Process Control and Instrumentation
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Lecture - 40
Instrumentation: General Principles of Measurement Systems (Contd.)

In our previous class, we have discussed how to combine different component errors to get an estimate of overall uncertainty in the measurement; in other words, since a particular measurement can be made of various experiments. And each experiment can have individual errors associated with it, so how do I combine this errors or uncertainties to get an overall estimate of the uncertainty in my measurement.

Similarly, a particular instrument can a varies components and this individual components may be associated with various uncertainties so how do I combine these uncertainties to get an estimate of overall uncertainty in my measurement. So, today let us take an example and see whatever we have discussed in the last class, how we can apply this.

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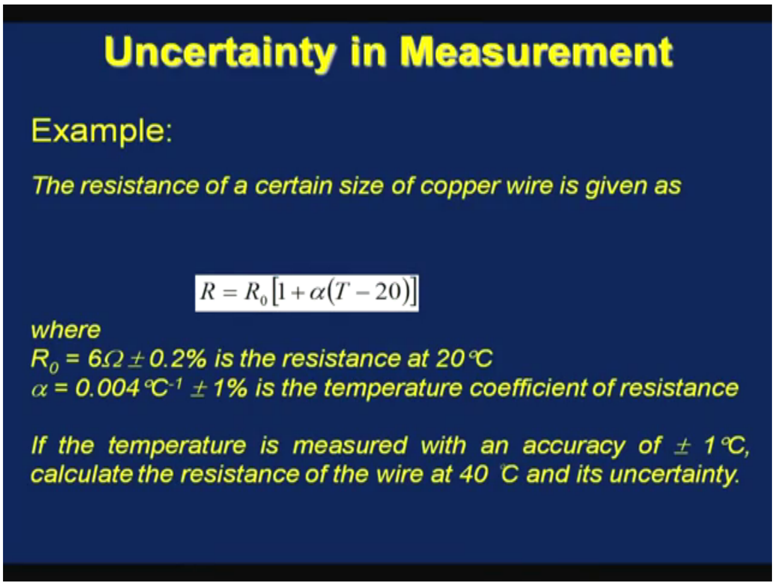

So, let us taken the example, the resistance of a certain size of copper wire is given as R equal to R 0 into 1 plus alpha into T minus 20, so is a familiar equation where R is the resistance of the copper wire, R 0 is the resistance at 20 degree Celsius. And given here as R 0 is 6 ohm plus minus 0.2 percent, so this is the error plus minus 0.2 percent is the error associated with R 0. Alpha which is temperature coefficient of resistance is given as 0.004 per degree Celsius plus minus 1 percent.

If the temperature is an measured with an accuracy of plus minus 1 degree Celsius, calculate the resistance of the wire at 40 degree Celsius and it is uncertainty. So, the resistance of a wire is given as R equal to R 0 into 1 plus T minus 20, so this equation tells us that if I know the resistance R 0 at T equal to temperature equal to 20 degree Celsius from the knowledge of alpha I can calculate the resistance at 40 degree Celsius.

We need to find out, what is the uncertainty in this computation, for resistance at temperature 40 degree Celsius if R 0 alpha and temperature are given with a specified uncertainty. So, let us try to solve this problem, so if you remember our previous days lecture, we calculated view overall as.

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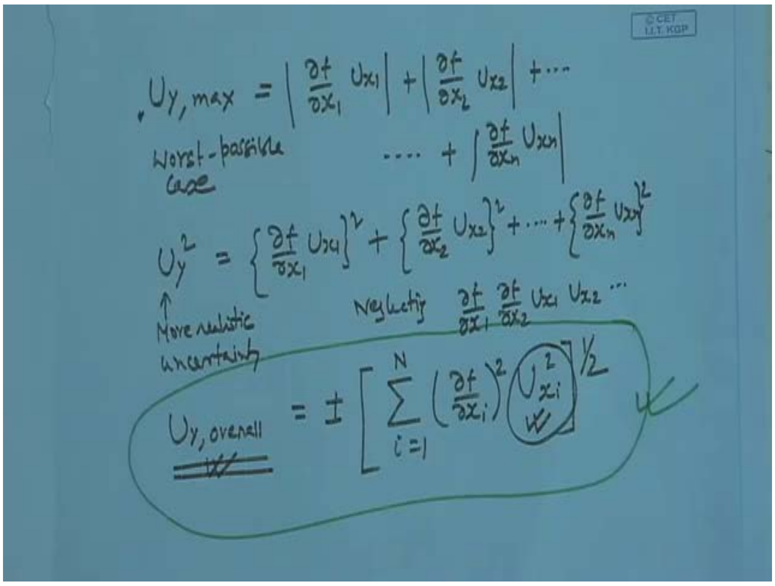

So, you will make use of this equation to find out what is the uncertainty in the computation of R, which is resistance at 40 degree Celsius.

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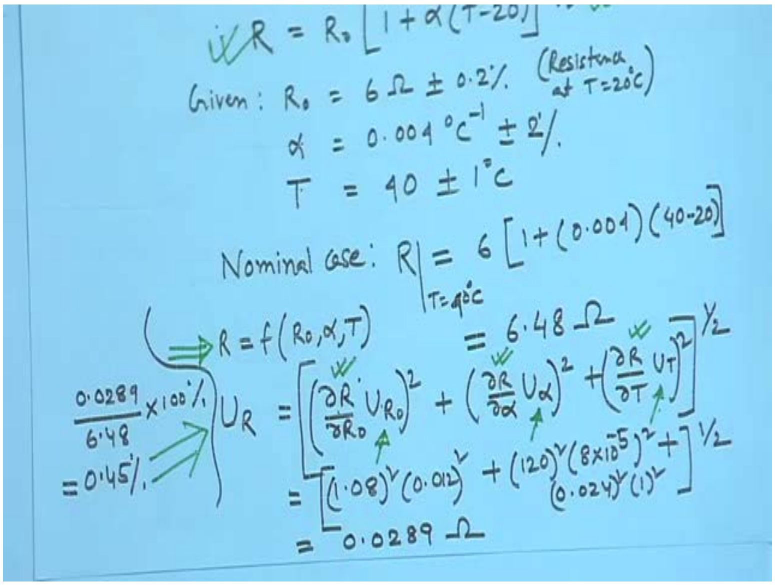

So, let us try to solve the problem, what is known is resistance at T equal to 20 degree Celsius, let us say I want I have alpha equal to 0.004 degree, 4 per degree Celsius plus minus 2 percent is the temperature co-efficient of the resistance. And temperature T is measured with accuracy of plus minus 1 degree Celsius, so by use of this equations, we can find out what is the value of the nominal resistance.

So, just we have to put values are R 0 alpha and T in this equation so if we do that R at T equal to 40 Celsius will be 6 plus 4, which is the value of alpha 40 minus 20, if you compute this it will come it is 6.48 ohm. Now, will make use of the equation that we discussed in our last class to find out the uncertainty in the computation of resistance, let us denote it by U R will be equal delta R delta R 0 U R 0 square plus delta R delta alpha U alpha square plus delta R delta T U T square, and square root of this sum.

We wrote this, because we know from here that R is the function of R 0 alpha and T, so while following whatever will be in our last class, we can obtain an expression for uncertainty in R as this, where U R 0 is the uncertainty in R 0. U alpha is the uncertainty in alpha and U T is the uncertainty in temperature, these values are given and this special derivatives, we can find out from the functional relationship of R equal to R 0 into 1 plus alpha into T minus 20, so let us do that now.

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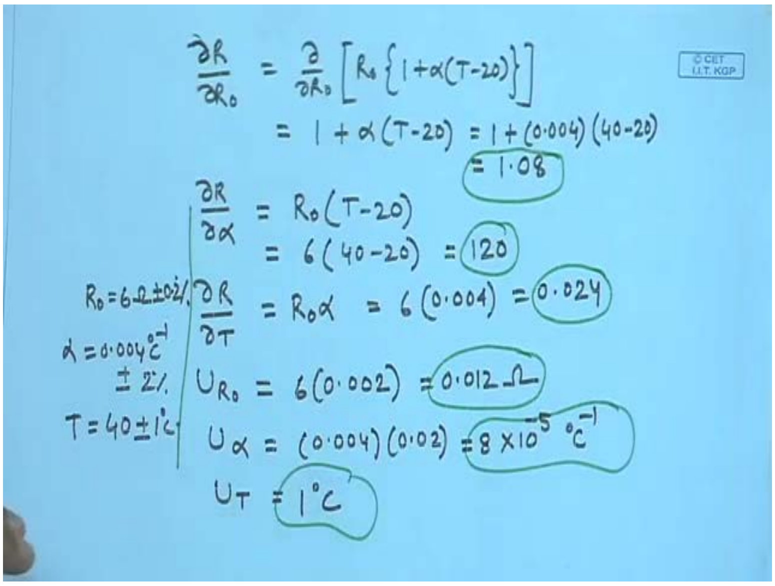

So, we need del R del R 0 which is nothing but, del del R 0 into R 0 into 1 plus alpha into T minus 20, which will come out is 1 plus alpha into T minus 20. So, if I put the numerical values 1 plus alpha is 0.004 and T is 40 minus 20, it will come as 1.08. Next we need delta R delta alpha, again if we compute this it will be R 0 T minus 20, after putting the numerical values, it will come is 20 into 6 120, finally del R delta T will be R 0 alpha.

So, 6 times 0.004 which is 0.024, so you now have values for delta R del R 0, delta R del alpha and del R del T, we now need to know U R 0 U alpha and U T. U R 0, you consider 0.2 percent R 0 is given as 6 ohm plus minus 0.2 percent. So, 0.2 percent of 6 ohm so 6 into 0.02 which is 0.012 ohm, U alpha is 2 percent of 0.004, so it will be 0.004 0.02, which will be 8 into 10 to the power minus 5 per degree Celsius.

If u remember, we said R 0 is given as 0.2 percent, alpha is given as 0.004 per degree Celsius plus minus 2 percent and temperature is measured with plus minus 1 degree Celsius accuracy, so 40 plus minus 1 degree Celsius. So, you obtain the values of U R 0, U alpha and U T from this given values, U T is straight away known as 1 degree Celsius. So, we now just kept to put all these values into the equation we wrote that means, in ((Refer Time: 13:30)) this equation.

So, now have values for all these terms, so if we put this values, it will be ((Refer Time: 13:56)) del R del R 0 is 1.08 and U R 0 is 0.012, del R del alpha and U alpha, we computed del R del alpha is 120 and U alpha is 8 into 10 to the power minus 5 plus delta del T and U T, we computed delta del T as 0.024 and U T as 1. So, 024 1 square, this will be 0.0289 ohm, so the uncertainty in the computation of resistance at 40 degree Celsius is 0.0289 ohm.

If you want to express this in terms of percentage, we can do that 0.0289 divided by 6.48, remember 6.48 is the resistance and the nominal case into 100 percent, this will come is 0.45 percent. So, we now know how to compute the overall uncertainty in the measurement, let us take an another example.

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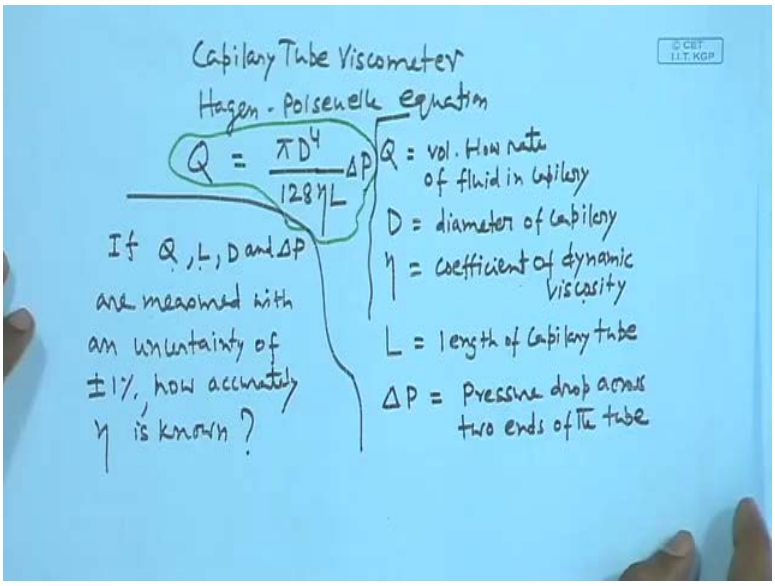

The governing equation for the capilary tube viscometer is well known, so let us consider a capilary tube viscometer. The governing equation is the well know Hagen-Poiseuely equation, which is Q pi D to the power 4 128 theta into L, where Q is volumetric flow rate of fluid in the capilary. D is diameter of the capilary, theta is coefficient of dynamic viscosity, L is length of capilary tube and delta P is pressure drop across two ends of the capilary tube.

Now, the question you ask is, if Q, L, D and P are measured with an uncertainty of plus minus 1 percent, how accurately the dynamic viscosity eta is known. So, you are considering the capilary tube viscometer and performing an experiment to determine the viscosity eta, the coefficient of dynamic viscosity eta, to determine eta you must have the knowledge of volumetric flow rate of the fluid in the capilary. The diameter of the capilary and the length of capillary tube, as well as the pressure drop is missing here ((refer Time: 20:05)), so this pressure drop.

So, this is the having Poiseuely equation, which can be used to determine the dynamic viscosity eta from the knowledge of volumetric flow rate, pressure drop diameter and length of the tube. So, the question you ask is, if the volumetric flow rate, the length of the tube, diameter of the tube and the pressure drop are measured with an uncertainty of plus minus of 1 percent, how accurately eta is known. We can follow the same approach that we just followed to solve the previous problem, to solve this problem as well.

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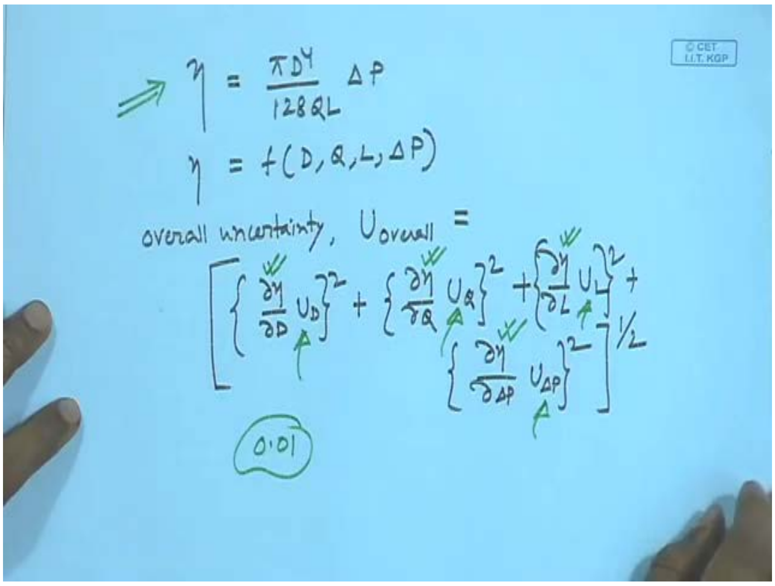

Eta is pi D to the power of 4 128 Q, L, delta P, just rearrange diagonal the Hagen-Poiseuely equation, so eta is the function of D, Q, L and delta P. So, the overall uncertainty can be computed as L, now we can find out this partial derivatives from this relationship; and all these values are specified as 1 percent that means, 0.01. So, following the same approach we can determine, the overall uncertainty in the measurement of dynamic viscosity eta.

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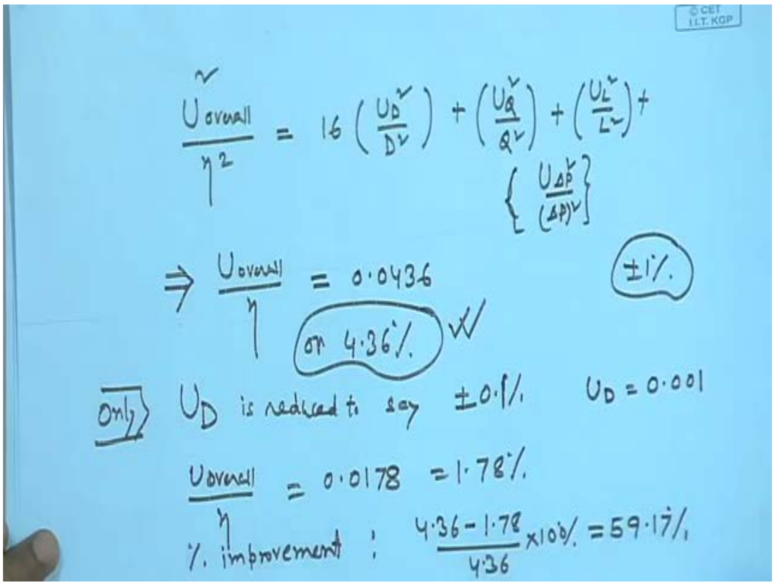

If you find out all these partial derivatives, the final equation will look like, after putting the numerical values you will get as or 4.36 percent. So, this is the uncertainty in the measurement of dynamic viscosity, now if we ask again that the uncertainty in the measurement of diameter is reduced to say plus minus 0.1 percent by using very improved instruments. So, what will be it is effect on the overall uncertainty, first we solved that when all the measurements of Q, D, L and delta P are available with plus minus 1 percent accuracy.

We have the overall accuracy as 4.36 percent, now we say only the diameter is measured with much more improved instrumentation, so that the uncertainty associated with the measurement of the diameter is reduced to 0.1 percent. So, what will be the effect on the overall uncertainty, if you do the same calculation while U D will now be 0.001, while all other U’s will be 0.1, so which is...

So, the uncertainty is reduced to 1.78 percent, if you calculate the percentage improvement will be it was 4.36, which is about 59.17 percent. So, more than 59 percent improvement in the overall measurement of dynamic viscosity, just by reducing the uncertainty in the diameter measurement from 1 percent to 0.1 percent. If you look at the relationship eta with the diameter Q L and delta P, you see heater where is with fourth power of diameter that is the reason we see here very sharp decrease in uncertainty, when we reduce the uncertainty in the diameter.

Now, let us close our discussion on error analysis by having some more discussion on the probability distribution function, that we talked about in our last class. We say that the measurement states that contain only random errors, conform to the Gaussian distribution, so measurements states that contain only random errors easily conformed to Gaussian distribution, also known as normal distribution.

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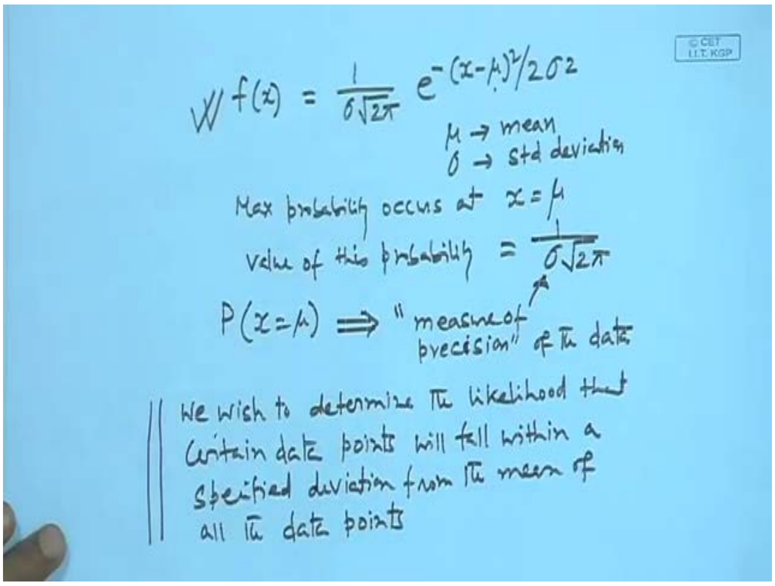

We wrote the probability density function, as the normal distribution function, where mu is the mean of the measurements and sigma is standard deviation, the maximum probability occur at x equal to mu, the mean. And what will be the value of this probability, you can obtain this if I put x equal to mu here, so the value of this probability will be 1 by sigma square root of pi, because this entered value will be equal to 1.

So, the smaller value of sigma will produce larger values of this probability, and this is expected intuitively also, so the probability x into mu is sometimes called measure of precision of the data. We now one to determine, the likelihood that a certain data points will fall within a specified deviation from the mean of all the data points. So, what you ask is, we wish to determine the likelihood that certain data points will fall within a specified deviation from the mean of all the data points. We can find this out using the definition of the distribution function, the probability that a measurement will fall within a certain range x 1 of the mean heading mu will be...

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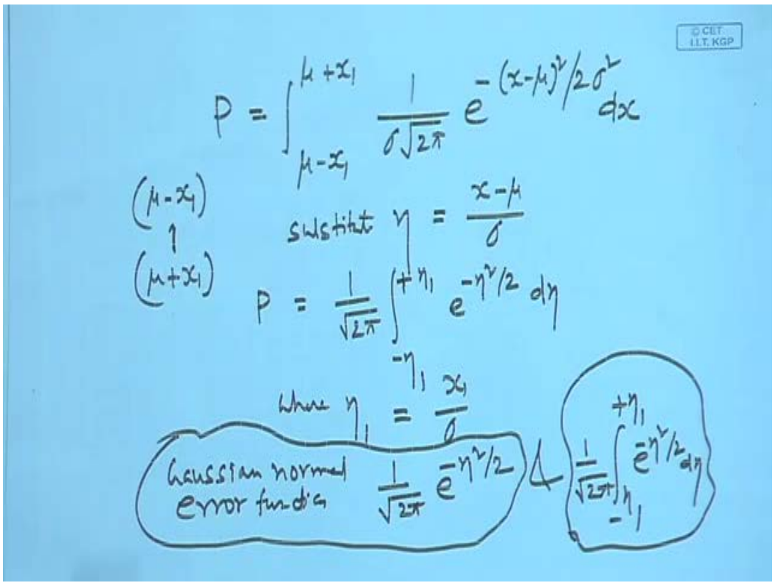

So, this is the probability that a measurement will fall within mu plus x 1 and mu minus x 1, so the probability that a particular measurement will fall between mu minus x 1 to mu plus x 1, can be obtained from the definition of the distribution function, so if I make this substitution, it can be shown that P is where... Now, the values of this Gaussian normal error function which is and the integral are given or available in the form of tables, the Gaussian normal error function.

And the integral of the Gaussian function are available as standard tables, we can make use of that table to find out what is the probability, the particular measurement will fall within say plus minus 1 standard deviation or plus minus 2 or plus minus 3 standard deviation.

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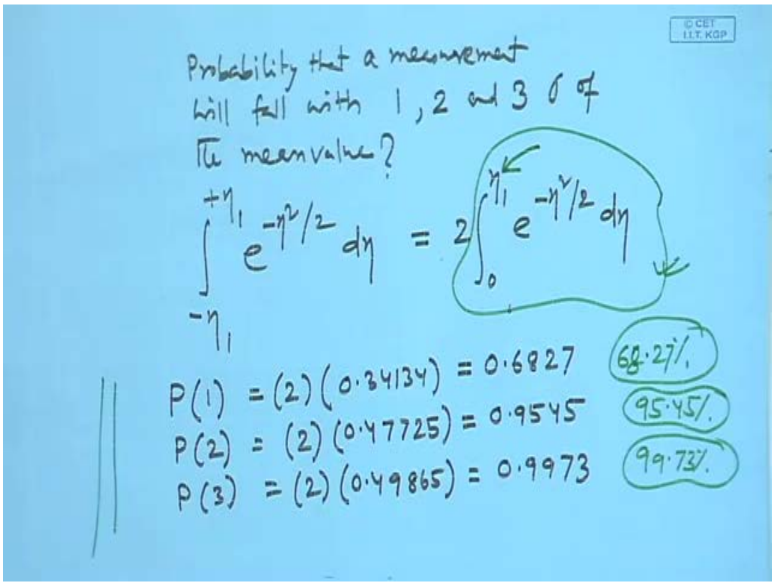

So, how do we do that, so let us ask ourselves, what is probability that are measured or a measurement will fall within 1, 2 and 3 standard deviation of the mean value. So, we need to determine what is the probability that a particular measurement will fall within mu plus minus sigma, sigma is standard deviation, so we can make use how about we learnt here. This is same as, so you want to find out what is the probability that a particular measurement will fall within 1 standard deviation of the mean value, let us denote that by this.

So, all you need to do is for eta 1 equal to 1, we need to look another table what is the value of this integral, if you look at the table this value is 0.34134, which will come is 0.6827. Similarly, the probability that a particular measurement will fall within plus minus 2 sigma of the mean value, can be computed as 2 into the value of these, when eta 1 equal to 2, which can be found from the table, whose value can be obtained as 0.47725, this will be 0.9545.

Finally, P 3 can be similarly obtained as value of this, when eta 1 equal to 3 from the table you can obtain the value as 0.49865, which will be 0.9973, so what we conclude is that the probability, that a measurement will fall within plus minus sigma of the mean value is 68.27 percent. Similarly, if the measurement the probability that a measurement will fall within 2 sigma of the mean value is 95.45 percent and the probability that a particular measurement will fall within 3 standard deviation of the mean value is 99.73 percent.

All these computations are of course, based on the fact that the measurement states conformed to normal distribution or Gaussian distribution, so this ends our discussion on error analysis, will now move on to transducers.

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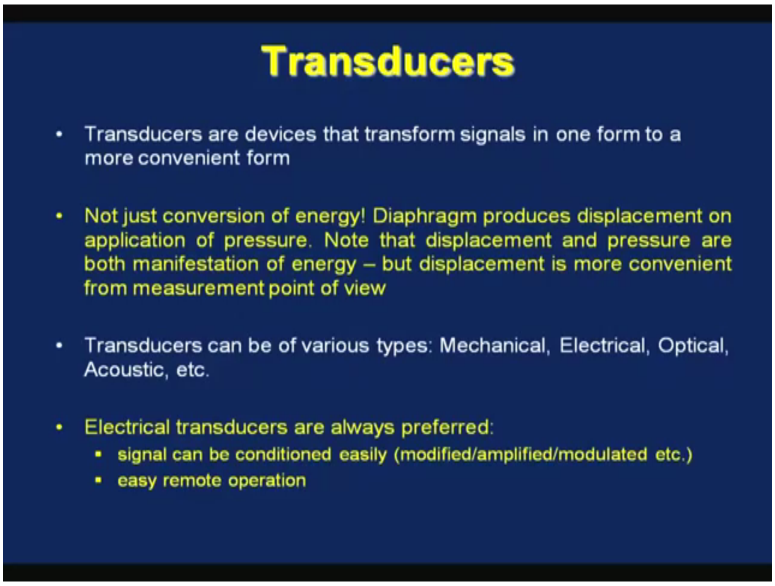

Transducers are devices that transform signals in one form to a more convenient form, so is a device which will receive a signal in one form, and will convert to another form which is more suitable for the purpose of measurement. Please note that is not just conversion of energy associated with that signal, it is not just conversion of energy in one form to another, we can take a... Let us now talk about transducers, transducers are devices that transform signals in one form to a more convenient form.

So, it is a device that receives signal in one form and converts that signal to another form, which is more suitable for the purpose of measurement, it is not just conversion of energy. Let us take example of diaphragm, you imagine a thin metallic diaphragm or thin non metallic diaphragm, which will produce a displacement on application of pressure. So, the diaphragm receives pressure as input and gives displacement as a output, so what is a transducers, note that both displacement and pressure are manifestation of the energy but displacement is more convenient from measurement point of view.

Transducers can be a various types, mechanical types, electrical types, optical types, acoustic types extra, obviously electrical transducers are always preferred, because there are a several advantages associated with electrical transducers. For example, the signal can be conditioned easily may be the signal can be modified or amplified or modulated very easily, it is also easy to transfer signal from one place to another, so electrical transducers will left easy remote operation.

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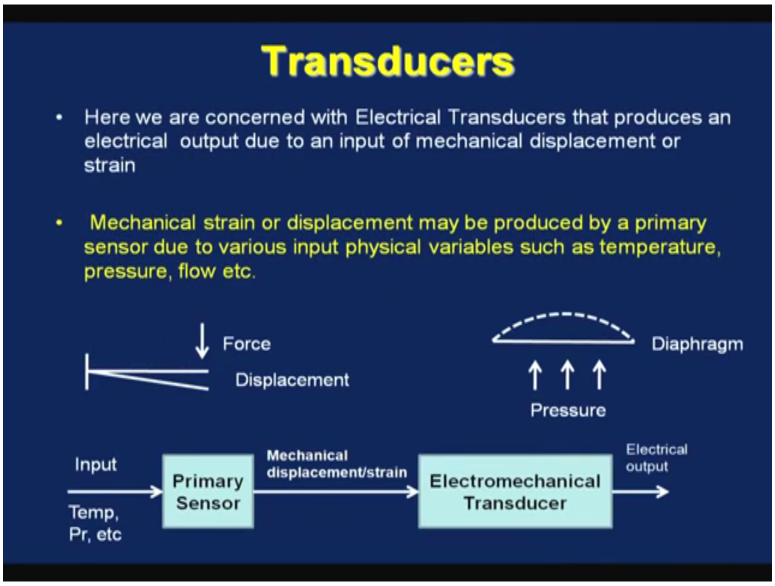

For the purpose of this course, we are concerned with electrical transducers or electromechanical transducers that producers an electrical output, due to an input of mechanical displacement or strain. So, in this course will mainly talk about few electrical transducers or electromechanical transducers that produces an electrical output, when it receives mechanical displacement or strain as input; additionally will also talk about just one pneumatic transducers briefly.

The mechanical strain or displacement may be produced by primary sensor due to various, input physical variables such as temperature, pressure, flow, extra. So, during act of measurement primary sensors that receives input signal, from physical variable such as temperature pressure flow extra may produce a mechanical strain or displacement. We are interested in converting these mechanical strain or displacement into an electrical signal, let us say this is a diaphragm, this is the diaphragm will talk about diaphragm in more detail later.

When this receives pressure as input it deflects, so there is a displacement, if we measure the displacement from the centre of this diaphragm, we have this distance as maximum deflection of the diaphragm. We are interested in converting this displacements signal to an electrical output, similarly of a force acts here it will defect, so this displacement has to be measured. We are interested in some electromechanical transducers, which receives these displacement as input and gives us an electrical output.

So, this mechanical displacement or strain can be an output from various primary sensors, that receives temperature, pressure, flow rate, extra as input, so this mechanical displacement or strain is received by some transducers as input and gives us an electrical output. So, will learn few such transducers, which takes mechanical displacement or strain as input and gives us electrical output.

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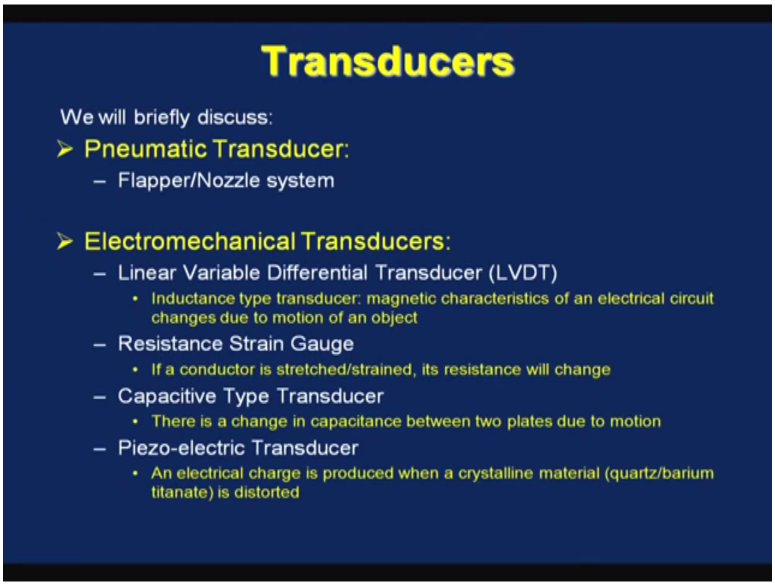

So, will briefly discuss mostly electro mechanical transducers, but will talk about one pneumatic transducers also and the pneumatic transducers will briefly talk about a flapper nozzle system, is an important pneumatic displacement measuring transducer, is an integral part of all pneumatic control systems. And electromechanical transducers will briefly discuss four different types of transducers, namely linear variable differential transducer, also known as LVDT, then resistance strain gauge, capacitive type transducer and piezo-electric transducer.

The linear variable differential transducer or LVDT is an inductance type transducer, it is working principle, is based on the fact the magnetic characteristics of an electrical circuit changes due to motion of an object. The working principle of resistance strain gauge is based on the fact, that if a conductor is stretched or strained it is resistance will change, and it is easy to measure this change in resistance, so you can measure the strain.

Similarly, capacitive type transducers is based on the fact that, there is a change in capacitance between two plates due to motion. Finally, piezo-electric transducer is based on the fact that, an electrical charge is produced when a crystalline material, such as quartz and barium titanate is distorted.

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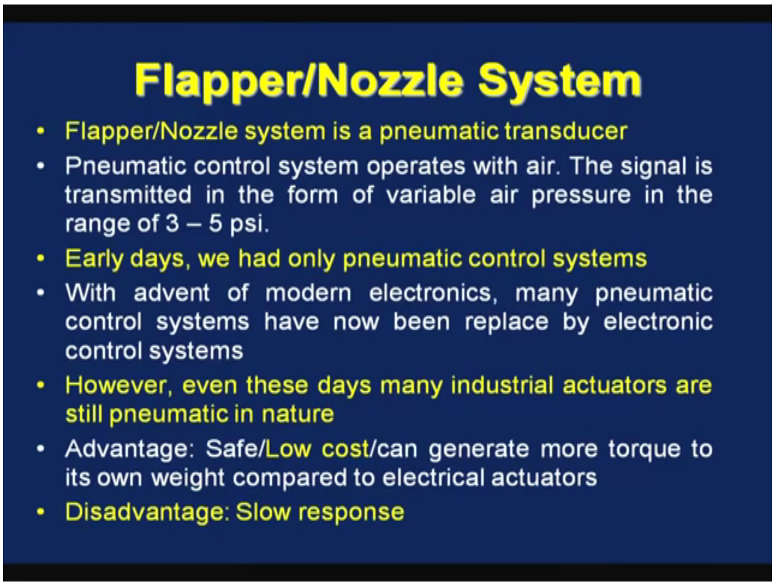

So, let us start with flapper nozzle system, flapper nozzle system is a pneumatic transducer, we know the pneumatic control system operates with air. The signal is transmitted in the form of variable air pressure, in the range of 3 to 5 psi, we have read more about this during discussion on control part of this course. Early day's we had only pneumatic control systems, but with the advent of modern electronics, many pneumatic control systems have now been replaced by electronic control systems.

However, even these days many industrial actuators are pneumatic in nature, there are certain advantages associated with pneumatic control system, they are very safe, because you handled layer only. It is chief, it also generates more torque to it is own weight compared to electrical actuators however, the disadvantage of pneumatic control systems is it is slower response.

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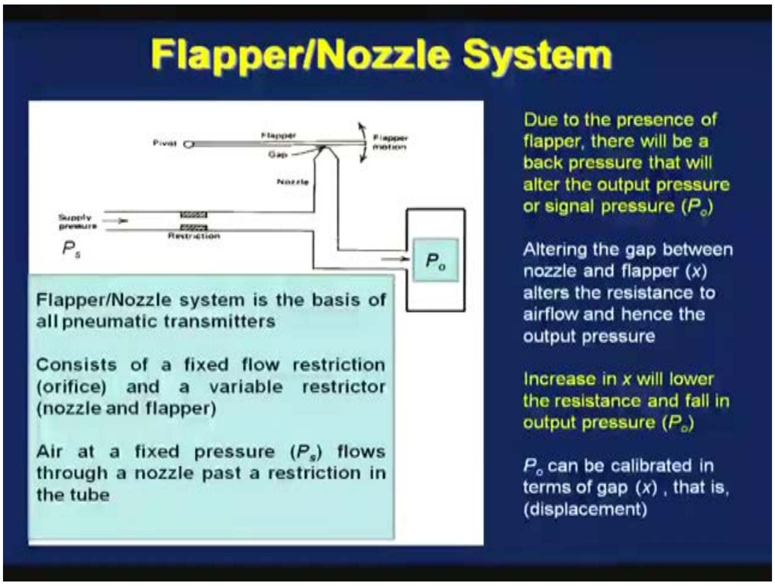

This is a flapper nozzle system is the basis of all pneumatic transmitters, the flapper nozzle system consist of a fixed flow restriction or orifice and a variable restrictor nozzle and flapper. So, it is a very simple system consist of an orifice, which provides a fixed flow restriction and a nozzle and flapper, which acts as a variable restrictor. Air at a fixed pressure, let us denote the pressure by P s, flows through a nozzle first a restriction in the tube.

So, the air at fixed pressure, flows through this restriction and through this nozzle, so due to the phrases of a flapper, there will be a back pressure that will alter the output pressure P 0, also called as signal pressure. Altering the gap between nozzle and flapper, alters the resistance to airflow and hence, the output pressure increase in x will lower the resistance that means, increasing the gap between the flapper and nozzle will lower, the flow resistance and fall in the output pressure.

So, thus P 0 which can be varies the measured using a good pressure measuring instrument and P 0 can be calibrated with the gap that exists between the flapper and the nozzle that means, the displacement can be calibrated with the output pressure P 0. So, let us repeat once again, the flapper nozzle system consists of a fixed flow resistance and a variable restrictor, orifice is the fixed flow resistance, and a nozzle and flapper works as a variable restrictor.

Here at fixed pressure, flows through this restriction and then, through the nozzle, because of the pressure of the flapper are back pressure will be developed and has the gap between the nozzle and the flapper changes this back pressure will change. Increase in the gap between nozzle and flapper will lower the resistance and fall the output pressure, so the output pressure can be directly calibrated, in terms of the gap between the flapper and the nozzle that means, distance.

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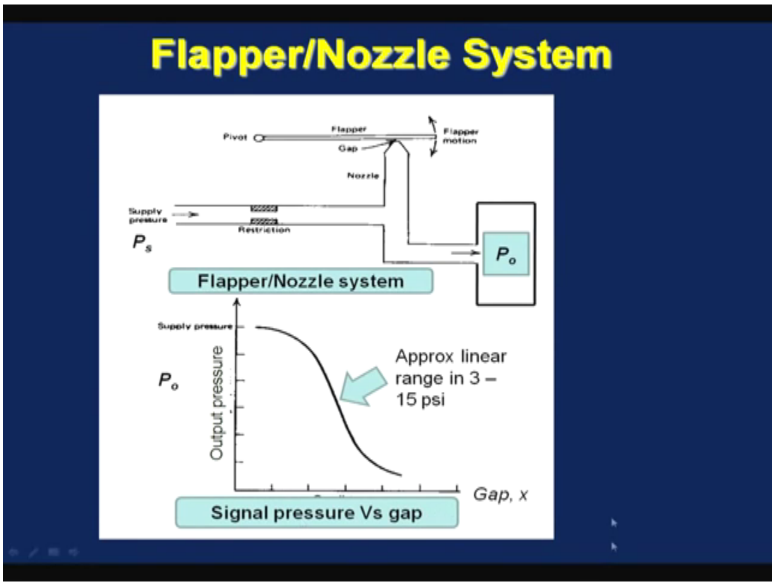

So, if I plot the output pressure verses the gap, you will see between the range of 3 to 15 psi pressure we have an approximately linear relationship, and that is the 3 to 15 psi pressure is the working range which is linear. So, will end our discussion today with the flapper nozzle system, and in the next class will talk about the other electromechanical transducers like LVDT, capacitive type, piezo-electric type and the resistance strain gauge.