Process Control and Instrumentation
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Lecture - 8
Dynamic Behavior of Chemical Processes (Contd.)

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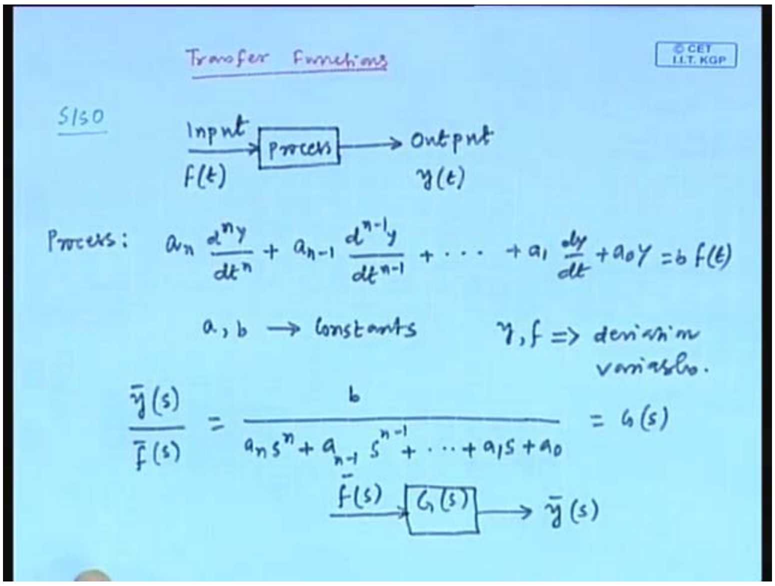

We will continue our discuss on transfer functions. Yesterday, we have considered a single input single output system and we have derived the transfer function for that SISO system. So, for a SISO system the process involves Single Input and Single Output, suppose this is a process, this is the input to the process and this is the output for the process, we have represented the input by f t and output by y t and the process is modeled by an nth order linear or linearized non linear equation.

So, we considered in the last class the nth order model represented by a n d n y d t n plus a n minus 1 d n minus y d t n minus 1 like this a 1 d y d t plus a naught y equals to b f t. A and b these are constant coefficients and both output and input these are deviation variables. In the next step, we took Laplace transform of the process model and finally, we got the expression in f S domain that is like this y S divided by f S equal to b divided by a n S to the power n plus a n minus 1 S to the power n minus 1 a 1 S plus a naught, which is basically the transfer function represented by G S.

Then in the next step, we made the block diagram is like this G S is representing the process the input to that process in Laplace domain in f bar S and output is y bar S, this is the block diagram for the SISO system. Now today, we will consider one multi input, single output system and we will try to derive the transfer function for that process.

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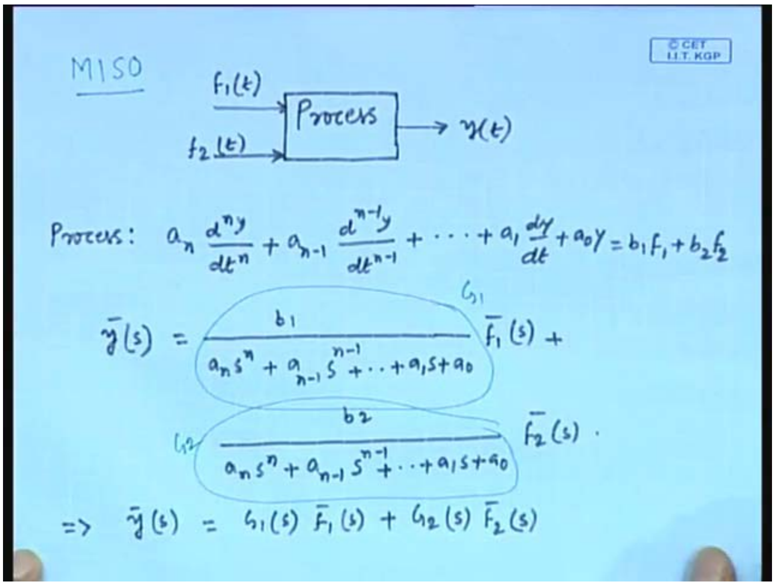

Today, we will derive the transfer function for a multi input single output system MISO. So, the process, we can represent by this. Suppose the process this is the process, this is one input represented by f 1 t, another input we will represent by f 2 t and single output that is y t. So, this is the representation of a multi input single output system.

So, next we have to consider the model for this process. So, we will represent the process by this equation a n d n y d t n plus n minus 1 d n minus y d t n minus 1 like this a 1 d y d t plus a naught y. And, right hand side includes 2 input variables one is b 1 f 1 t and another one is b 2 f 2 t. The process model is quite similar with that of the SISO system only new addition is the input variable, in the previous case, we had 2 input variables, here one input variable.

So, next we have to take the Laplace transform, if we take the Laplace transform and if we rearrange then, we will get the expression like this b 1 a n S to the power n, a n minus 1 S to the power n minus 1, a n S plus a naught multiplied by f 1 bar S. Another expression will be b 2 divided by a n S to the power n, a n minus 1 S to the power n minus 1 a 1 S plus a naught f 2 bar S. In this discussion, you remember that y and f both are the deviation variables.

So, this is the representation in S domain of the linear model or linearized non linear model. So, we can write this equation in this form y bar S equal to G 1 S f 1 bar S plus G 2 S f 2 bar S, we can rearrange the equation, I mean the equation in Laplace domain by this equation, here this part represent G 1 and this 1 is G 2.

In the next step, we need to develop the block diagram, what will be the block diagram of this system. So, we have basically 2 transfer function one is G 1 another one is G 2, 2 inputs in Laplace domain one is f 1 S another one is f 2 S and single output that is y S.

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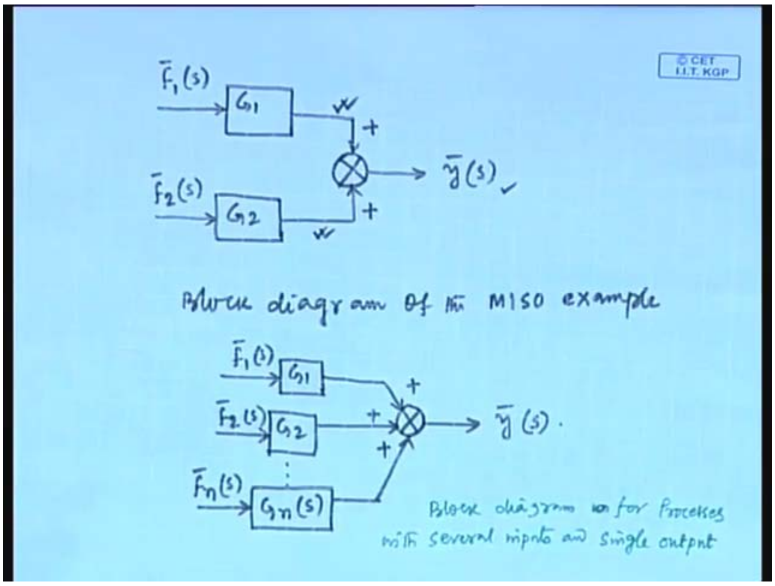

So, one block for G 1 input to this G 1 is f 1 bar S another block, we need to use for transfer function G 2 input to this block is f 2 bar S. Now the output of these 2 blocks, we need to add suppose, we are using this symbol to add this 2 signals, one is this one another one is this one. So, this output is basically f 1 G 1 and this is f 2 G 2. So, f 1 G 1 plus f 2 G 2 that is nothing, but y S agree f 1 multiplied by G 1 that is this signal and this signal is basically multiplied by G 2. So, if we add this 2, we get y S.

So, this is the block diagram, of the multiple input single output example. Now, if we consider n number of input variables, but, a single output variable what will be the block diagram for that. If we consider n number of input variables f 1 f 2 up to f n and if we consider only one output variable y, what will be the block diagram that you draw.

So, this is G 1, input to this G 1 is f 1, next 1 is G 2 input 2 G 2 is f 2 like this way last term will be suppose, last transfer function is suppose G n S input to this is m f n S by adding all these output, we get y S. So, this is for several input and single output, this is the block diagram with block diagram for process with several inputs and one output or single output.

Now, to find the transfer function of a process of a non linear process, we need to note 2 important points, one is if the model is non linear, we need to linearize that equation. This is the first point second point is we need to represent the linearized equation in terms of deviation variables.

So, these are 2 important points, which we need to follow to derive the transfer function, one is if the process model is non linear, we need to linearize that equation, if that is linearize that is, second point is we need to represent the linearized equation in terms of deviation variables. Another question comes that is what is the use of this transfer function? We will consider one single input, single output system.

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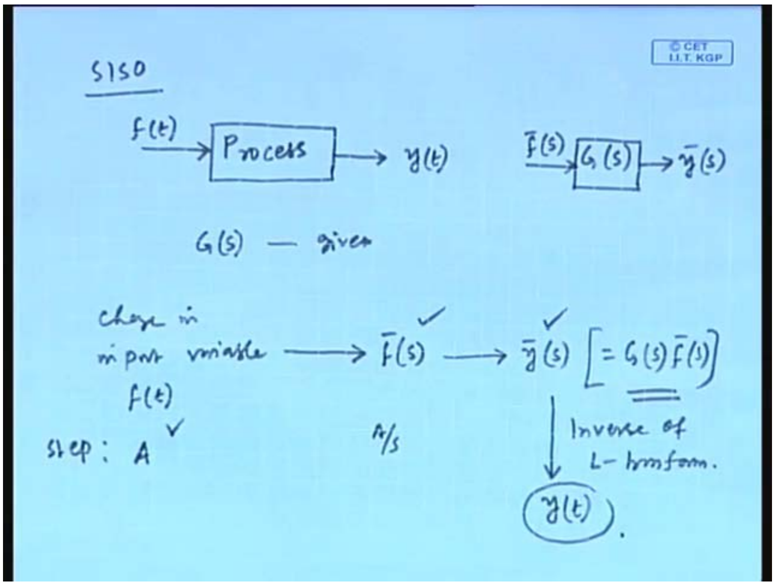

So, for a S I S O system, we consider one process and input to this process is f t output to this process is y t for this S I S O system, the block diagram is G S input is f bar S output is y bar S.

Now, G S is known to us I mean, if we have the process model, we can easily derive the transfer function for that process. So, G S is derived I mean, G S is known to us. So, G S is suppose given now what we can do, we can give a change in input variable ultimate answer is to observe the process dynamics, we use the transfer function fine by the use the transfer function, we know the transient behavior or dynamic behavior of the process, but how we can know that we have the process initially at steady state.

Now, we are introducing some change in input variable and that is represented by f t, we are introducing some change in f t then we can easily transform this, f t to f S by taking Laplace transform. Say for example, if we introduce a step change with magnitude a suppose this is a step change with magnitude A, we can change it in S domain, how much that will be A by S. In the next step, we can determined y bar S, how we can determined y bar S y bar S is G S f bar S.

So, it is straight forward to determine y bar S, now we need to take the inverse of Laplace transform. So, that we get y t, if we can determined y S then, we can get y t by taking inverse of Laplace transform. So, we have a particular process, we are introducing some change in input variable in terms of f t. Now, we can change to that input in S domain represented by f bar S, in the next step, we can calculate y bar S, because y bar S is equal to G S f bar S then by taking inverse of Laplace transform, we can know y t.

So, if we give a change in input variable f t, we can know the transient behavior of that process in terms of y t. So, that is a use of transfer function. Now, you will consider some chemical engineer examples and we will try to find the transfer functions first we will consider a simple example that is liquid tank system.

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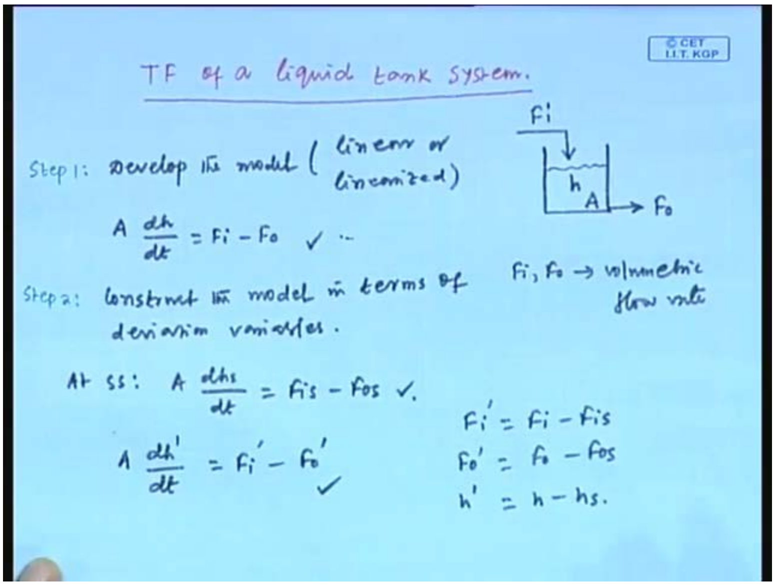

So, we will derive the transfer function of a liquid tank system. So, this is a schematic of a liquid tank system, a liquid is entering at a flow rate of F i and the output is coming out at a flow rate of F naught. The liquid height is h cross sectional area of tank is suppose A. This is a schematic of a liquid tank system, F i and F naught both are in terms of volumetric flow rate.

So, F i and F naught are the volumetric flow rate, I hope, we have consider this example previously. So, will discuss step wise, in the first step, we need to develop the model in the first step, the model should be linear or linearized non linear model, if the model is linear it is fine, if the model is non linear, we have to linearize that using Taylor series.

So, what is the model of this system, I mean what is the modeling equation, if d h d t equal to F i minus F naught by assuming the liquid density constant. So, this is the modeling equation for the example liquid tank system in step 2, we need to construct the model in terms of deviation variables.

So, to construct the model in terms of deviation variables, we have to consider the model at steady state. So, what will be the modeling equation at steady state condition, at steady state, the modeling equation becomes F i S minus F naught S, we have just added suffix S to represent the steady state anyway. This model is the linear equation or non linear equation, this is the linear equation. So, there is no need of linearization.

So, this is the equation modeling equation at steady state condition. So, how we can represent the modeling equation in terms of deviation variable, we need to subtract this steady state equation from this dynamic equation. So, the model in terms of deviation variables has this form A d h prime d h equal to F i prime minus F naught prime, here F i prime is F i minus F i S, F naught prime is F naught minus F naught S and h prime is h minus h S. So, this is the mathematical model in terms of deviation variables.

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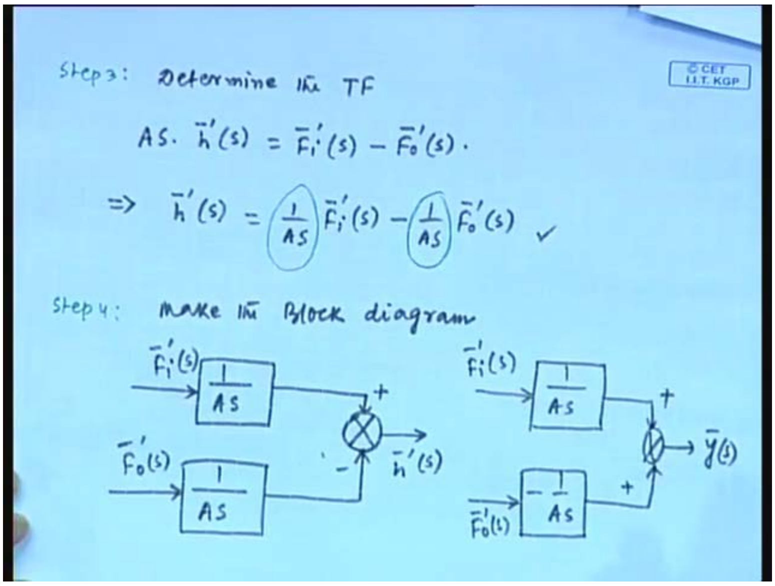

In step 3, we need to take Laplace transform and we need to derive the transfer function. So, in step 3, we will determined, the transfer function. So, for that, we have to take the Laplace transform, if we take the Laplace transform, we get A S h bar prime S equal to F i bar prime S minus F naught bar prime S. This is the representation of the model in S domain.

Now, if we rearrange this equation, we get h bar prime S equal to 1 by A S F i bar prime S minus 1 by A S F naught bar prime S. So, this is the equation, which we got finally, in Laplace domain or S domain in the next step, we need to make the block diagram based on this equation. So, in step 4, we need to make the block diagram can you make the block diagram based on this equation.

So, one transfer function is 1 by A S another transfer function is 1 by A S, you can consider or you can consider minus 1 by A S. So, the block diagram will be 1 by A S input to this block is F i bar prime S. The another transfer function is 1 by A S input is f naught bar prime S. So, we will use here positive sign and here negative sign, output is h bar prime S, we are just considering 1 by A S F i bar prime S minus 1 by A S F naught bar prime S equals to h bar S, why negative sign is there, because F naught is the output and F i is the input.

So, we can say that F naught is the negative input fine or you can do one thing, if you want to consider minus 1 by A S as the transfer function then we can get this block diagram. F i bar prime S and another transfer function is minus 1 by A S F naught bar prime S, we are adding both of these output and we can get y bar S, this block diagram also we can get. In the next, we will consider another example that is a C S T R.

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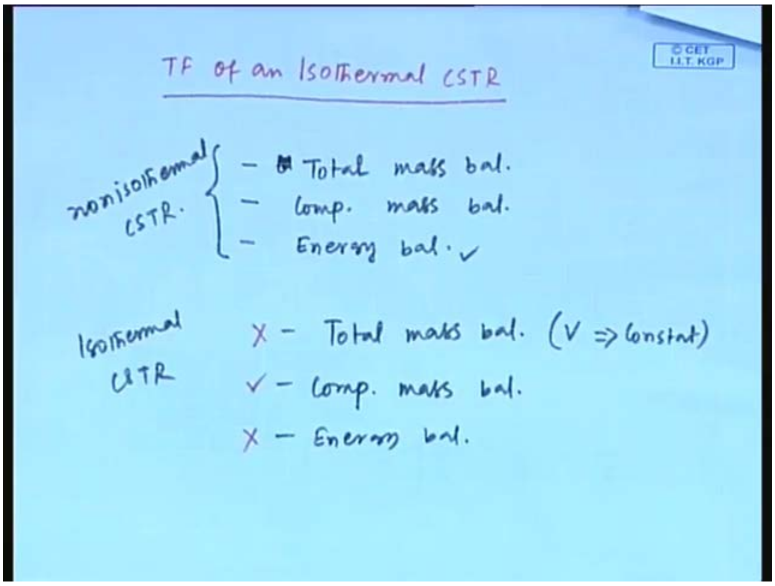

So, we will derive the transfer function of an isothermal C S T R, previously we have consider one C S T R, I mean, we have develop the model structure of a C S T R previously and the model structure of that C S T R includes 3 equations, can you remember this 3 equations.

One is based on mass balance, I mean total mass balance, one equation we got based on total mass balance, second equation we got based on component mass balance and another equation, I mean third equation we got based on energy balance, now isothermal means constant temperature. So, for the case of isothermal C S T R the operation, I mean the reactor operates at constant temperature. So, there is no variation of temperature, if that is the case then we have considered one C S T R and we derived modeling equations based on these 3 balances. So, what is that C S T R.

Yes that is a non-isothermal C S T R, because temperature varies in that C S T R with time. So, the C S T R, which we have modeled earlier that is the non isothermal C S T R, so for a non-isothermal C S T R the energy balance equation, we usually derive. Now here will consider the isothermal C S T R total mass balance, there will not be any total mass balance equations. How we can avoid this, if we assume the reaction mixture volume constant, I mean, if we consider the volume of the reaction mixture constant in that case, we can avoid total mass balance equation. So, we are assuming that.

Second is component mass balance, we will consider fine and third one is energy balance energy balance does not exist for isothermal case. So, presently we will consider the considering the isothermal C S T R that includes only the component mass balance equation.

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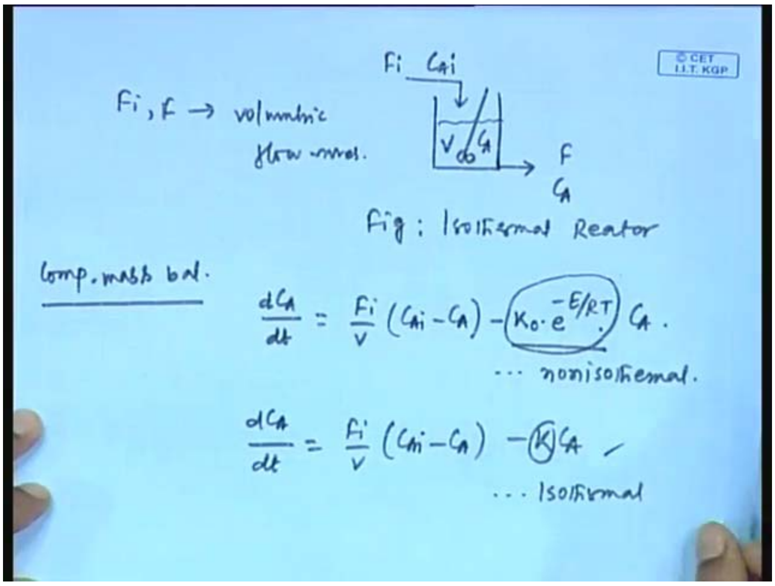

Now, the schematic representation of an isothermal C S T R is like this inlet flow rate is F i concentration is C A i, the output is coming out with a flow rate of F and A concentration of C A. So, the concentration in the vessel is also C A and will assume the value is V, since there is no variation of temperature. So, there is no need of any external heating or cooling medium.

So, what will be the component balance equation then F i and F both are volumetric flow rates, this is a schematic representation of an isothermal reactor, what will be the component balance equation. Component mass balance equation previously, we got this equation d C A d t equal to F i divided by V C A i minus C A minus K naught exponential of minus E R T into C A. This equation, we derived for non isothermal C S T R, how we can represent this equation for the case of isothermal C S T R.

See in this is basically, K is not it, this whole is basically K and in this term, you see temperature is there, since the case is non isothermal, if we consider the isothermal C S T R, the temperature is constant. So, this whole term becomes constant. So, we can rewrite this equation for the case of isothermal C A i minus C A minus K C A for the case of isothermal, there is no variation of temperature, that is way this whole terms becomes constant and that, we are representing by K.

So, this is the modeling equation for the isothermal C S T R, now we need to represent this equation in terms of deviation variables anyway before that, we can simplify this equation like this is out modeling equation d C A d t equal to F i divided by V C A i minus C A into K C A. Now, we can write this equation in this way.

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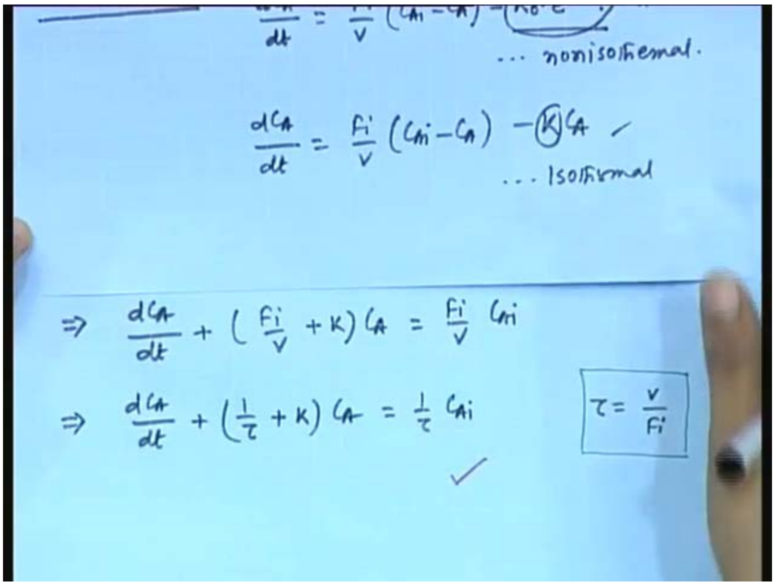

D C A d t plus F i divided by V plus K multiplied by C A equal to F i divided by V multiplied by C A i. It gives d C A d t plus 1 by tau plus K multiplied by C A equal to 1 by tau C A i where, tau is V divided by F i tau is V divided by F i. Next, we will represent this equation in terms of deviation variables to find the transfer function, you directly write the equation in terms of deviation variables.

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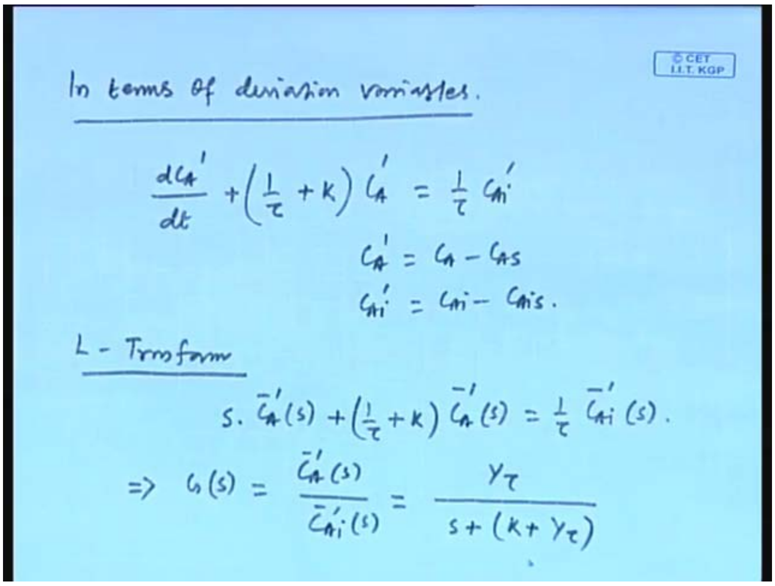

Write the equation in terms of deviation or perturbation variables d C A prime d t plus 1 by tau plus K multiplied by C A prime equal to 1 by tau C A i prime. This will be the model equation in terms of deviation variables here, C A prime is C A minus C A S and C A i prime is C A i minus C A i S to get the transfer function, we need to take Laplace transform, take Laplace transform of this equation. If, we take the Laplace transform of this equation, we get S C A bar prime S plus 1 by tau plus K C A bar prime S equal to 1 by tau C A i bar prime S.

Now, the transfer function represented by G S equal to output divided by input equal to 1 by tau S plus K plus 1 by tau. This is the final representation of the transfer function for the example isothermal C S T R, this is the transfer function. Now, the block diagram is quite straight forward.

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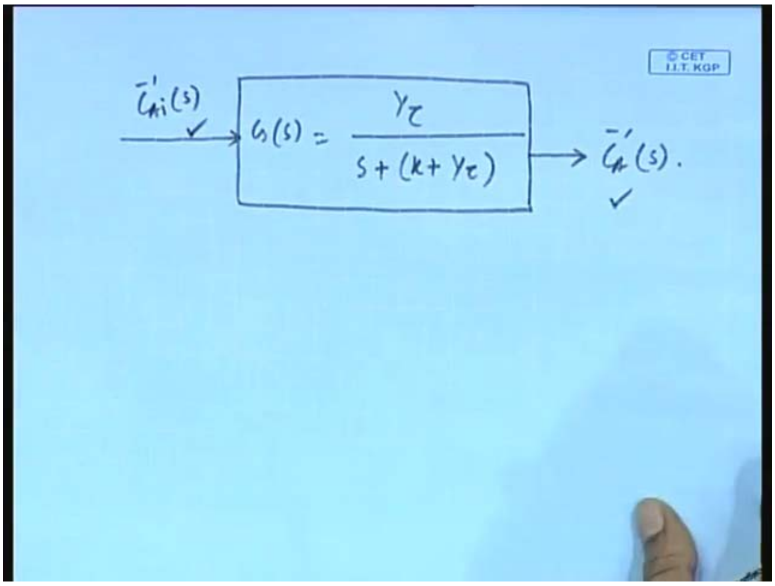

We can directly write G S equal to 1 by tau divided by S plus K plus 1 by tau input is C A i bar S output is C A bar prime S. So, this block diagram is represented in terms of concentration not in terms of flow rates, because we have already assume the volume of the reactor is constant. So, this is the input concentration C A i and this is the output concentration. So, this is the block diagram for the example C S T R.

Now, So, for we have discussed the transfer function of single output although, we have consider multiple input, but our discussion is restrict to the single output. So, in the next, we will discuss with the consideration of multiple outputs. So, we will consider multiple input and multiple output systems.

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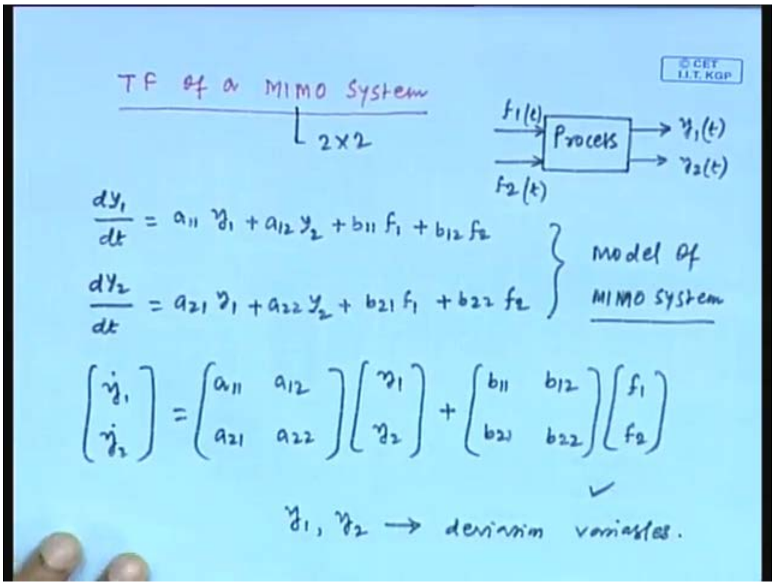

So, transfer function of a multi input multi output system. Now, the process, we will represent in this way, this is the process suppose, 1 input is f 1, another input is f 2, they are in time domain and we will consider 2 outputs, one is y 1 another one is y 2. So, in this M I M O system, I mean multiple input multiple output system, basically we are considering 2 by 2 system, 2 input 2 outputs.

Now, first we need to write the input output model for this process. So, input output model is d y 1 d t a 1 1 y 1 plus a 1 2 y 2 plus b 1 1 f 1 plus b 1 2 f 2, this is our linear equation and this equation is representing the variation of y 1.

Similarly, for the second output y 2, we have the linear model represented by this equation d y to d t equal to a 2 1 y 1 plus a 2 2 y 2 plus b 2 1 f 1 plus b 2 2 f 2. So, this is the model of the 2 by 2 system, I mean M I M O system, we can represent this 2 modeling equations in matrix form and that is represented in this way equal to a 1 1 a 1 2 a 2 1 a 2 2 y 1 y 2 and another matrix for input b 1 1 b 1 2 b 2 1 b 2 2 multiplied by f 1 f 2. Just, we have representing this 2 modeling equations in this matrix form.

Now, here y 1 and y 2 both are perturbation variables or deviation variables. So, y 1 and y 2 both are deviation variables. Now, if we take the Laplace transform of this 2 equations and if we rearrange, we get finally, the equations, which I am writing next, I am writing the final expressions.

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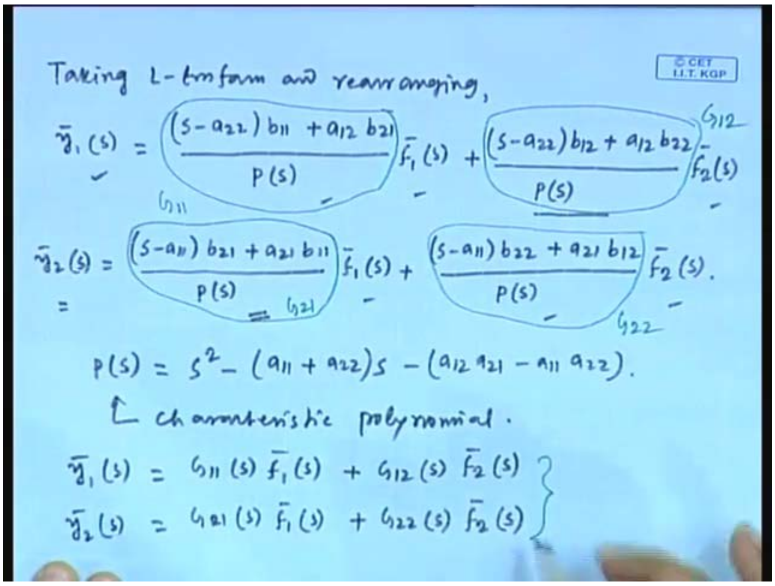

So, if we take Laplace transform, taking Laplace transform and rearranging, we get y 1 bar S equal to S minus a 2 2 b 1 1 plus a 1 2 b 2 1 divided by P S is a polynomial, we will write that expression for P S later.

So, this is the first right hand term and another term related to f 2 that is S minus a 2 2 b 1 2 plus a 1 2 b 2 2 divided by P S multiplied by f 2 bar S. This is a final expression for y 1, similar expression for another output y 2, we get that is y 2 S equal to S minus a 1 1 b 2 1 plus a 2 1 b 1 1 divided by P S, f 1 S plus S minus a 1 1 b 2 2 plus a 2 1 b 1 2 divided by P S f 2 bar S. These are the 2 expressions and a expression for the characteristics polynomial P S is like this the P S is the characteristics polynomial represented by a 1 1 a 2 2 S minus a 1 2 a 2 1 minus a 1 1 a 2 2, this P S is a characteristic polynomial.

Now, you see the output is output equal to 1 transfer function multiplied by first input variable another transfer function multiplied by second input variable. Similarly, for the case of second output one transfer function multiplied by first input variable second transfer function multiplied by second input variable.

So, we can write y 1 S equal to G 1 1 S f 1 bar S plus G 1 2 S f 2 bar S, similarly if y 2 bar S equal to G 2 1 S f 1 bar S plus G 2 2 S f 2 bar S. So, we can say that this term is basically G 1 1, this is another transfer function related to first output that is G 1 2, this transfer function is involved with y 2 that is G 2 1 and this is G 2 2. So, 4 transfer functions is involved and the final expressions, we have written here and we can represent them in matrix form. So, if we represent then matrix form.

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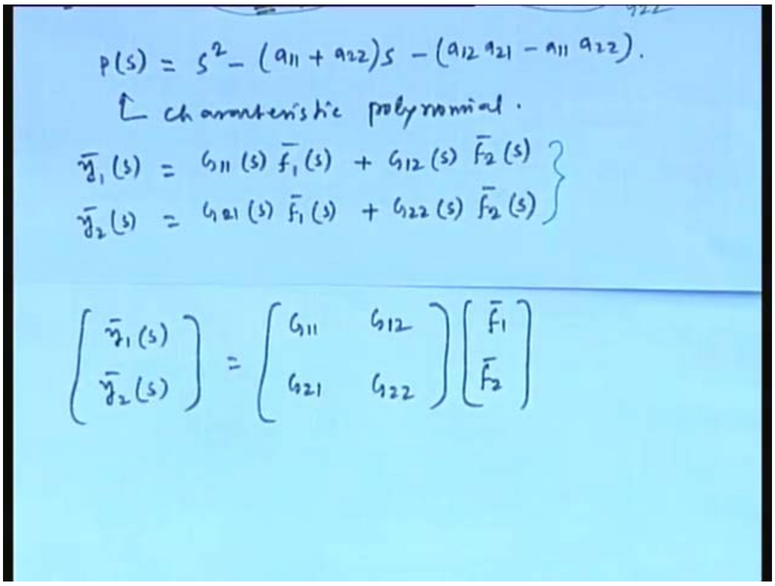

We will get y 1 bar S y 2 bar S equal to G 1 1 G 1 2 G 2 1 G 2 2 another matrix is for 2 input variables f 1 and f 2. So, this is about the derivational transfer function for the M I M O system. In the next step, we need to make the block diagram, what will be the block diagram for this system.

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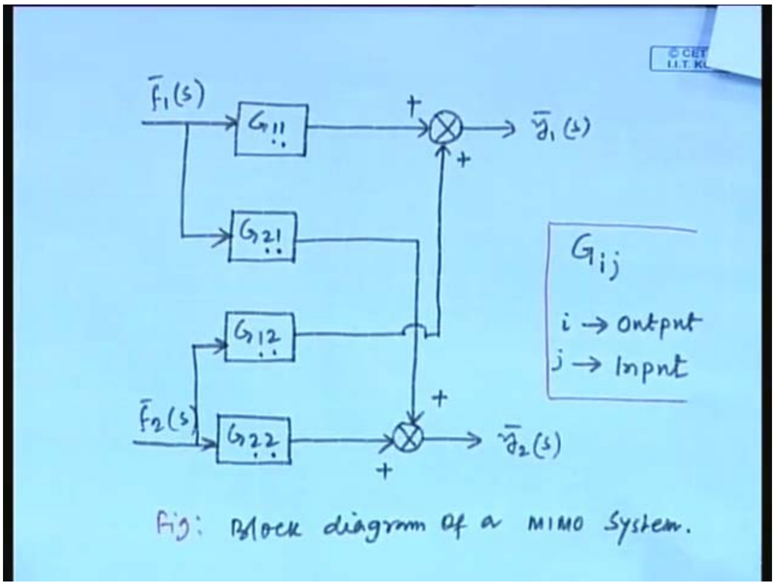

See there are 4 transfer functions, one is G 1 1, second one is suppose G 2 1, third one is G 1 2 and fourth one is G 2 2, basically the transfer function G has 2 suffix one is i another one is j, i represents output and another suffix j represents input. So, this is the nomenclature of the transfer function.

So, G 1 1 this first one means output. So, this is the output is y 1 second suffix 1, that indicates the input and first input agree, similarly for the last transfer function, first suffix to indicates the output. So, output will be y 2 second suffix 2, that indicates input, second input is basically f 2 bar S.

So, how we can add the input and output with this G 2 1, you see G 2 1 first suffix is 2. So, output second output. So, this is the second output, second suffix indicates the input that is first input that means f 1. So, we will add f 1, similarly for G 1 2 first suffix is 1 that means, y 1 and second suffix is 2 that indicates second input that means, f 2. So, this is the block diagram of the multi input multi output system. in the next class, we will discuss about the quantitative analysis of this transfer function.

Thank you.