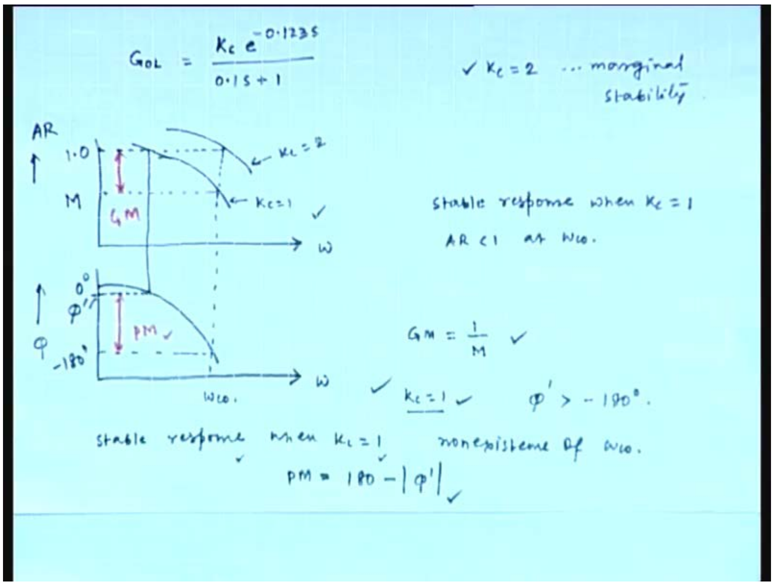
Process Control and Instrumentation

Prof. A.K. Jana
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 28
Feedback Control Schemes (Contd.)

In the last class we discussed the topic gain margin and phase margin, we will continue that topic.

(Refer Slide Time: 00:58)



Gain Margin and phase Margin. And we considered the example having open loop transfer function, equal to Kc exponential minus 0.123 is divided by 0.1s plus.. Now, when K c if equal to 2 we obtain marginal stability, this is the marginal stability condition. We developed in the last class the bode plot like this. This is omega this is phi and this is a semi log graph paper. Suppose this is the phi verses omega plot in semi log graph paper. Now, cross over frequency we obtain considering phi equals minus 180 degree, So, this is cross over frequency.

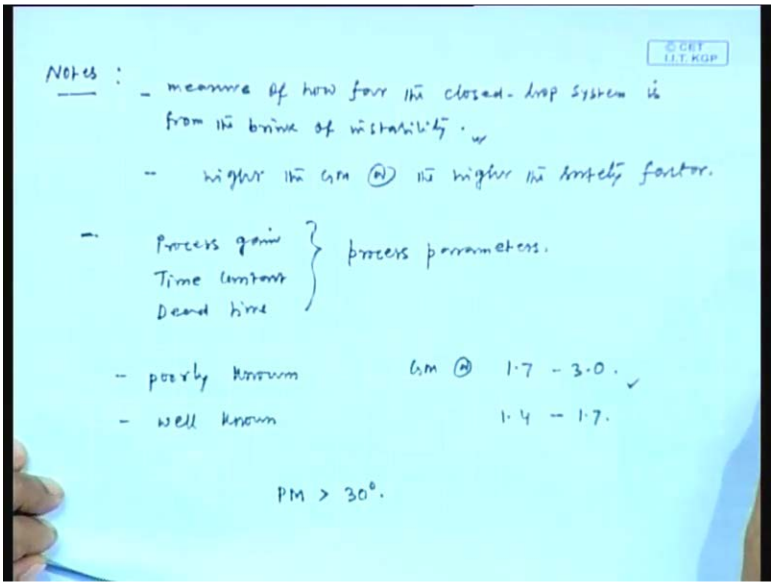
Now, above this we consider amplitude ratio verses omega and this plot is produced in a log graph paper. Suppose this is amplitude ratio verses omega plot when Kc equal to 1. So, at cross over frequency the amplitude ratio we obtain suppose that is M. Now, as mentioned the marginal stability corresponds to Kc equal to 2. Suppose this is a amplitude ratio verses omega plot, when Kc equal to 2.

That means at cross over frequency, we obtain the amplitude ratio equal to 1.So, this is for Kcc equals 1 so, we can say that we obtain stable response and Kc equal to 1 because at cross over frequency amplitude ratio is less than 1. So, we obtain stable response when Kcc equal to 1. Because, amplitude ratio is less than 1 at cross over frequency. Now, this difference is called as gain margin. So, it is obvious that it is a safety factor. Now, How can we represent the gain margin mathematically? Gain margin we can represent as 1 divided by M, difference is log 1 minus log M. So, we can write gain margin as 1 divided by M..

Now, when amplitude ratio equals 1 suppose this is I am just extending this, like this when amplitude ratio equals 1. How much is the corresponding phi? See we are considering Kcc 1. Previously also we considered Kcc 1 and we obtained amplitude ratio M. Similarly here, we are considering Kcc equals 1. And the corresponding value of phi is this, quantity suppose this is, phi prime this 1. So, when amplitude ratio equals 1. The corresponding phi, we are indicating by phi prime agree. And this phi prime is greater than you see minus 180 degree agree.

So, we can say that we obtain stable response when Kcc equals 1 because of the non-existence of cross over frequency. We obtain stable response when Kcc equals 1 because of the non-existence of cross over frequency. See this observation is based on this plot, and the previous observation was based on this plot. And this difference is called phase margin. You see this, phi prime is higher than 180. So, similarly we can say that this phase margin is also a safety factor, and this phase margin is represented as 180 minus, the absolute value of phi prime. The phase margin we can represent or we can calculate by this correlation. So, these are 2 safety factors and based on the value of gain margin and phase margin we can tune the control parameters.

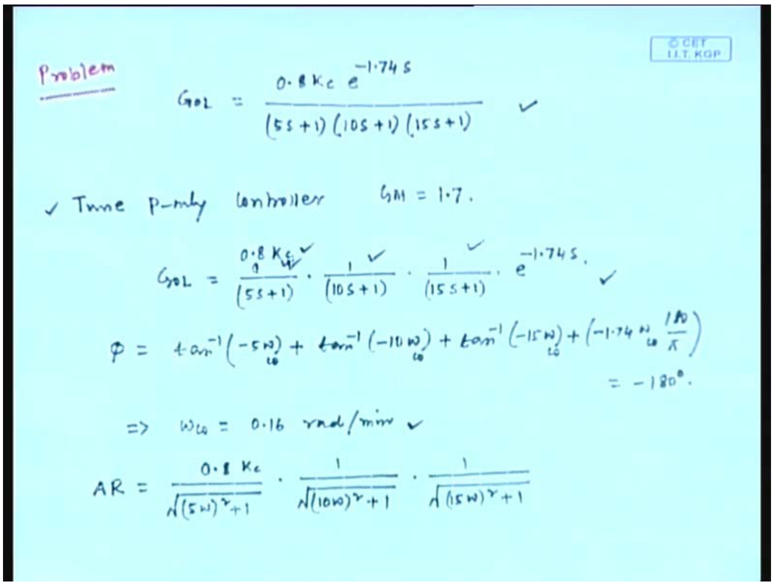
(Refer Slide Time: 09:43)



Now, next we will note down few points. So, Gain Margin is a measure of how for the close loop process is from the brink of instability. This is also true, for phase margin. So, they are the measure of how for the closed loop system is from the brink of instability. Now, the higher the gain margin implies the higher the safety factor. Now, question is. How we can select the values of gain margin and phase margin? First we will discuss the selection of gain margin, then we will discuss the selection of phase margin. The heuristics is that, when the process parameter values are poorly known…First we have to know the process parameters.

Process parameters mean process gain, time constant, and dead time etcetera. These are the process parameters, so, process gain is 1 parameter. Then time constant then dead time and so on. These are the process parameters, when these parameters are poorly known imprecisely known the gain margin is taken in between 1.7 to 3.This is the heuristics. When there well known, they are known quite accurately in that case the gain margin is taken in between 1.4 and 1.7. These are the safety limits. And for phase margin it is usually taken as a greater than 30 degree phase margin is usually taken larger than 30 degree. So, these are the heuristics for selecting the gain margin and phase margin values.

(Refer Slide Time: 14:08)



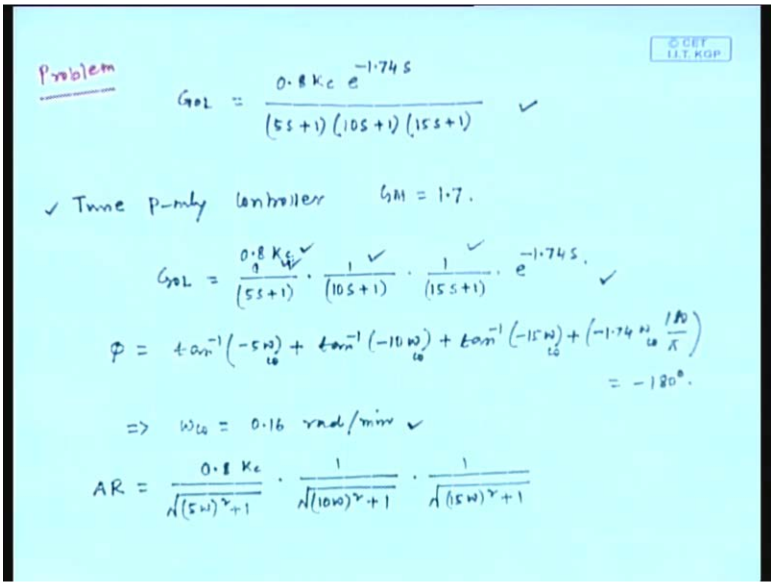
In the next you will solve one problem, based on this phase margin and gain margin concept. The open loop transfer function is given has 0 point Kc exponential of minus 1.74 s, divided by 5 s plus 1, 10 s plus 1, 15 s plus 1. So, this is basically the transfer function of a third order plus dead time system. And only P controller is employed for the system. So, this is the open loop transfer function of the example system. Now, we want to tune the controller p only controller, tune P-only controller. Taking gain margin equal to 1.7 tune the P-only controller I mean find the Kc value considering gain margin equal to 1.7. Now, the open loop transfer function we can write as 1 divided by 5 s plus 1, 1 divided by 10 s plus 1, 1 divided by 15 s plus 1.

So, first order systems connected in series next part is the dead time part. We can rewrite the open loop transfer function in this form. Now, in the first step we need to calculate the cross over frequency, for finding the cross over frequency we need to write the expression for phi and we need to consider phi equals minus 180 degree.

So, what will be the expression for phi tan inverse minus tau p omega is the expression for phi for First order system. So, for this First order system tan inverse minus 5 omega for the second First order system tan inverse minus 10 omega, and for this First order system tan inverse minus 15 omega, and for the dead time part minus 1.74 omega multiplied by 180 divided by phi. And we can write this is equal to, minus 180 degree agree. See when we write phi equals minus 180 degree then, that omega is crossed over frequency

So, we can include the suffix co omega co. Here also omega co everywhere omega co. Now, solving this, we obtain the cross over frequency 0.16 radian per minute. Here, time is in minute solving this equation we obtain cross over frequency 0.16 radian per minute. Now, we need to write in the next the expression for amplitude ratio. So, for the First order system I mean for this system for this transfer function, what would be the amplitude ratio 5 omega whole square plus 1. Now, one thing we missed in this open loop transfer function here that is 0.8 Kc we missed this numerator term 0.8 Kc. So, we can write here that term 0.8 Kc. For the next transfer function 10 omega whole square plus 1 for Third 1 we can write 1 divided by 15 omega whole square plus 1.

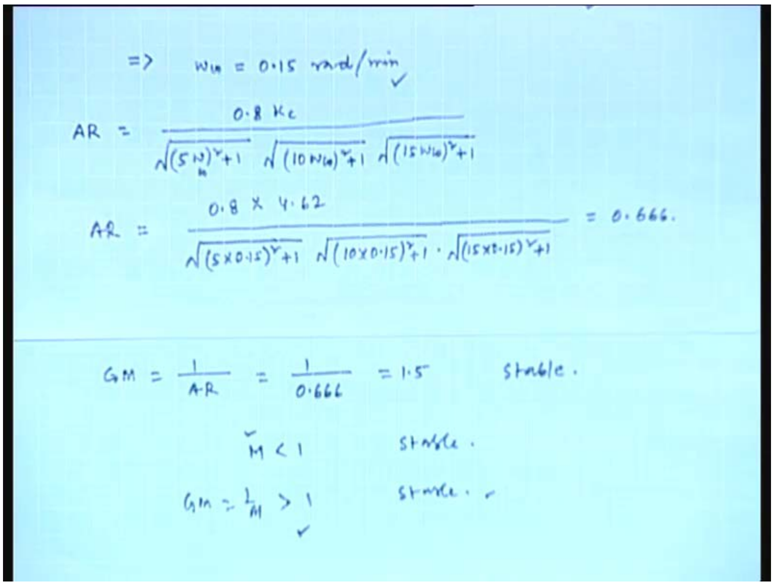
(Refer Slide Time: 20:41)



Now, we will substitute omega equals omega co then, we can write the expression for amplitude ratio as 0.8 Kc divided by 5 into 0.16 whole square plus 1, 10 multiplied by 0.16 whole square plus 1, 15 multiplied by 0.16 whole square plus 1. This is equal to 0.8 Kc divided by the first term yields 1.28 next term gives 1.89 and third one gives 2.6 which is equal to 0.127 Kc. So, at cross over frequency we obtain amplitude ratio this much, suppose this is equal to M now gain margin is given 1.7 this is given. In the problem gain margin of 1.7 is given. Now, how much is gain margin 1 by M this is gain margin; that means, we can write 1 divided by 0.127 Kc equal to 1.7. Then how much is Kc, Kc becomes 4.62.

So, for the example system if, we consider k c equal to 4.62 then amplitude ratio at cross over frequency is lower than 1 . So, this is a answer for the 1st part. Now, in this example see we calculated this Kc value considering the dead time 1.74 suppose, that value is wrong the actual dead time value is suppose 2.4 minute.

(Refer Slide Time: 24:01)

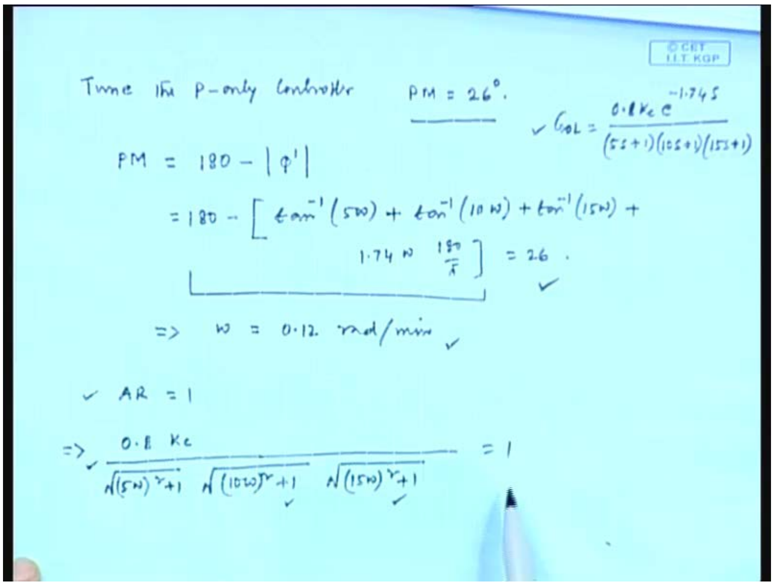


True value of dead time is 2.42 minute instead of 1.74 minute. What about stability if, we use Kc equals 4.69. Now, question is what about, stability if we use Kc equals 4.62 and original td is 2.42 minute. This is a second question we found Kc value 4.62 based on the wrong dead time value. Now, question is, What about stability if we use Kc equals 4.62 and the original dead time is 2.42 minute? Now, in this situation 1st we will found the cross over frequency, considering this dead time.

So, again we need to write the phi expression tan inverse minus 5 omega C naught, plus tan inverse minus 10 omega C naught plus tan inverse minus 15 omega C naught minus of 2.42 omega C naught multiplied by 180 divided by phi, and this is equal to minus 180 degree. In this phi expression we have changed only this td value. Solving this we obtain cross over frequency 0.15 radian per minute. Next we will calculate the gain margin our aim is to check the gain margin value. So, further we need to determine the amplitude ratio amplitude ratio expression we know 0.8 Kc divided by root over of 5 omega C naught whole square plus 1, 10 omega C naught whole square plus 1, 15 omega C naught whole square plus 1 f 0.8 Kc we determined 4.62 and we are still using that, 4.62 value divided by 5 multiplied by 0.15 square plus 1, 10 multiplied by 0.15 whole square plus 1. Next is 15, 0.15 whole square plus 1, and we obtain this is equal to 0.666 this is the amplitude ratio value considering td equals 2.42.

Then how much is gain margin? 1 divided by amplitude ratio I mean, 1 divided by M 0.666 and this is equal to 0.5. What about stability? This is stable, because when M is less than 1. We say that the system is stable. So, gain margin is basically 1 by M. So, if 1 by M is greater than 1 than, the system is stable when M is less than 1 definitely gain margin should be greater than 1. So, this is the condition for stability if, gain margin is greater then 1 than we can say that the system is stable. So, the problem we have solved based on the gain margin value. We will continue this problem considering the phase margin of 26 degree.

(Refer Slide Time: 30:35)



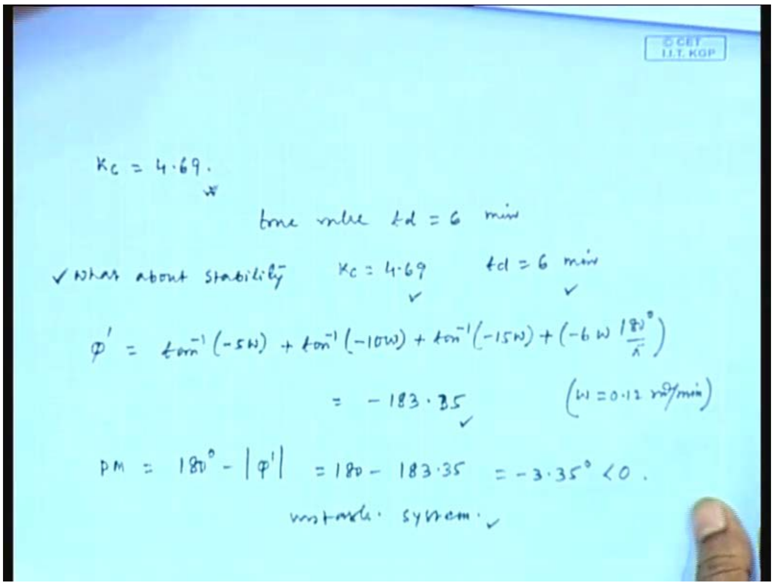
So, next question is, tune the P-only controller, when the phase margin is 26 degree find the Kc value when this phase margin is 26 degree. Now, we know the expression for phase margin. Phase margin we express as 180 minus phi prime this is the expiration for phase margin 180 minus phi prime 180 minus phi prime. So, what is the expiration for phi? That we will write here tan inverse phi omega, absolute term we are writing I mean this is absolute value no, plus tan inverse 10 omega plus tan inverse 15 omega plus 1.74

omega 180 divided by phi equal to 26. And this we have return based on this transfer function Gol equals 0.8 Kc exponential minus 1.74 s divided by 5 s plus 1, 10 s plus 1, 15 s plus 1.

We are solving we are solving this problem considering this open loop transfer function I mean, using this open loop transfer function fine the value of Kc when phase margin is taken as 26 degree. Now, if we solved this, tan we obtain omega as 0.12 radian per minute, If, we considered this part equal to 26 degree and if, we solved then we have obtained omega equals 0.12 radian per minute. Now, amplitude ratio equals 1 basically phi prim is the phi value at amplitude ratio is equals 1.

So, we will considered this here now what is the expiration for amplitude ratio, 0.8 Kc divided by root over of 5 omega whole squire plus 1 10 omega whole squire plus 1, 15 omega whole squire plus 1 and this is equal to 1 this is the expiration for amplitude ratio 0 point 8 Kc divided by 5 omega hole squire plus 1 root over of tan omega whole square plus 1 root over of 15 omega whole squire plus 1 and that is equal to 1.

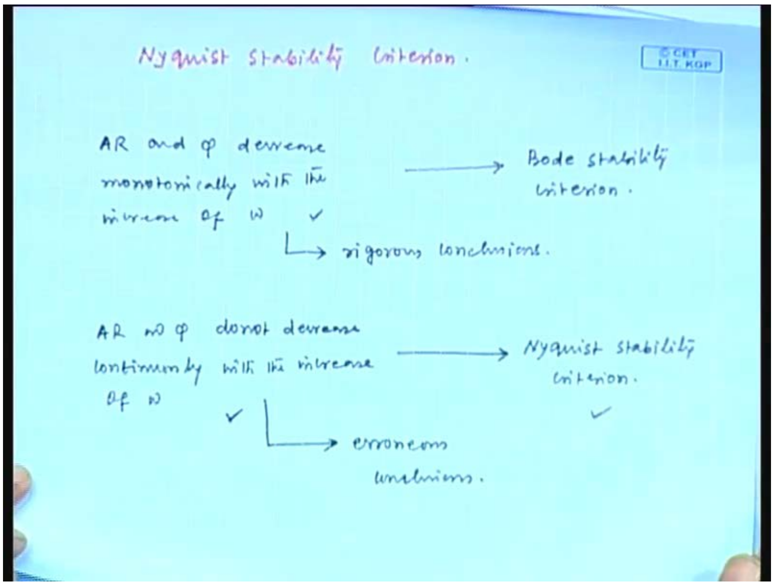
(Refer Slide Time: 35:16)



Now, if we solve this, we obtain Kc equal to 4.69 here, only if Kc is unknown if, we substitute omega equal to 0.12. So, finally, we obtain k c equal to 4.69. So, previously we obtain Kc equal to 4.62, considering gain margin 1.7. Now, here were getting Kc equal to 4.69 considering phase margin 1.26 degree. Now next question is the dead time we have taken wrongly. So, the original dead time is the true value of dead time d is suppose 6 minute, instead of 1.74 minute. Now question is. What about stability? If, we use Kc equal to 4.69 when the actual dead time is 6 minute. .

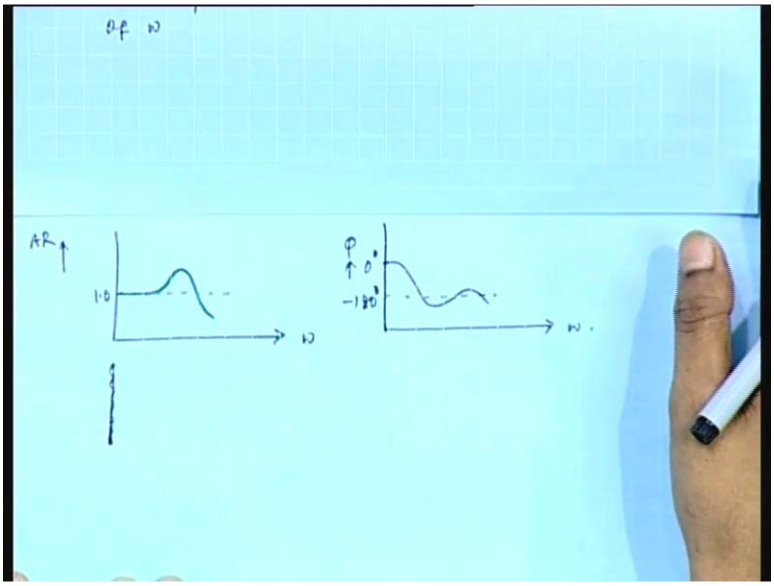
First we need to calculate phi prime, expiration is 10 inverse minus phi omega tan inverse minus 10 omega than 10 inverse minus 15 omega, dead time is minus 6. So, minus 6 omega 180 degree divided by phi. This is the expiration for phi, and considering omega equals 0.1 to radian per minute. We obtain this as 183.35 substituting omega equals 0.12 radian per minute we obtain the phi prime as minus 183.35. What about phase margin then? 180 degree minus absolute value of phi prime so, 180 minus 183.35 minus 3.35 degree which is less than 0z. So, what about stability? Unstable. So, this is unstable problem I mean unstable system. If the dead time is 6 minute and if, we use Kc equals 4.69 than the system becomes unstable fine. So, this is the problem related to this, gain margin and phase margin. So, first we if constructed the bode plot then, by the use of the bode plot we discussed the bode stability criteria under which we determined the control parameter values. Similarly we discussed the construction of Nyquist plot are polar plot.

(Refer Slide Time: 40:14)



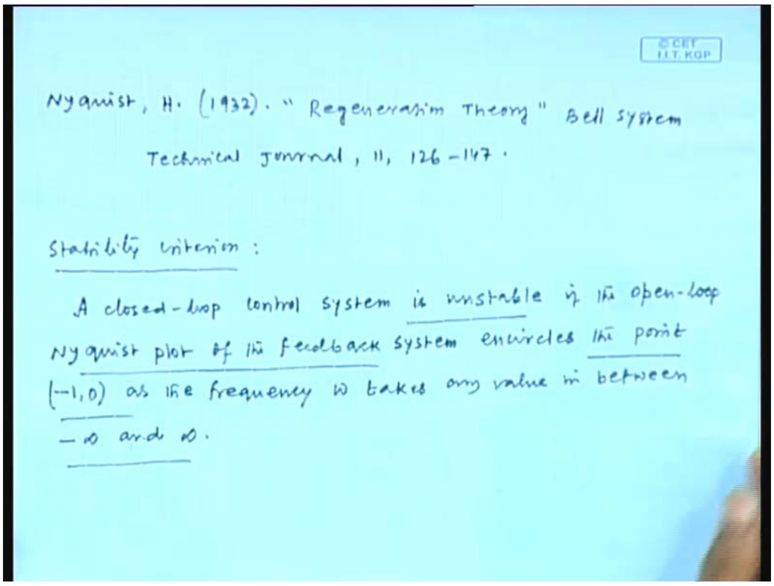
So, now we will use that plot for stability analysis. So, next topic is Nyquist stability criteria. So, we discussed a number of examples, under this frequency response analysis. And we observe that, with the increase of omega both amplitude ratio and phi decrease. So, far we have discussed the frequency response analysis taking a number of examples, and it is observed that with the increase of omega both amplitude ratio and phi decrease, this is our observation amplitude ratio and phi decrease monotonically with the increase of omega. If, this is the situation then, it is recommended to use the bode stability criteria If, this is the situation the bode stability criteria leads to rigorous conclusions. But, there are few systems for which amplitude ratio and phi do not decrease continuously with the increase of omega. There are very few systems in chemical engineering for which the amplitude ratio and phi do not decrease monotonically with the increase of omega.

(Refer Slide Time: 43:29)



And the plot looks somewhat like this. This is suppose amplitude ratio verses omega plot, in log graph paper here, suppose the value is 1. So, it goes like this , similarly the phi verses omega plot not the corresponding plot I, want to say for few systems the phi verses omega plot is somewhat like this. So, it is very clear that the amplitude ratio and phi do not decrease monotonically with the increase of omega. And in this situation it is not recommended to use bode stability criteria. And it is recommended to use Nyquist stability criteria which to be discussed in the next. In this situation the bode plot bode criteria leads to erroneous conclusions and it recommended to use the bode Nyquist stability criterion.

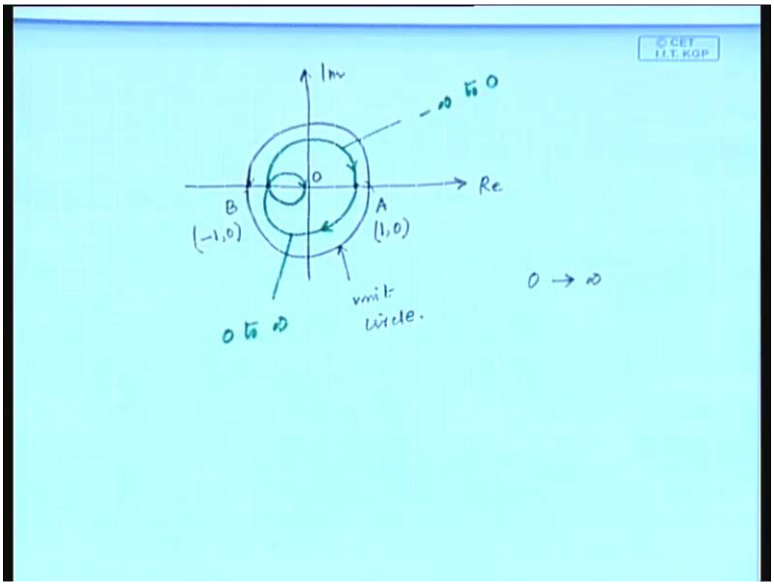
(Refer Slide Time: 46:27)



This criteria was proposed by Nyquist in 1932, and which has been published in journal namely Bell Systems Technical Journal. The Nyquist stability criteria was proposed in 1932 the title of the work was Regeneration theory, and this work was published in the journal namely Bell System Technical Journal no theorem, no proof of the theorem will be discussed, in this course and you are strongly recommended to go through this paper. If, you are interested to know more about the Nyquist stability criteria.

So, what is that criteria? Nyquist stability criteria we, can write has a closed loop control system is unstable. If the open loop Nyquist plot of the feedback system, encircles the point minus 10 as the frequency omega takes any value, in between minus infinity and infinity.

(Refer Slide Time: 50:01)



So, previously we construct the polar plot in the complex plane. So, this is the complex plane first we draw 1, unit circle. this, is a unit circle. So, this point represent suppose this o is the origin, this is point A and this is point B, A is basically 10 and B is minus 10. Now, if, the Nyquist plot… Do not encircle this, then the system is said to be stable it is somewhat like this, suppose for a particular system we are getting say for a Third order system we are getting this polar plot. And this plot we obtain, varying suppose omega 2 from 0 to infinity.

Another plot we are getting suppose, this 1.So, this plot we can obtain varying omega minus infinity to 0. And this we obtain varying omega from 0 to infinity. Now, you see this Nyquist plot do not encircle this minus 10. So, we can say that the example system is stabled.