Process Control and Instrumentation
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Lecture - 27
Feedback Control Schemes (Contd.)

So, in the last class we started the topic Bode Stability Criterion we will continue that topic and we took one example also. So, today we will continue Bode Stability Criterion with that example.

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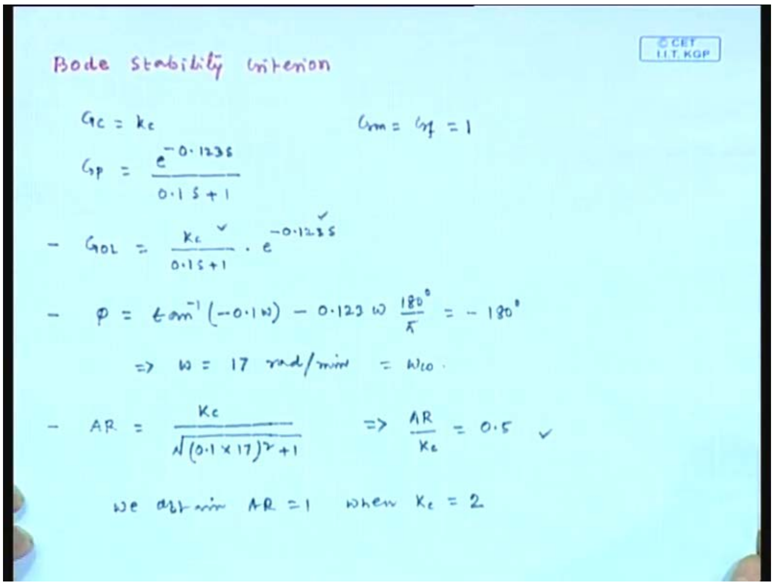

Bode Stability Criterion we considered ap only controller having transfer function kc. We considered the 1st order process dead time system for the process and the transfer function is exponential of minus 0.123S divided by 0.1S plus 1 for simplicity we considered Gm Gf both equal to 1. First we need to write the open loop transfer function.

That we can write as kc divided by 0.1S plus 1 exponential of minus 0.123S this is first order and this is dead time part. So, the expression for phi we can write as tan inverse minus 0.1 omega minus 0.123 omega multiplied by 180 degree divided by phi this is the expression for phi. If, we consider phi equals minus 180 degree then we obtain omega equals 17 radian per minute here time is in minute. Now, this is basically cross over frequency cross over frequency or critical frequency is the frequency which is calculated at phi equals minus 180 degree.

In the next step we can write the expression for Amplitude Ratio that we can write as K c divided by tau square omega square plus 1 here tau is 0.1 omega we are considering cross over frequency whole square plus 1 then we obtain Amplitude Ratio divided by Kc equal to 0.5. We obtain Amplitude Ratio divided by Kc equal to 0.5 now, we obtain Amplitude Ratio 1 when Kc equal to 2. We obtain amplitude ratio equal to 1 when Kc equal to 2.

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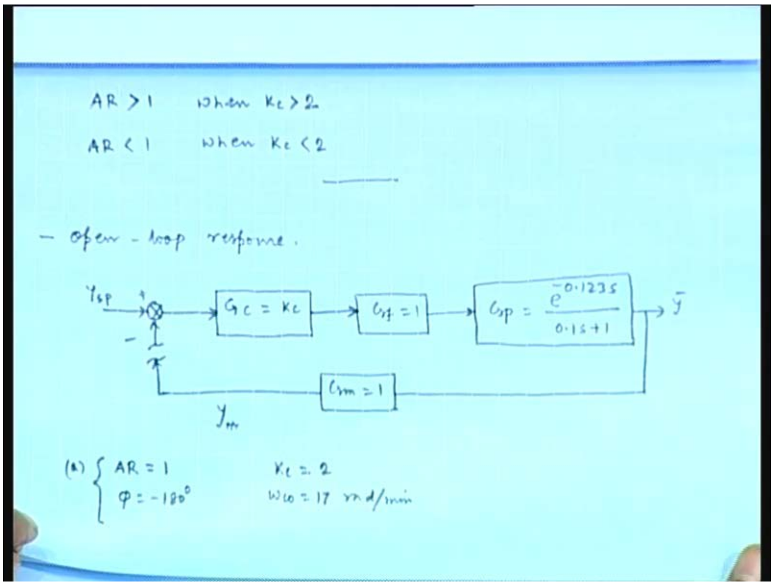

So, we can say that Amplitude Ratio we can obtain greater than 1 considering Kc greater than 2 similarly, we obtain amplitude ratio less than 1 when Kc less than 2. So, Kc equals to 2 that is the critical value of Kc 2 is the critical value for Kc. I hope up to this we discussed in the last class. So, today we will start with the open loop response next we will find the open loop response of the example system. Now, how we can open the loop? Previously we considered the closed loop. We want to open that than we need to disconnect something and that is shown here. This is the block for first order plus dead time system output is y bar this is the comparator this is set point and this is measuring device negative this is positive.

Now, we are disconnecting the measurement signal from this comparator then we can say that this is open loop. We use to know the open loop response by disconnecting the measurement signal from this comparator in addition we will consider the followings: 1st will consider Amplitude Ratio equal to 1 phi equal to minus 180 degree if, we consider phi equal to minus 180 degree we obtain cross over frequency 17 radian per minute agreed if we consider phi equal to minus 180 degree we obtain cross over frequency 17 radian per minute. Now, at this cross over frequency we obtain Kc equal to 2 at Amplitude Ratio equals 1. So, this is our first consideration Amplitude Ratio equal to 1 and phi equal to minus 180 degree.

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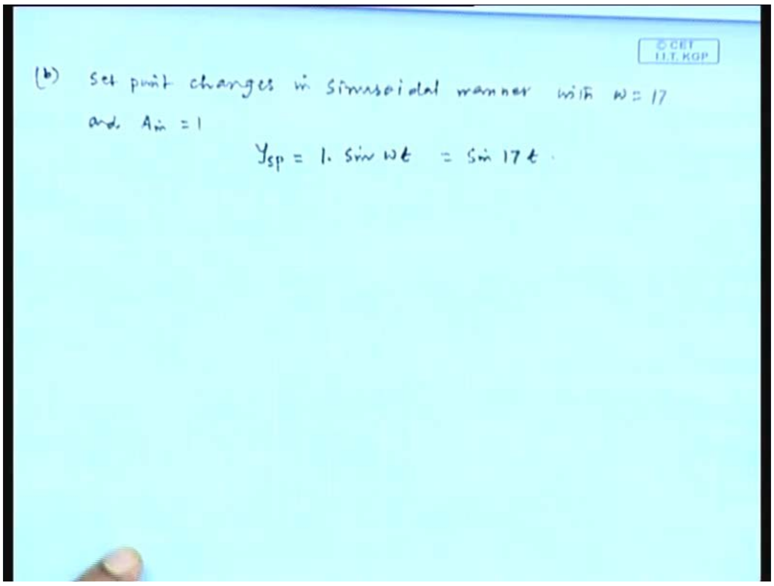

Secondly, set point changes in a Sinusoidal manner with omega equals 17 radian per minute and input amplitude equals to 1. This is the second consideration set point changes in a Sinusoidal manner with omega equals 17t and input amplitude equals 1 that means, this is sin 17t.

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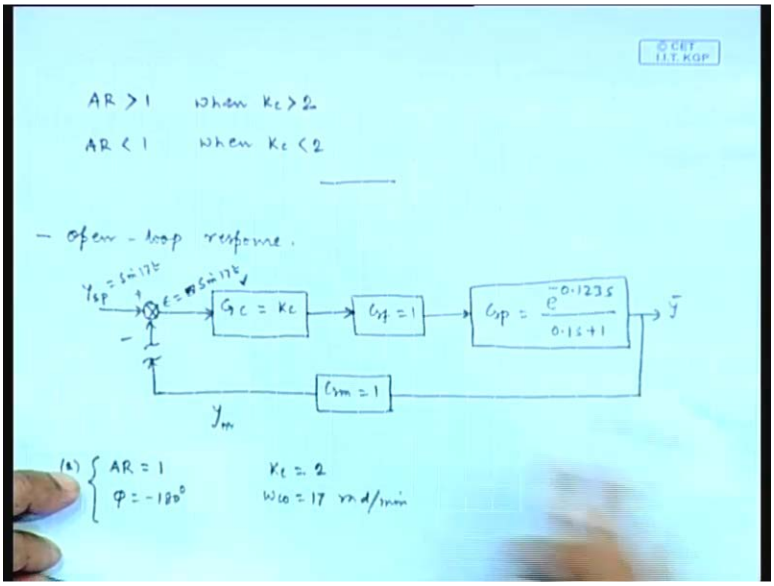

So, in time domain I can write this y set point equals sin 17 t. Here, we can write y set p equals sin 17t than how much is the error signal error? Error is also sin 17t because, we have disconnected this measurement signal from the comparator. So, error signal is also sin 17t which is equal to y set point and there written in time domain. Now, our first consideration is Amplitude Ratio equals 1.

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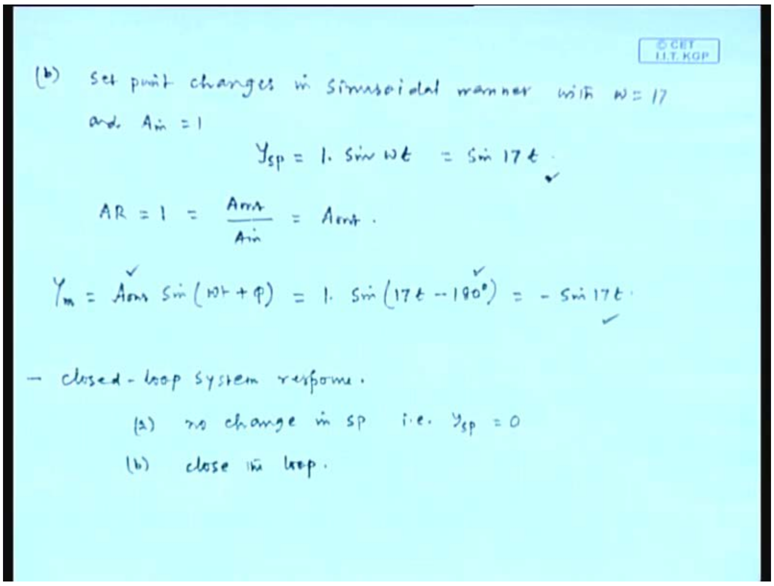

Amplitude ratio equal to 1 Amplitude Ratio is nothing, but output amplitude divided by input amplitude again we consider in the second consideration input amplitude equals 1.

So, how much is output amplitude? Output amplitude becomes 1. So, if the input is changed in Sinusoidal manner. What is the output? Output is a out sin omega t plus phi this is the output. So, amplitude of this output ais1 omega is 17 and phi is minus 180.

We have considered in our first point phi is minus 180 that means, this is minus sin 17t agree. So, if we introduce set point change in Sinusoidal manner the output is minus sin 17t this is also Sinusoidal. So, we can write Ym equals sin 17t this is the open loop response find the process is subjected to a change in set point in Sinusoidal manner.

In the next we will discuss the close loop system response. So, we will considered 2 events at a time I mean we will consider at the same time two events they are no change in set point that means, Y set point equal to 0 at the same time we consider another point that is close the loop this is the second point.

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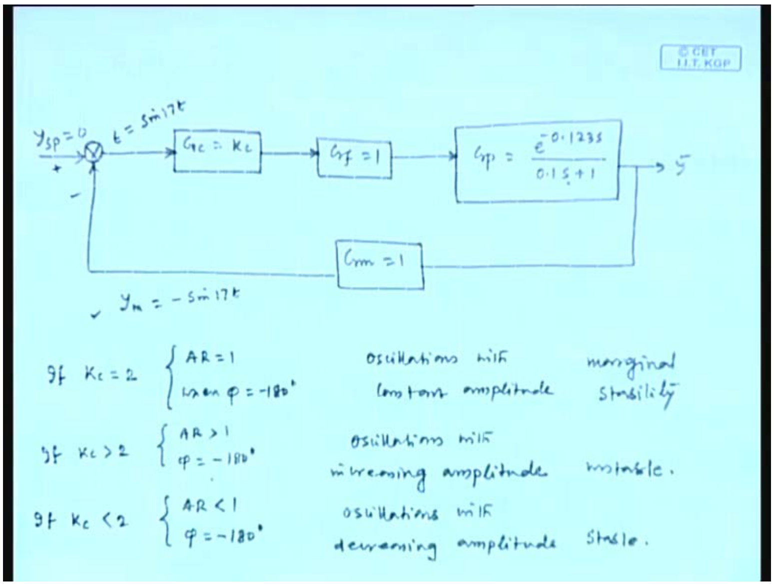

So, what would be the corresponding block diagram then? Controller, is this one final control element is final control element has transfer function of unity process transfer function is exponential of minus 0.123S is divided by 0 point 1S plus 1 y bar Gm equal to 1 comparator y set point equal to 0. This, we are simply connecting the measurement signal with this comparator. So, this is the close loop system. Here, we are considering no set point change and we are closing this. The output signal was minus sin 17t in the previous block diagram the output was minus sin 17t and we are suddenly closing it and at the same time we are not introducing any change in set point. So, this is negative this is positive. So, how much will be error now? Sin17t.

So, you notice thsat the error remains in the open loop system error vise sin 17t and when you are closing this loop and we are introducing no change in set point we are getting same error signal that is sin 17t. So, error is sin 17t and we are getting output minus sin17t What it indicates? Sustained oscillation I mean if we consider Kcequal to 2 then Amplitude Ratio is equal to 1.

When phi is equal to minus 180 degree if Kc is equal to 2 which is a critical Kc value then Amplitude Ratio equal to 1 when phi equal to minus 180 degree and in this situation we obtain sustained oscillation I mean oscillation with constant amplitude.

What about stability? If, we obtain sustained oscillation what about stability marginally stable we can use this term marginal stability. Now, similarly we can conclude that if Kc is greater than 2 then Amplitude Ratio greater than 1 when phi is minus 180 degree. So, What type of response we can obtain in this condition? Oscillations with increasing amplitude then, what about stability unstable similarly if Kc is less than 2 then Amplitude Ratio less than 1 and phi is minus 180 degree if Kc is less than 2 then Amplitude Ratio becomes less than 1.Whenphi is minus 180 degree.

So, in this condition we obtain oscillations with decreasing amplitude and this is stable. Now, based on this observation we can write the Bode Stability Criteria that is a feedback control system is unstable if the Amplitude Ratio of the corresponding open loop transfer function is greater than 1 at cross over frequency.

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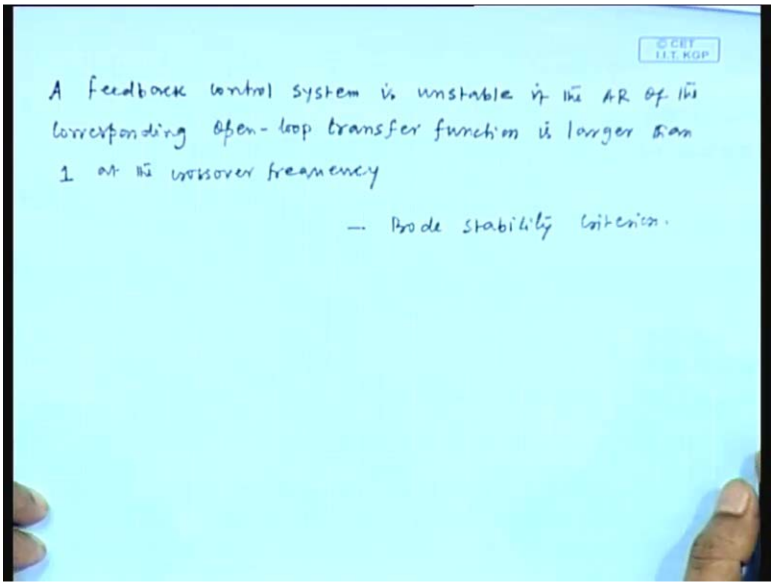

Bode Stability Criteria is stated as a feedback control system is unstable if the amplitude ratio of the corresponding open loop transfer function is larger than 1 at the cross over frequency this is the Bode Stability Criteria . A feedback control system is unstable if the Amplitude Ratio of the open loop transfer function is larger than 1 at the cross over frequency. Next we will discuss the Stability Analysis based on this Bode Stability Criteria we will take a number of examples and we will apply this Bode Stability Criteria.

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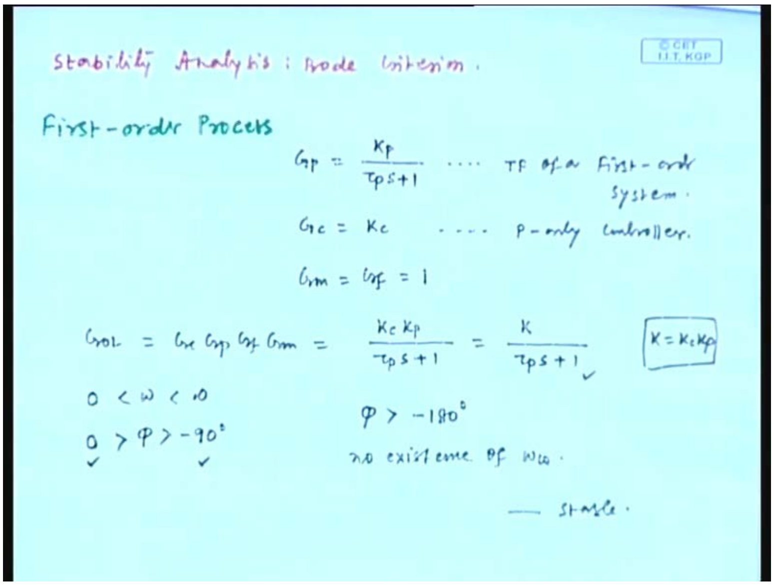

So, our topic is Stability Analysis based on Bode Criteria. First[t] we will consider a First order process. For a First order process we can consider the transfer function as Kp divided by tau pS plus 1 this is the transfer function of a First order system. We consider p only controller. So, Gc equals Kc this is the transfer function of p only controller for simplicity we consider Gm and Gf both equal to 1. Now, at the first step we need to find the open loop transfer function the open loop transfer function for this example system we can write as Gc Gp Gf Gm and that is equal to Kc Kp divided by tau pS plus 1 and we can write this as K divided by tau pS plus 1 here K is Kc Kp.

Now, what about the phi if we consider omega equal to 0? How much will be phi? Phi will be 0 this is a First order system I mean this is the transfer of a First order system when omega is infinity, How much is phi for a first order system? Minus 90 when omega is 0 phi is 0 when omega is infinity phi is minus 90. For a first order system phi remains in between 0 and minus 90. So, what it indicates? Phi is always greater than minus 180 agreed phi is always greater than minus 180 degree that means, there is no existence of cross over frequency.

So, what about stability stable if there is no cross over frequency then the system always remains as stable. So, for a First order system the process always remains at stable state irrespective of the Kc value only thing of you that if you considered large K c than oscillations will be more, but there is no instability problem.

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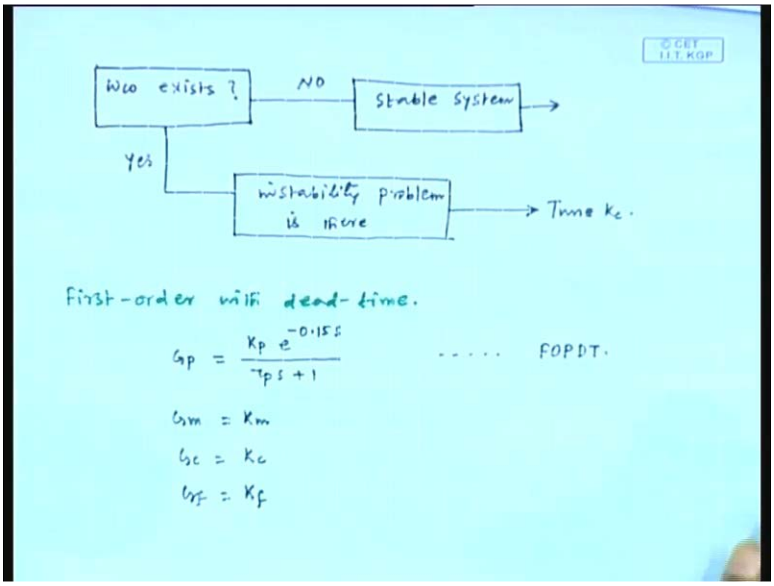

So, we can say that first we need to check weather cross over frequency exists or not. If, the answer is no then the system is stable if the cross over frequency does not exists then the system is stable system. So, you can use any Kc value at least there will not be any instability problem, but as I mentioned with the increase of Kc value now oscillation also the response also becomes more oscillatory now, if the answer is yes cross over frequency exists then instability problem is there. So, how we can make it stables this unstable system? We can make it stable by tuning the controller we can make this unstable systems stable by tuning the controller. So, if there is any instability problem Tune Kc. So, initially we considered the first order process in the next we will consider a First order process dead time system. Next we will discuss a First order with dead time process transfer function of a First order process a dead time system we can write as Kp exponential minus 0.15S divided by tau pS plus 1 here dead time is 0.15.

So, this is a transfer function of a First order plus dead time system. Here, we are considering Gm equal to Km controller is p only controller. So, Gc equals Kc and for the final control element Gf is equal to suppose Kf. So, these are the individual transfer function for the examples system.

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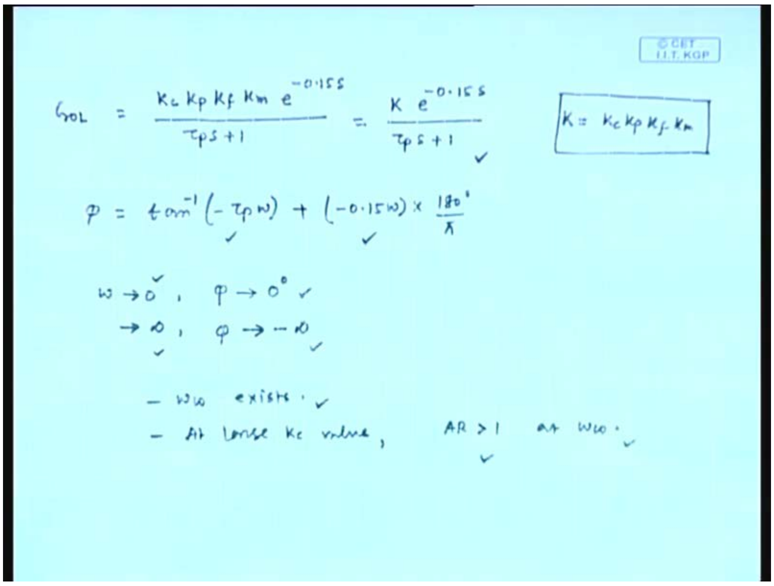

Now, we need to write the open loop transfer function. What will be the open loop transfer function? Kc Kp Kf Km exponential minus 0.15S divided by tau pS plus 1 this is the open loop transfer function. So, this transfer function we can rewrite as K exponential of minus 0.15S divided by tau pS plus 1 where K is Kc Kp Kf Km. So, this is overall the transfer function of a first order plus dead time system. Now, what will be the expression for phi? Ppi will be tan inverse minus tau p omega this is for the dead time free part and for dead time part the phi will be minus 0.15 omega this is for dead time free part and this is for dead time part.

Now, we can multiply with 180 degree divided by phi. Now when omega tense to 0 what about phi if omega tense to 0 then phi tense to 0 degree when omega tense to infinity What about phi? Phi will be minus infinity agree if omega tense to 0 then we can write phi tense to 0 degree and when omega tense to infinity phi tense to minus infinity. So, what about cross over frequency? Cross over frequency exists now at large value of Kc Amplitude Ratio may become greater than 1 at cross over frequency at large Kc value Amplitude Ratio may become greater than 1 at cross over frequency it clearly indicates there is an instability problem.

So, first point is the cross over frequency exists and at large Kc value the Amplitude Ratio may become greater than 1 at cross over frequency that means, the there is some instability problem. So, overall we can say that dead time is a principle source of this stabilizing effect in chemical process control systems we can conclude based on this example that dead time is the principle source of this, stabilizing effects in chemical process control systems.

Now, when we started the Coherence technique it was mentioned that the response of most of the chemical processes can be approximated by the response of First order plus dead time system when we started the Coherence technique it was mentioned that the response of most of the chemical process is can be approximated by the response of First order plus dead time that means, almost all chemical processes have instability problem due to the inclusion of dead time part. So, we can say that almost all chemical processes include the dead time part.

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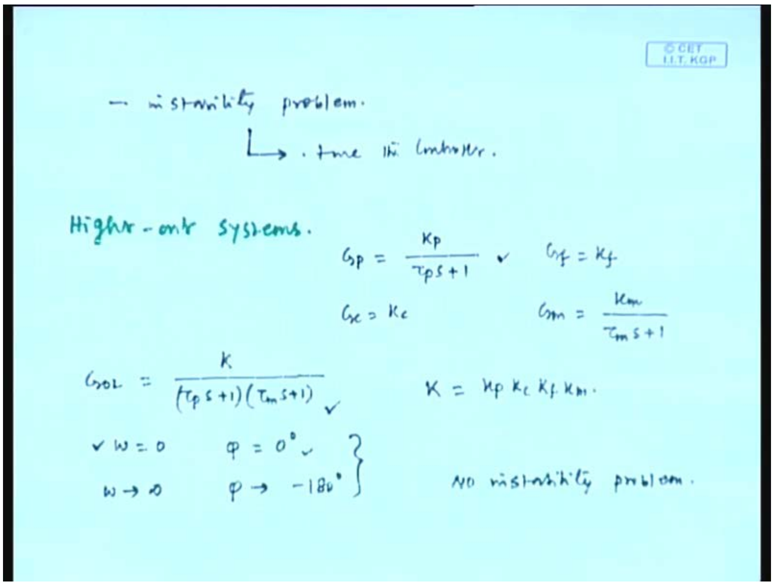

That means almost all chemical processes have instability problem. Now as mentioned before to make it stabilize we need to tune the controller. So, controller tuning becomes a crucial task. Next we will consider a third order system and will apply the Bode Stability Criteria. Higher order systems here before Third order system we will consider second order system will consider next second order system then third order system. So, the transfer function for a second order system we can write as Kp divided by tau pS plus 1 now this is the transfer function of a first order system.

Now, we are considering Gf equal to Kf we are considering Gc equal to Kc and Gm equal to Km divided by tau mS plus 1. These are four individual transfer function now, overall transfer function we can write as K divided by tau pS plus 1 tau mS plus 1 can we write this? Overall transfer function becomes K divided by tau pS plus 1 multiplied by tau mS plus 1 this is the second order system I mean this is the transfer function of a Second order system and obviously, here K equal to Kp Kc Kf Km.

Now, if we consider omega equal to 0. How much is phi for the Second order system? 0 degree we are not writing the expression for phi because, we discuss this several times when omega is equal to 0 for the Second order system phi becomes 0 degree when omega tense to infinity phi tense to minus 180 degree for the second order system the phi varies from 0 to minus 180 degree.

So, what about the existence of cross over frequency there is no existence of cross over frequency at finite value of omega that means, there is no instability problem. So, see First order system without dead time no instability problem Second order system without dead time no instability problem next we will consider the third order system.

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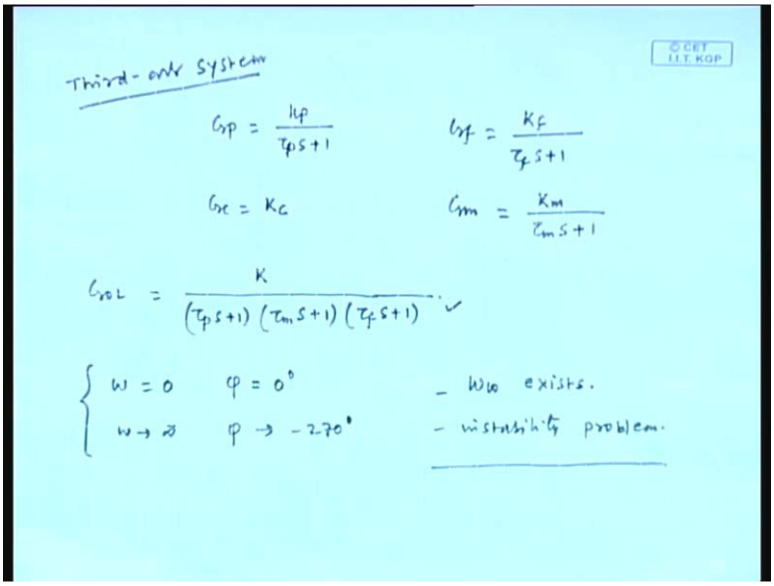

So, next we will consider the third order system. Suppose the transfer function of the process is Kp divided by tau pS plus 1 transfer function for the final control element is considered as Kp divided by tau fS plus 1 controller is p only controller and the transfer function for the measuring device is considered as Km divided by tau mS plus1. These are the four individual transfer function so obviously, the open loop transfer function becomes Third order transfer function. I mean we can write it as K divided by tau pS plus 1 tau mS plus 1 tau fS plus 1 this is the open loop transfer function for the example system.

Now, when omega is equal to 0 phi becomes 0 degree and when omega tense to infinity phi tense to minus 270 degree. So, for a Third order system the phi where is in between 0 and minus 270. So, what about the cross over frequency cross over frequency exists. So, there is instability problem. So, we can note 1 point that up to Second order system if there is no dead time the systems do not have any instability problem First order system without dead time no instability problem Second order system without dead time no instability problem Third order system without dead time instability problem is there. So, up to Second order there is no instability problem if there is no dead time, but, if dead time is there from First order system itself stability instability problem arises.

Next we will discuss another topic under this frequency response analysis that is Gain margin and Phase margin.

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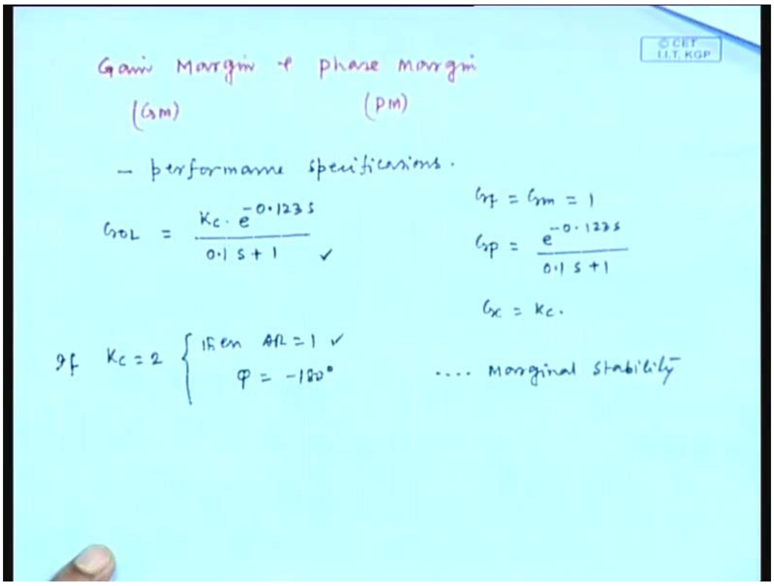

This gain margin and phase margin of the typical performance specifications associated with the frequency response analysis. Gain margin and Phase margin are the performance specifications associated with the frequency response analysis. So, for discussing this Gain margin and Phase margin it is better to consider an example and we will continue the example which we considered earlier having the open loop transfer function equals Kc exponential minus 0.123S divided by 0.1S plus 1.

To discuss the Gain margin and Phase margin concept we considered this example where Gf and Gm both are equal to 1 and the transfer function for the process we consider exponential minus 0.123S divided by 0.1S plus 1 and Gc equals Kc recall the fact that if Kc equal to 1 if Kc equals 2 then Amplitude Ratio becomes 1. When phi equals minus 180 degree. We concluded these for this example are here if Kc is equal 2 than Amplitude Ratio becomes 1 when phi equals minus 180 degree and this corresponds to Marginal stability.

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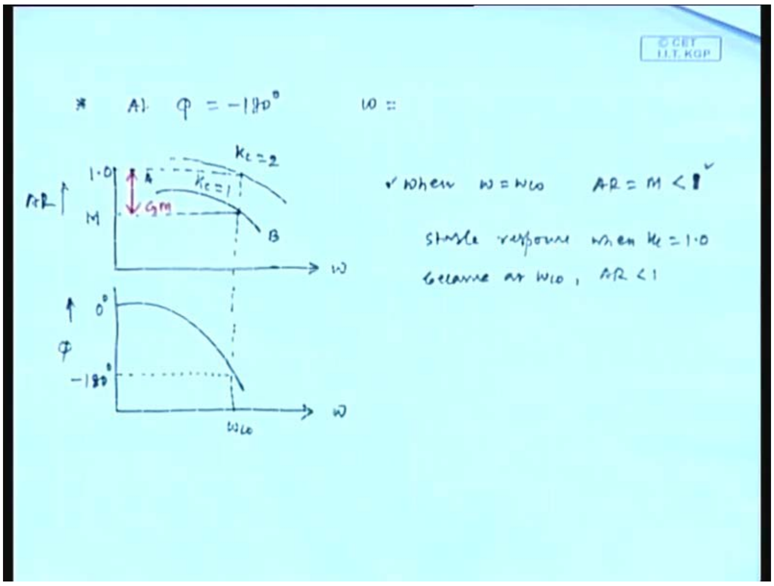

Now, at phi equals minus 180 degree we know that at phi equals minus 180 degree we obtain the cross over frequency. So, we will try to show it in the bodeplot this is omega this is phi have the plot is say like this. This is suppose 0 degree and this is suppose minus 180 degree. So, at minus phi equals minus 180 degree we obtain cross over frequency agree.

So, the frequency corresponding to this phi equals minus 180 degree is cross over frequency which is shown in this figure. Now, above this we will draw Amplitude Ratio verses omega. Suppose this A B curve we obtain for Kc equals 1 this A B curve I mean this is the Amplitude Ratio verses omega plot we obtain considering K c equals one that means, the Kc value we have taken lower than the critical Kc value now at this cross over frequency, How much is this value? At cross over frequency Amplitude Ratio is suppose m agree and critical value of Kc is 2 suppose this is the Amplitude Ratio verses omega plot when Kc equal to 2.

So, at cross over frequency how much is this Amplitude Ratio 1. So, when omega equals omega co we obtain Amplitude Ratio equal to M which is lower than 1 agree. If, we obtain 1 than marginal stability if this is lower than 1 that means, stable system. So, when omega equals omega co I mean at cross over frequency if Amplitude Ratio is less than 1 than that is the indication of stability.

So, we can say that the response is stable response when Kc equal to 1 because, at cross over frequency Amplitude Ratio is less than 1. I am repeating again the steps this is phi verses omega plot for this example system this now cross over frequency is the frequency which is obtained at phi equals minus 180.

So, we consider phi equals minus 180 corresponding omega is cross over frequency now for the example system we know the critical value of Kc is 2 here, we first consider Kc equals 1. I mean it indicates stability. Now, A B is the amplitude verses omega plot when Kc equals 1 at cross over frequency it is clear from this bode plot that Amplitude Ratio is M then, we can make another amplitude verses omega plot at the critical value of Kc that is 2 and at cross over frequency.

The corresponding Amplitude Ratio is 1. So, it says that when we consider Kc equals 1 Amplitude Ratio less than 1 at cross over frequency now this difference is the Gain margin you see this is nothing, but a safety factor this difference is called the Gain margin and this is nothing, but a safety factor in the next class we will discuss the Phase margin.