**Advanced Mathematical Techniques in Chemical Engineering**

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**Module No. # 01**

**Lecture No. # 10**

**Eigenvalue Problem in Discrete Domain (Contd.)**

Good morning everyone. So, we are looking into the matrices, determinants, Eigen value problems and their applications in various chemical engineering problems. So, in the last class we looked into the standard Eigen value problems. And there are various properties of an Eigen values and Eigen vectors of a square matrix would satisfy and we are looking into some of these properties and we have derived several theorems for that. The last theorem as I remember is that, we have proved that for a symmetric matrix the Eigen vectors are orthogonal to each other if the Eigen values are simples; that means, all the Eigen values are distinct.

Now, in this class we will look into one more theorem that is that will be quite appropriate and it will be quite useful as we go along the along this course, various classes of this course. This theorem is the last theorem that we are going to look into the series of theorems of an Eigen value problem should satisfy and after this will be looking into some of the typical major applications of Eigen value problems in chemical engineering applications.

Now, this theorem is that, for the eigenvectors for a matrix and its transpose matrix, they are orthogonal to each other, they form a bi-orthogonal set. We have already seen earlier that the Eigen values of a matrix and its transpose matrix are identical, but their Eigen vectors may not be identical. So, in this theorem we will be proving that Eigen vectors of A and A transpose, therefore the matrix is A, they are mutually orthogonal to each other and form a bi-orthogonal set.

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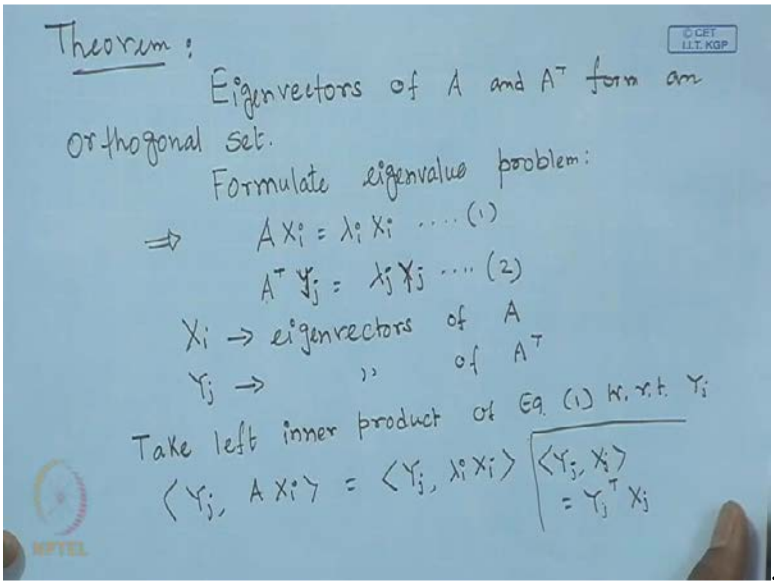


So, the last theorem that we are going to prove, which deals with properties of the Eigen vectors is that, Eigen vectors of a matrix a and its transpose matrix a transpose, they form an orthogonal set. So, we formulate the corresponding Eigen value problem. This problem is A X i is lambda i X i, that is number one. Next one is a transpose Y j is lambda j Y j. So, this is Y. This is the Eigen value problem for the matrix A and this is the Eigen value problem for the matrix A transpose, where X i's are Eigen vectors of matrix A and Y j's are the Eigen vectors of matrix A transpose.

So, next what we do? We take the left inner product of equation one with respect to Y j and see what we get. Take left inner product of equation one with respect to Y j, and what will be getting is that Y inner product of Y j A X i is nothing but, inner product of Y j lambda i X i.

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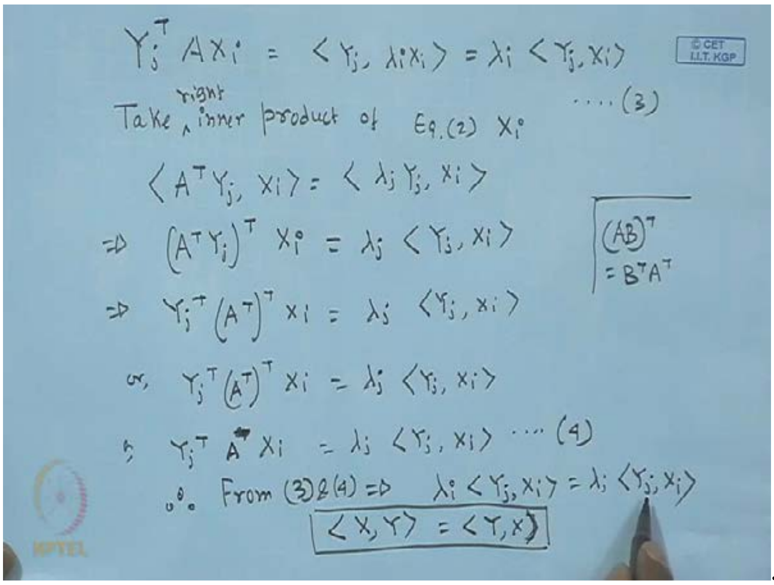


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So, if you remember the property of the of the inner product that we have already proved this earlier, that is inner product of Y j and X j will be nothing but, multiplication of Y j transpose and X j. That we have already proved earlier, so utilizing this will be simplifying the left hand side of this equation. So, this equation, so it will be nothing but, Y j transpose multiplied by A X i. So, if we get that you will be writing Y j multiplied by A X i should be is equal to inner product of Y j and lambda i X i.

Now, we have already looked into the property of inner product, since lambda i being an Eigen value, it is a scalar. So, this scalar will be coming out of the inner product sign and it will be Y j and inner product of Y j and X i. So, this is equation number three. So, next we take the inner product of equation number two, that is the Eigen value problem of A transpose with respect to X i. So, we take right inner product of equation two with respect to X i. If you do that what you will be getting is that, A trans inner product of A transpose Y j and X i should be is equal to inner product of lambda j Y j and X i.

Now, using the same rule that we have already theorem, that we have already proved earlier, these will be nothing but, multiplication of transpose of A transpose Y j and X i. So, if you write it that way, you will be getting A transpose Y j transpose of that X i is equal to, again here the lambda j being a scalar, so it will be coming out of the inner product. So, it will be Y j comma X i.

Now, we invoke the property of the matrix operation, that is A B transpose is nothing but, B transpose A transpose. Invoking this property and utilizing it over here, what we will be getting is Y j transpose and A transpose transpose of that X i lambda j inner product of Y j and X i. So, you will be getting Y j transpose, transpose of transpose, so it will be A transpose and enter A transpose should be is equal to lambda j Y j X i. So, this will be Y j transpose, transpose of transpose will be the same. So, it will be Y j transpose A X i is equal to lambda j Y j comma X i.

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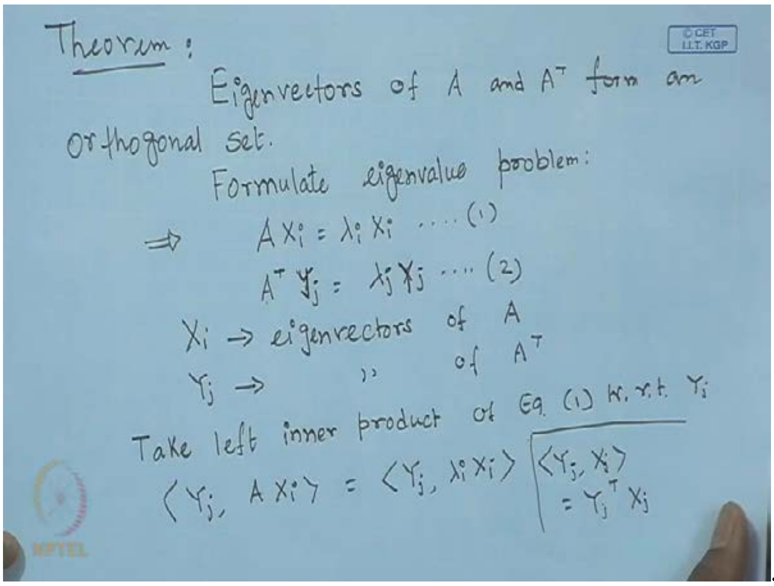


So, if you look into the earlier derivation, that when we take the left inner product equation number one with respect to Y j. So, it will be inner product of Y j and A X i is equal to Y j inner product between Y j and lambda X i, we invoke this property inner product of Y j and X j should be Y j transpose and X j. So, in this equation we will be having a transpose over here. So, that is equation number three and this is equation number four.

Now, if you compare the equation between the equation number three and four, you will be seeing that the left hand side is identical. So therefore, comparing the two we can from three and four, equation three and four, we can say that lambda i inner product of Y j X i is equal to lambda j inner product of Y j X i.

So, and we have already seen that inner product of X and Y is identical as inner product of Y and X. So, we utilize this property and the come to conclusion, when you get this on the left hand side, what you will be getting as. So, basically inner product of Y j and X i is identical to the inner product of Y j and X i.

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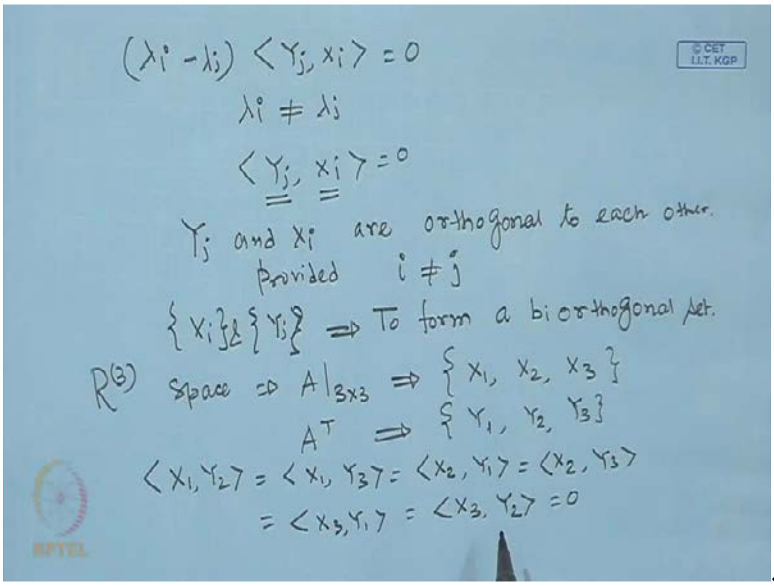

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So, you will be getting lambda i minus lambda j inner product of Y j and X i is equal to 0. Now, this lambda i is not equal to lambda j therefore, will be having to satisfy this equation, the only option we have that inner product of Y j and X i is equal to 0; that means, the Eigen vectors of A transpose and Eigen vectors of A they are orthogonal to each other. So, Y j and X i are orthogonal to each other, provided i is not equal to j. So, this will be satisfied if i is not equal j. So, this set of the Eigen vectors of A and Eigen vectors of A transpose, they are called to form a bi-orthogonal set.

So, if we talk about a vector in R three space, a matrix in R three space. So, it will be a three into three matrix, A is basically have a size three into three, and it will be having the Eigen vectors X 1 X 2 and X 3. Similarly, A transpose will be having Eigen vectors Y 1, Y 2 and Y 3, then by using this theorem one can say that inner product of X 1 and Y 2 is equal to inner product of X 1 and Y 3 is equal to inner product of X 2 and Y 1 is equal to inner product of X 2 and Y 3 is equal to inner product of X 3and Y 1 is equal to inner product of X 3 and Y 2 will be is equal to zero. That means, unless and until i is not equal to j, all the eigenvectors they form the bi-orthogonal set and they will be orthogonal to each other.

So, that completes this proof that A and A transpose, the Eigen vectors of A and A transpose will form a bi-orthogonal set and these Eigen vectors will be orthogonal to each other. So, that completes various theorems of a standard Eigen value problems, where the Eigen values and Eigen vectors will obey these rules or axioms. Now, next what we will be doing, we will be looking into several chemical engineering applications of this Eigen values and Eigen vector problems.

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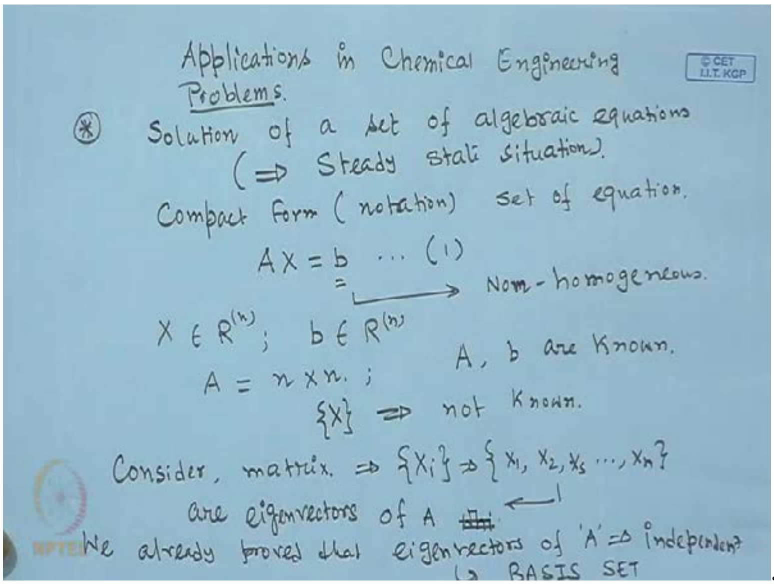


So, let us look into the applications in chemical engineering problems. The first application I will be talking about is the solution to non-homogenous set of algebraic equations. Whenever we will be writing the mathematical models for any chemical engineering systems, typically for a steady state problem, you will be landing up with a set of algebraic equation. In case of transient problems, you will be landing up with a set of ordinary differential equations.

Now, so both of these set of equations either algebraic equations or ordinary differential equations, they are typical for any chemical engineering processes depending on the steady state and unsteady state. Now, using these Eigen value problems, one can elegantly solve these set of equations. So, we will be using this method or the Eigen value method as a tool to solve various chemical engineering problems.

Now, we will be looking into the solution of set of algebraic equation. Algebraic equations, these are typically correspond to steady state problems, steady state situation.

So, we need to solve a set of equation in a compact form, the compact notation, compact form or matrix notation, these set of equations is written in this form A X is equal to b. And this term b on the right hand side is the source of non-homogeneity and therefore, these equations are called non-homogeneous algebraic equations.

Now, the vector X belongs to n-dimensional real space. It would formulate the whole problem in a more generalized fashion, so that it can be reduce to solve solution of any dimensional, finite dimensional problem, may be three-dimensional problem, may be ten-dimensional problem. And similarly, the vector b belongs to n-dimensional real space, where the matrix A is having a size n into n.

Now, in this equation one, A b are known, both A b are known, because whenever you are writing, you are modeling the chemical engineering process, what you are basically doing, you are writing some mathematical expressions to express the chemical engineering processes. So, that is called the modeling, solution of them is the simulation.

Now, the matrix, the various elements of the matrix A will be determine from the processes and we will be looking into specific example of this. And similarly, from the model equations, the various elements of this vector b will be known. So, A and b are known, they are fixed by the process of the chemical engineering process. On the other hand, the solution matrix, the solution vectors X is not known.

So, therefore, where our aim is to obtain this vector X. Now, we consider, so what is the method that we are going to adopt? We consider matrix A and the set of vector X, say set of vectors X i, which are basically constitute X 1, X 2, X 3 up to X n, they are the Eigen vectors of A which are. So, set of vectors A, X 1 to X n, are the Eigen vectors of A. And of course, we have already proved that Eigen vectors of a matrix are independent to each other and they form a basis set.

So, therefore, we already proved that Eigen vectors of a matrix A are always independent, they form independent set of vectors so therefore, they form a basis set. That means, any other vector in the space can be written as a linear combination of this Eigen vectors which are independent vectors. Therefore, this the vector b in equation number one can be expressed as a linear combination of Eigen vectors, which are nothing but, Eigen vectors of the matrix A.

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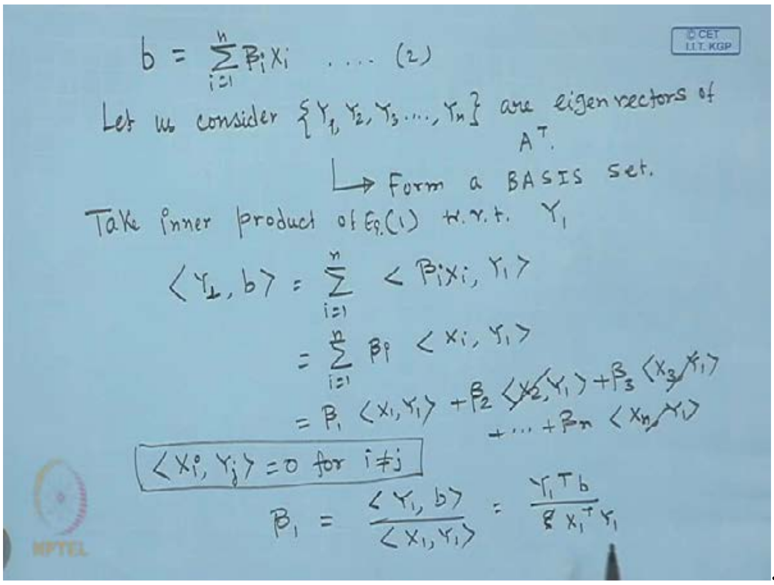


So, let us express the vector b as a linear combination of Eigen vectors i is equal to 1 to n. So, since X i's form the basis set vectors or independent set vectors therefore, the vector b can be expressed as a linear combination of this.

So, the coefficients beta i are not known to us, we are going to determinate. Now, let us consider another case, let us consider the set of vectors, Y 1, Y 2, Y 3 up to Y n are Eigen vectors of A transpose, A transpose, and since these are also Eigen vectors of a matrix A transpose, they form a set, they form a basis set.

Now, next what we do, we take the inner product of equation a with respect to, lets say any Eigen vector of A transpose, lets say with respect to Y 1, take inner product of equation one with respect to vector Y 1. So, what we will be getting is that, Y 1, inner product of Y 1 and b will be nothing but, summation if you just express it, you just open up the summation term wise, then it will be inner product of each term with respect to Y 1. So, we can put the summation outside and this will be simply i is equal to 1 to n inner product of beta i X i and Y 1. So, this will be nothing but, i is equal to 1 and beta i being a scalar, so by the property of inner product, it will be taken out of the inner product sign and this will be simply X i and Y 1.

Now, we open up this summation. So, this if you open up this summation, this will be become beta 1 inner product of X 1 Y 1 plus beta 2 inner product of X 2 Y 1 plus beta 3 inner product of X 3 Y 1 likewise. And finally, you will be having beta n inner product of X n and Y 1. And we have already seen that for the two matrices, in this class is the first theorem that we have proved in this class, that for the matrix A and A transpose the corresponding Eigen vector will form a bi-orthogonal set.

So, therefore, X i and Y j inner product of that will be always equal to 0, for i is not equal to j, where X i's are the Eigen vectors of a and Y j's are the Eigen vectors of A transpose. Therefore, all these terms will be equal to 0, X 2 inner product of X 2 and Y 1 X 3 and Y 1 X n and Y 1, only one term will survive out of these summation series and that will be beta X 1 and Y 1,. so inner product of X 1 and Y 1. So, you will be able to calculate the value of beta 1, that will be a simply Y 1 inner product of Y 1 and b and inner product of X 1 and Y 1. So, you can write it in more compact invitation, Y 1 transpose b and X 1 transpose Y 1. So, it will be matrix multiplication.

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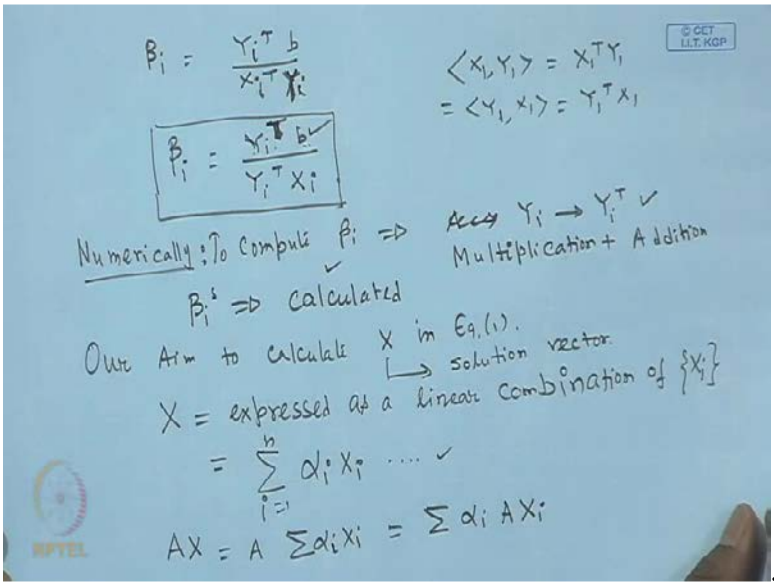

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So therefore, this whole thing can be calculated as beta 1. So, you can generalize it and we will be able to get the form of beta 1 as Y i transpose b and you can write it as X 1 transpose Y 1. You can put it in the notation i and you can always argue that inner product of X 1 and Y 1 is equal to inner product of Y 1 and X 1, so what is X 1 transpose Y 1 that will be identical to Y 1 transpose X 1. So therefore, since this is in the form of Y j transpose, we can write it in this form in a more generalized fashion, Y i transpose b divided by Y i transpose X I, this will be Y.

So, they are identical inner product of X 1 Y 1 is equal to inner product of Y 1 X 1. So, inner product of X i and Y 1 is identical to inner product of Y i X i. So, they are basically identical. Why we have written in this form? Because here it appears this transpose ,here it is appearing as Y i, so I would like to put Y i t and X i.

Why I will be doing that? I will just comment in a minute that, whenever you will be having a large number of system, large dimensional system, for example, a 10 into 10 dimensional system or may be 20 into 20 dimensional system, it is a very complicated system. So, you will be tackling with so many equations simultaneously, then you cannot calculate all these things by using a calculator or by hand. So, you have to take recourse to the numerical techniques.

So, if you put it in this form the numerical operations will be more. So, for numerical, if you talk about the numerical calculations, what is basically you will be doing that, you will be taking recruits of simple assumptions, simple operations in order to compute beta i. To compute beta I, what the operations you have to take, you have to basically get, once you have a vector, you have to get the transpose of the vector. How the transpose of vectors will be done? It is a very simple operation, it is basically the elements in the rows will be or columns will be assign to those in the rows.

So therefore, by changing the indices, one can get a transpose vector, may be Y i to Y transpose is basically it is a one line program. Next is the multiplication of the two vectors or two matrices, this is 1 into n this is n into 1. So, you can have a matrix multiplication and this matrix multiplication, if you remember, it is basically multiplying the corresponding terms and add then up. You multiply the corresponding terms and add them up and that will be giving the value of the multiplication of the matrices.

So, it is basically, the basic operation is multiplication, first is multiplication, second is addition. You multiply the corresponding elements and add them up. So therefore, once you get this operation, Y i to Y transpose. So, by writing one simple one or two lines simple subroutines, you need not to compute X i transpose. So, what is the advantage you are going to get by writing it in this form? The advantage is that, once you are writing a program for getting the value of and elements of Y i transpose ,you need not to compute the elements of X i transpose, same Y i transpose will be appearing in the denominator and the numerator. So, you avoid one more subroutine which will be calculating X i transpose.

So, that is not require, that is why we have written this equation keeping the denominator in the form of Y i transpose, so that same transpose routine, since all the elements of Y i transpose are already been computed that can be directly used here, and we are we have reduced one step of computation. So therefore, it is a known vector b and X i's are known because they are the Eigen vectors of the matrix A. Y i's are known because they are the Eigen vectors of Y transpose. So, we will be able to compute the values of beta i. So, beta i's will be calculated from this equation.

Now, if you remember that in our original problem our aim is to calculate X in equation number one. Our aim was not to calculate beta i but, our aim was to calculate X, the solution vector in equation one. And what is X? It is nothing but, the solution vector.

Now, since again this vector belongs to n-dimensional space, again this vector can be written as a linear combination of Eigen vectors of A trans of the matrix A because they form a basis set of vectors. So therefore, X can be expressed as a linear combination of basis set vectors which are nothing but, the Eigen vectors of matrix A. So therefore, we can write, X can be written as i is equal to 1 to n alpha i X i. And so, this is one more equation and then what do you do since matrix is an operation, we take, we operate this equation by A. So, whatever we will be getting is that A X is nothing but, summation of A alpha i X i. So, alpha i can be taken as, alpha is being a scalar, so it will be operated on every vector. So, it will be nothing but, summation alpha i a X i. And if you remember what these X i's are? These X i's are nothing but, the Eigen vectors of the matrix A, so they will satisfy the equation A X i is equal to lambda i X i.

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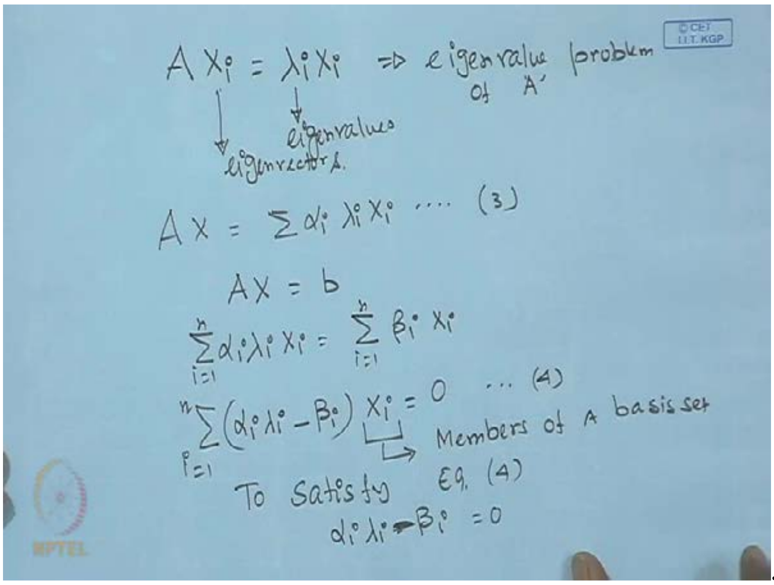

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So, A X i’s will always satisfy the corresponding Eigen value problem. This is the Eigen value problem of A, of matrix A, where lambda i are the Eigen values and X i’s are Eigen vectors. So therefore, we can write this equation as A X is equal to summation alpha i lambda i X i. So, this is equation number three. Similarly, we have the, if you remember what was the original problem, the original problem was a set of non-homogeneous equations. So, A X is equal to b. So therefore, we just substitute A X by this equation. So, this will be summation alpha i lambda i X i and this b, if you remember from equation number two, we have written as b as a linear combination of the Eigen vectors X i. So, this will be nothing but, beta i X I, where the index i runs from 1 to n one and 1 to n. So therefore, we can take this on this side. So, this will be alpha i lambda i minus beta i is equal to multiplied by the vector X i is equal to 0, where the index i runs from 1 to n.

Now, since X i forms an independent set of equation or independent set of vectors. So, they are members of a basis set, they are all independent. So therefore, to satisfy this equation to be 0, all the coefficients have to be identical equal to 0 and we have already proved this earlier. So therefore, to satisfy equation number four, one has to have the corresponding coefficient should be equal to 0. So, alpha i lambda i minus beta i should be equal to -.

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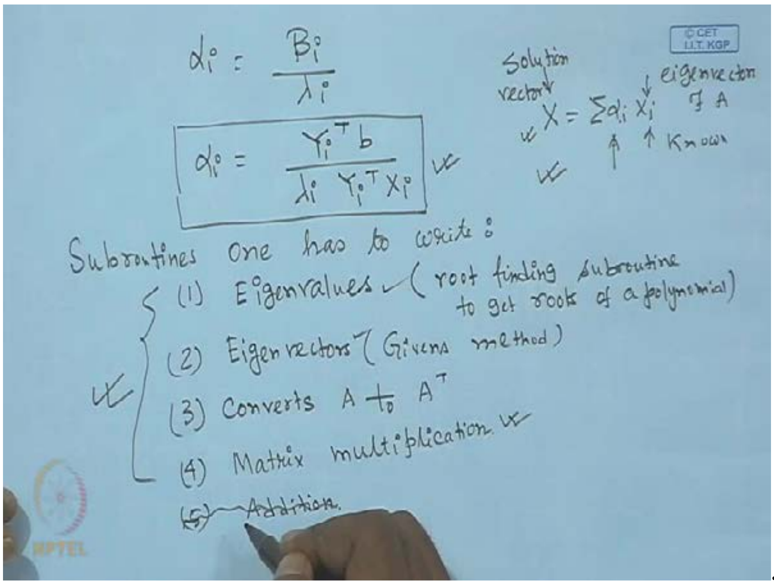


So, therefore, the condition that we are going to have to get this equation correctly, we have the alpha i is equal to nothing but, beta i by lambda i. So therefore, we will be getting, what is beta i? Beta i we have already proved that it is nothing but, Y i transpose b divided by lambda i, lambda i will be there, Y i transpose X i. So, this will be the expression of alpha i. And if you remember, what this alpha is at there, solution vector is formed by alpha i X i. So, this is the solution vector that we are looking for. These are the Eigen vectors of A. So, these are known, alpha i at the corresponding, alpha i is obtained from this equation. So, where in this equation b is a known vector, Y i at the Eigen vectors of A transpose, so again this is a known vector. So, the numerator can be computed, lambda i at the corresponding Eigen values of A, Eigen values of A and A transpose both are identical, we have already proved that. So, this is also known, numerator is known and X i's are basically the Eigen vectors of A, they are also known. So, alpha i can be computed. Once alpha i is known, by using this equation, you can X i's are known. So, alpha i is unknown, you can get the solution vector X.

So, therefore, this is an elegant way of getting the solution of set of algebraic equation. So, let us look into what are the different subroutines, one has to make to compute this problem, for solution of this problem numerically, subroutines one has to write are as follows.

First one is that, you have to have a subroutine to get A trans[pose]- first you have to get the subroutine to get the Eigen values, subroutine to compute the Eigen values of a matrix. And to get the Eigen values, you will be getting a characteristic equation and from the characteristic equation you can find out the, using the honors method, one can find the number of roots in a polynomial. So, basically you will be getting a root finding subroutine, get roots or zeros of a polynomial. Next, corresponding to these Eigen values we have to have a subroutine for Eigen vectors. May be there are several method, Givens method is one of them. So, you can compute the Eigen vectors, then only these two are the major things, then you have to write a program which will get a transpose matrix from a matrix, convert a program that converts A to A transpose, that is again a small program, couple of lines program may be. Then second a program for matrix multiplication.

Next, you have to get a program for addition because whenever you multiply, you have to multiply the corresponding elements and add them up. So, basically this will be included here itself. So, you require to have a matrix multiplication, multiplication includes the multiplication of the corresponding elements and then add them up.

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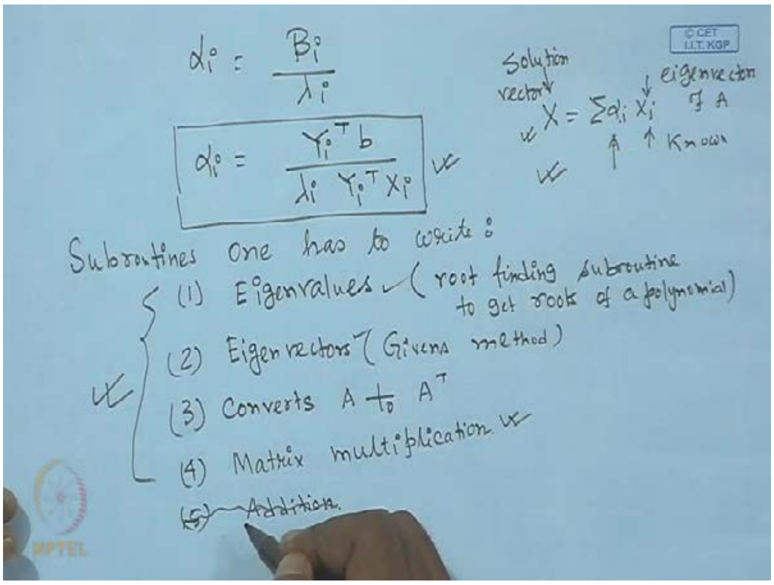


So therefore, it does not matter. So, the addition operation is included into the matrix multiplications subroutine. So, you require only these four operations, four subroutines to compute the solution vector X from a set of algebraic equations. Now, that completes the method how to compute the solution of set of non-homogeneous algebraic equation using the Eigen value problem, whatever we have discussed in the last few classes.

Now, if you remember how will you solve a set of non-homogeneous algebraic equation, probably if you remember Gauss Seidel algorithm or by Gauss Elimination method, one can compute either analytically or numerically. If it is a large dimensional problem, it is easier to go for numerical techniques. Now, for using and for implementing Gauss Seidel algorithm, the major problem or the major barrier one will face is getting the inverse of the matrix.

So, that becomes a very problem, problematic thing, if the matrix is not well behaved or well posed, then the determinant, the matrix may be a singular matrix, the determinant may be tending to 0. So, matrix inversion becomes a very big problem, it is a big challenge numerically for solving such set of algebraic equations. So, but, if you adopt to this method, one can do away with the matrix inversion and just using the algorithm for evaluation of Eigen values and Eigen vectors and simply matrix multiplication and transpose of the matrix, that will give you the solution vector. And one can just avoid the matrix inversion in this particular method, and you will be getting an elegant solution by using the Eigen value problem for the solution of set of linear algebraic equation.

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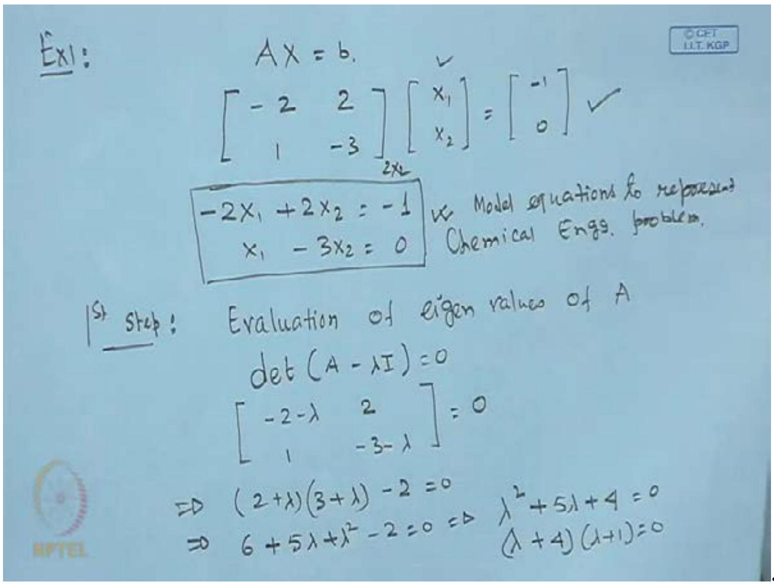


Now, I will takeoff couple of examples to demonstrate this method. The first example that I will be talking about is, solution of the problem a X is equal to b, where the, I will be taking a very simple problem in order to demonstrate, otherwise one if it is a, I will be taking a two-dimensional problem for demonstration purposes. For hard dimensional problem, one has to go for the numerical methods to get this.

Now, all these subroutines that I was just talking about in order to solve this set of equations, the Eigen values, Eigen vectors, conversion from A to A transpose, multiplication of the matrices, all these subroutines are, you need not to write your own code, these subroutines are always available in the numerical recipes, they are available either in Fortran or in c plus plus, or one can use the MATLAB code to connect all these routines together and can get the complete solution numerically. So, you need not to write your own code, you can invoke these subroutines either from libraries or MATLAB or you can join them up to get a complete solution.

So, for the demonstration purpose, I will be taking up this example, A X is equal to b, where the elements of A are minus 2 and 2 1 and minus 3 and solution matrix, the solution vector X will be composing of two elements X 1 and X 2, and b will be composing of minus 1 and 0. So, basically the chemical engineering system will be described by 2 X 1 plus 2 X 2 is equal to minus 1 and X 1 minus 3 X 2 is equal to 0. The chemical engineering process will be having two unknowns and these two unknowns will be requiring two equations to solve them uniquely. So therefore, these are the model equations to represent the chemical engineering problem and probably this chemical engineering system is operating under steady state, so we model the equation, model the system by writing these two equations. So, these two equations can be written in a compact matrix form, in the form of X equal to b in this fashion.

Now, our aim is to find out what are the solution vector X 1 X 2 like that. In this case, since it is 2 into 2 matrix, will be having only two solution, two elements in the solution vector X 1 and X 2. So, our aim is to obtain X 1 and X 2. So, first step lambda i is ,we find out the Eigen values of matrix A. Evolution of Eigen values of matrix A. So, how to evaluate them? You can evaluate by computing determinant of A minus lambda i is equal to 0. And if you remember that minus two minus lambda, minus two minus lambda two one minus three minus lambda, that is the determinant of A minus lambda I, that should be is equal to 0. So, if you open up this determinant, this into this minus this into this. So, it will be 2 plus lambda, minus minus plus, so it will be 3 plus lambda minus 2 will be is equal to 0. So, it will be a quadratic, it will be 3 into 6 plus 3 lambda plus 2 lambda 5 lambda plus lambda square minus 2 is equal to 0. So, you will be getting lambda square plus 5 lambda plus 4 is equal to 0. This is a quadratic equation, you can get the roots as lambda plus 4 into lambda plus 1 is equal to 0. So, two Eigen values are obtained as minus 4 and minus 1 in this case.

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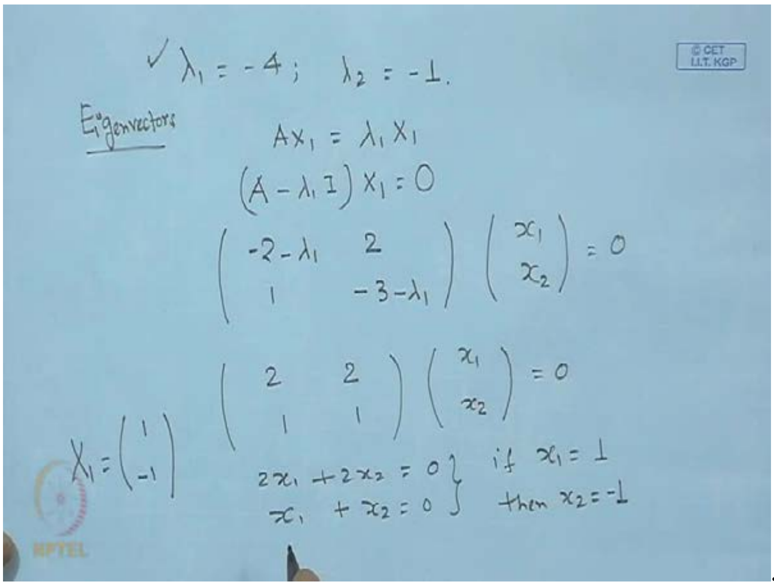


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So, the Eigen values are lambda 1 is equal minus 4 and lambda 2 is equal to minus 1. Now, we get the Eigen vectors corresponding to lambda 1, you will be having the Eigen vector X one, so A X 1 will be nothing but, lambda 1 X 1. So, A minus lambda 1 i X 1 should be is equal to 0. So, what you will be getting is that, I am writing the elements of this matrix, minus 2 minus lambda 1 2 1 minus 3 minus lambda 1 and the A means of the Eigen vectors are X 1 and X 2 that will be equal to 0.

Now, if you put lambda 1 is equal to minus 4 you just write, so let us write these elements of this equation of this matrix, 2 2 1 1 X 1 X 2 will be is equal to 0. So therefore, we constitute the corresponding two equations 2 X 1 plus 2 X 2 is equal to 0, X 1 plus X 2 is equal to 0. So, they are giving basically identical solution. So, we have to just assume one value. So, if X 1 is equal 1, then X 2 is equal to minus 1. So, 1 minus 1 will be one of the Eigen vectors corresponding to the Eigen value, lambda 1 is equal to minus 4.

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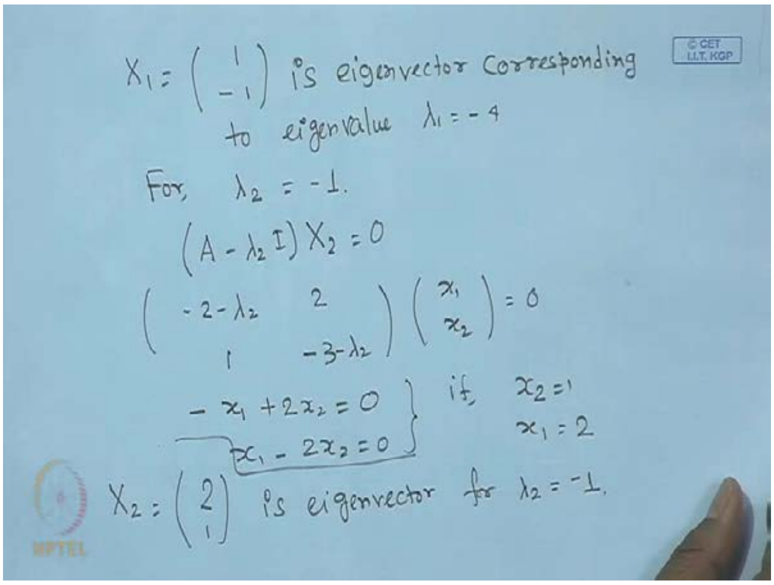


So therefore, X 1 is nothing but, 1 minus 1, that is the solution of this set of equation. So, 1 minus 1 will be the Eigen vector corresponding to lambda 1 is equal to minus 4. So, we write it down more explicitly. So, X 1 is equal to 1 minus 1 is Eigen vector corresponding to Eigen value lambda 1 is equal to minus 4.

Then, we compute X 2, the Eigen vector corresponding to lambda 2 is equal to minus 1. For lambda 2, next Eigen value lambda 2 is equal to minus 1. So, again we formulate A minus lambda 2 I, formulate the Eigen value problem, I X two is equal to 0. So, this will be minus 2 minus lambda 2 2 1 minus 3 minus lambda 2. And again the elements will be, let us say X 1 X 2 that will be equal to 0. So, you will be getting minus X 1 plus 2 X 2. You put the value of lambda 2 as minus 1. So, you will be getting this minus 1 plus 2 X 2 will be equal to 0, X 1 minus 2 X 2 is equal to 0.

So, again they are giving identical, they are identical. So, if X 2 is equal to 1, then X ` will be is equal to 2. So therefore, 2 1 is Eigen vector corresponding to lambda 2 is equal to minus 1. For the Eigen value as minus 1, the corresponding Eigen vector is 2 1. So, we have found out the Eigen values and Eigen vectors of A. Next step is evaluation of Eigen vectors of A transpose.

Now, we have already seen that Eigen values are identical for a matrix A and its transpose matrix A and A transpose, but the Eigen vectors are different. So therefore, in the next class we will be looking into how the Eigen vectors are evaluated for A transpose and the rest of the solution follows. And we will closely look into the solution step by step and complete the problem in the next class We stop this class here and we will start the next class after few minutes. Thank you very much.