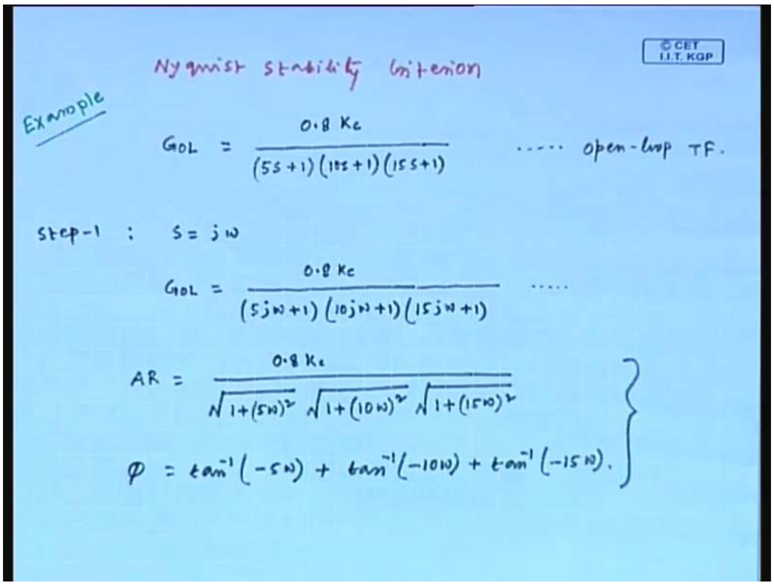
Process Control and Instrumentation
Prof. A. K. Jana
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 29
Feedback Control Schemes (Contd.)

So in the last class we discussed Nyquist Stability Criteria. So, today we will continue that topic with an example.

(Refer Slide Time: 01:02)

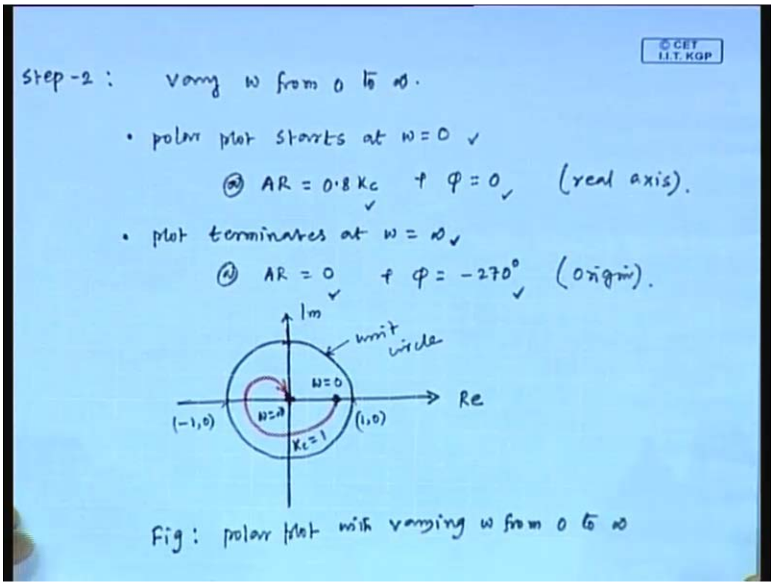


So, today we will continue the Nyquist Stability Criteria with an example. Now, the close look transfer function is given as 0.8Kc divided by 5s plus 1 multiplied by 10s plus 1 multiplied by 15s plus 1. The open look transfer function is given in this form open loop transfer function here, Kc is the proportional gain controller gain now, we will perform the stability analysis by developing the polar plot with varying omega from minus infinity to infinity.

So, in first step we will replace s by j omega. P Accordingly we get the frequency response transfer function as GOL equals 0.8 Kc divided by 5 j 5 omega plus 1 multiplied by 10j omega plus 1 multiplied by 15j omega plus 1 this is the frequency response transfer function.

Now, we can write the expression for Amplitude Ratio and phase angle 5 5. So, we can say that this is the I mean 3 First order systems are connected in series this indicates that the 3 First order systems are connected in series. So, it is straight forward to write the expression for Amplitude Ratio 0.8Kc divided by 1 plus 5 5 omega square multiplied by 1 plus 10 omega square multiplied by 1 plus 15 omega square. T his is the expression for Amplitude Ratio similarly we can write the equation for phase angle 5 that is tan inverse minus 5 omega plus tan inverse minus 10 omega plus tan inverse minus 15omega these are the 2 expressions 1 is for Amplitude Ratio and another is for 5.

(Refer Slide Time: 05:48)



Next we will go to the 2nd step. In this vary omega from 0 to infinity I have mentioned that we will develop the polar plot by varing omega minus infinity to infinity. So, that range we will divide into 2 ranges 1 is from minus infinity to 0 another 1 is from 0 to infinity. So, in step 2 we will vary omega from 0 to infinity under this we can say that the polar plot starts at omega equals to 0.

Now, at omega equals 0. What is the corresponding Amplitude Ratio that we can get from that expression which we have written when omega equals 0 then Amplitude Ratio becomes 0.8 Kc. Similarly if we can determine the values of phase angle. How much is the phase angle when omega equals 0? 0.

So, when omega equals 0 we can obtain Amplitude Ratio 0.8 Kc and 5 0 degree now, plot polar plot terminates at omega equals infinity when omega approaches infinity the polar plot ends. So, when omega equals infinity we obtain the corresponding Amplitude

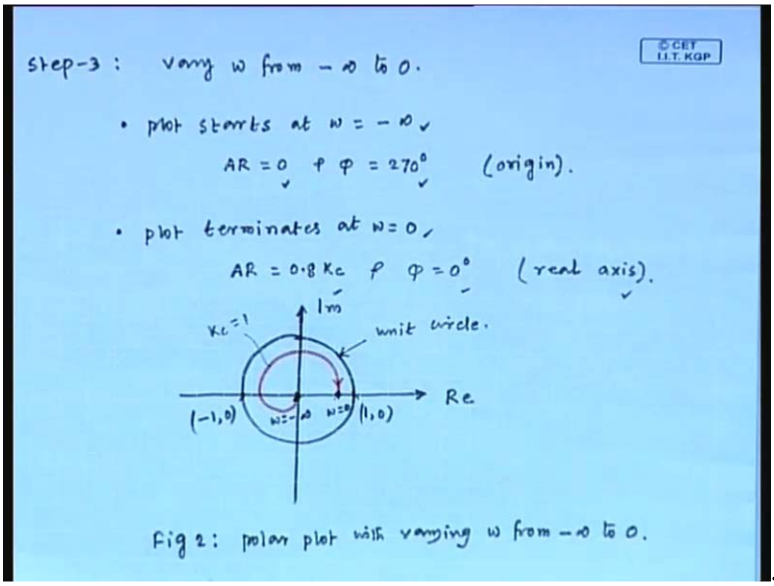
Ratio as 0. Similarly if we obtain 5 minus 270 degree when omega approaches infinity we obtain Amplitude Ratio is 0 and 5 minus 270 .

So, an Amplitude Ratio is 0.8 Kc and 5 0 the starting point on real axis you see 5 is 0. So the starting point will be on real axis similarly when Amplitude Ratio is 0 and 5 minus 270 the end point will be on in origin this indicates the origin agreed.

Now, we will draw the polar plot varying omega from 0 to infinity. So, this is the complex plane real axis imaginary axis this is the unit circle. Now, this point indicates 1, 0 and this point indicates minus 1, 0 and this is the origin now the plot starts at omega equals to 0 when Amplitude Ratio 0.8 Kc.

So, we will consider Kc values unity 1 that means, this will be 0.8 Amplitude Ratio is 0.8 if Kc is 1 and 5 is 0. So, we can locate the point say here this is the point starting point. So, this starting point we obtain and omega equals to 0 this is starting point which indicates Amplitude Ratio is 0.8 and 5 is 0 what about the end point? At end point Amplitude Ratio is 0 and 5 is minus 270 that means, this. So, if we join then we obtain the plot like this. So this is the polar plot and deduction is this, this is the deduction of this plot definitely we obtain polar plot considering Kc equal to 1 this the starting point at omega equals 0 this is the end point origin when omega equals infinity and this is the polar plot and Kc is equals 1. So, this is polar plot with varying omega from 0 to infinity.

(Refer Slide Time: 12:49)



In next step PI mean in step 3 varying omega from minus infinity to 0 in the third step we will vary omega from minus infinity to 0. Now plot starts at omega equals to minus infinity polar plot at omega equal to minus infinity. What will be the corresponding Amplitude Ratio? Corresponding Amplitude Ratio is 0. What about phi? Phi is 270 degree. So, at omega equal to minus infinity we obtain Amplitude Ratio as 0 and phase angle phi phi as plus 270 degree. So, what about point location of point starting point origin. So, location of point starting point is origin.

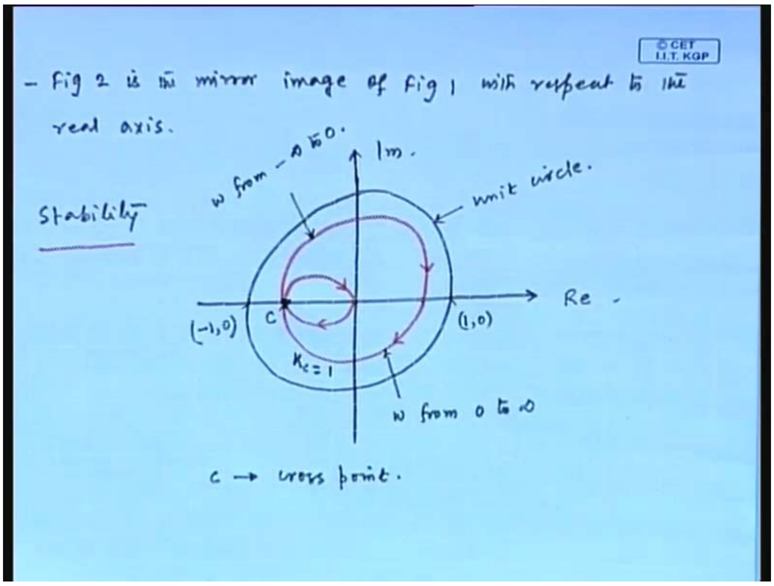
Similarly if plot terminates the plot terminates at omega equals to 0. What about the corresponding Amplitude Ratio? Amplitude Ratio is 0.8 Kc k c. What about phi 0 degree. So, when omega equal to 0 we obtain Amplitude Ratio as 0.8 Kc and phi 0 degree what about the location on the real axis the location of the end point should be on real axis.

Now, we will draw the polar plot for this case I mean from varying from varying omega from minus infinity to 0 this is real axis in complex plane this is imaginary axis similarly we need to draw the unit circle this is the unit circle. So, this is 1, 0 and this point is minus 1, 0 and this is the origin. So, plot starts at origin and Amplitude Ratio 0 and phi is minus 270 and plot ends when Amplitude Ratio is 0.8 Kc and phi 0.

So, for this case in this case we will consider Kc 1. So this is the starting point and this is the end point starting point we obtain considering omega minus infinity end point we obtain considering omega 0. So, the polar plot is obtained as this is the deduction clock wise. So, this plot we obtain when Kc equal to 1 this is suppose figure 2 polar plot with varying omega from minus infinity to 0 this is figure 2 which is basically the polar plot with varying omega from minus infinity to infinity and previously we obtain polar plot with varying omega from 0 to infinity.

Suppose this is figure 1 in figure 1 obtain polar plot with varying omega from 0 to infinity and in figure 2 we obtain polar plot with varying omega from minus infinity to 0. Now, you see figure 2 is the mirror image of figure 1 it can be observed that this figure 2 is mirror image of the figure 1 with respect to the real axis.

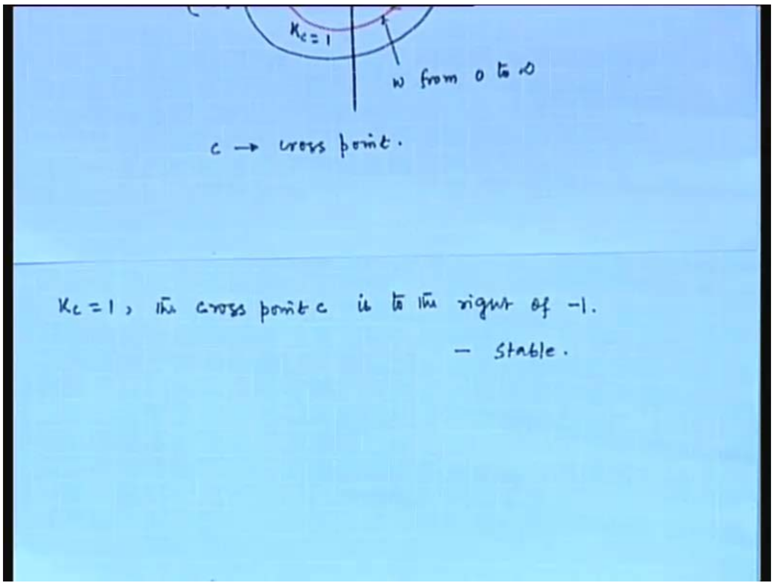
(Refer Slide Time: 19:10)



Previously it was mentioned that we will produce the polar plot by varying omega from minus infinity to infinity, but, we have drawn 2 plots in 1 case we have varied from 0 to infinity and in another case we have varied from minus infinity to 0. Now, we will combine this 2 plots. So, in the next step we perform Stability analysis and for that we will combine 2 figures I mean figure 1 and figure 2. So, that we can show the polar plot with varying omega from minus infinity to infinity. So, this is complex plane this is real axis this is imaginary axis this is the unit circle this point indicates 1, 0 and this point indicates minus 1, 0. So, we will consider Kc equal to 1 then when we varied omega from 0 to infinity then we obtain this plot you can say this is figure 1.

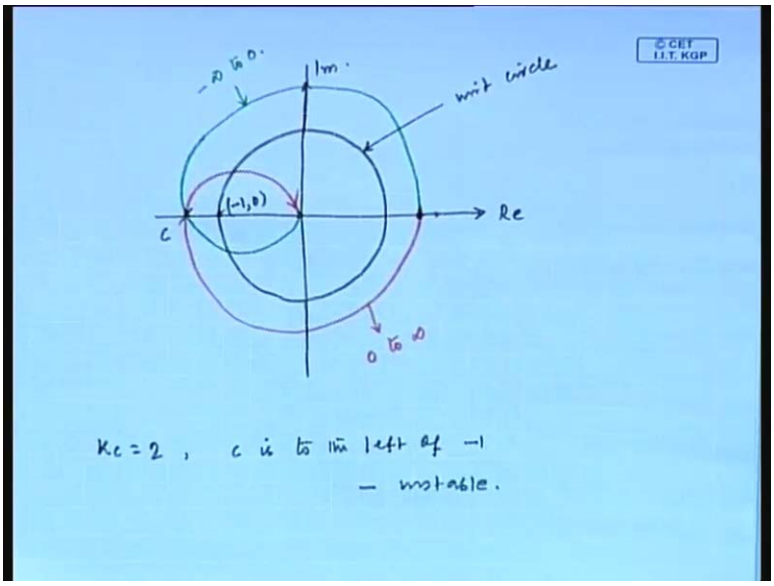
Similarly if as we varied omega from minus infinity to 0 we obtain this plot agree. So, this plot we obtained by varying omega from 0 to infinity and this plot we obtain by varying omega from minus infinity to 0 and this polar plot corresponds Kc equal to 1 now, this point is called cross point the point C is called the cross point C is called the cross point. Now, we will note down some important points on stability. You see when Kc equals to 1 the cross point is to the right of minus 1 Kc equals to 1 the cross point is to the right of minus 1 it indicates stability and when the cross point is to left of the minus 1 that indicates in stability I mean the system is unstable .

(Refer Slide Time: 24:23)



So, when Kc if equal to 1 the cross point C is to right of minus 1 that means, the system is stable we can define it in another way you see the plot corresponding to Kc equal to 1 does not encircle minus 1, 0 therefore, the system with Kc equal to 1 is stable the polar plot corresponding to Kc equal to 1 does not encircle minus 1, 0 that means, the system is stable. If, we consider Kc equals to 2 what happens?

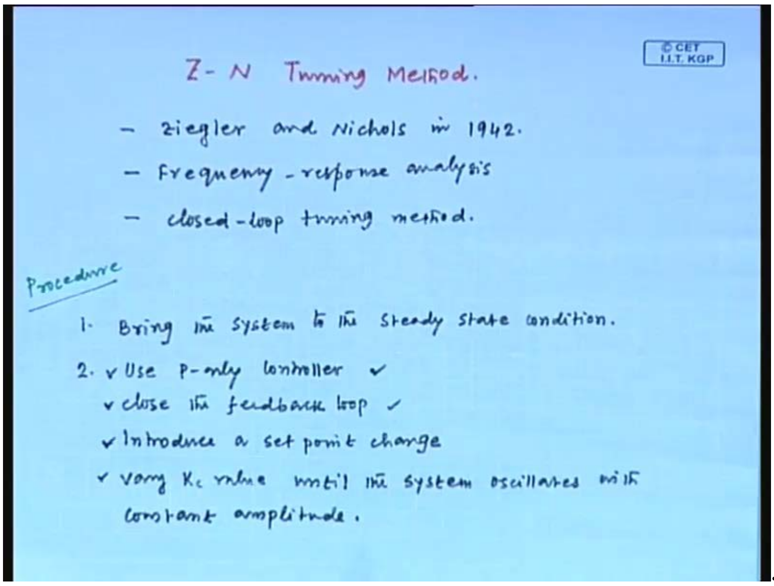
(Refer Slide Time: 25:49)



Suppose this is the unit circle, unit circle in complex plane. If we consider Kc equals to 2 then this becomes 0.8 I mean 0.8 multiplied by 2 is 1.6 . So, it will be like this the polar plot will be if suppose this is indicating 1.6 this is the polar plot by varying omega from 0 to infinity and this the polar plot by varying omega from minus infinity to 0. This the polar plot when varying omega from 0 to infinity and this the polar plot when we vary omega from minus infinity to 0. Which 1 is the cross point? This point and this is the minus 1, 0 point you see when Kc is equals to 0 the cross point C is to the left of minus 1.

So, when Kc equals to 2 the cross point C is to the left of minus 1. So, we say that the system is unstable by this way we can apply the Nyquist plot for stability analysis and this criteria can be used for tuning the controllers. So, this is all about Nyquist plot next we will discuss another Tuning Method that is Zn method Ziegler Nicolas technique.

(Refer Slide Time: 28:58)



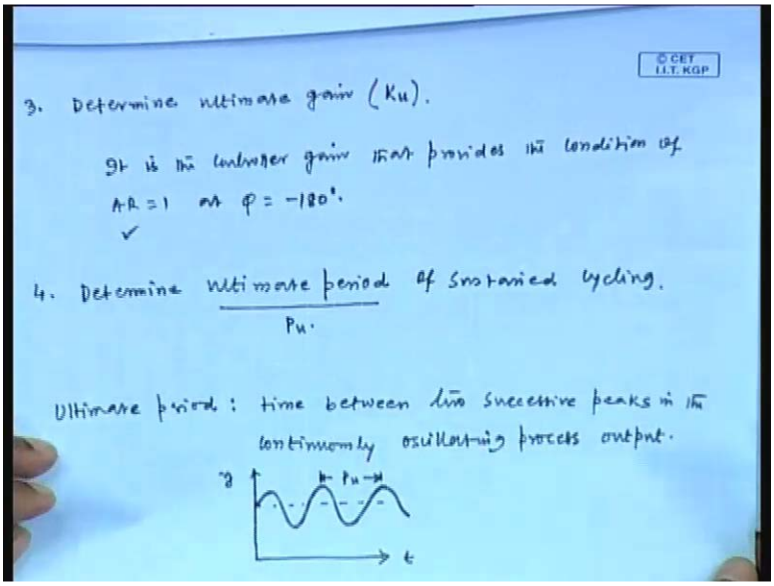
In the next we will discuss another Tuning Method that is Zn tuning method Ziegler Nicolas Tuning Method. This was proposed by Ziegler and Nicolas in 1942. This technique is also based on Frequency response technique analysis and this technique is known as closed loop tuning method. This technique can also used online I mean when the process is in the operation we can use this technique for tuning purpose therefore, this technique is also sometimes called online tuning method. So, this technique is called closed loop as well as online tuning method.

So, in the next we will discuss how we can use this technique for tuning purpose? We will know the several sequences states for applying this technique for tuning purpose. So, we will discuss the step wise procedure at first you bring this process to the steady state condition. In first set we need to bring the system to the steady state condition.

So, bring the system to the steady state condition this is the first step. Secondly, use the P only controller you cannot use PI PPI or PID . So, first you use P only controller at the same time you close the loop. So, close the feedback loop apply the Ponly controller and close the feedback loop. Then, introduce a set point change. So, first we need to use we need employ Ponly controller not PPI or PPID at the same time you close the feedback loop P then introduce a set point change then we can tune the controller I mean we will vary the Kc values. So, that we can obtain sustain oscillation.

So, next keep on varying Kc value. So, I am writing vary Kc value which is proportional gain until the system oscillates with constant amplitude. So, under this second step use P only controller at the same time close the feedback loop then introduce a set point change then vary Kc values. So, that we can obtain sustain oscillations.

(Refer Slide Time: 35:19)

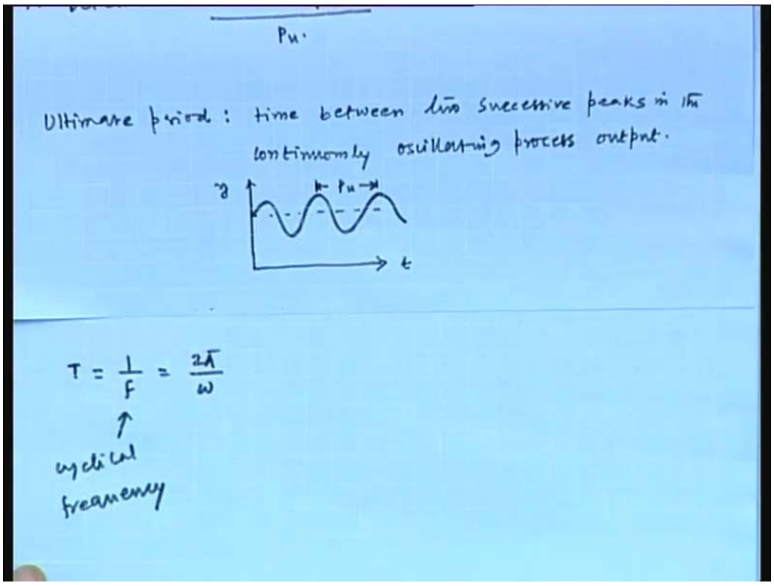


This is step 2 in the procedure next Determine ultimate gain. You see last point under second step PPI we have mentioned that vary Kc value. So, that we can obtain sustain oscillations that Kc values is basically ultimate gain. So, ultimate gain means the process gain that provides the conditions of Amplitude Ratio equal to 1at phi equals to minus 180 degree. We can say that the values of Kc at which we obtain sustain oscillations and this is represented by Kc suffix u Ku is the values of Kc at which we obtain sustain oscillations and this corresponds to the condition of Amplitude Ratio equal to 1 at phi equal to minus 180 degree.

In the next step we need to determine the ultimate period in 4 steps. We need to determine the ultimate period of sustained cycling and this ultimate period is represented by suffix u. What is ultimate period? Ultimate period is time between two successive peaks in continuously oscillating process output continuously it is somewhat like this.

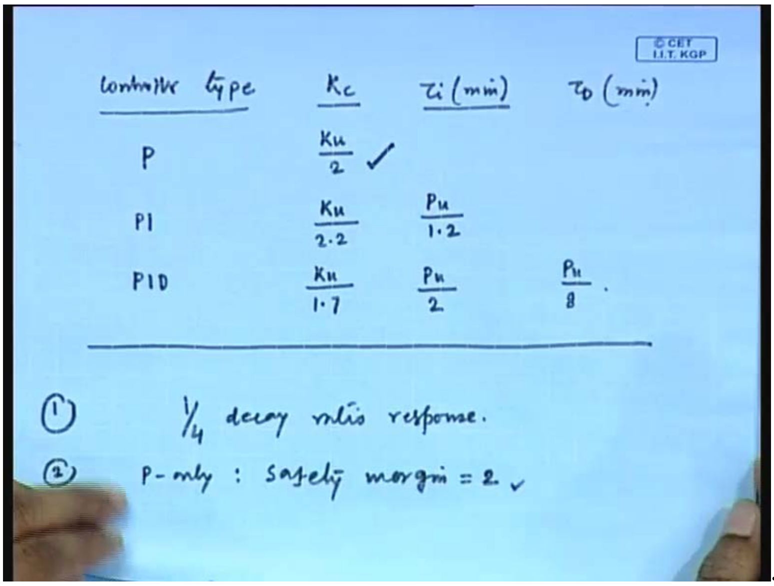
Suppose, this is y this is t it is continuously oscillating like this. So, this is the ultimate period Pu you see this is the continuous oscillation. So, the time between 2 successive peaks that is Pu now, we can determine this mathematically. What is period of oscillations which we represented earlier by t? period of oscillations t equal to1 byF. F is the cyclical frequency this is equal to 2 phi by omega basically omega is 2 phi A by omega.

(Refer Slide Time: 39:52)



If, we consider sustain oscillation that means, what omega equal to omega co cross over frequency therefore, we can write this Pu equal to 2 phi divided by omega co. So, this is the ultimate period of sustained cycling. So, in step 3 we knew the calculation of ultimate gain and in the next step we discussed the determination of ultimate period now, knowing the values of Ku and Pu we need to use some correlations recommended by Ziegler and Nicolas.

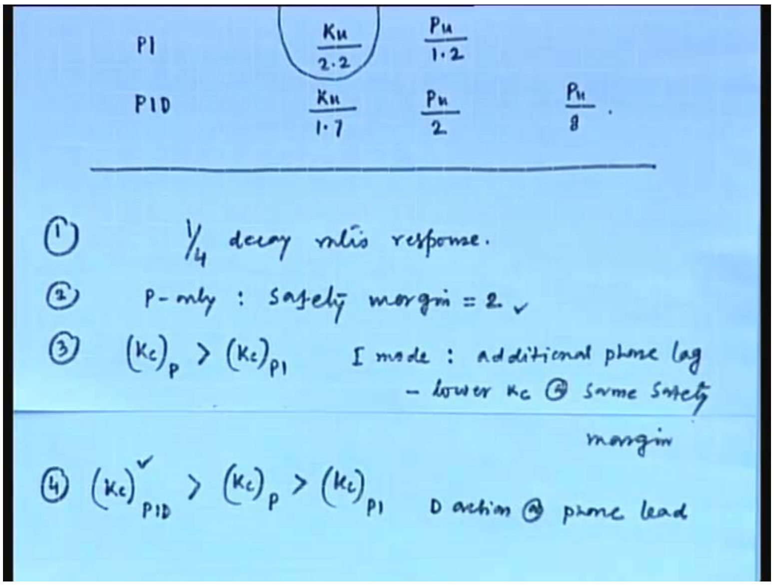
(Refer Slide Time: 41:50)



So, we will know the different correlations which have been proposed by Ziegler and Nicolas controller type proportional gain Kc time constant tow y in minute derivative tow d in minute. For P only controller Ziegler and Nicolas proposed the expression for Kc which is equal to Ku divided by 2 for PI controller the correlations is Ku divided by 2.2 for Kc Pu divided by 1.2 for tow I. Next is PID controller for PID controller Kc is Ku divided by 1.7 tow I equals Pu divided by 2 and tow d is Pu divided by 8 you see all the correlations depend on Ku and Pu. So, knowing the values of ultimate gain Ku and ultimate period Pu we can use this correlations recommended by Ziegler and Nicolas for finding the controller parameter values.

Now, important points we need to note down now. First point is the tuning relations are determined empirically to provide 1 quarter decay ratio response this correlations are determined to provide 1 quarter decay ratio response this is the first point this correlations are have been determined to provide 1 quarter decay ratio response. I mean decay ration should be 1 by 4. Secondly, for the P only controller the safety margin is 2 P because, Kc is equal to half of the stability limit. You see here because Kc is equal to half of the stability limit that is Ku divided by 2 therefore, the safety margin for P only controller is 2.

(Refer Slide Time: 45:36)

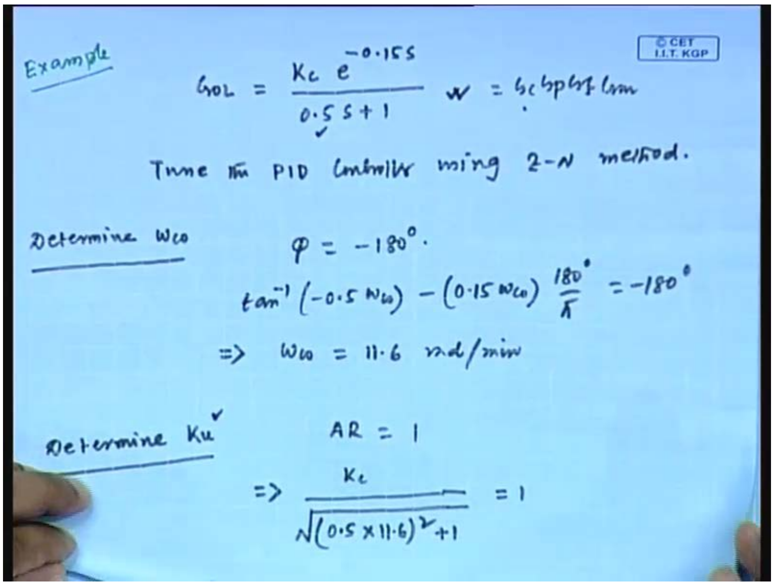


Next if you see the correlations we observe that the proportional gain of P only controller is greater than proportional gain of PI controller if you see the correlations we observe that the Kc of the P only controller is higher than the Kc of PI. What is the reason? Because integral mode introduces additional phase leg with destabilizing effect on the system.

Another important point is under this lower Kc maintains approximately the same safety margins, in the last point we will discuss we will compare the Kc values of all 3 controllers you see the Kc values of PID controller is greater than Kc of the P only that is again greater than Kc of PI.

So, the Kc values of PID controller is the largest 1 because the derivative action introduces the phase lead with stabilizing effect on the system for PID controller we consider large Kc values because derivative action introduces additional phase lead you write phase lead derivative action introduces phase lead with stabilizing effect on the system. So, these are the steps related to Ziegler and Nicolas technique next we will solve 1 problem in which will apply this Ziegler and Nicolas technique for finding the tuning parameters values.

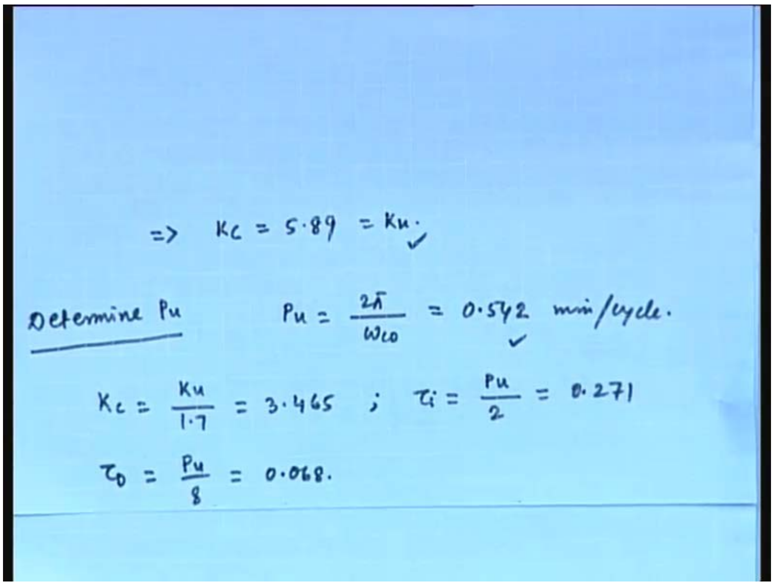
(Refer Slide Time: 49:03)



Example open loop transfer function is given as k c exponential of minus 0.15s divided y 0.5s plus 1 open loop transfer function is given as Kc exponential of minus 0.15s divided y 0.5s plus 1 here, time constant is 0.5 and dead time is 0.15. point. Now, tune the PID controller using Zn technique. So, first you determine cross over frequency see in this open loop transfer function we have considered P controller only. So, this basically Gc GpGf Gm know here we considered Gc equals to Kc.

First you determine cross over frequency for determining cross over frequency we write phi equals minus 180 degree What is the phi expression for open loop transfer function given? Tan inverse minus 0.5 omega co 0.15 omega co multiplied by 180 divided by phi that is equals minus 180 degree. If, you solve we will obtain cross over frequency as 11.6 radian per minute this is the values for cross over frequency. Then, determine ultimate gain we can obtain ultimate gain if we consider Amplitude Ratio equal to 1 at cross over frequency Ku is the values of Kc at which we obtain sustained oscillation and that corresponds to the condition of Amplitude Ratio equal to 1 at cross over frequency. So, What is the expression for Amplitude Ratio? Kc divided by 0.5 multiplied by 11.6 whole square plus 1 equal to 1. This is the expression for Amplitude Ratio and that is equal to 1.

(Refer Slide Time: 53:30)



So, if we solve it we obtain Kc values equal 5.89 and this is nothing, but ultimate period ultimate gain Ku. Next determine ultimate period. What is the expression for Pu? Pu is 2 phi divided by omega co that is equals to 0.542 min per cycle. So, ultimate period is ultimate gain is 5.89 ultimate period is 0.542 now you can easily use the correlations to determine the 3 tuning parameter values for PPID . So, for PPID controller Kc is Ku divided by 1.7 using the value of Ku we obtain 3.465 this is Kc integral time constant has the form Pu divided by 2 using the value of Pu we obtain 0.271.

Now, derivative time is Pu divided by 8 and this is equal to 0.068. So, by this way we can tune P only and PPI controller also. So, today we discussed the Nyqumist Stability Criteria we can example then we discussed the theory involved Ziegler Nicolas technique then we discussed 1 example to apply the Zn technique.