Process Control and Instrumentation
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Lecture - 26
Feedback Control Schemes (Contd.)

So, presently we are discussing the frequency response analysis. Under frequency response analysis, today we will cover the construction of polar plot, previously we started the construction of polar plot.

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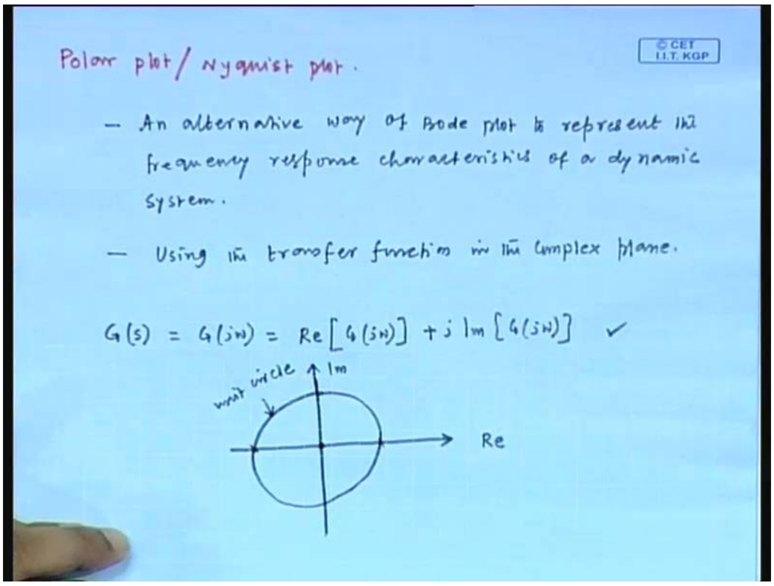

Now, we will discuss the construction of polar plot, this plot is also called Nyquist plot, because this plot is used for the stability analysis using Nyquist criteria. So, the polar plot is also called Nyquist plot. Now in Nyquist plot is an alternative way of polar plot to represent the frequency response characteristics of dynamic systems. Nyquist plot is an alternative way of prode plot to represent the frequency response characteristics to represent the frequency response characteristics of a dynamic system.

So, this is basically a alternative to prode plot and this can be used for stability analysis later. This plot is construction using transfer function in the complex plane, fine. Now, we will take the transfer function in general form I mean you can take the transfer function as G s now replacing S by G omega you can write as G j omega which is frequency response transfer function, and this you can write again in Cartesian form as real G j omega plus j imaginary part, fine.

We can write the transfer function like this now you will produce 1 plot in the complex plane. So, this is real axis this is imaginary axis, and this is a unit circle which as the radius of 1 this is a unit circle fine; that means, this point corresponds to 10 this is the origin, and this point correspond to minus 10 this 1 minus 10 this is 10, and this is the origin. Now from the frequency response transfer function we can write.

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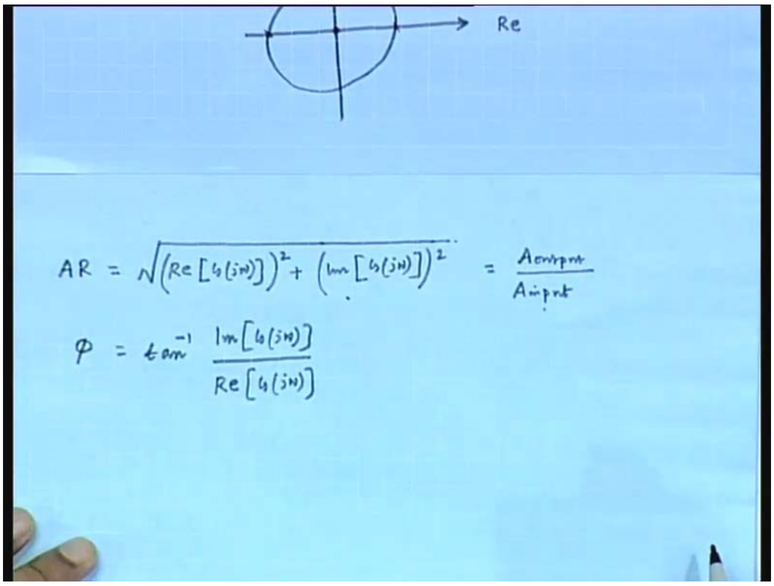

Expression for amplitude ratio is root over of the real part whole square plus imaginary part whole square fine amplitude ratio we know is represented by output amplitude divided by the input amplitude I mean amplitude of the output divided by amplitude of the input, and this transverve function G.

Which is represented as output divided by input agree here what is the amplitude of f S , because we can write this term divided by 1 I can write this term divided by 1 plus G multiplied by 0; that means, amplitude of output is this amplitude of input is 1 agree. So, we can say that here amplitude ratio is becoming output amplitude now. What will be th fails it phi phi we can write as tan inverse imaginary part divided by the real part, fine. If we consider this as complex number.

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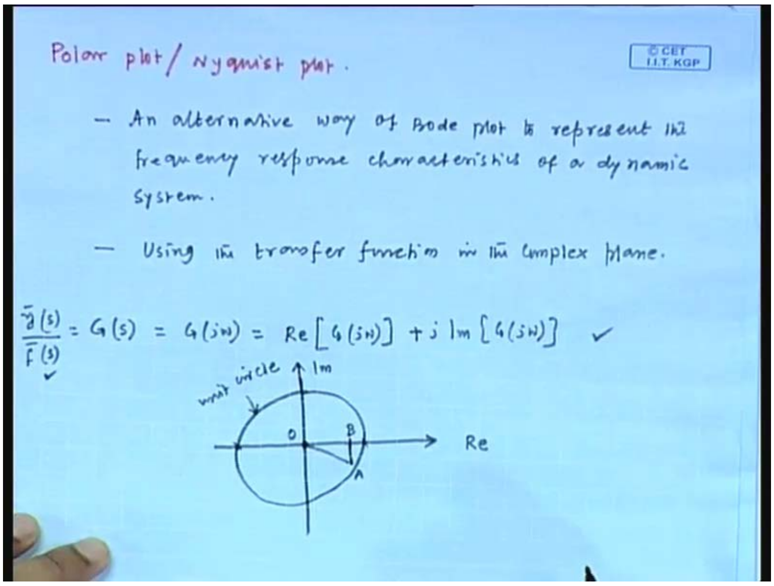

Then suppose this part OB represent the real part BA represent the imaginary part. So, how much is the OA this is the amplitude ratio agree, if we consider G s is this 1 and amplitude ratio is this 1 square root of real part square plus imaginary part square then this OA represent the amplitude ratio.

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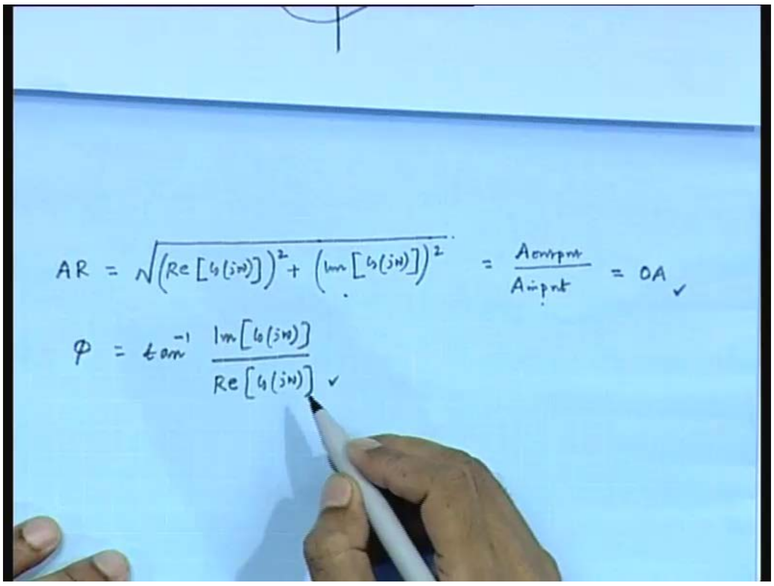

So, I can write this amplitude ratio is basically, OA this is basically the magnitude of this G j omega how much is argument this 1 phi.

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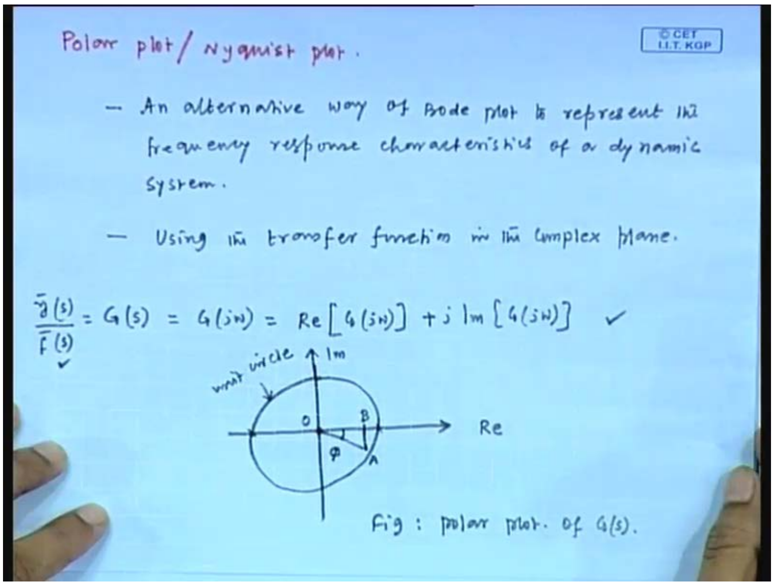

That means this angle is phi agree, and this OA is representing the amplitude ratio. So, this is the polar plot of transverve function G s. So, this is the polar plot of transverve function G s next we will discuss the cons traction of polar plot for first order system second order system extra. So, in the next we will consider the first order system.

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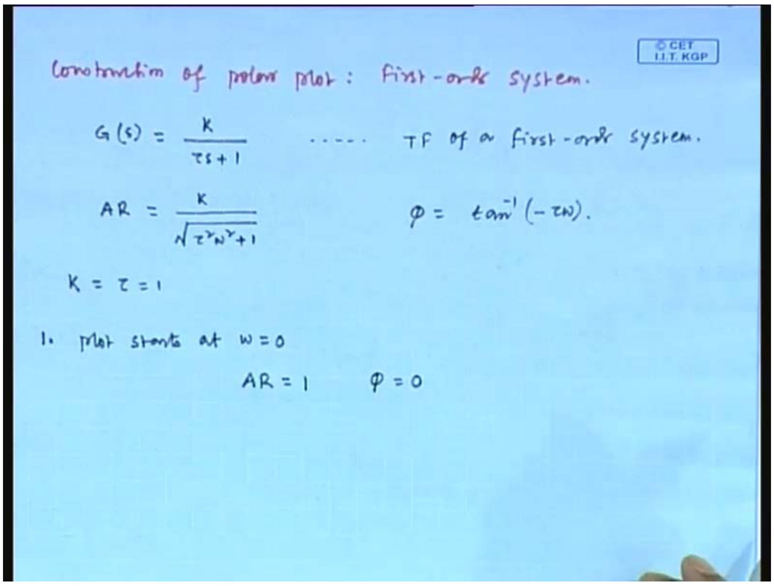

I mean the constraction of polar plot and we will consider first order system the transverve function of first order system we can write as K divided by tau S plus 1, fine. This the transverve function of a first order system we determine the expression for first order system we determine the expression for amplitude ratio that is K divided by root over of tau square omega plus 1 we determine the expression for faze angle phi that is tan inverse minus tau omega this 2 expressions we have determine earlier.

Now, we will consider both the process gain and time constant are equal to 1, we will consider both K, and tau are equal to 1, fine. Now bode plots sorry polar plots polar plot starts at omega equal to 0 fine the polar plot starts at omega equal to 0, if we consider omega equal to 0 what is amplitude ratio I mean how much is amplitude ratio amplitude ratio is 1 how much is phi; phi is 0, fine. So, if we consider this point I mean omega equals 0.

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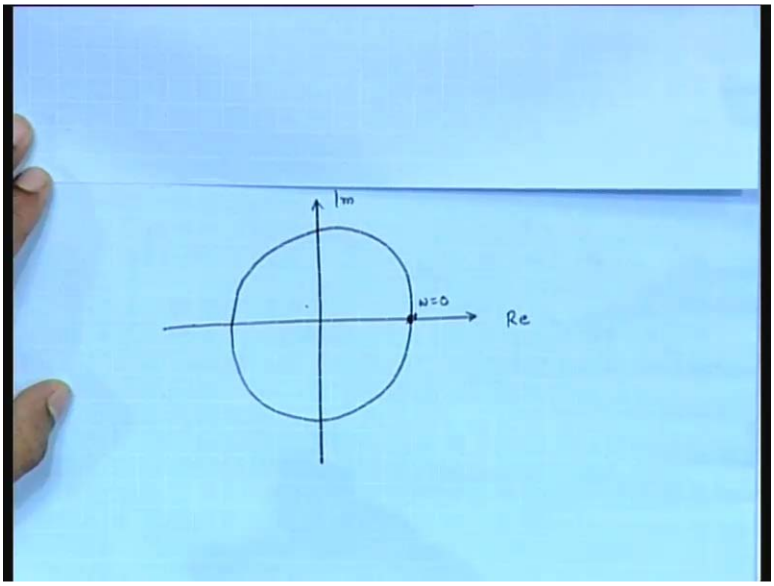

What will be the polar plot this is real axis this is imaginary axis. So, first we will draw the unit circle this is the unit circle amplitude ratio 1 means this is basically 1 I mean this point represent 1 agree when phi equals to phi is equals to 0. So, amplitude ratio 1, and phi equal to 0 represent this point agree. So, we will write here and omega equal to 0 the starting point we get this 1.

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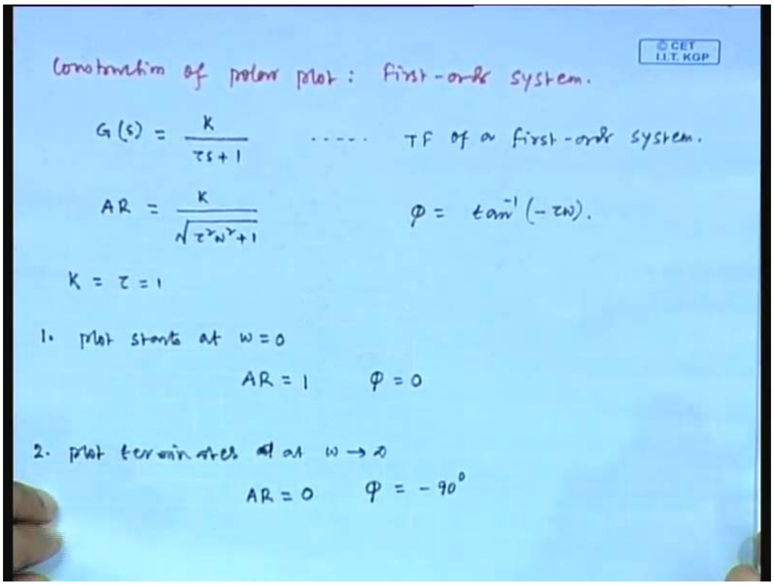

Next we will consider the end point the plot terminates at omega equals infinity plot. Terminates an omega is equal to infinity if omega is equal to infinity how much is amplitude ratio amplitude ratio is 0 how much is phi; phi is minus 90, fine. Because phi minus tan inverse tau omega. So, can you locate end point.

End point is this 1 here phi is equal to minus 90 and amplitude ratio is 0. So, this is the end point next we need to consider the intermediate points to draw this polar plots now when omega is in between 0 and infinity how much will be amplitude ratio when we consider omega 0 we got amplitude ratio 1 when we consider infinity we got amplitude ratio 0. So, we can write like this agree when we consider omega 0 we obtain amplitude ratio 1 when we consider omega infinity we obtain amplitude ratio 0.

So, we can write this accordingly similarly we can write for phi when omega equals 0 phi is 0 when omega is infinity phi is minus 90 degree. So, we can write this 0 greater than phi greater than minus 90 if use intermediate points in this complex plane then we will get the polar plot like this and this is the deduction this is starting point, and this is the end point and we need take few intermediate points. Then we will get this type of plot. So, this is the polar plot for the example first order system this is the polar plot for the example first order system, fine.

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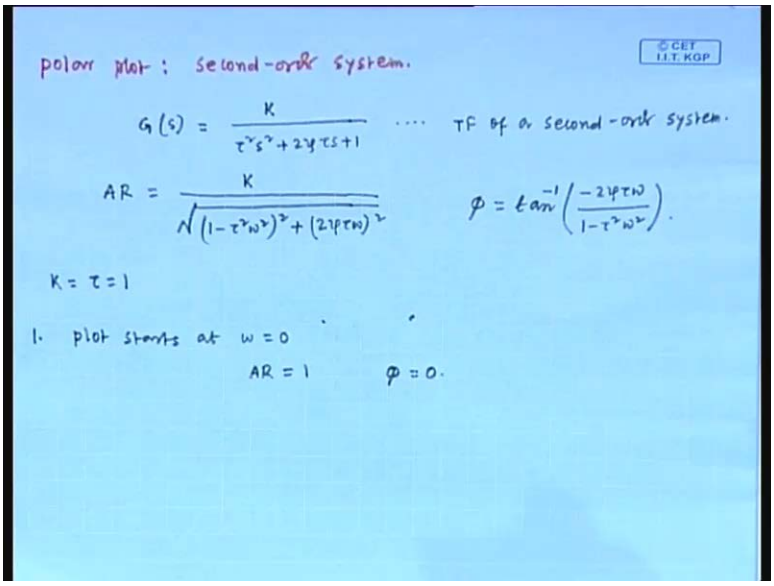

Similarly, we will try to construct the polar plot for second order system in the next polar plot for second order system we know the transverse function is represented in general form as K divided by tau square S square plus 2 zeta tau S plus 1 this is the transverse function of a second order system, fine. Previously we determine the expressions for amplitude ratio and phi for second order system they are amplitude ratio equals K divided by root over 1 minus tau square omega square whole square plus 2 zeta tau omega whole square this expression we determine earlier.

Similarly, if we determine the expression for phi as tan inverse minus 2 zeta tau omega divided by 1 minus 2 square omega square, fine. Now we will construct the polar plot and for that we will consider K and tau both are equal to 1 for simplicity we are considering the K and tau equal to 1 now the polar plot starts at omega equal to 0. So, plot start at omega equal to 0 what is the corresponding amplitude ratio 1 how much is phi; phi is 0, when omega equal to is equal to 0 amplitude ratio becomes 1 and phi becomes 0.

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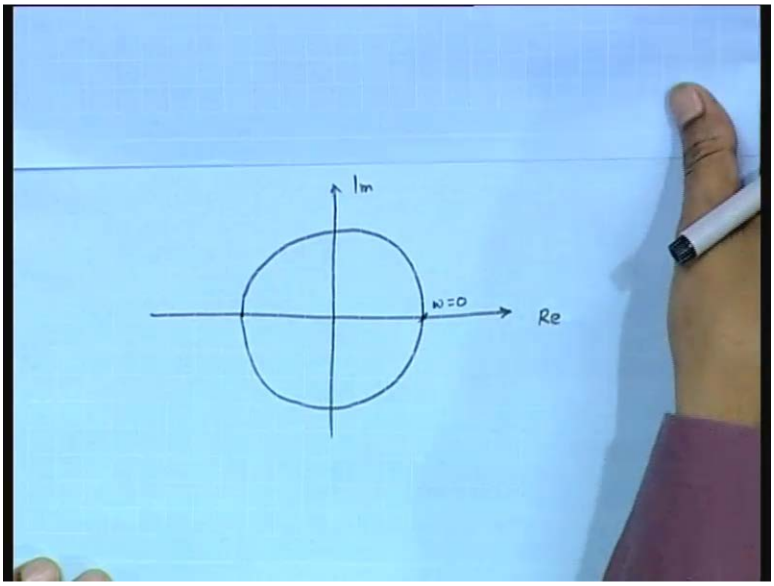

Now, we will locate the starting point in the complex plane. This is the unit circle when amplitude ratio is equal to 1 and phi is equal to 0 we get this point. So, when omega equal to 0 the polar plot should start from this point.

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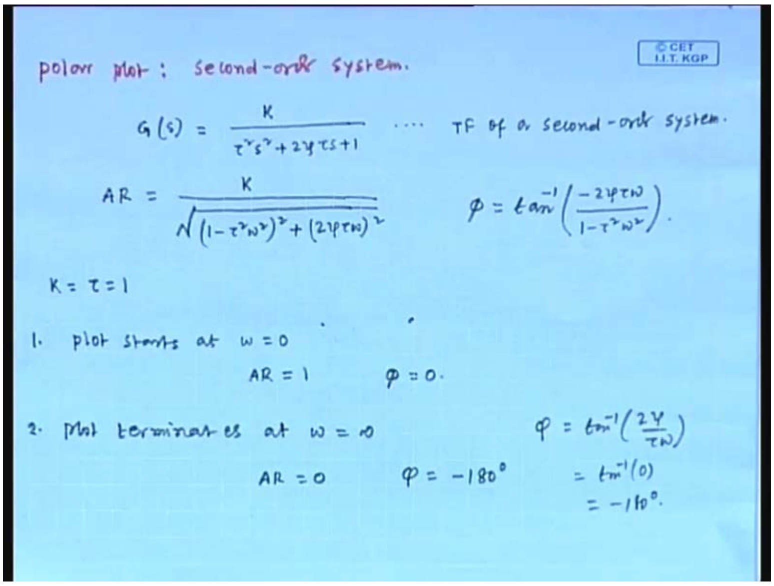

Secondly if we will consider the we will try to find the end point this polar plot terminates at omega is equal to infinity now if this the case then how much is the amplitude ratio 0 how much is phi; phi is minus 180 pi, we can write as tan inverse 2 zeta y tau omega tan inverse 0 this we consider minus. So, what will be the end point in this complex plane.

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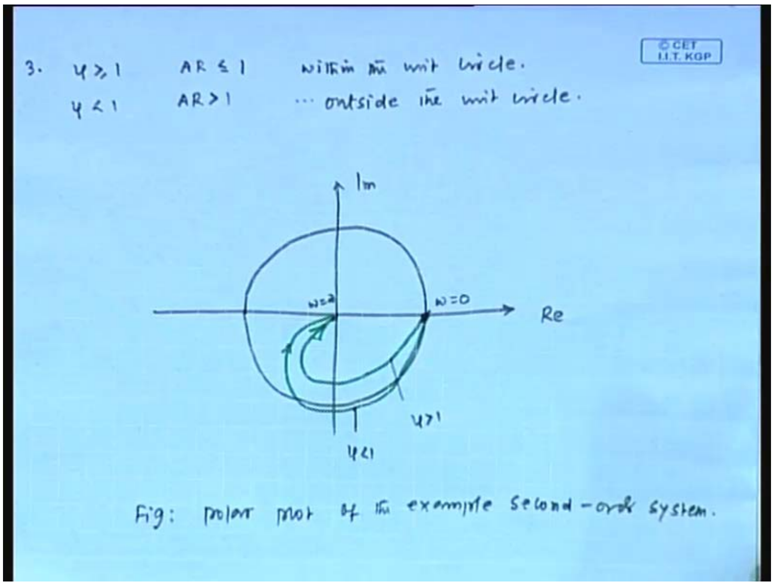

Here, but the polar plot should go through this quadrant next we need to generates some intermediate data see here another parameter is involved that is zeta, fine. So, when zeta is greater or equal to 1 amplitude ratio becomes less than or equals to 1 it is easy to test that when we consider zeta is greater than or equal to 1 the amplitude ratio becomes less than or equals to 1 what it indicates the polar plot will be within the unit circle amplitude ratio less than 1 means the polar plot should be within the unit circle.

Similarly, if we consider zeta less than 1 it is easy to investigate the amplitude ratio becomes greater than 1 when zeta is less than 1 amplitude ratio becomes greater than 1; that means, this outside of the unit circle the polar plot goes outside of the unit circle. So, this is an omega equals infinity this point an omega equals 0. So, when zeta greater than 1 we obtain this plot, because it is within the unit circle this we obtain an zeta greater than 1 similarly when zeta less than 1 we obtain this plot I mean this polar plot is outside of the unit circle, fine. So, it is clear that the polar plot depends on the value of zeta this is the polar plot of the example. Second order system, fine. Next we will consider the third order system.

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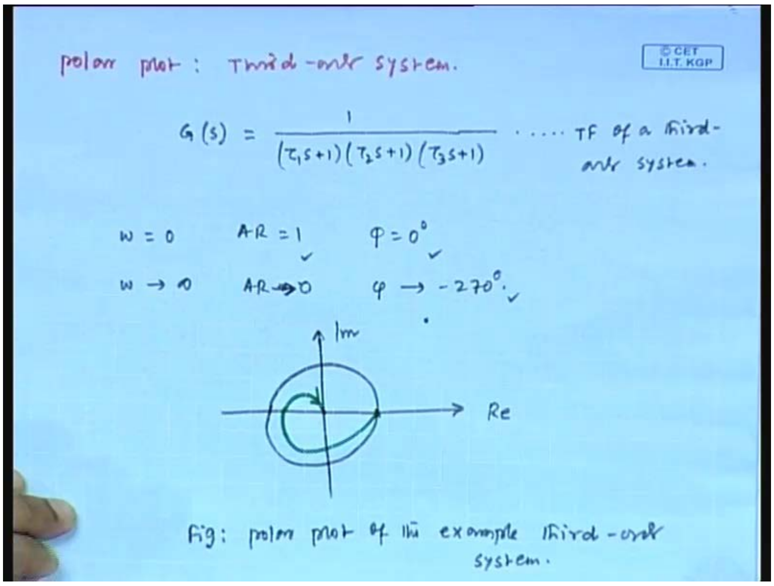

Construction of polar plot for a third order system the transverse function of third order system can be written as 1 divided by tau 1 S plus 1 tau 2 S plus 1 tau three S plus 1, fine. This is the transverse function of a third order system. Now, it is easy to show that when omega equal to 0 amplitude ratio is 1 and phi is 0 degree, similarly when omega tends to infinity amplitude ratio tends to 0 and phi tends to minus 270 degree for the first order system we obtain minus 90 for second order minus 180 for third order system minus 270, fine. Similarly we can generate some intermediate data this is the unit circle in complex plane starting point is this 1 and end point is this 1 and polar plot we obtain like this. So, this is the polar plot of the example third order system, fine. This is the polar plot of a third order system we can take another example like if pure delay system how we can construct.

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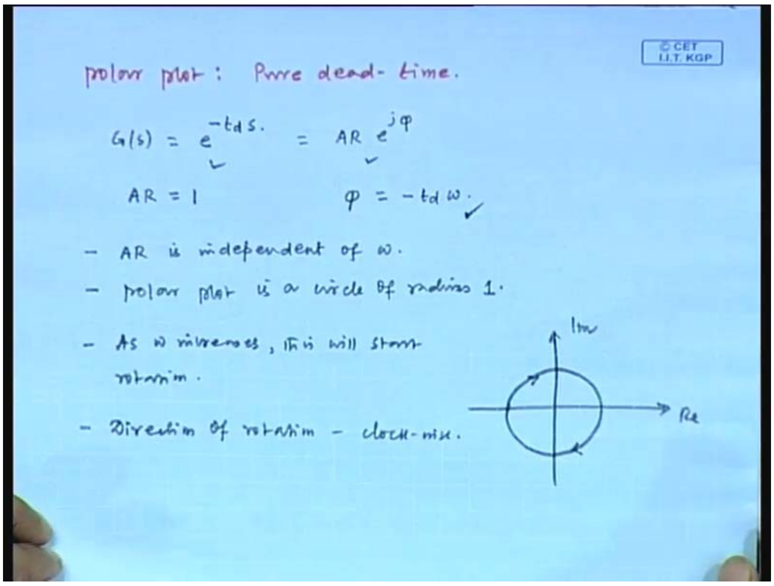

Polar plot of a pure dead time the transverse function of this element pure dead time element we can write as exponential minus t d S this is the transverse function of the pure time delay system how much is amplitude ratio you can compare this with the standard polar form that is amplitude ratio exponential of j phi compare this with this how much is amplitude ratio 1 how much is phi minus t d omega fine minus t d omega.

So, it is obvious that amplitude ratio is independent of omega it is obvious that amplitude ratio is independent of omega that is always 1. So, the polar plot is if circle of radius 1 it clearly indicates that the polar plot should be a circle of radius 1. So, the polar plot is if circle of radius 1 this is real axis this is imaginary axis amplitude ratio 1 means the polar plot this is the polar plot we can say the polar plot is a circle of radius 1 as omega increases what happens as omega increases this will start rotating it is obvious from this expression if you keep on increasing the omega holus we will see this phi will changing like this that means.

As omega increases this will start rotating now next question is what is the deduction of that rotation? I mean we have to show the deduction of this polar plot also that will be clock wise. So, deduction of rotation will be clock wise. So, deduction of rotation will be clock wise clock wise rotation. So, this is the polar plot of pure dead time system. Now what is the purpose of this frequency response analysis started. So, far what is the purpose of this.

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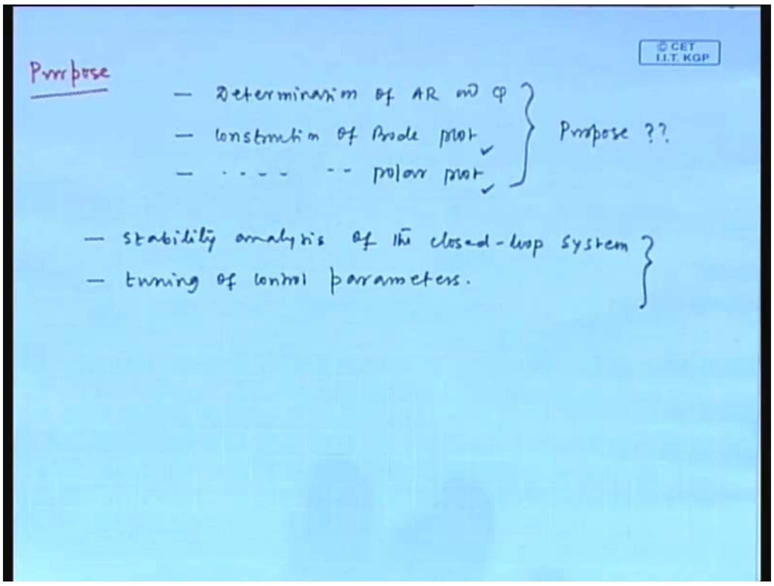

Frequency response analysis. So, far we have started under frequency response analysis the determination of amplitude ratio and phi. So, we have started under frequency response analysis the determination of amplitude ratio and phi. So, we have started the cons traction of polar plot, but what is the purpose of doing all this what is the purpose of doing all this the frequency response analysis is performed for two reasons; oner is for stability analysis another one for controller tuning frequency response analysis is performed for two reasons one is for stability analysis of the closed loop system.

Frequency response analysis is performed for tuning of control parameters and it is interesting to note that for the stability analysis and for control tuning we will use thisbode plot and polar plot. So, far we have just started construction those plots in the next we will use those plots for stability analysis, and for control tuning. So, first we will discuss thebode stability criteria, and we will use for that purpose thebode plot.

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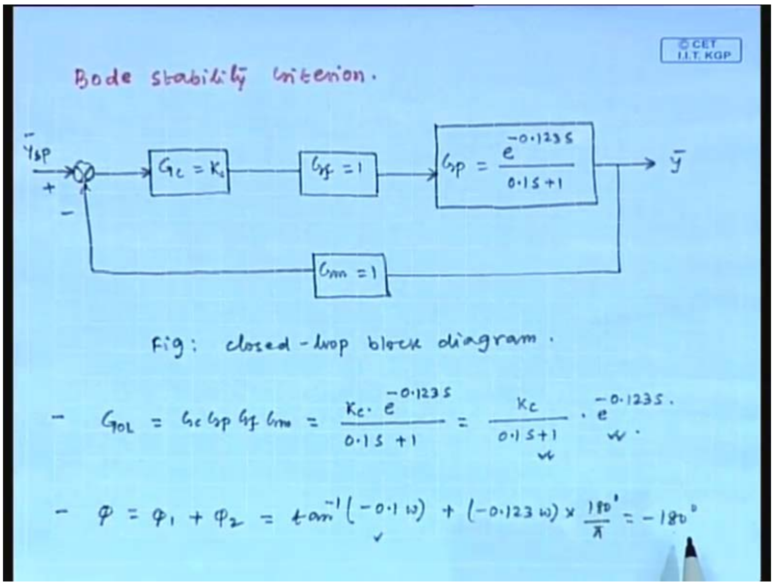

So, first we will discuss the bode stability criteria, fine. Now for this purpose will consider 1 closed loop system, because it is mentioned that the bode stability criteria is used for stability analysis of closed loop system. So, we will consider a closed loop system and presently we are making that close look block diagram having p only controller then considering G f equals 1 and G p equals exponential of minus 0.123 S divided by 0.1 S plus 1 and considering G m equals 1, fine.

This is a close loop block diagram of a sharp o problem this close loop block diagram of a sir loop proble, fine. Now you know that for constricting in the bode plot we need the expression for amplitude ratio and we need the expression for phi for constricting in the polar plot we need the expression for amplitude ratio and phi now for deriving the expression for amplitude ratio and phi what we need we need the open loop transverse function open loop transverve function we denote by G suffix o l which is nothing, but a multiplication of all four tansverve function G c G p G f G m now all the tansverve function are defined in this block diagram accordingly we obtain K c exponential of minus 0.123 S divided by 0.1 S plus 1.

Now, we can write it like this K c divided by 0.1 S plus 1 and separately this exponential term. So, this is the transverse function of first order system and this is a dead time part in the next we will try to find the expression for faze angle phi. So, if we consider first order system and dead time element then we can write as phi equals phi 1 plus phi 2 this is representing the first system I mean 1 and this the second. So, how much this will be for a first order system we know the phi is tan inverse minus tau omega. So, tan inverse how much is the tau 0.1 omega, fine. And how much is phi for this dead time element minus 0.123 omega. So, these in degrees this is in radians. So, we will multiply this with 180 degree divided by pi defiantly we write in the right hand side minus 180 degree you can write minus phi also accordingly you write the left hand side presently we are representing all in degrees fine can you calculate from this expression the holus of omega we are considering phi equals minus 180 degree.

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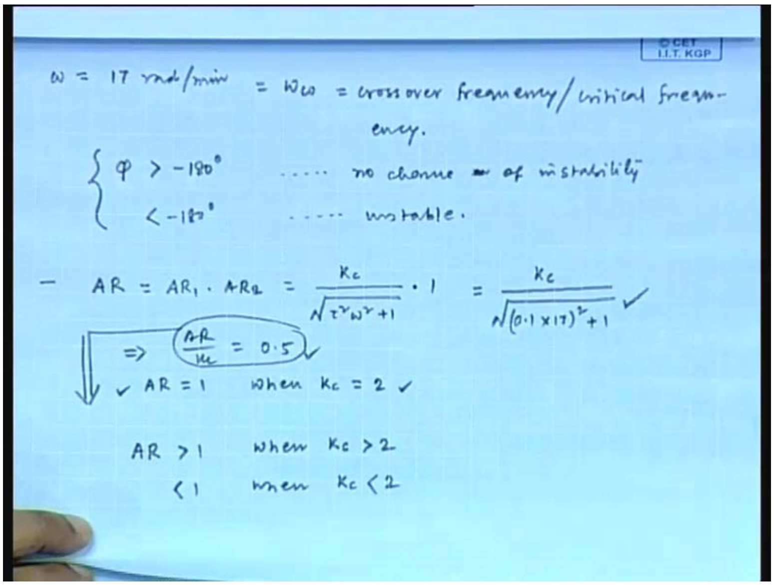

Now, you calculate from this the value of omega that we obtain 17 radian per minute the value of omega at phi equals minus 180 degree is called cross over frequency the frequency which is calculated at phi equals minus 180 degree that omega is called cross over frequency and which is represented by omega suffix c o cross over frequency this also called as critical frequency the frequency which is calculated at phi equals minus 180 degree is called cross over frequency or critical frequency.

Now phi if phi is always greater minus 180 degree, then the there is no chance of instability if phi is always greater minus 180 degree then the there is no chance of instability no chance when phi is less than minus 180 degree, then there is probability of instability I mean that is in unstable system. Therefore, we can say that phi equals minus 180 degree that is critical value, fine. So, we got cross over frequency equal to 70 radian per minute here the dead time is given in minute and time constant is also in minute what how we can calculate the amplitude ratio?.

If we consider the first order system and dead time system; that means, there are 2 system. So, we can write the overall amplitude ratio equal to the multiplication of the 2 individual amplitude ratios for the first order system we know the amplitude ratio is written as process gain divided by tau square omega square plus 1 this amplitude ratio for a first order system and for the time dead system the amplitude ratio is 1, now we will try to determine the value of this several amplitude ratio at cross over frequency tau is 0.1 cross over frequency we determine 17 accordingly we obtain this.

Now, we obtain amplitude ration 1 when K c equal to 2, I mean if we write this amplitude ratio by K c we obtain 0.5 fine amplitude ratio is this calculating this denominator we can write amplitude ratio divided by K c equals to 0.5 then we can say that we obtain amplitude ratio equals to 1 when K c equal to 2, fine in this line we can say that amplitude ratio is greater than 1 and K c is greater than 2 we can conclude from this expression amplitude ratio becomes greater than 1 when K c is greater than 2 similarly we can write amplitude ratio less than 1 when K c is less than 2 fine can we draw the bode plot.

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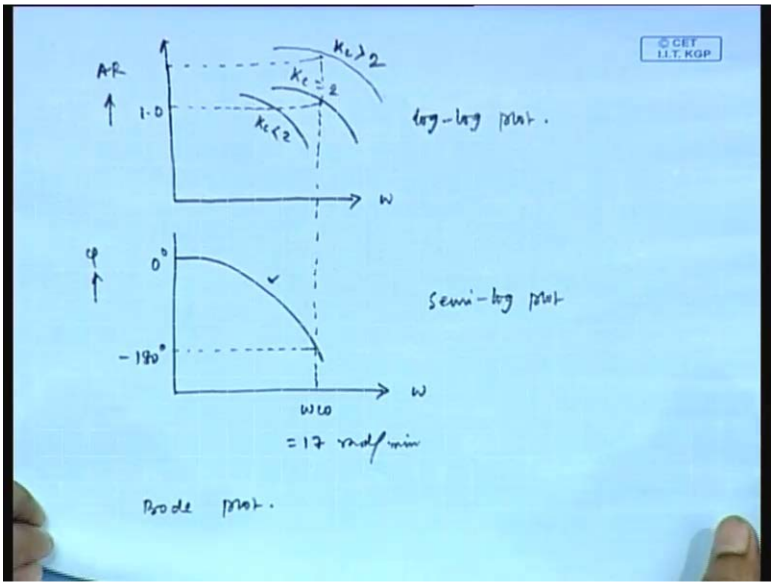

Now, we will construct the polar plot based on this information this omega phi omega amplitude ratio this is a log plot and this is a semi log plot this is 0 degree now phi equals minus 180 degree we obtain the omega which is omega c o this is cross over frequency the frequency at phi equals minus 180 is cross over frequency, and this we obtain 17 radian per minute correct. Now this is the plot when k c equal to 2 at cross over frequency how much is amplitude ratio 1 definitely when K c equal to 2 agree see that definition I mean we have derived cross over frequency as we consider phi equals minus 180 degree we obtain this now we obtain amplitude ratio 1 and K c is 2 at cross over frequency we obtain amplitude ratio 1 when K c equal to 2 and omega equal to omega c o.

Therefore, we have drawn this I mean this is the 1 at cross over frequency and K c is equal to 2 and K c is greater than 2 this is indicating K c is greater than 2 how much will be amplitude ratio greater than 1 it is clearly indicating that and K c is less than 2 how much is amplitude ratio less than 1 fine. So, this is phi verses log omega plot at phi equals minus 180 degree we obtain cross over frequency that is minus that is 17 radian per minute amplitude ratio is equal to 1 and K c is equal to 2 at omega c o that is shown here other 2 cases are also shown here they are K c is less than 2 and K c greater than 2 it is obvious from this plot that and K c less than 2 and amplitude ratio less than 1 and when K c greater than 2 amplitude ratio become becomes greater than 1.So, this is the bode plot. So, in the next day we will discuss the stability criteria continuing this portion thank you I hope presently what we have discussing that is most difficult part of process control. So, do not sleep sitting in the first bench.