Process Control and Instrumentation
Prof. A. K. Jana
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 34
Advanced Control Schemes (Contd.)

In the last class, we discussed about the configuration of feed forward control scheme. And today, we will discuss the design of feed forward control schemes.

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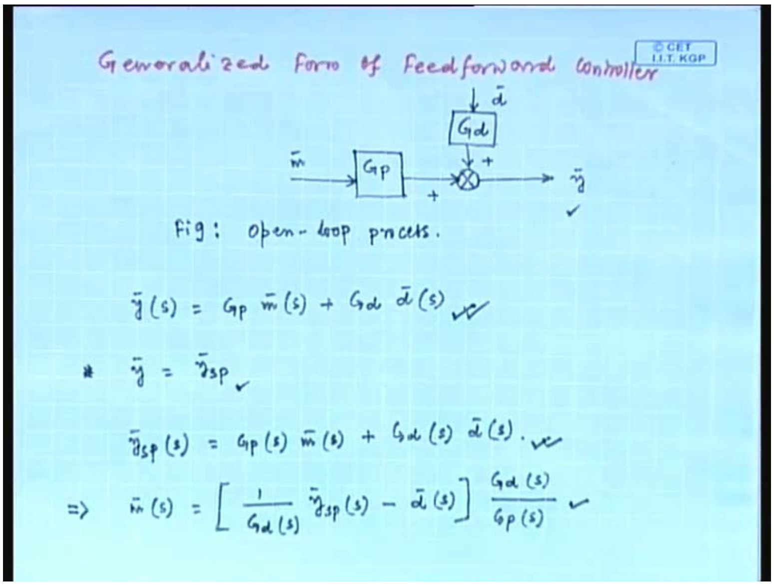

Generalized Form of Feedforward Controller

Fig: open-loop process.

$\bar{y}(s) = G_p \bar{m}(s) + G_d \bar{d}(s)$

* $\bar{y} = \bar{y}_{sp}$

$\bar{y}_{sp}(s) = G_p(s) \bar{m}(s) + G_d(s) \bar{d}(s).$

$\Rightarrow \bar{m}(s) = \left[\frac{1}{G_d(s)} \bar{y}_{sp}(s) - \bar{d}(s) \right] \frac{G_d(s)}{G_p(s)}$

So, first we will discuss a generalized form of feed forward controller fine. So, for this we first draw the process. So, there are 2 transfer; functions one is G p, another one is G d d is the disturbance and n is the input to the process output is y fine. So, this is block diagram of the open loop process this is the block diagram of the open loop process. Now, we can represent this process mathematically as output y bar s which is equal to G p m bar s plus G d d bar s. This is the representation of the open loop process shown here fine. Now, our target is to keep with the y at its set point value our target is to keep y at it is set point valued this is our target fine. So, accordingly the feedback controller sorry feed forward controller is require to take the control action.

Feed forward controller needs to take action to keep y at its desired value that is y set point. So, if we substitute y equal to y set point in this equation then we obtain y set point s equal to G p s m bar s plus G d s d bar s. This expression we obtain just substituting y equal to y set point in this open loop expression Now, if we rearrange this equation, we obtain m bar s which is equal to one by G d s y set point bar s minus d bar s multiplied by G d s divided by G p s. If we rearrange this equation, we get this expression in the form of in which is the input to the process. Now, we can write this expression as.

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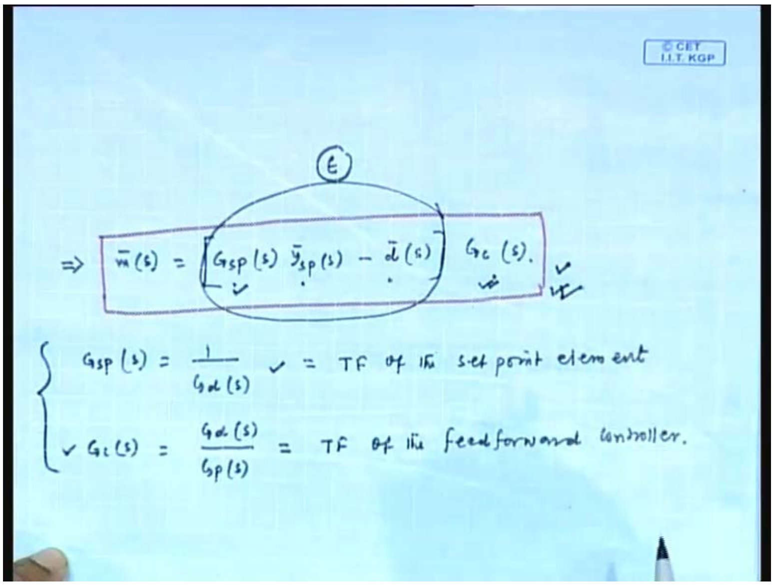

Ⓔ

$$\Rightarrow \bar{m}(s) = \left[G_{sp}(s)\,\bar{y}_{sp}(s) - \bar{d}(s)\right] G_c(s).$$

$$G_{sp}(s) = \frac{1}{G_d(s)} \checkmark = \text{TF of the set point element}$$

$$\checkmark\ G_c(s) = \frac{G_d(s)}{G_p(s)} = \text{TF of the feedforward controller.}$$

We can write this expression as m bar s equal to G set point s y set point bar s minus d bar s multiplied by G c s. G c is the transfer function of the controller and G s p is the transfer function of the set point element. G suffix s p is the transfer function of the set point element and G c is the transfer function of the controller. Now, if we compare the last 2 equations if we compare the last 2 equations, we obtain G s p equal to 1 by G d s. If we compare the last 2 equations, we obtain G s p equals to 1 divided by G d s; this is transfer function of the set point element.

Similarly, we obtain G c s as G d s divided by G p s if we compare the last 2 equations we obtain similarly, the expression for controller. This is the transfer function of the feed forward controller feed forward controller fine. So, these 2 transfer functions, we obtain in the next we will develop the block diagram of these feed forward control scheme. We will develop the block diagram for the feed forward control scheme.

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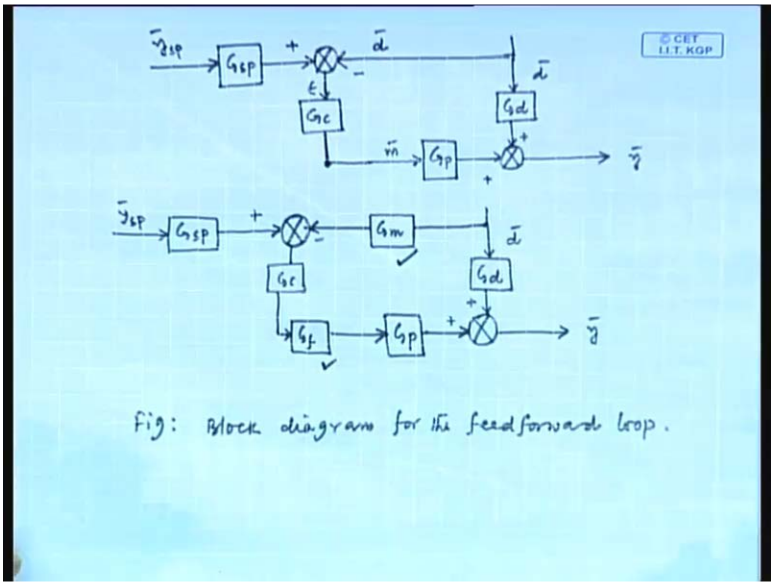

Fig: Block diagram for the feedforward loop.

First we draw the open loop process; the open loop process includes 2 transfer functions; one is G p, another one is G d. So, this is a open loop transfer fine next we use to include the feed forward control scheme. With this open loop transfer function, you see the expression this is a expression of the feed forward control scheme, this is a comparator this is d see here minus d is involved. So, we can put here minus sign next G s p y s p. So, we can put here set point element and input to this set point element is y set point. This is positive fine the output of this comparator is suppose error and this is a error G set point y set point minus d. That is the error; this error is this is suppose the error epsilon this error is multiplied with G c. So, we can put here one block for G c fine output is m this is output.

So, first this disturbance is measured and this disturbance is subtracted from G s p multiplied by y s p. So, we can consider G s p multiplied by y s p minus d as error signal. So, this is basically the error then this error is multiplied with G c therefore, G c is placed here and this output signal of this controller is m. So, this is the block diagram of the feed forward control scheme fine Now, in this block diagram you see the final control element and measuring device are not included the final control and measuring device are not included if we use to include that then we obtain the close loop block diagram somewhat like this; this is disturbance. So, first we measure this disturbance using one measuring device which has transfer function of G m.

Then this measure disturbance is compared in the comparator; this is a set point element then here controller here final control element fine. So, this is the block diagram for the feed forward loop block diagram; for the feed forward loop fine we have just included here one measuring device and final control element. Now, we will note down few important points on this feed forward control scheme say if we see the equation of the feed forward control scheme this one. It is obvious, that the feed forward controller is not like P or PI or PID controller. If you compare this equation with any expression of P or PI or PID it is obvious that the feed forward controller is not like P PI or PID controller.

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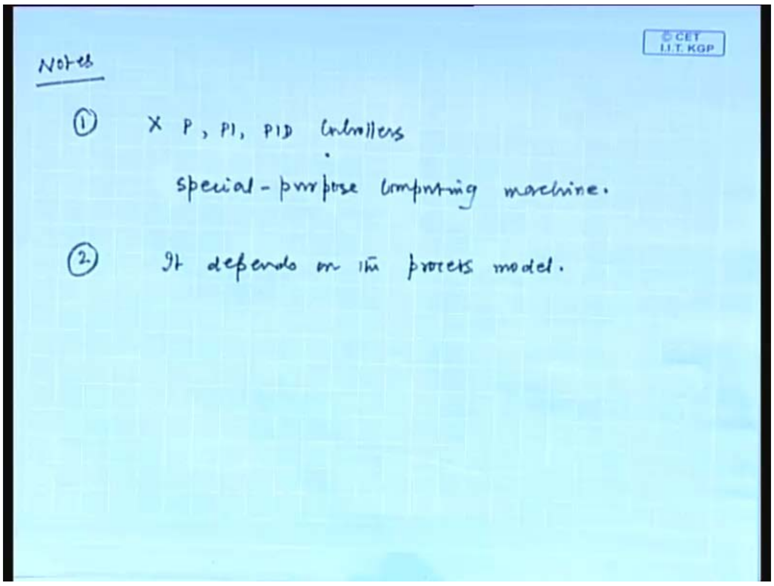

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Notes

① X P, PI, PID Controllers
special-purpose computing machine.

② It depends on the process model.

This is the first important point from the equation of feed forward controller it is obvious, that it is not like P PI or PID controller. It is a special purpose computing machine, it can be viewed as a special purpose computing machine, it can be viewed as a special purpose computing machine. Second important point is if you see this equation it is obvious, that the feed forward controller heavily depends on the process model you see in this equation G d G p transfer functions are involved. That means the feed forward controller depends heavily on the process model. It depends on the process model fine it is impossible to develop the accurate model in practice practically it is not possible to develop the accurate model. So, we cannot achieve perfect performance from this feed from this feed forward controller practically it is not possible to develop the accurate model. And we can consider this is a drawback of feed forward controller fine. Next, we will take one example to discuss to derive the design equation for this feed forward controller.

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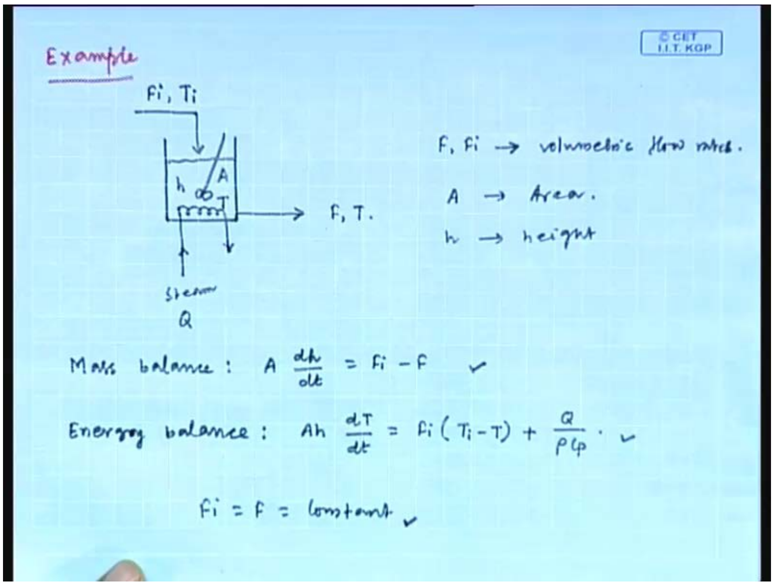

Example

$F, F_i \rightarrow$ volumetric flow rates.

$A \rightarrow$ Area.

$h \rightarrow$ height

Mass balance : $A\frac{dh}{dt} = F_i - F$ ✓

Energy balance : $Ah\frac{dT}{dt} = F_i(T_i - T) + \frac{Q}{\rho c_p}$. ✓

$F_i = F =$ constant ✓

We will consider an example of this feed forward control scheme. So, we will take if hitting tank system which we discuss earlier F i is the input flow rate, T i is the temperature, F is the outlet flow rate. And T is the temperature liquid height in the tank is h and T is the temperature. Now, the heating medium is passed through this heating coil suppose steam is the heating medium. And it has the flow rate of Q I mean it carries Q amount of heat fine here F F i both are volumetric flow rate volumetric. Flow rate h is the height of liquid in the tank a is the cross-sectional area of the tank a is the cross-sectional area of the tank h is the liquid height in the tank fine.

Now, for this process we developed earlier 2 equations one is based on mass balance another one is based on energy balance. So, we obtain the mass balance equation as a d h d T equal to F i minus F this is a mass balance equation. Similarly, we obtain the energy balance equation as a h d T d T equal to F i T i minus T plus Q divided by rho c p. This is a energy balance equation; this is a mass balance equation to discuss a feed forward control scheme. We assume that F i and F both are equal and they are constant here. We are assuming that F i equal to F and that is constant quantity, accordingly from the mass balance.

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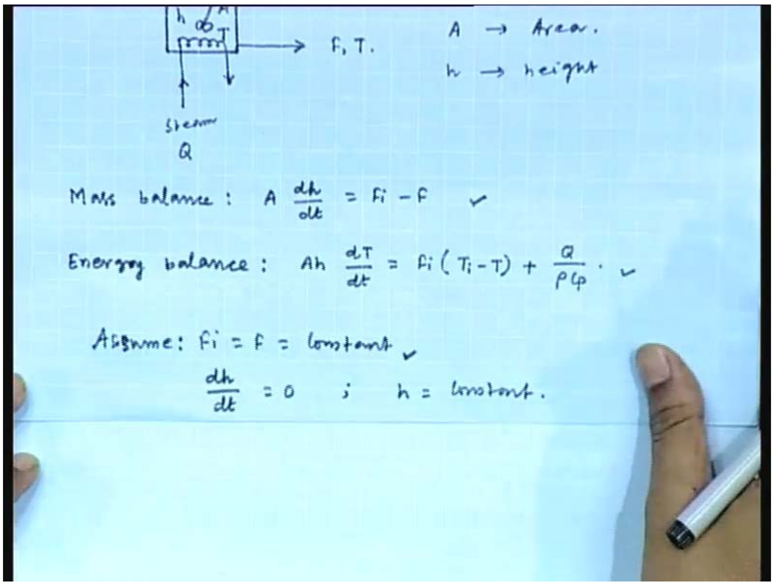

A → Area.

h → height

Mass balance : $A\frac{dh}{dt} = F_i - F$ ✓

Energy balance : $Ah\frac{dT}{dt} = F_i(T_i - T) + \frac{Q}{\rho C_p}$. ✓

Assume: $F_i = F$ = constant ✓

$\frac{dh}{dt} = 0$; h = constant.

We obtain d h d T equal to 0; that means, h is again a constant quantity. So, what is our control objective?

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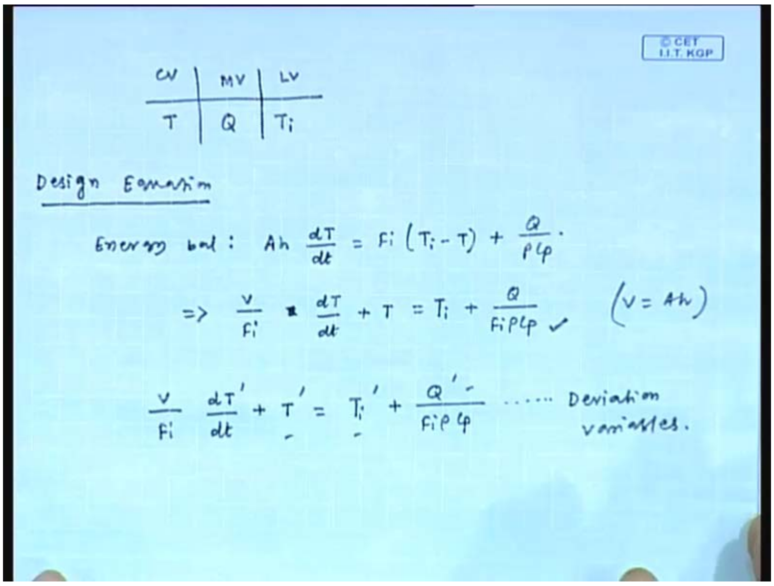
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CV	MV	LV
T	Q	T_i

Design Equation

Energy bal : $Ah\frac{dT}{dt} = F_i(T_i - T) + \frac{Q}{\rho C_p}$.

$\Rightarrow \frac{V}{F_i} \times \frac{dT}{dt} + T = T_i + \frac{Q}{F_i \rho C_p}$ ✓ $(V = Ah)$

$\frac{V}{F_i}\frac{dT'}{dt} + T' = T_i' + \frac{Q'}{F_i \rho C_p}$ ...... Deviation variables.

Our control objective is to maintain the temperature. So, temperature is our control variable what is the corresponding manipulated variable Q we have assume that F i is a constant quantity. Therefore, T i is the lowered variable therefore, T i is the lowered variable. So, next we will derive the design equation for the feed forward control scheme next we will derive the design equation for the feed forward control scheme. So, we will start from a energy balance equation which is a h d capital T d small T equal to F i T i minus T plus Q divided by rho c p. This is the energy balance equation, we have written earlier.

Now, we can write this equations as v divided by F i we are just dividing both sides by F i. We are just dividing both sides by F i then we obtain v divided by F i d T d T plus T equal to T i plus Q divided by F i rho c p here we assume v equal to A h fine. So, by dividing both sides by F i and rearranging, we obtain this equation. In the next step, we can write this equation in terms of deviation variables as v divided by F i d T prime d T plus T prime equal to T i prime plus Q prime divided by F i rho c p. This equation is written in terms of deviation variables fine here 3 deviation variable are involved; one is T T i and third one is Q.

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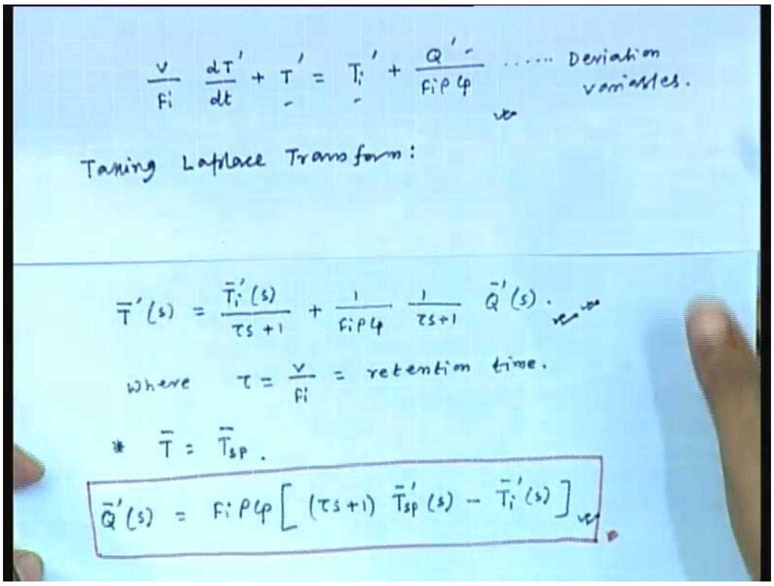

$$\frac{v}{F_i}\frac{dT'}{dt} + T' = T_i' + \frac{Q'}{F_i \rho c_p} \quad \ldots\ldots \text{Deviation variables.}$$

Taking Laplace Transform:

$$\bar{T}'(s) = \frac{\bar{T}_i'(s)}{\tau s + 1} + \frac{1}{F_i \rho c_p}\,\frac{1}{\tau s + 1}\,\bar{Q}'(s).$$

where $\tau = \frac{v}{F_i}$ = retention time.

* $\bar{T} = \bar{T}_{sp}$.

$$\boxed{\bar{Q}'(s) = F_i \rho c_p \left[(\tau s + 1)\,\bar{T}_{sp}'(s) - \bar{T}_i'(s) \right]}$$

Now, if we take the Laplace transform, we obtain T bar prime s T i bar prime s divided by tau s plus 1 plus 1 divided by F i rho c p 1 divided by tau s plus 1 Q bar prime s where tau is equal to v divided by F i. This is the retention time of liquid in the tank retention time of liquid in the tank, if we take the Laplace transform of this equation. And after rearranging, we obtain this expression where tau is the retention time of liquid in the tank. As mention earlier the objective is to keep the temperature T at its set point value this is the objective.

Accordingly the feed forward controller computes the control action the objective is to maintain the temperature T at its desired value that is T set point and accordingly the feed forward controller takes action. So, substituting T s p in place of T in this equation and rearranging finally, we obtain Q bar prime s equal to F i rho c p tau s plus 1 T set point bar prime s multiplied by T i bar prime s fine. Substituting T equal to T s p in this expression and rearranging finally, we obtain this expression the expression of Q fine. So, this is the design equation of the feed forward control scheme; this is the design equation of the feed forward control scheme. Now, can we configure the feed forward control scheme, considering this equation for the heating tank system? In the next, we will try to configure the feed forward control scheme.

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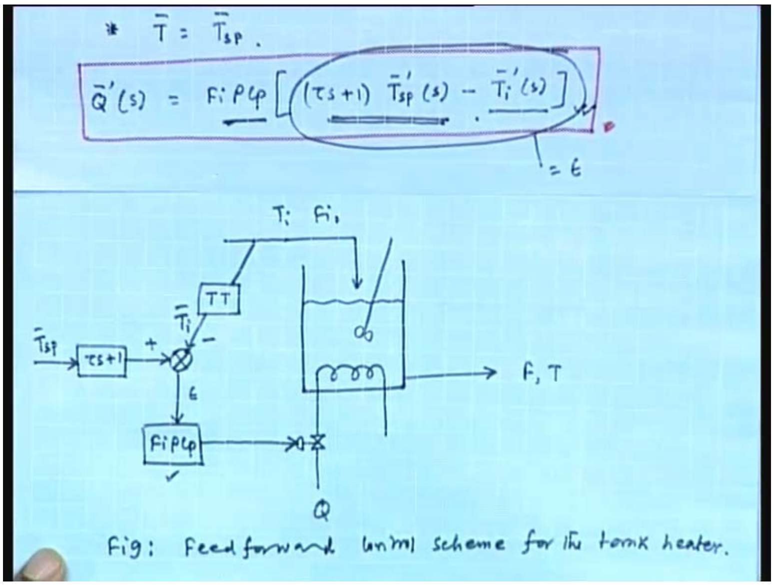

$* \; \bar{T} = \bar{T}_{sp}$.

$$\bar{Q}'(s) = F_i \rho c_p \left[(\tau s + 1)\, \bar{T}'_{sp}(s) - \bar{T}'_i(s) \right]$$

$= \epsilon$

Fig: Feedforward control scheme for the tank heater.

Incorporating this expression I mean applying this expression. So, this is the example system fine in the feed forward control scheme the disturbance is measured here disturbance is T i. So, we can measure this T i using one temperature measuring device fine. Then this T i is compared with the set point value fine you see T s plus 1 this 1 minus T i this is T i. Then the output of this error signal is multiplied with F i rho c p. So, this is F i rho c p and that is basically Q prime tau s plus 1 multiplied by T s p that is this 1 minus T i minus T i.

So, this part we can consider as error. So, this is error then this error is multiplied with F i rho c p this is F i rho c p this is F i rho c p and output is Q. And that is written here in terms of deviation variable agree. So, this is a feed forward control scheme for the heating tank system this is a feed forward control scheme for the tank heater fine. Now, if see this feed forward control scheme, we have develop considering the dynamic equation dynamic energy balance equation. Therefore, this feed forward controller is sometimes called as dynamic state feed forward controller. Since, we have used the dynamic energy balance equation for deriving the feed forward controller. Therefore, this is sometimes called as dynamic state feed forward controller fine.

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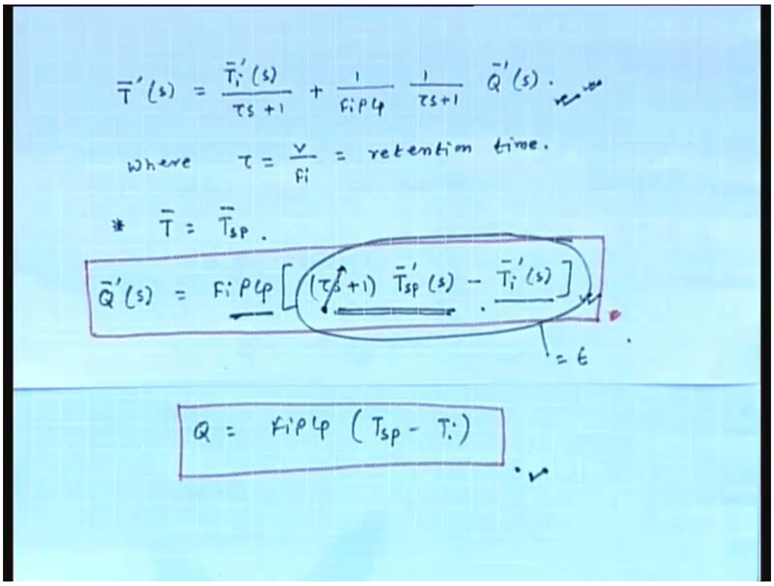

$$\bar{T}'(s) = \frac{\bar{T}_i'(s)}{\tau s + 1} + \frac{1}{F_i \rho c_p} \frac{1}{\tau s + 1} \bar{Q}'(s).$$

where $\tau = \frac{V}{F_i}$ = retentim time.

* $\bar{T} = \bar{T}_{sp}$.

$$\boxed{\bar{Q}'(s) = F_i \rho c_p \left[(\tau \cancel{s} + 1) \bar{T}_{sp}'(s) - \bar{T}_i'(s) \right]}$$

$= \varepsilon$

$$\boxed{Q = F_i \rho c_p (T_{sp} - T_i)}$$

Similarly, we can derive the steady-state feed forward controller. Similarly, we can derive the steady-state feed forward controller. And for steady-state feed forward controller, we write the energy balance as this we have just considered a h d T d T equal to 0 left hand side of the energy balance equation. We consider zero then we obtain this steady-state equation fine. Now, if we rearrange it we obtain Q equal to F i rho c p multiplied by T minus T i if we rearrange this steady-state form of energy balance equation we obtain this fine. Now, this is the design equation sorry we just have to put T equal to T set point. Then finally, we obtain this and this is the design equation for the steady-state feed forward controller. This is the design equation for the steady-state feed forward controller.

Now, if you compare the dynamic state feed forward controller and steady-state feed forward controller. We see that only this term does not exist for steady-state feed forward controller this is equal to 0 fine. We can get the steady-state feed forward controller just substituting tau is equal to 0 in the dynamic state feed forward controller. So, we have configure the dynamic state feed forward controller with the sitting tank system shown here for steady-state feed forward controller. You will just write inside this block one for steady-state feed forward controller; you replace tau s plus 1 by 1 fine. So, that is the steady-state feed forward controller.

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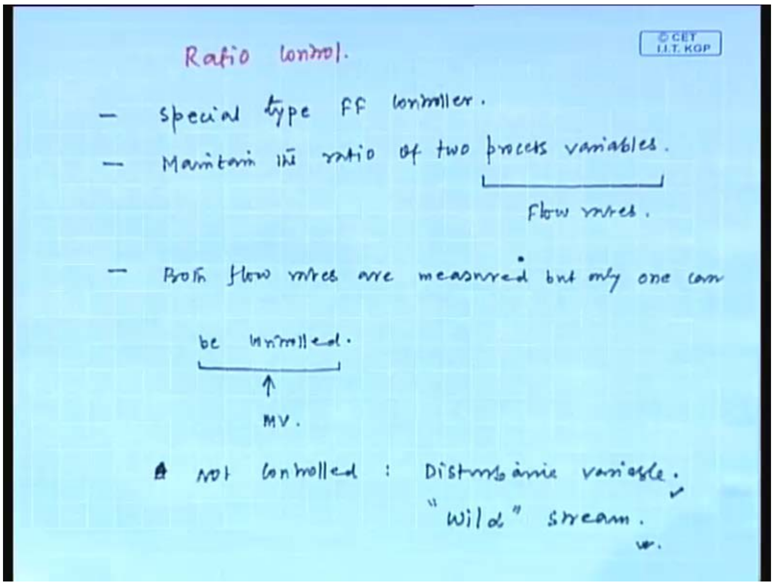

Ratio control.

- special type FF controller.
- Maintain the ratio of two process variables.

Flow rates.

- Both flow rates are measured but only one can be controlled.

MV.

Not controlled : Disturbance variable.

"wild" stream.

In the next, we will discuss another advanced control scheme that is ratio control. We will discuss another advance control scheme that is the ratio control scheme. This ratio control scheme is a special type of feed forward controller. Ratio controller is a special type of feed forward controller its objective is to maintain the ratio of 2 variables. Therefore, the n m its objective is to maintain the ratio of 2 process variables the ratio of 2 process variables at a specified value. Its objective is to maintain the ratio of 2 process variables at its specified value and these 2 process variables are usually flow rates. So, the objective of the ratio control scheme is to maintain the ratio of 2 flow rates fine both flow rates are measured.

But only one is controlled both flow rates are measured, but only one can be controlled, only one can be controlled fine now which is controlled that is called manipulated variable. So, this is manipulated variable and another flow rate is not controlled that is basically disturbance variable the variable which is not controlled that is basically disturbance variable. So, the objective of the ratio controller is to maintain the ratio of 2 flow rates, both the flow rates are measured. But only the manipulated input is controlled the flow rate which is not under controlled is the disturbance variable. And since this is not controlled it is called as wild stream fine since the manipulated variable is not under controlled it is called as wild stream.

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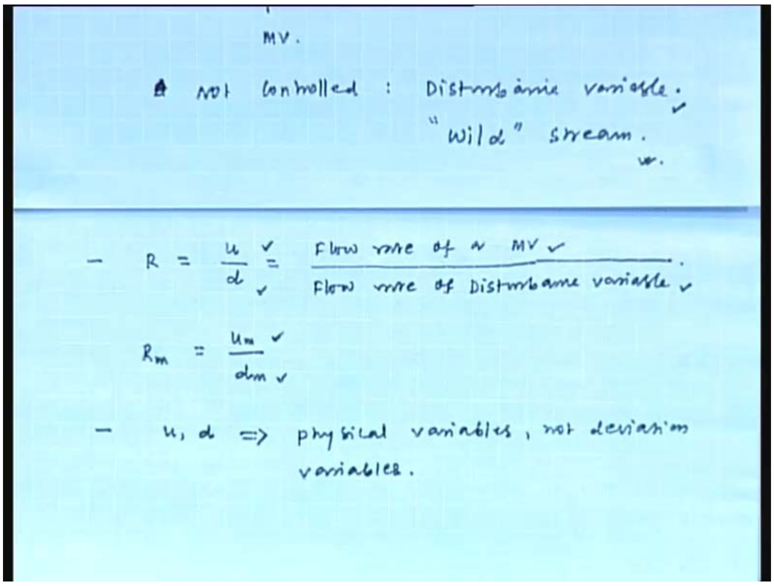

MV.

Not controlled : Disturbance variable. ✓

"Wild" stream. ✓

$$- \quad R = \frac{u}{d} \; = \; \frac{\text{Flow rate of a MV}}{\text{Flow rate of Disturbance variable}}.$$

$$R_m = \frac{u_m}{d_m}$$

\- $u, d \Rightarrow$ physical variables, not deviation variables.

And the ratio is the ratio denoted by R is equal to u divided by d u is the flow rate of the manipulated input flow rate of a manipulated variable and d is a flow rate of disturbance variable fine. Since the flow rate of manipulated variable and flow rate of disturbance variable the both are measured. So, sometimes they are written as u measured divided by d measured and accordingly the ratio is denoted by R suffix m since the manipulated input. And the disturbance variable both are measured quantity. Therefore, they are sometimes denoted by u suffix m and d suffix m. Accordingly the ratio R is denoted by R suffix m and another important thing is that both u and d are physical variable not deviation variables u and d. Both are physical variables not deviation variables fine. In the next, we will try to develop the ratio control configuration, in the next, we will develop we will discuss the ratio control configurations.

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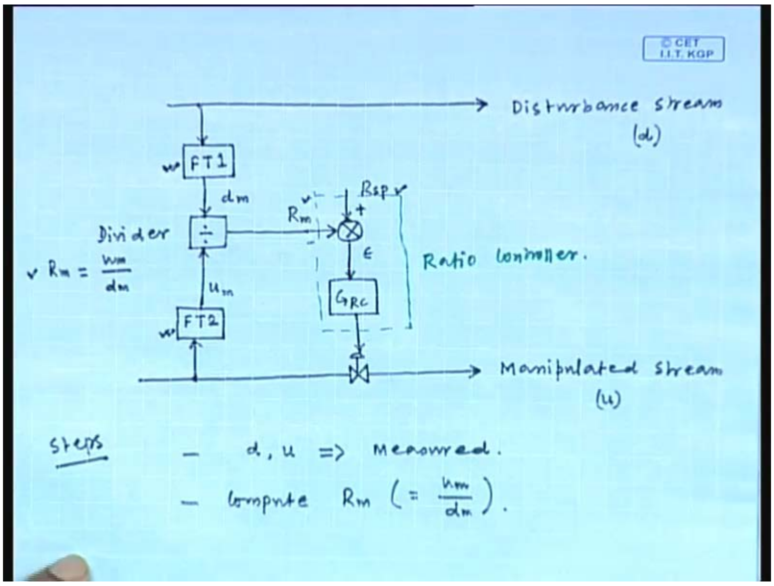

So, this is one stream say this is the disturbance stream fine and this is another stream which we can call as manipulated stream u and this is d now it is mention that both the flow rates are measured. So, we can employ one flow measuring device that is suppose F T one for measuring the flow rate of disturbance stream. Similarly, we can measure this manipulated variable using another flow measure in defines that is say F T 2. So, this is said d suffix m and this is suppose u suffix m it is mention that both the flow rates are measured accordingly. We can employ one flow measuring device for disturbance stream another flow measuring device for manipulated input. Now, in the next it is required to compute the ratio.

So, this is representing the divider this is the divider which is computing the ratio u m by d m in this divider r m is calculated using this expression u m by d m. So, the output of this divider is basically r suffix m measured ratio fine this measured ratio is compared with the set point value of reflux ratio R R s p is the set point value of ratio. So, the output of this comparator is basically the error signal output of the comparator is error represented by epsilon. And this output goes to the ratio controller G R c is the transfer function of the ratio controller. Then the output of this ratio controller is implemented through the control bulb fine. It is mention that both the flow rates are measured accordingly F T one can be employed to measured disturbance variable F T 2 can be employed to measure this manipulated variable.

Then this is the divider element in which the reflux the ratio r m is computed considering this expression u m by d m. So, output of this divider is r m which is compared by the set point value of this ratio r s p. Then this error signal goes to the controller G R c is the transfer function of the ratio controller and the controller action is use to control this manipulated input. So, this is a ratio controllers this is a ratio controller fine. So, I am repeating again the steps both the flow rates d and u are measured. Then compute the measured ratio compute the measured ratio r m which is equal to u m divided by d m fine. Now, the output of the divider element is sent to the ratio controller.

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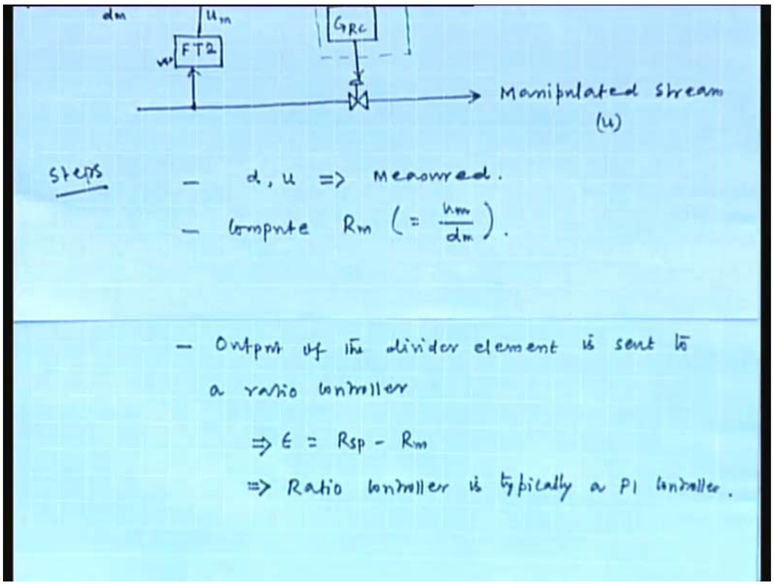

Steps

- d, u ⇒ Measured.
- Compute R_m $\left(= \frac{u_m}{d_m}\right)$.

- Output of the divider element is sent to a ratio controller

  $\Rightarrow \epsilon = R_{sp} - R_m$

  ⇒ Ratio controller is typically a PI controller.

Output of the divider element is sent to a ratio controller fine. In this ratio controller first the error is calculated, first the error is calculated by comparing R m with R s p fine and then the ratio controller takes action. Now, the ratio controller is takes typically a PI controller ratio controller is typically a PI controller fine. Now, at the beginning I have mention that the ratio controller is a special type of feed forward controller. Now, we are saying that the ratio controller is a PI controller which is feedback controller. So, what is the justification behind this? Actually in this ratio control scheme the disturbance is measured. Therefore, the ratio control is in essence is a special type of feed forward controller fine. In this ratio control scheme the disturbance is measured therefore, the ratio control is in essence is a special type of feed forward controller.

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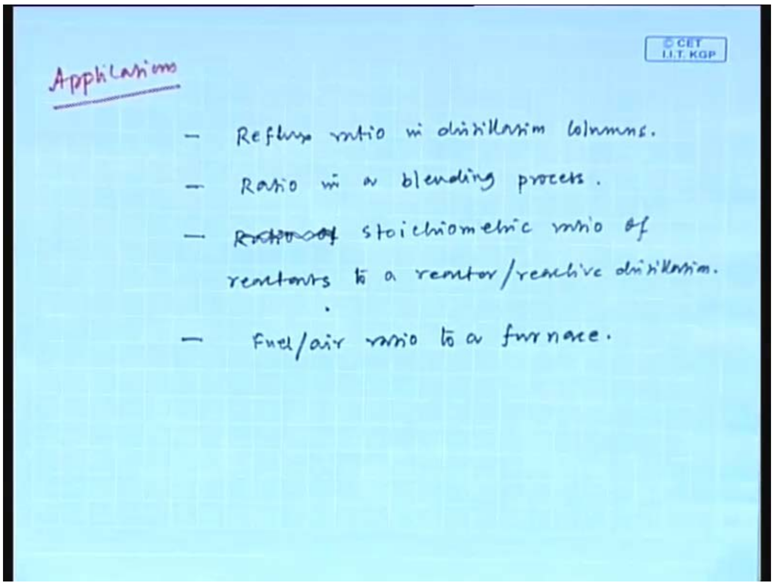

Applications

- Reflux ratio in distillation columns.
- Ratio in a blending process.
- ~~Ratio of~~ stoichiometric ratio of reactants to a reactor/reactive distillation.
- Fuel/air ratio to a furnace.

Next, we will discuss the application of this ratio controller. Can you give few examples where we can employ this ratio controller where basically the ratio is involved? Like one example is reflux ratio in this relation. So, to control the reflux ratio in distillation columns the ratio controller is used to maintain the reflux ratio in distillation columns. The ratio controller is used to control the ratio up to blended streams maintain the ratio in a blending process. The ratio controller can be employed to maintain the stoichiometric ratio of reactance in the reactor or reactive distillation. Maintain the ratio of stoichiometric to maintain the stoichiomatric ratio of reactance to a reactor or reactive distillation.

The ratio controller can be used if any reactant is in access that can come out as the product in the product and that degrades the product quality. Therefore, it is require to maintain the ratio stiochiomatric ratio of reactants which are fate to the reactor or reactive distillation to maintain the fuel air ratio to a furnace. The ratio controller can be employed to maintain the fuel air ratio to a furnace the ratio controller can be employed. So, these are the applications of ratio controller and ratio controller is extensively used in chemical engineering processes. So, today we discussed the derivation of design equation for the feed forward controller scheme. We discuss the feed forward controller with taking one example that is tank heater. And finally, we discussed a special type of feed forward controller that is ratio controller. Thank you.