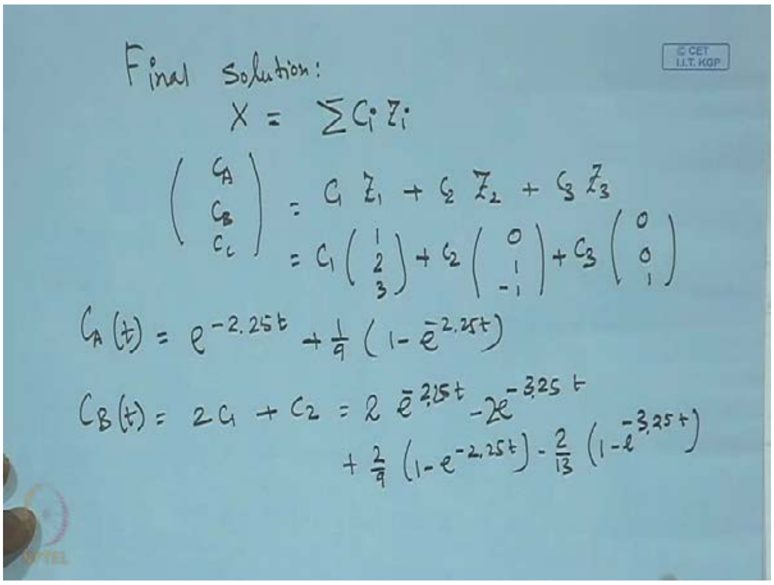
(Refer Slide Time: 47:55)



Similarly, we can have beta 3, beta 3 is Y 3 transpose b Y 3 transpose Z 3, so this will be 1 1 1 1 by 4 0 0 divided by 1 1 1 that is Y 3 transpose it will be 0 0 1; and this will be 1 by 4 divided by 1so it will be 1 by 4. Now, you are in a position to get the coefficient C 1 t, so C 1 t will be e to the power minus 2.25 t plus e to the power minus 2.25 t, if we look into the solution this becomes 0 to t 1 by 4 e to the power 2.25 t d t.

So, ultimately after getting into the integration carried out, the solution becomes e to the power minus 2.25 t plus 1 by 9 1 minus e to the power minus 2.25 t. And you will be getting C 2 t as minus 2 e to the power minus 3.25 t minus 2 by 13 1 minus e to the power minus 3.25 t. And C 3 t will be e to the power minus 0.25 t plus 1 minus e to the power minus 0.25 t.

(Refer Slide Time: 49:55)

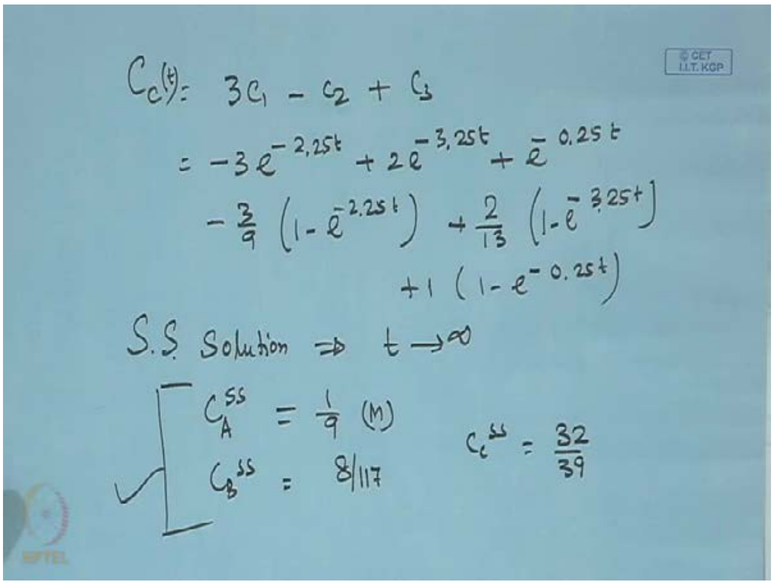


So, we are in a position to get the coefficients C 1, C 2, C 3, next set we are going to get the final solution. If you remember the final solution was given by the solution that vector X is summation C i Z I; so you will be getting C A C B C C is equal to C 1 Z 1 plus C 2 Z 2 plus C 3 Z 3. So, you have the coefficient that we have already evaluated C 1 as a function of time, you have just evaluated just now, just put the value of Z 1 1 2 and 3 plus, C 2 you have already evaluated that, 0 1 minus 1 plus C 3, we have already evaluated the expression of C 3 and Z 3 will be 0 0 1.

So, therefore, you are in a position to get the value of final solution. Now, we will be putting C 1 is e to the powe,r actually both it will be, C 1 times one plus C 2 times 0 plus C 3 times 0, so they will contribute nothing, so it will minus 2.25 t plus 1 by 9 1 minus e to the power minus 2.25 t, that is the expression of C 1.

And if you look into the expression of C B, it will be 2 times C 1 plus 1 times C 2 plus C 3 into 0, so we have already evaluated to C 1 and C 1 and C 2 put them here, 2 e to the power minus 2.25 t minus e to the power 2 minus e 2 e power minus 3.25 t plus 2 by 9 1 minus e to the power minus 2.25 t minus 2 by 13 1 minus e to the power minus 3.25 t; so, we can just simplify this equation and get the final expression of C C.

(Refer Slide Time: 52:14)



The C C will be simply, 3 times C 1 minus C 2 1 times C 2 plus 1 times C 3; so ultimately you will be getting minus three e to the power minus 2.25 t plus 2 e to the power minus 3.25 t plus e to the power minus 0.25 t minus 3 by 9 1 minus e to the power minus 2.25 t plus 2 by 13 1 minus e to the power minus 3.25 t plus 1 multiplied by 1 minus e to the power minus 0.25 t; so that gives the final solution of C C. So, we get the expression of concentration of A B and C as a function of time.

So, we can get the steady state solution also, steady state solution is obtained when t tends to infinity; so, once we just put t tends to infinity in the solution, we get the steady state solution. And the steady state values of C A at the steady steadies 1 by 9 molar gram mole per liter. And C B steady state is 8 by 117, and C C steady steadies 32 by 39.

So, one can get the transient or non-homogenous ordinary differential equation and the Eigenvalue - Eigenvector method can be fundamentally utilized, to solve elegantly the set of ordinary differential equation homogenous and non-homogenous both and also there should be the eigen, Initial Value Problem; so, IVP homogeneous and non-homogeneous, ordinary differential equation can be solved by this using this a Eigenvalue - Eigenvector methods.

Next, in the, we will be, we can see that since this the solution contains the form e to the power minus lambda t; now depending on the sign of lambda one can expect the solution, how you get the solution grows in time or decays with time, because the solution will be in the form e to the power lambda t; so sign of lambda plays a crucial role and so therefore this Eigenvalue or Eigenvector method plays a very crucial role in determining the stability of ordinary differential equations. And which are basically representing the chemical engineering system; so, chemical engineering systems can also be directly related to the signs of the Eigenvalues.

So, therefore, in the, so we stop in this class and the next class we will start looking into the stability of the chemical engineering system depending on the sign of the Eigenvalues. And how these will be quantified and put in a more formulized mathematical way to evaluate the stability of the system. Thank you very much.