Process Control and Instrumentation
Prof. D. Sarkar
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 39
Instrumentation: General Principles of Measurement Systems (Contd.)

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In today's class we will talk about Error Analysis. While talking about performance characteristics of instruments we have talked about static error and dynamic error. Now, since no measurement can be perfect and may particular measurement will always been error this error can be a various forms. So, we should have some estimate of how much error there can be in our measurement.

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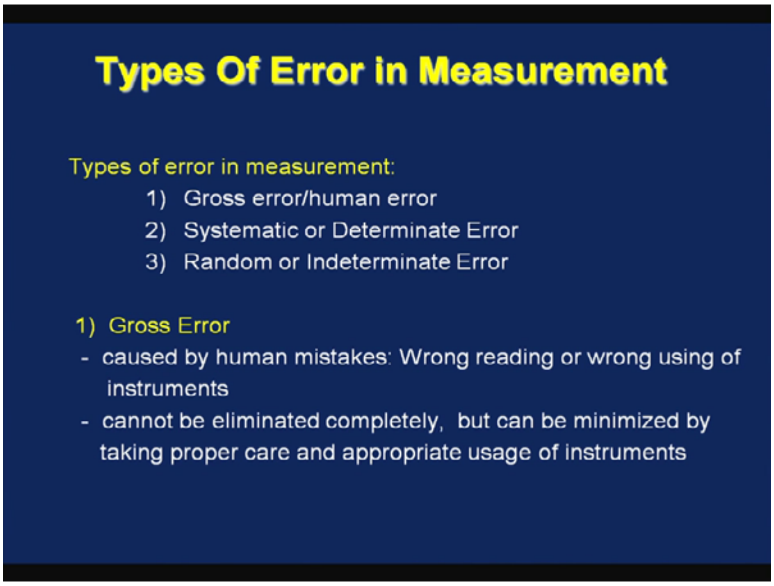

So, let us now talk about error analysis in some detail Types of Error in Measurements. The Types of Error in Measurement can be broadly classified into 3 categories 1 the Gross error or human error 2 Systematic or Determinate Error and 3hree Random or Indeterminate Error.

So, all the errors that can be involved on in our measurement can broadly be classified intothis3 categories namely Gross error Systematic error and Random error. Now, Gross Error is caused by human mistake. So, this is Wrong reading or wrong using of instruments gross error which is caused by human mistakes cannot be eliminated completely, but it can be minimized by taking proper care and appropriate uses of instruments.

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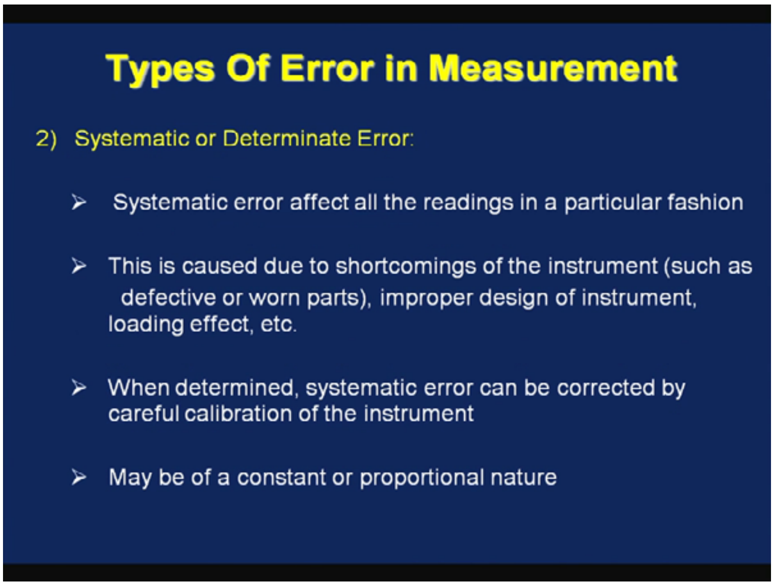

Next let us talk about Systematic or Determinate Error as the name suggests Systematic error affect all the readings in a particular fashion. This is caused due to shortcomings of the instrument such as defective or worn parts in the instrument improper design of instrument loading effect etcetera.

However, systematic errors are usually recognizable. So, when determined systematic error can be corrected by careful calibration of the instrument. So, let us repeat again systematic error are those error which affect all the readings in a particular fashion this is caused due to shortcomings of the instrument such as defective or worn parts improper design of instrument loading effect etcetera. When you can recognize a determined systematic error this type of error can be corrected by careful calibration of the instrument systematic error May be of constant or proportional nature.

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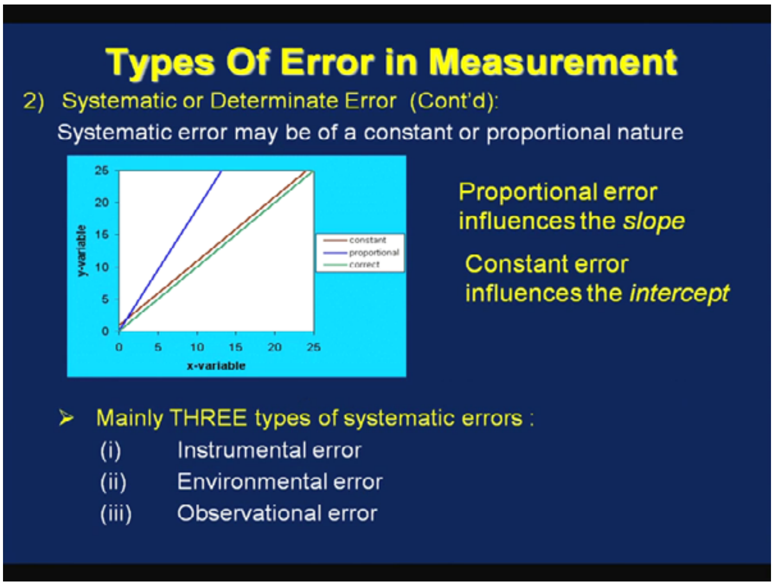

So, if we look at this figure is a float of x variable versus y variable. We can consider x variable to be the input to the instrument and y variable as the output of the instrument. So, this can be considered as a calibration curve. So, if we look at the blue line let us say blue line should be the correct characteristic of the instrument. If, we get the behavior of the x variable and y variable that is the input and output relationship is represented by a red line. We see that there is an error associated with the ready but, these error is a constant type whereas, if the input output relationship is characterized by the blue line will have a Proportional error. However, as the graph suggest that if, we know how much of constant error or how much of proportional error is associated within instrument we can always introduce a suitable correction factor.

So, it is rewriteable easy to handle systematic error. We can classify the Systematic errors under three different categories: 1 Instrumental error 2 Environmental error and 3 observational error. This can also be considered as source of systematic errors. So, systematic errors can be of Instrumental error Environmental error or Observational error.

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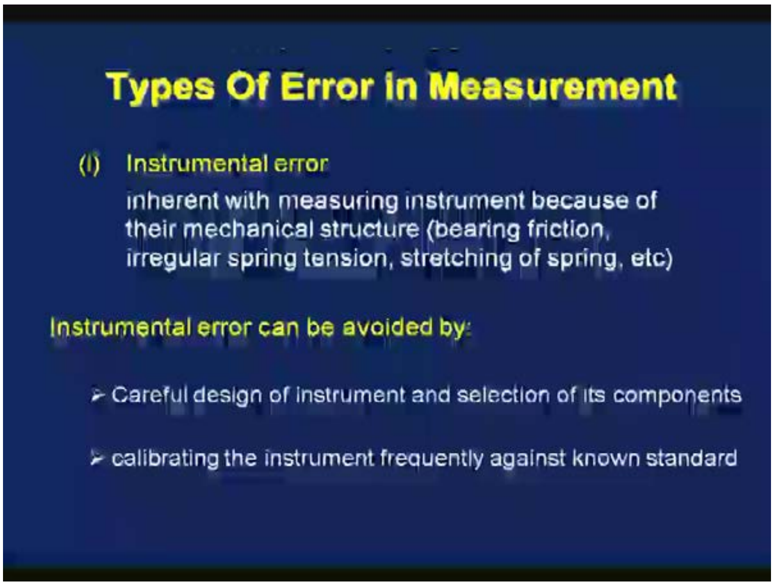

Now, let us look at Instrumental error. The Instrumental error is inherent with the measuring instrument because, of their mechanical structure like bearing friction, irregular spring tension, stretching of spring, etcetera. So, instrumental error is inherent with the measuring device. Instrumental error can be avoided by Careful design of instrument and by careful selection of the components of the instruments it can also be avoided by calibrating the instrument frequently against known standards.

So, Instrumental error is inherent with the measuring instrument because, of their mechanical structure, so just bearing fraction, irregular spring tension, stretching of spring, etcetera. And the best way to avoid Instrumental error will be the Careful design of the instrument and proper selection of the components of the instrument.

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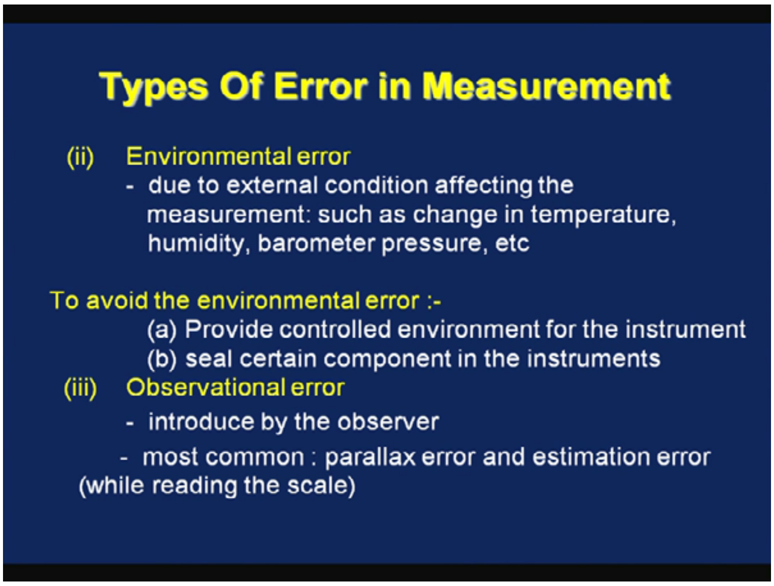

Environmental error this is due to external condition affecting the measurement such as change in temperature, change in humidity, change in atmospheric pressure, etcetera. So, Environmental error is due to external conditions effecting the measurement such as change in temperature, change in pressure, change in humidity, etcetera. To avoid the environmental error we should provide suitable control environment for the instrument. So, providing controlled environment for the instrument is necessary to avoid the environmental error. Environmental error can also be avoided by sealing certain components in the instruments. Observational error is introduced by the observer the most common observational error is parallax error and estimation error while reading the scale.

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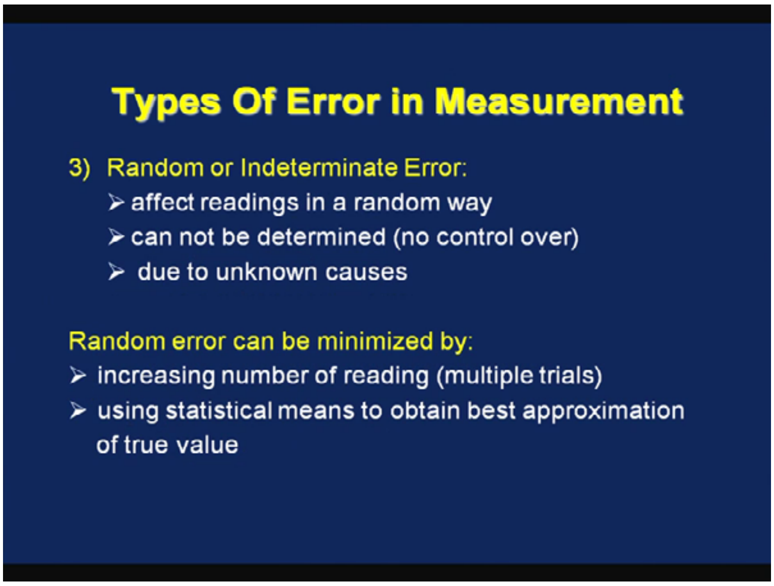

Next let us come to Random or Indeterminate Error. As the name suggests random errors affect readings in a random way. So, unlike systematic error which effects all the readings in systematic fashion random errors affect readings in a random way. It cannot be determined easily we do not have any control over random errors and these are due to unknown causes.

So, causes for random error are not always easily known. Random errors cannot be eliminated completely it is difficult to eliminate completely the random errors because, the sources of random error are usually not always known. However, the random errors can be minimized by increasing the number of readings that means, multiple trials we take several readings of the same value of the measuring variable. So, we repeat the number of measurements and then use statistical means to obtain best approximation of the true value. So, will take a particular reading several times this is call multiple trials and then we will use some statistical principles to obtain best approximation of the true value. This way we can minimize the random error in our measurements, but we can never completely eliminated.

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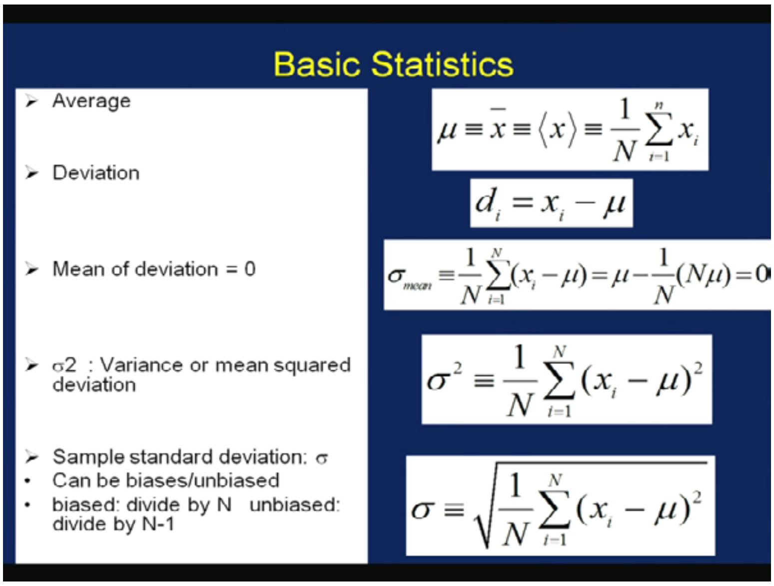

Let us now, talk about some Basic Statistics 1st Average. So, if we take a particular reading N times and each time I have the major values in the carried as xi. So, I have values x1, x2, x3, up to xN, and then some them up and then divide by the total number of measurements. So, this is known as average of n readings or mean frequently represented as mu or x bar. Deviation is defined as difference between a particular value and the mean.

So, we have to statistics as of now average which is the sum of N readings divided by N. It is frequently stated that, the mean value is the most probable value of a set of readings and that is why it is a very important role in statistical error analysis the deviation of the individual readings from the mean value is obtain as di equal to xi minus mu where xi is the i-th reading and mu is the mean. We know one to have an idea about the deviation that is why the individual readings are far away from the mean value or not. So, if we one to have some idea whether the individual readings are far away from the mean value or not can you use Mean of deviation.

So, if this is xi minus mu is mean. So, this will represent Mean of deviations. Unfortunately we cannot use the mean of deviation, because if you follow this the mean of deviation is always equal to 0. So, instead we use Variance or mean squared deviation which is defined as this. So, Variance is mean squared deviation. So, we find out the individual deviation square sum of over N readings and divide by number of readings N.

So, Variance or mean square deviation can be used to find out how far away from the a particular reading is how far away from the mean value a closely related statistic is Sample standard deviation which is square root of variance. We can have a biased estimate of standard deviation or unbiased estimate of standard deviation. If, we want to have a biased estimate of standard deviation we divide this by N if we want to use unbiased estimate we should divide it by N minus 1. If, we have a large number of readings typically more than 30 or so, it does not make much difference whether we divide this by N or N minus 1 the term standard deviation which is square root of variance is often used as a measure of uncertainty in a set of measurements.

So, we have Average or mean value then, the individual Deviation which is difference between a particular value and the mean then the Variance or means square deviation and then standard deviation which is square root of sigma square. Now, let us talk about the probability distribution function.

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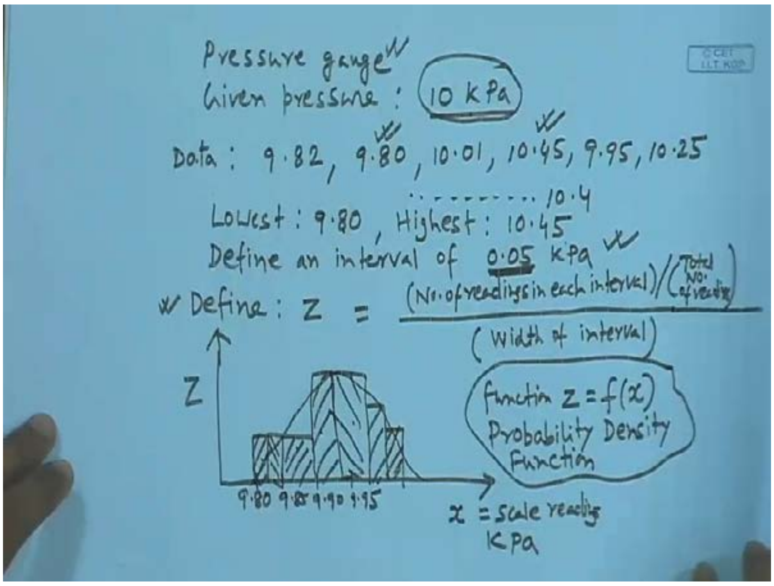

Suppose we have a sufficiently accurate Pressure gauge, and where measuring a certain pressure with this pressure gauge several times. Let us say the given pressure gauge the given pressure is say 10 kilopascal. So, a measuring a 10 kilopascal pressure using a pressure gauge and let us say for example, the Data looks like this, you can have even more number already let us say we end like this. So, we are measuring a 10 kilopascal pressure with a pressure gauge which, can be consider as a sufficiently accurate instrument and I measure the same pressure several times and I get the readings like this.

So, the Lowest is 9.80 and the Highest is 10. 45. Now, let us Define an interval of let us say 0.05 kilopascal and we want to determine, How many readings fall in each interval of 0.05 kilopascal? So, let us Define a quantity called Z which is number of readings in each interval divided by Total Number of readings divided by width of interval.

So, we have these data from repeated measurement of a given pressure obtain kilopascal using this Pressure gauge and now, if I plot the values using an interval of 105 and calculate Z which is define as follows Number of readings in each interval by Total number of readings divided by width of interval. We can plot bar graph with high Z for each interval which will be a histogram. So, maybe I can get a histogram like this, may be something like this where this is and this is Z. So, here plot it x versus Z where x is the readings scale readings.

Now, area of a particular bar is numerically equal to the probability that is specific reading will fall in the associated interval. the area of the entire histogram has to be 1 or 100 percent because, there is definite probability or there is a probability 1 that a particular reading will fall anywhere, within the histogram. That is it will fall somewhere between the lower value and the upper value that is somewhere between 9.80 and 10.45 it this belongs to if this is 10. 45. So, it will lie between anywhere, between here to here that you can say with probability 1.

Now, if I take infinite number of readings here, I have taken a finite number of readings each with an infinite number of significant t digits we can make chosen intervals as small as we want. And still have each interval contain a finite number of readings, in this particular plot we have taken an interval width of 0.5. Now, in this example as small as we want, and still can expect to have definite number of readings that will fall in each interval.

Now, if we do this then these steps in this graph will be smaller and smaller and the graph will approach a smooth curve in the limiting case. So, all we are saying is if we, increase the number of readings that it will large value if we repeat this experiments infinite number of times. We can make the intervals extremely small and even then we can expect that some readings will fall in each interval. In that case the steps will be smaller and smaller and this plot this plot will be like, a smooth graph or smooth curve in the limiting case. Now, if we take this limiting abstract case as a mathematical model for the real physical situation then the function Z equal to fx is call the probability density function. So, if we take large number of readings this histogram can be represented by smooth curve and the functional relationship Z equal to fx can be considered as the probability density function.

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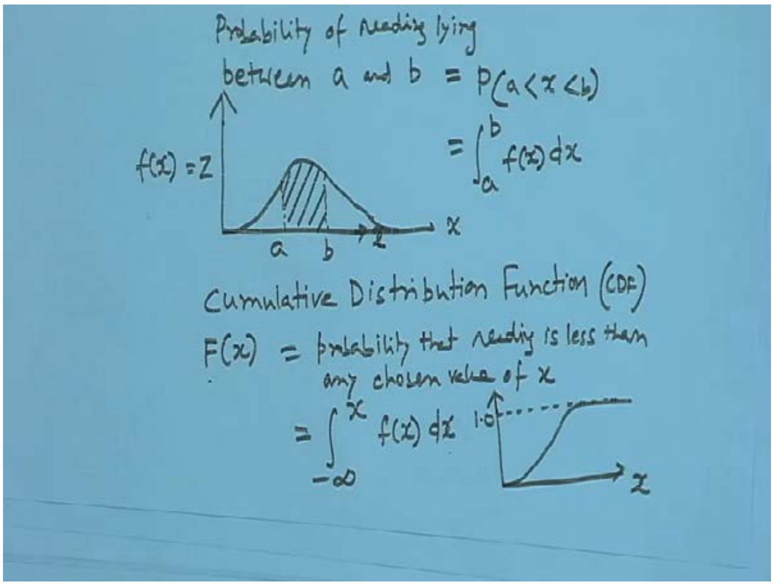

So, From the definition of the Z, the Probability of reading lying between a and b which can be written as the Probability that x lies between a and b, is the area under the curve so. So, the probability of reading lying between a and b is the area under the curve. So, this said region this probability information is sometimes given in terms of Cumulative Distribution Function the probability information can also be given in terms of cumulative distribution function. Which is defined as, the Cumulative Distribution Function capital F x equal to probability that reading is less than, any chosen value of x.

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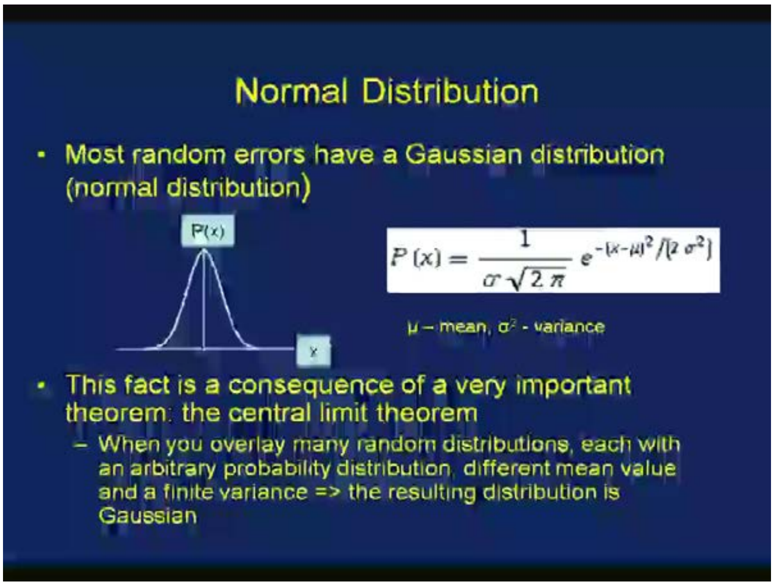

So, this will be represented as, now most random errors have a Gaussian distribution or normal distribution. Since, Normal Distribution or Gaussian distribution is one of the most use full density function. There are other probability density functions as well. In fact, there are many, but the normal distribution function or the Gaussian distribution function, is one of the most useful probability density function.

The normal distribution function or the Gaussian distribution function is define as this, 1 over sigma into square root of 2 pi into e 2 the power minus x minus mu square divided by 2 sigma square. Where mu is the mean value and sigma square is the variance which we define previously. So, Gaussian distribution can look like this, the fact that most random errors may have a Gaussian distribution is the consequence of a very important theorem, call the central limit theorem. When you overlay many random distributions, each with an arbitrary probability distribution different mean value and a finite variance the resulting distribution is Gaussian.

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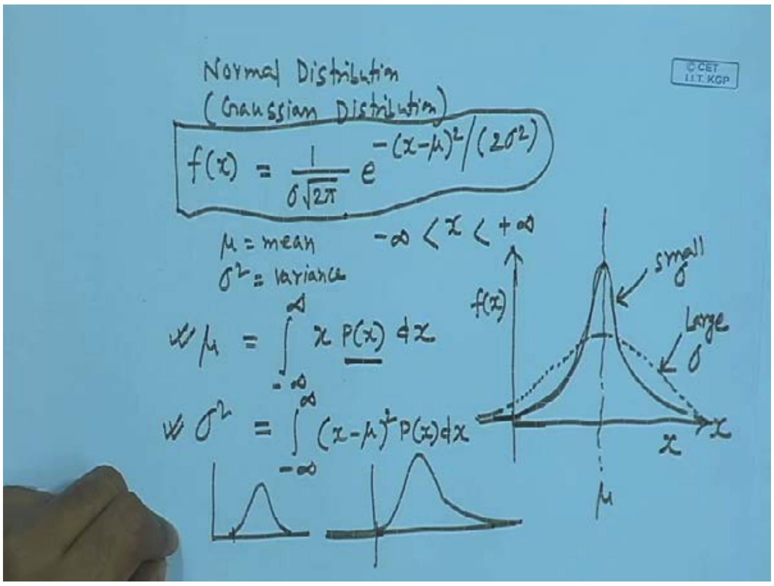

So, the Normal Distribution or Gaussian distribution which is defined as, where mu is mean and sigma square is variance. So mu can be obtained as, and the variance. So, given the probability density function P x mean is, defined as follows and the variances is defined as this equation. So, if you look at the Gaussian or Normal Distribution function it has two characteristic parameters. One is sigma which is variance; another is mu which is mean. So, the shape of the Gaussian or normal distribution function is determine by mu and sigma.

Now, we can have a Gaussian Distribution like this we can also have a Gaussian Distribution like this. Obviously, both has same mean but, different variance or different standard deviation. This distribution has small standard deviation whereas, this has large standard deviation both has same mean. So, when the standard deviation is small it indicates that, there is a high probability a reading will be close to mu or mean.

So, we have a distribution with small standard deviation it indicates that, there is a high probability that the reading will be close to mean value mu, while if, the standard deviation is large, the readings will be more scattered along the mean. It may be pointed out here, that if you look at this mathematical definition of the Gaussian Distribution it says that, there is a small probability that even a very large reading will also occur. But, a real distribution will always have their tails cut off. So, instead of this a real distribution will have will be, where their tails will be cut off.

However, most of the or many measurements the errors the random errors associated with many measurements can be represented by a Normal Distribution Function. Now, having defined some of the basic statistics let us, now see. How the error can be propagated in a particular measurements? A particular instrument can have various components. So, a particular instrument can be chain of components, the question we ask is if we know the error or uncertainty associated with each component.

What will be the overall error or what will be the overall uncertainty measurement? You can also think of a situation where, a particular measurement involves various other measurements. So, particular experiment can involve various measurements. And we calculate a value of a particular variable, from the measure from these measure values. Now, if I know the uncertainty or error associated with each measurement. Can I compute the overall error or uncertainty measurements.

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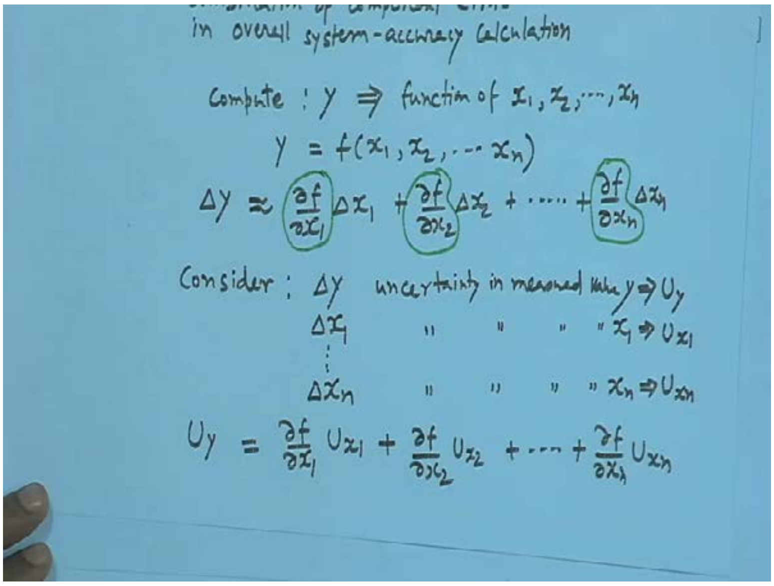

So, let us now see how we do that? So, the question you ask is, if an experiment involves several measurements. Say, using several instruments and if, I know the individually inaccuracy or individual error uncertainty. How do I combine this individual uncertainties or errors to get an estimate of overall accuracy? So, now, consider the problem of Computing a variable Y which is known to be a function of n independent variables. So, we want to compute Y which is a function of n independent variables x1, x2 up to xn.

So, we may are measuring x1 x2 up to xn. If we know the errors or uncertainties associated with x1 x2 up to xn we want to compute the overall accuracy in Y. So, we say Y is a function of x1 up to xn. For a small change in the independent variable x1 x2 up to xn from a given say operating point. A Taylor series expansion will give an good approximation for the corresponding change in Y. So, which can be represented as where you can only the First-order terms. So, if there are change is small change is in the independent variables x1 x2 up to xn around a given operating point. The corresponding change in Y can be obtain from a Taylor series expansion of these function, which is written as this where we are written only the First-order terms.

We can think of these partial derivatives, as the sensitivity of Y to changes in the particular x. So, these are sensitivity of Y to changes in particular x. So, this is sensitivity to change sensitivity of Y to change in x1 sensitivity of Y to change in x2 and. So, on and so 4^{th}. If partial derivative is large we say Y is very sensitive to that particular x. Now, Consider delta Y to be the uncertainty in measured value Y. Which we represent as let us, say similarly delta x1 to be uncertainty in measured value x1 which we represented as u x1. Similarly, delta xn is to be considered as uncertainty in measured value xn which will represent as U xn.

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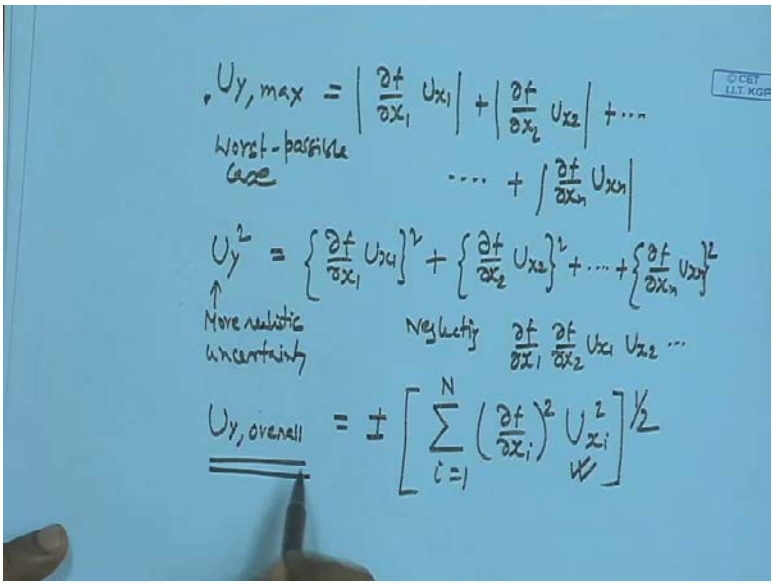

Then we can write as, Uy is we have just, use this definition in this equation. Now, the maximum value of uncertainty will be when all the uncertainties happen to have the same sign. So, the maximum value of uncertainty can be taken as, would be obtained when all the uncertainties happen to have the same sign. And this would be the worst possible case; however, the probability of such an occurrence is generally very small. Therefore, the more realistic way is to square both sides to give equal weight age to both positive and negative values of uncertainties. So, more realistic way of representing uncertainty will be, to square the terms, here we have Neglected terms like cross terms like.

So, this will be More realistic uncertainty. So, we can now take overall uncertainty as, where N is this x1 up to x2 up to xn, an number of independent variables. So, if you have an estimate of the individual uncertainties, we can determine the overall uncertainty.

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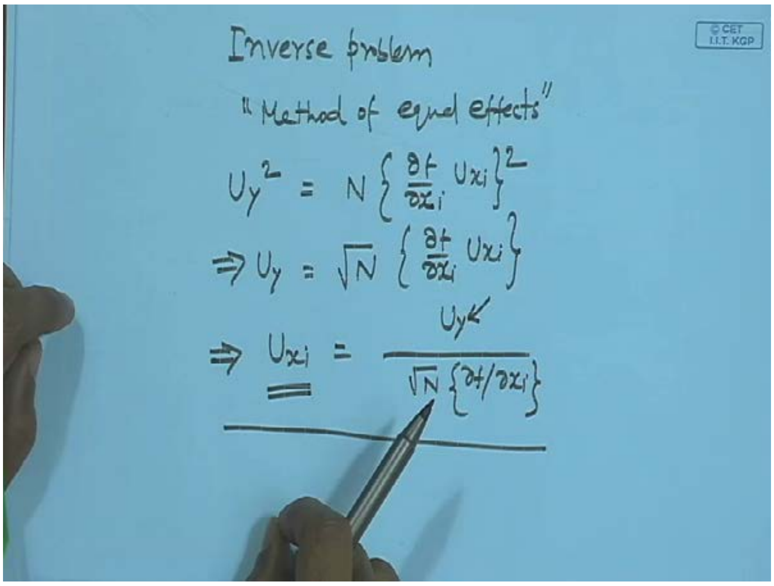

But, is the inverse problem which is more important. What do you mean by inverse problem is as follows? When you originally plan an experiment we must decide, how much accurate our measurement will be. So, we set our goal that our measurement must meet this much of accuracy, and accordingly we need to know what will be the allowable uncertainties in individual measurements.

Now, this problem is not. So, easy to solve because there can be various combinations of these uncertainties which will give the same over all accuracy. So, we have to get started somewhere. So, at this stage we can make use of something called Method of equal effects, where we assume that all the instruments or all the individual components contribute equally to the overall error. In other words the individual components contribute equally to the overall error.

So, if we look at this equation, under the assumption of Method of equal effects. We can now, write or so we can now, have an estimate of; what should be the accuracy of individual component to meet the requirement of overall accuracy? If you can find instruments which meet all these nets we have at least one solution to a problem. If one or more requirements cannot be meet we should check if some of our instruments are better than that the above equation requires. If, that is the case, then you can relax the requirement that we cannot meet ultimately the overall accuracy must be meet.