Process Control and Instrumentation
Prof. D. Sarkar
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 38
Instrumentation: General Principle of Measurement Systems (Contd.)

In our previous discussion we started with Static Characteristics of Instruments. We discuss that there are set of Desirable Characteristics of Instruments and there are set of Undesirable Characteristics of the Instruments. The desirable characteristics like Accuracy, Sensitivity, Precision and the undesirable static characteristics we talked about our Static Error, Dead zone and Drift.

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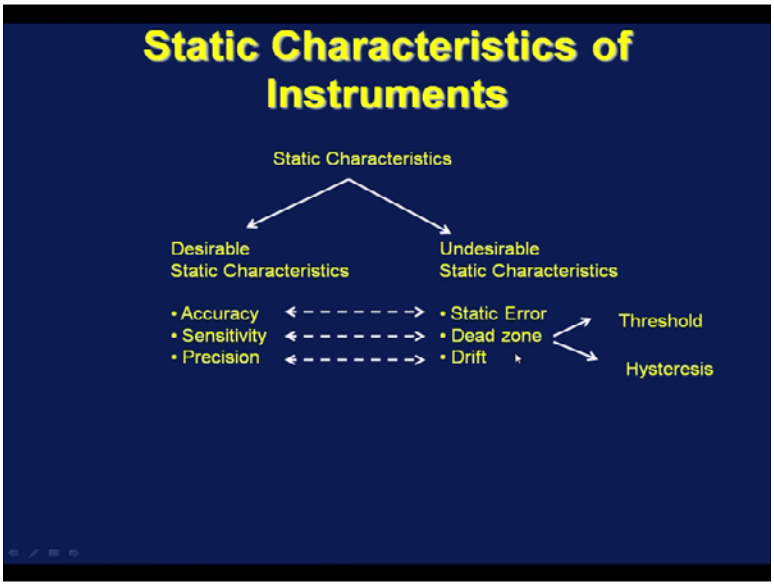

Today, let us talk more about the Static Characteristics of Instruments. So, we have set of desirable static characteristics and we have set of undesirable static characteristics. The desirable static characteristics accuracy has a corresponding undesirable static characteristic static error similarly, sensitivity and dead zone precision on the desirable static characteristic list and corresponding undesirable static characteristic is drift. Dead zone can come from Threshold or Hysteresis.

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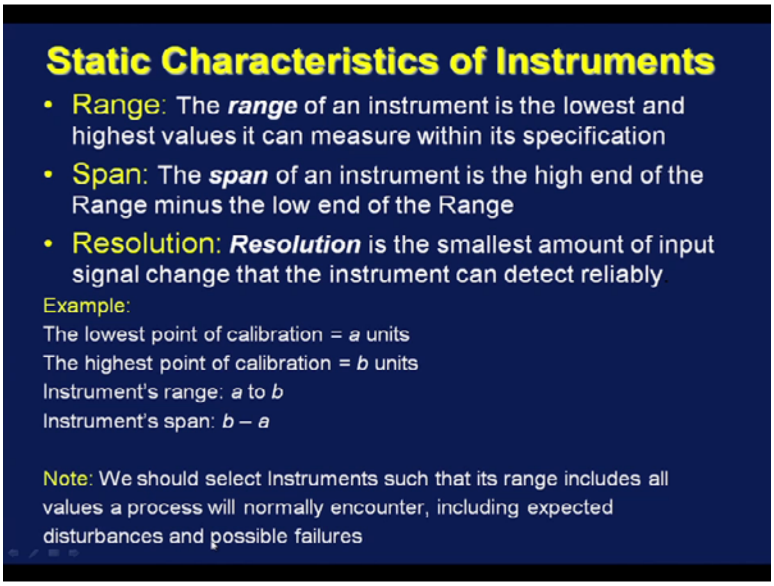

We define these terms in our previous lecture let us do a quick review. The range of an instrument is the lowest and highest values it can measure within its specification, while the span of an instrument is the high end of the range minus the low end of the range. For example, the lowest point of calibration say a units and the highest point of calibration is b units that the instruments range is specified as a to b while instrument span is b minus a.

Resolution is defined as the smallest amount of input signal change that the instrument can detect reliably. It should be noted that we should select instruments such that, its range includes all values a process will normally encounter and that we should include expected disturbances as well as possible failures.

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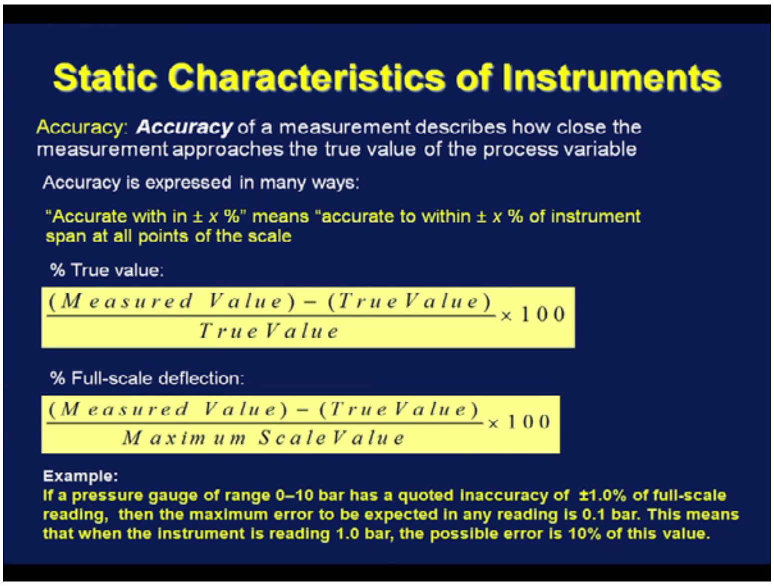

Accuracy of a measurement describes how close the measurement approaches the true value of the process variable. Accuracy can be expressed in many ways when you say accurate within plus minus X percent we mean accurate to within plus minus X percent of instrument span at all points of the scale. Accuracy can also be expressed as percentage True Value, percentage true value can be defined as Measured Value minus True Value divided by True Value multiplied by 100.

Accuracy can also be expressed as percent Full-scale deflection. Percent full-scale deflection can be described as, Measured Value minus True Value divided by Maximum Scale Value multiplied by 100. For example, if a pressure gauge of range 0 to 10 bar has a quoted inaccuracy of plus minus 1 percent of full-scale reading then, the maximum error to be expected in any reading is 0.1 bar. This means, that when the instrument is reading 1 bar the possible error is 10 percent of this value.

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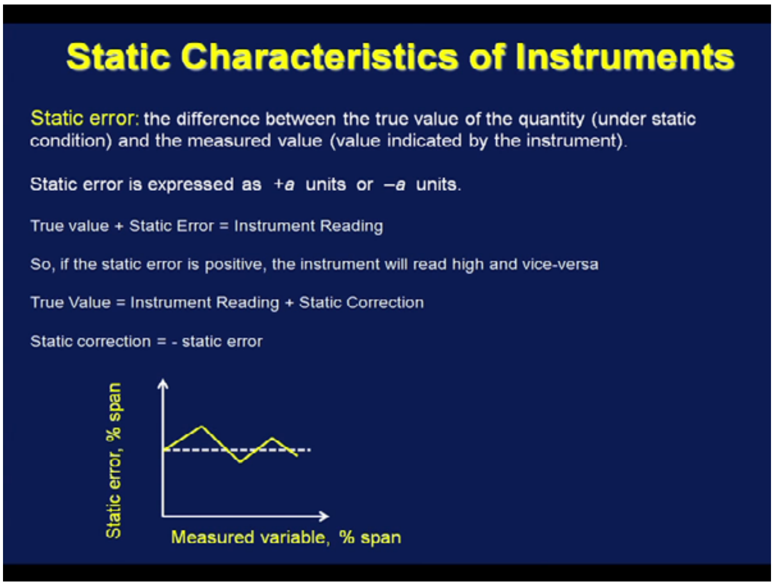

Corresponding to desirable static characteristic accuracy we have undesirable static characteristics static error. Static error is defined as the difference between the true value of the quantity under static condition and the measured value that is value indicated by the instrument. So, it is a difference between the true value of the quantity and the value indicated by the instrument and, we are talking about a condition which is not changing with time or changing very slowly. So, that it can be considered static. Static error is expressed as plus a units or minus a units.

So, True Value plus Static Error is Instruments Reading. If, true value plus static error is equal to instrument reading and if the static error is positive the instrument will read high and if the static error is negative the instrument will read low. So, True Value can be equated to Instrument Reading plus Static Correction. In other words Static Correction is equated to minus of static error.

A typical static error curve may look like this. While, on x axis is the Measured variable percent span and, on y variable I have plotted static error again expressed as percent span. So, corresponding to different value of Measured variable I can have an estimate of Static error if, I have this Static error curve.

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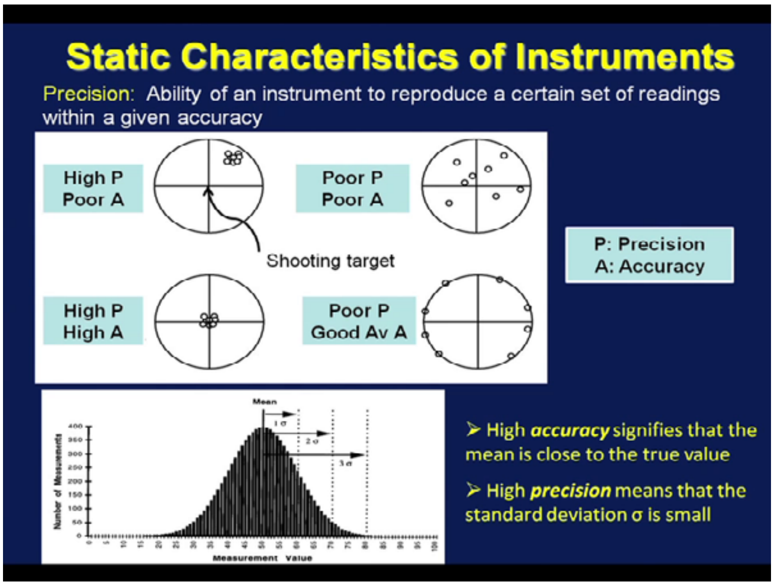

Next, let us talk about another desirable static characteristic Precision. Precision is defined as ability of an instrument to reproduce a certain set of readings within a given accuracy. Say it is the ability of an instruments to reproduce a certain set of readings within a given accuracy this, can be best explained by an example. Let us consider a person is doing a shooting practice. So, the person has few bullets in hand. In fact, 8 in number and the Shooting target is this boy which is intersection of these two lines.

Now, here is a set of outcomes of the persons shooting practice. In this case he miss the target, but all the bullets are here. So, this is a case of High Precision, but Poor accuracy Precision is high because, all the bullets are together. So, there is ability to reproduce, but accuracy is very poor because the person could even hit the target once. So, it is not necessary that an instrument is highly precise will also be highly accurate.

Let us consider this scenario. Here, the bullets are all scattered. So, this is a case of Poor Precision Poor Accuracy. In this example, each time the person has been able to hit the target exactly all the 8 bullets have a this centre. So, this is obviously, a case of High Precision and High Accuracy.

Now, let us consider the last scenario. Here, the person has not been able to exactly hit the target even for once, but all the bullets are around here. So, we can say that this is a case where Good Average Accuracy is there but, there is no precision. So, it is a case of Poor Precision, but Good Average Accuracy.

Now, if I plot the measurement values verses number of measurements I will get this frequency plot. This represents the mean. So, High accuracy will signify that the mean of the measurements is close to that true value. So, high accuracy will mean the mean is very close to the true value. While, High precision will mean that there is less scattered in other words the standard deviation is low.

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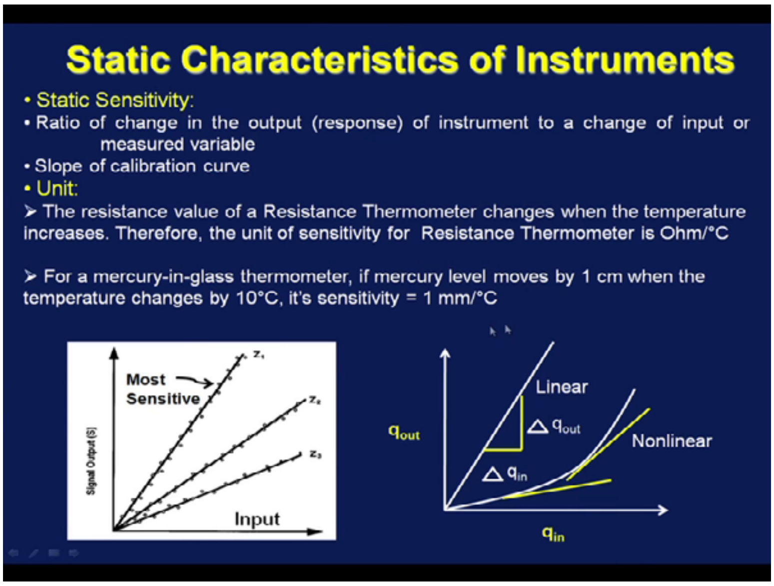

Let us define another static characteristic call Static Sensitivity. This is defined as Ratio of change in the input or response of instrument to a change of input or measured variable. So, this is change in output or response divided by change in input or measured variable. So, this can be defined as delta output divided by delta input where, delta output is change in output and delta input is change in input. So, it is easy to recognize that this is nothing, but slop of calibration curve.

So, let us look at this calibration curve. This is a calibration curve for a Linear instrument where, the input output follows a linear relationship. So, the Slope of this curve or slope of this straight line which is given by delta output divided by delta input will be the Static Sensitivity of the instrument. This is a calibration curve for a Nonlinear instrument where, the instruments input and instruments output follows a nonlinear relationship. Here, the static sensitivity can be defined at different points by the slope at that point.

Let us take this plot. Let us say these are all calibration curve for different instruments. It is easy to recognize that the sensitivity of this instrument is highest next sensitive is this instrument and last is this one because, the gradient is stiff here compare to these two. So, delta output by delta input value is maximum here compare to this and then this. So, What should be the unit of static sensitivity? It is defined as delta output by delta input. So, its unit will also be unit of output by unit of input.

Let us say The resistance value of a resistance thermometer changes with temperature changes when the temperature increases. See if, we change the temperature the resistance of a Resistance Thermometer will change so obviously, the unit of sensitivity for Resistance Thermometer will be unit of resistance divided by unit of temperature. So, it may be Ohmper degree Celsius. So, for a mercuryinglass thermometer if, mercury level moves by 1 centimeter when the temperature changes by 10 degree Celsius its sensitivity will be 1 centimeter divided by 10 degree Celsius. In other words it is 1 millimeter per degree Celsius.

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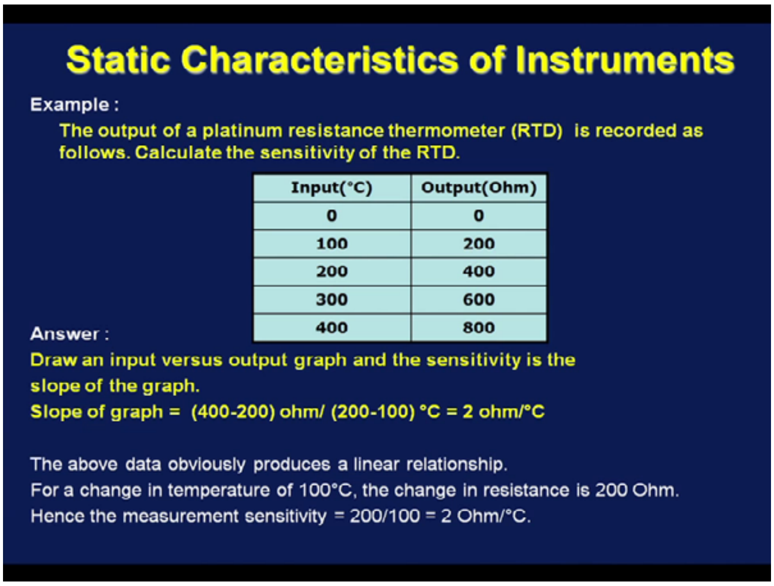

Let us take an Example. The output of a platinum resistance thermometer which is called RTD in brief is recorded as follows. On the left I have temperature column and in right we have resistance corresponding resistance column. So, at 0 degree Celsius the resistance of the RTD is 0. At 100 degree Celsius the resistance of RTD is 200 Ohm, at 200 degree Celsius the resistance of RTD is 400 Ohm, at 300 Celsius the RTD has resistance of 600 Ohm and at 400 degree Celsius RTD has resistance 800 Ohm.

We need to calculate the sensitivity of the resistance thermometer device or RTD. It can be easily done if we just Draw an input verses output graph. So, I will make a graph of temperature on x axis and the resistance in Ohm on y axis. So, the Slope of the graph will give me the value of static sensitivity. If, you look at the data table it is obvious that the input which is temperature in degree Celsius and the output which is resistance in Ohm follows a linear relationship. So obviously, we are going to get a straight line. So, is easy to calculate the slope from the data given.

So, slope of the graph will be if we consider these pair of points 400 minus 200 divided by 200 minus100. So, this is 200 divided by 100 which is equal to 2 Ohm per degree Celsius. Also simply by looking at the table as I just discuss that the data obviously, produces a linear relationship and for every 100 degree Celsius increase in temperature there is 200 Ohm increase in the resistance. So, the measurement sensitivity is 200 divided by 100 is 2 Ohm per degree Celsius.

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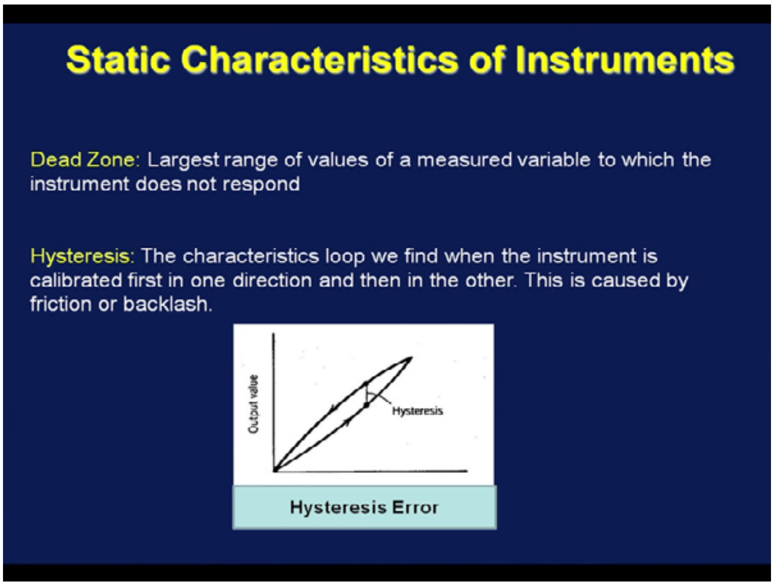

Next, static characteristic we will discuss is Dead Zone. Dead Zone is the Largest range of values of a measured variable to which the instrument does not respond. So, dead zone is the largest range of values of a measured variable to which the instrument does not respond. The Hysteresis is the characteristic loop we find when the instrument is calibrated first in one direction and then, in the other direction this is caused by friction or backlash.

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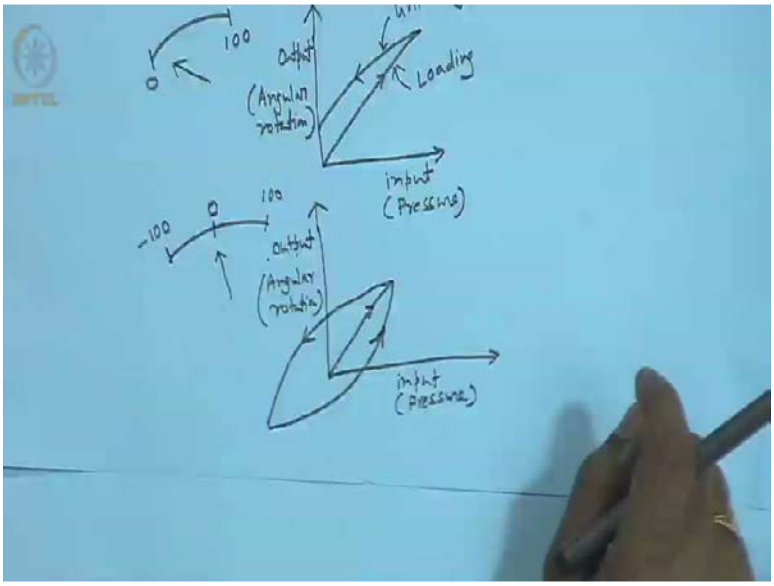

Let us consider an instrument with a scale as follows. We have 0 at lower end and let us say 100 at high end. So, is the point. Let us say this is the pressure gradation we go on increasing the pressure from 0 degree from 0 say atmosphere to 100 atmosphere and then, slowly realize the pressure from 100 atmosphere to 0 atmosphere. So, now if I make a plot of input which for this case is Pressure verses output which can be considered as the Angular rotation of the point at against this scale which is of course, calibrated in terms of pressure units. So, if while increasing the pressure from 0 units to 100 unit I follow this path while, lowering down the pressure from 100 units to 0 units I may follow another path as follows. So, Hysteresis is this loop this is the non coincidence of this Loading and Unloading curve. So, while I am increasing pressure or where in I am putting more energy to the instrument and while I am reducing the pressure from 100 units to 0 units, I am recovering that energy. That energy is being released.

So obviously, I am unable to recover the entire amount of energy. So, this non coincidence of this loading and this unloading curve is called Hysteresis which is a result of friction or backlash. If, we have a scale like say 0 at the middle and we have say minus 100 units at this end and 100 units at the other end, if I do the same experiment. So, if we start from 0 go on increasing up to 100 units then comeback to minus 100 units and then finally, return to my initial starting point, if I now make the same plot. The plot will now look like or may look like start from 0, I reach here, comeback to this unit and then this. So, again this is the Hysteresis loop.

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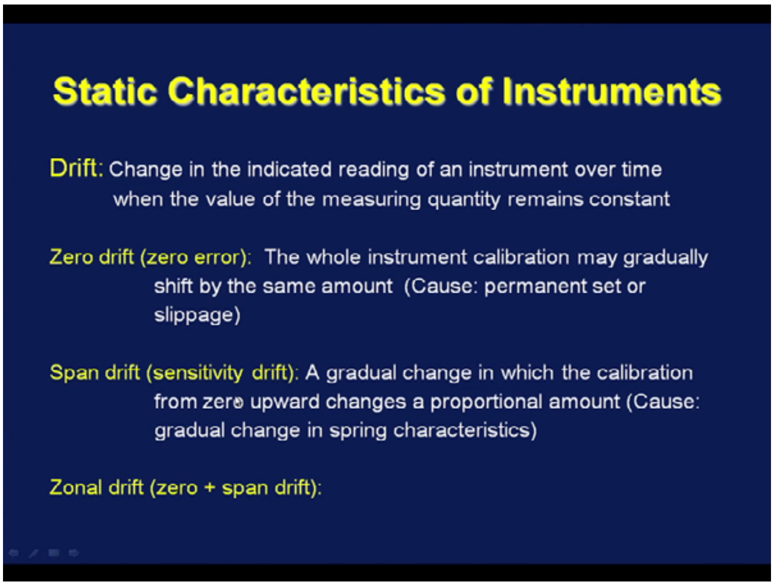

Next, let us define a static characteristic called Drift. Drift is the Change in the indicated reading of an instrument over time when the value of the measuring quantity remains constant. So, drift is defined as the change in the indicated reading of an instrument over time when the value of the measuring quantity remains constant. The drift can be classified as Zero drift or zero error, Span drift or sensitivity drift and finally, Zonal drift which is a combination of zero and span drift.

In case of Zero drift or zero error the instrument calibration may gradually shift by the same amount. You can see that in some of the instruments let us say a pressure gauge when there is no pressure input to the instrument, the instrument still shows of small positive value which, is a zero error we should take a note of this zero error, so that we can apply the correction factor to the measured value being indicated by the instrument. So, Zero drift or zero error The whole instrument calibration gradually shift by the same amount it may be cause by permanent set or slippage.

In case of Span drift or sensitivity drift A gradual change in which the calibration from zero upward changes a proportional amount. This may be cause by gradual change in the spring characteristics of the instruments. Well, Zonal drift can be viewed as a combination of zero drift as well as span drift or sensitivity drift.

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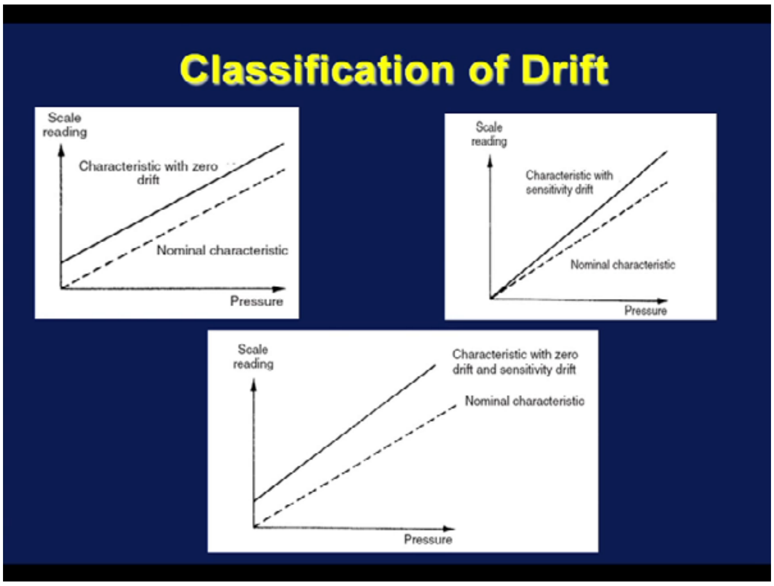

So, this is an example while the instrument shows zero drift. If, this is the Nominal characteristic between input for this particular case let us we are talking about a Pressure gauge. So, pressure is the input and this is the output Scale reading. So, if this is the nominal characteristic or the calibration curve when there was no zero error this may be the case when the instrument has develop a zero error or zero drift. So, the whole calibration has shifted by this amount which is the magnitude of zero error.

This is the case of span drift or sensitivity drift. This is the Nominal Characteristic while the characteristic with the sensitivity drift or the span drift shows a gradual change from zero upward. So, this is a case of sensitivity drift or span drift. Finally, this is the case where we have zero drift as well as sensitivity drift. You can see that if the instrument had no zero error this would have this point should have coincided with the origin and, if the instrument had no sensitivity drift, but only zero drift this line would have been parallel to this Nominal characteristic line.

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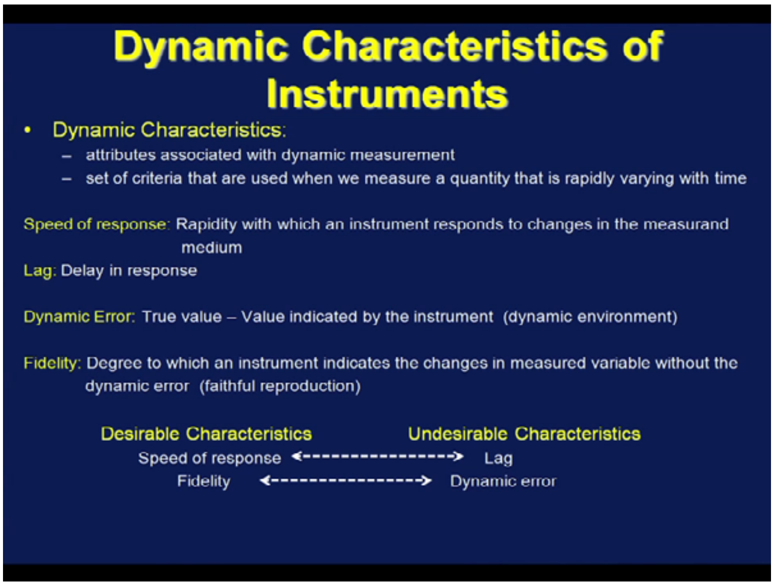

Now, let us talk about the Dynamic Characteristics of Instruments. We have defined Dynamic Characteristics as the characteristics that you must consider when the instrument is measuring a medium which changes with time. So, these are the attributes associated with dynamic measurements and, we must consider these whole to just the performance of an instruments or the quantity of measurement when the measuring medium is changing rapidly with time.

Similar to static characteristic we have few Desirable Dynamic Characteristics and we have few corresponding Undesirable Dynamic Characteristics. The Desirable dynamic Characteristics are Speed of response which, is defined as the Rapidity with which an instrument responds to changes in the measured medium or measure input. So, speed of response is the rapidity with which an instrument response to changes in the measuring medium, while, Lag is Delay in response.

So, speed of response is a Desirable dynamic Characteristics while obviously, Lag which represents delay in response is Undesirable dynamic Characteristic. Similar to static error, Dynamic Error is defined as True Value minus Value indicated by the instrument and of course, it is in dynamic environment. So, Dynamic Error is the difference between the True Value and the Value indicated by the instrument when the instrument is being used to measure a dynamic medium. And Fidelity is the Degree to which an instrument indicates the changes in measured variable without the dynamic error. So, this is a faithful reproduction.

So, Fidelity is defined as degree to which an instrument indicates the changes in measured variable without the dynamic error. So, we have Speed of Response and Fidelity under Desirable static Characteristics and you have Lag and Dynamic error under Undesirable static Characteristics. So, the corresponding Undesirable Characteristic to Speed of Response is Lag and similarly, the Dynamic error is corresponding to Desirable Characteristic Fidelity.

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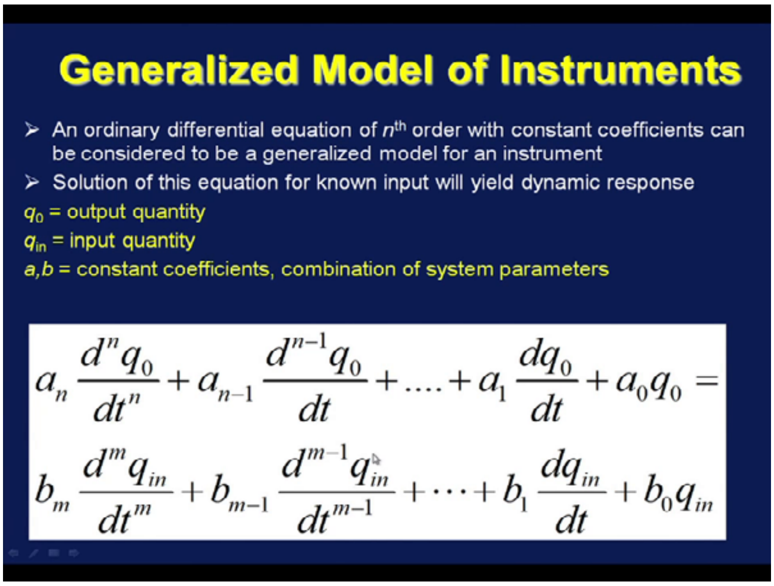

Now, let us talk about Generalized Model of Instruments. We need a mathematical model to describe the behavior of an instrument. If, we develop a mathematical model that relates the input output relationships then you can solve that equation for known inputs to get what will be the output of the instrument. In other words the dynamic response of the instrument can be studied, if we have a suitable mathematical model for my instrument. An ordinary differential equation of nth order with constant coefficients can be considered to be a generalized model for any instrument.

So, we can consider an nth order differential equation ordinary differential equation with constant coefficients to be a generalized model for an instrument. So, this is an nth order in output and nth order in input differential equation that can be considered to be a Generalized Model for an Instrument. Let us define q0 as output quantity, q in as input quantity and a, b are constant coefficients which are essentially combination of system parameters may be combination of physical parameters which are all being assumed to be constant. So, that all the a's and b's are constants.

So, and this is an equation where on the left we have an nth order OD in output q0 and nth order OD in input qin. This can be taken as a very general model for any instrument. Say, if I solve this equation for known inputs I can obtained q0 or output. So, you can study the dynamic response of this instrument with help of this model.

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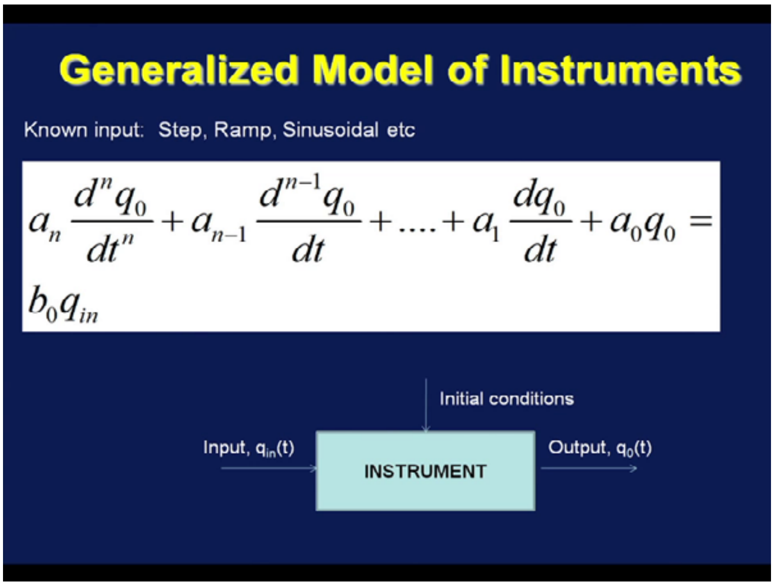

Now, it is not really necessary to represent the input as nth order OD. We can express the output the unknown output as nth order OD and we will know the functional form of the input. Because, it may be a Step input, it may be a Ramp input, it may be a Sinusoidal input it may be a combination of these inputs. So, we will consider a generalized model of an instrument which is as follows. We have an nth order OD in output or in q0 and this is the input with known functional form. So, if I solve these equations with known Input and Initial condition I will get output. So, the dynamic response can be studied with help of such equation.

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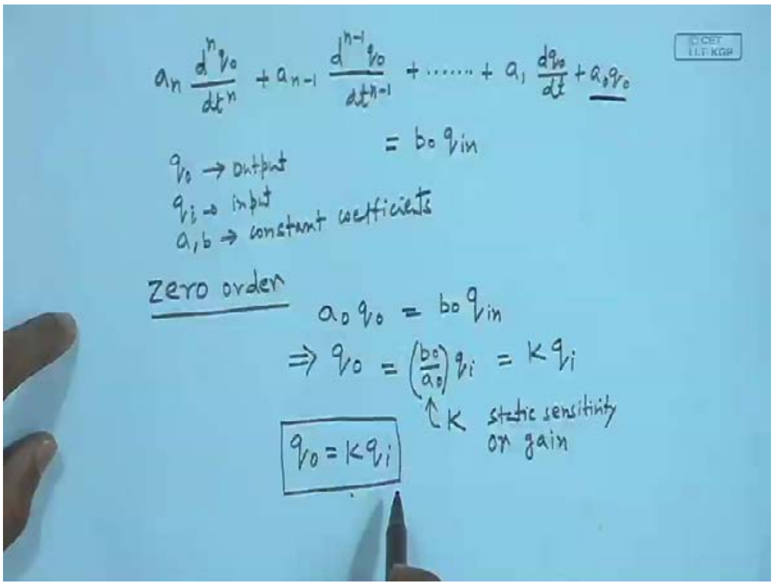

Now, let us rewrite this equation here. We have an nth order OD in q0. So, this is the general mathematical model for my instrument where, q0 is output quantity qi is input quantity and a, b are constant coefficients which are combination of system parameters. Now, if an instrument is described by nth order differential equation we say that the order of the instrument is n. So, if the instrument is described by second order OD or ordinary differential equation we call the order of the instrument to be 2 for first order instrument the instrument will be described by a first order OD.

Similarly, at zeroth order OD which is nothing, but an algebraic equation will represent a zero order instrument. So, let us first see, how the zero order instruments model will look like. So, when we are talking about zero order instruments let us retain only the zero order term in this equation. So, we have we will retain only this term from the left hand side of the model equation.

So, the zero order instruments will be described by if, we rearrange call this K. Which is static sensitivity also called gain. So, this becomes K into qi. So, q0 K qi is the model for a zero order instrument. Look at this equation this is an algebraic equation the output can be found out for known functional form of input. Let us say Step input or Ramp if we know the value of the gain or static sensitivity. It may be noted that this zero order instrument will show perfect dynamics the output will immediately follow they will not be any lag any distortion there is no time term involved here.

So, we can say zero order instrument, if we can have a zero order instrument it will follow perfect dynamics they will not be any lag at all. It is not easy to get a zero order instrument, but let us consider a Displacement measuring potentiometer which can be approximated as a zero order instrument let us draw the diagram.

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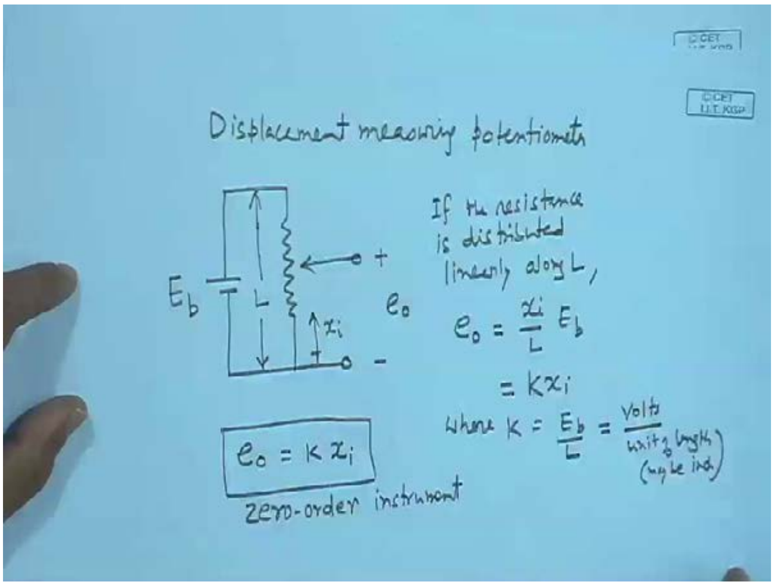

Let this length be L. If, the resistance is distributed linearly along length L, so if the resistance is distributed linearly we can write which is again gain its unit will be say volts per unit of length may be inch. So, you see the displacement measuring potentiometer is model by output which is a zero order instrument. Similarly, a first order instrument will be described by a first order OD. So, again if we look at the previous differential equation just say, generalize model for an instrument. We will retain only the First order term. So, we will retain this as well as this for a first order instrument.

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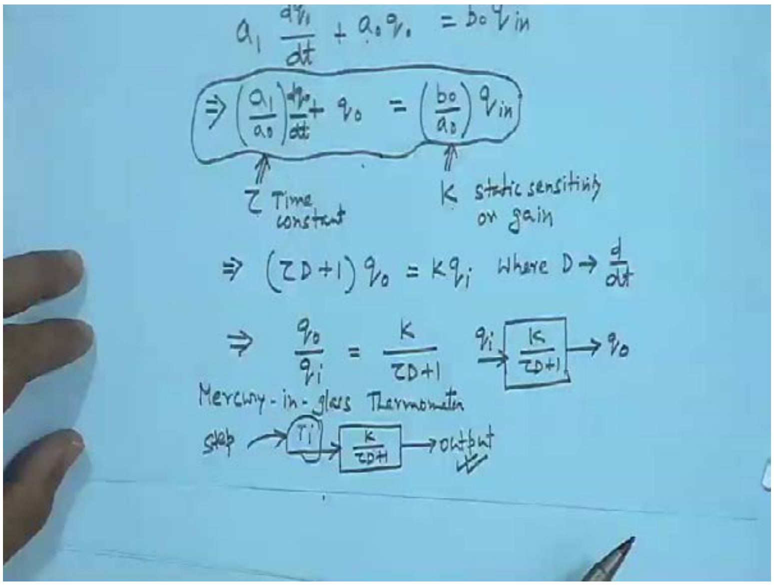

So, if we retain the First order terms which is a first order differential equation first order ordinary differential equation. If, we rearrange the equation as follows divide this equation throughout by the a0. So, let us make this term coefficient less. We have forgot the dq0 dt term here. If, you remember your class on process control this actually gives Time constant term whereas, this is static sensitivity or gain.

So, we can write the equation as where. So, if we rearrange this we will get your familiar transfer function. So, first order system will follow this model. For example, a Mercury-in-glass Thermometer to be more specific there ordinary Mercury-in-glass Thermometer that means, a Mercury-in-glass Thermometer without any cover will behave like a first order instruments. So, it can be shown by writing an energy balance equation or heat balance equation that the governing equation is first order and we will look like this.

So, if I want to know. If, I put my Mercury-in-glass Thermometer suddenly from say a temperature of 30 degree Celsius to a temperature of 40 degree Celsius. How the instrument will respond to the this step input of magnitude 40 minus 30 equal to Ti? All I need to do is solve this, so given the step input of magnitude Ti. I now have the transfer function or I have now the ordinary differential equations they describes the mercury-in-glass thermometer provided this tau and K which are combination of system parameters are known. I can determine what will be the output of the Mercury-in-glass Thermometer where the mercury level will rest in the scale.

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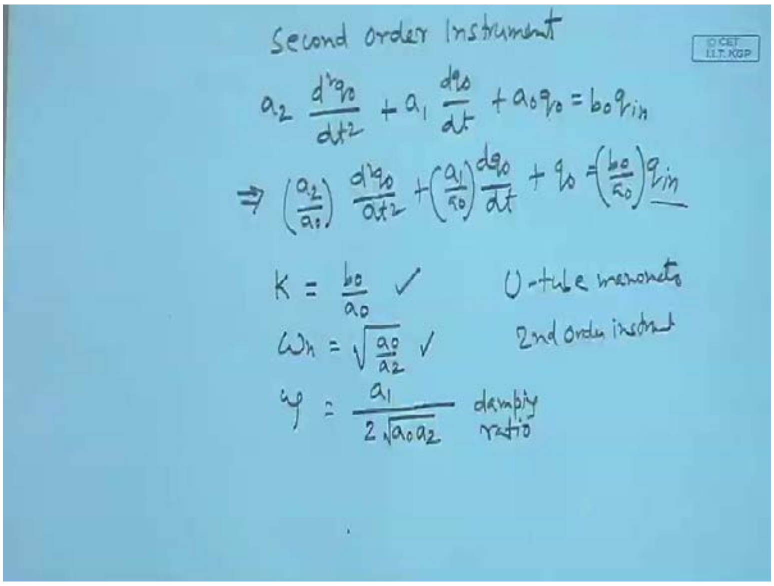

Similarly, a Second order Instrument will be described by a second order OD. So, again let us make this term coefficient less. We define the terms you are familiar with when you read the process control part of the course usual static sensitivity, undammped natural frequency damping ratio. So, these are three parameters which will if we know then a second order instrument can be solved for any given input. U-tube manometer is a good example of 2nd order instrument. How these instruments whether it is first order instruments or second order instruments will behave when these instruments are subjected to a known input.

You are familiar with those responses because, the response of first order systems response of second order systems for varies inputs have been discussed in detail in the process control part of the course. So, the same methodologies same principles can be apply to determine what will be the dynamic response of an instrument whether whatever its order be very easily.