Process Control and Instrumentation
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Lecture - 25
Feedback Control Schemes (Contd.)

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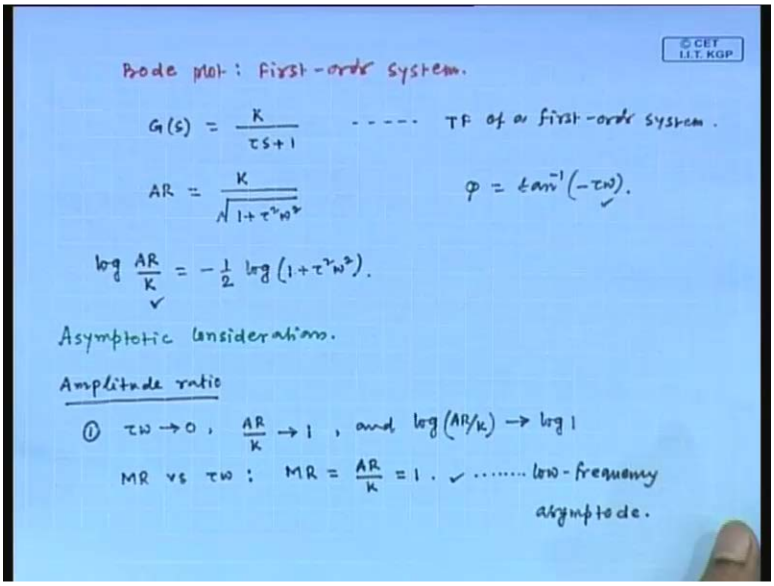

So, we will continued discussion on the constructions of prode plot in the last class we discuss the construction of prode plot with taking three examples, fine. So, today will discuss the construction of prode plot for a first order system. So, the transfer function of a first order system is return as K divided by tau s plus 1 this is the transfer function of a first order system now we untainted the expirations for amplitude ration and phi in the previous class.

As amplitude ratio equals K divided by root over of 1 plus tau squire omega squire and we obtained the expiration for phase angle as 5 equals ten inverse minus tau omega now we will take the logarithm of this amplitude ratio expiration in both said then we can write as amplitude ratio by K log amplitude ratio by K equals minus half log 1 plus tau squire omega squire fine

So, this is the magnitude ratio basically next we will considered the asymptotic considerations before making the plot we need to discuss the asymptotic considerations first we will considered the amplitude ratio first we will discuss the amplitude ratio then we will discuss the phase angle fine now if we considered tau omega tends two 0.

If we considered tau omega tens two 0 then amplitude ratio by K tens to 1 here the amplitude ratio is multiply do it 1 by K that mean this is scaled amplitude ratio and here wear considering scaled omega I mean tau omega we are considering now if we considered tau omega tends to 0 then amplitude ratio by k tends to 1. So, we can write that log amplitude ratio by K tends to log 1 fine. So, at the very low sequence is the log log plot of magnitude ratio verses tau omega tends to the horizontal line.

At very low sequence is the log log plot of magnitude ratio verses tau omega tends to a horizontal line represented by magnitude ratio equals amplitude ratio by K in equals 1 fine since this situations arise is at low frequence is this line is cold low frequency asymptode since this situation arise is act low frequencies this line is cold low frequency asymptode fine now if we if produce the plot considering this equation I mean this line then what we obtained.

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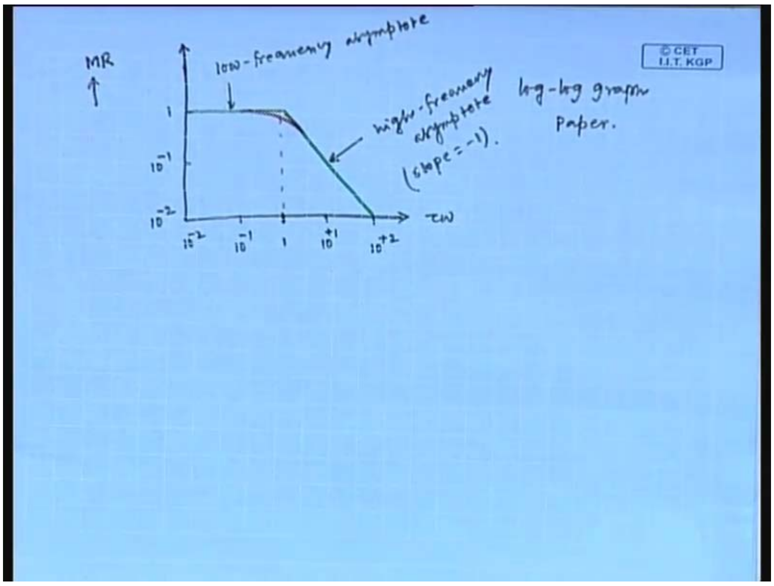

. So, here wear considering tau omega and here magnitude ratio this is a log log plot. So, wear using log log graph paper this is ten to the power minus 2 10 to the power minus 1 1 10 and this is suppose 10 to the power plus 2 and here wear considering 10 to the power minus 2 10 to the power minus 1 than suppose this is 1, fine.

Now, we obtained this form log amplitude ratio by K approximately equal to log 1 when we considered low frequencies; that means, it is clearly a horizontal line fine. So, this is and this is the low frequency asymptote this is low frequency asymptote fine now under asymptotic consideration next we will consider tau omega tends to infinity.

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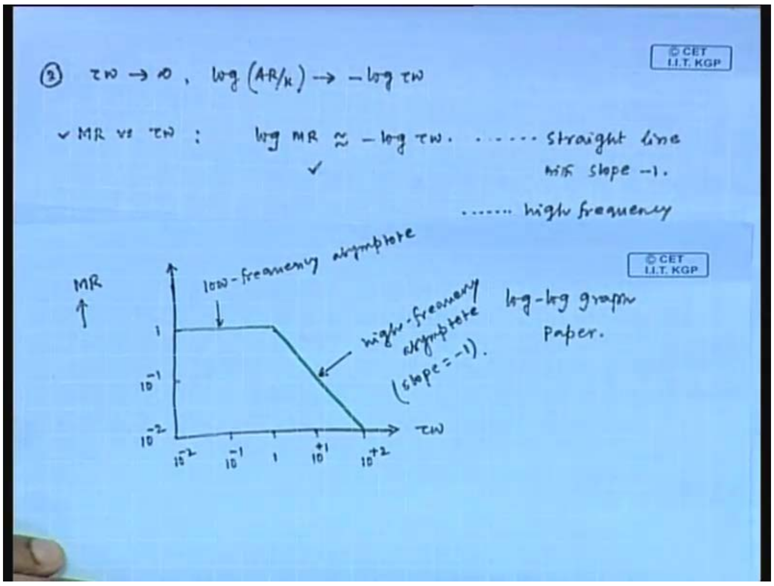

In the next we will considered tau omega tends to infinity as tau omega tends to infinity then we can write log amplitude ratio by K tends to minus log tau omega when tau omega is much much better one we can neglect this one term I mean this is the general form when tau omega tens to infinity we can neglect this one; that means, this right hand said becomes minus log tau omega agree. So, when tau omega tends to infinity we can write log amplitude ratio by K tends to minus log tau omega. So, the log log plot of magnitude ratio verses tau omega tends to the line represented log magnitude ratio approximately equal to minus log tau omega.

in the log log plot of magnitude ratio verses tau omega we obtain we need to considered this equations log magnitude ratio approximately equals to minus log tau omega this is nothing, but a straight line with slope minus one this is a straight line with slope minus one and this situation arise is act high frequencies there for this straight line is cold as high frequency asymptote since this situation arise is act high frequencies this straight line is cold as high frequency asymptote fine. So, using I mean we will next considered this equation in the prode plot. So, it is basically straight line with slope minus 1. So, this is representing the high frequency asymptote this is representing high frequency asymptote.

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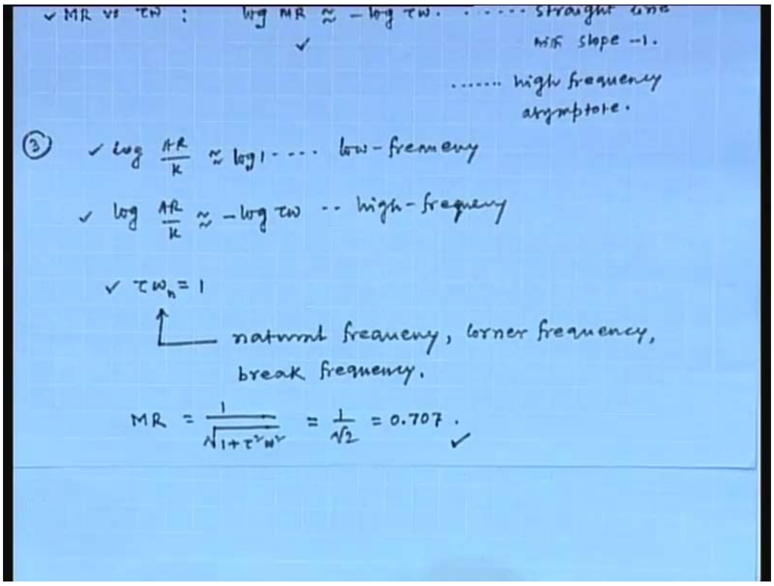

It has slope of minus 1 fine in the next step we need to determined the point of intersection in the third step we need to determined the point at which the two asymptote intersect when we considered tau omega tends to 0, we got log amplitude ratio by K is nearly equal to 1. This we considered low frequency asymptote when we considered tau omega tends to infinity we got now this we got log 1. So, when we considered tau omega tends to infinity we got log amplitude ratio by K which is approximately equal to minus log tau omega fine now if we equate these two equations equating these two equations we obtain.

Tau omega equals one agree equating these two equations this 1 and this 1 we obtain tau omega equals one; that means, they inter set act tau omega equals one, now in this situation we usually use one suffix for this omega that is in act this situations we use the suffix in for omega; that means, we can write tau omega n equals 1 the value of omega act which this two frequency to asymptote inter sets that is cold natural frequency. So, here omega in is the natural frequency.

And most popularly it is cold as corner frequency it is also cold as break frequency fine now can we determined the exact values of the magnitude receive add this corner frequency what is the exact value of magnitude ratio act this corner frequency we have the expiration of magnitude ratio that is magnitude ratio equals 1 divided by root over of 1 plus tau squire omega squire fine. So, this is the exact value of magnitude ratio act corner frequency according the we need to construct the prode diagrams I mean this magnitude ratio verses tau omega plot should lops like this like this and the corresponding tau omega is 1 here.

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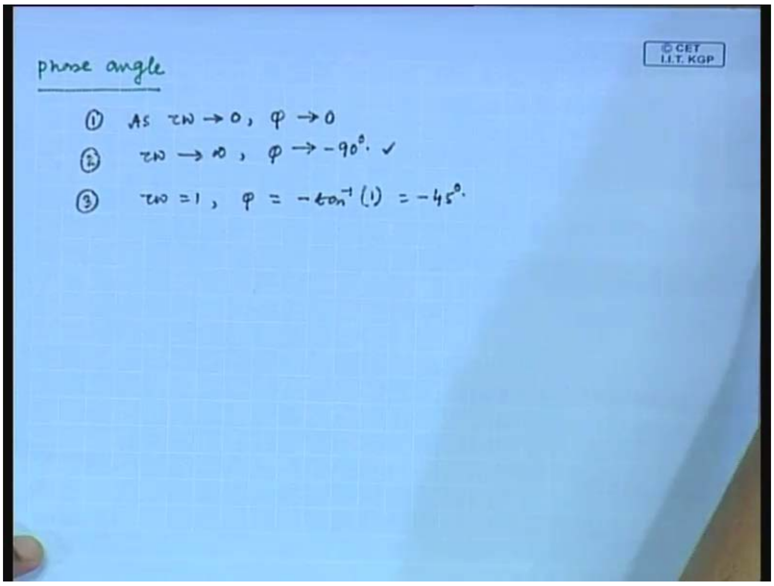

This is the corresponding natural frequency in this first plot. So, this is the asymptotic considerations in terms of amplitude ratio and we obtained this plot similarly we need to considered the asymptote consideration in term of phase angle. So, next we will considered the phase angle as tau omega tends to 0 phi tends to what zero fine in the second as tau omega tends to infinity how much is phi [vocalized- noise] minus 90; that means, phi tends to minus 90.

So, phi never we lay less then minus 90 fine, it clearly indicates that phi never we less then minus 90 and in the third situation, we will considered the corner frequency at the corner frequency; that means, tau omega equals 1 how much is phi phi is minus 45 minus tan inverse 1.

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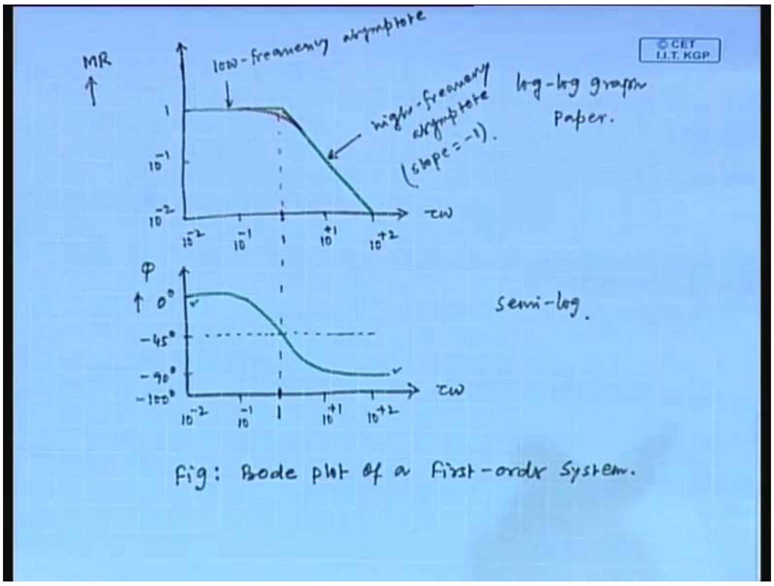

So, this is minus forty five. So, these are the asymptote considerations in terms of phase angle now we will drove the of phi verses log tau omega plot this is the semi log graph paper and this is 1 which correspond to the corner frequency ten to the power minus 2 10 to the power minus 1 this is ten to the power plus 1 and 10 to the power plus 2, this is suppose minus 100 degree minus 90 degree this is a minus 45 degree this is a 0 degree at low frequencies at low frequencies the tau omega the five is 0 fine. So, it is somewhat like this at low frequencies phi is 0 and when tau omega tends to infinity we obtain 5 minus 90.

And when tau omega equals 1 we obtained minus 45 agree at low frequencies this phi is near to near 2 0 and act as tau omega tends to infinity this is minus 90 and act corner frequency this is minus 45. So, this is the prode plot for a first order system this is the prode plot of a first order system in the next we will discuss the construction of prode plot for a second order system.

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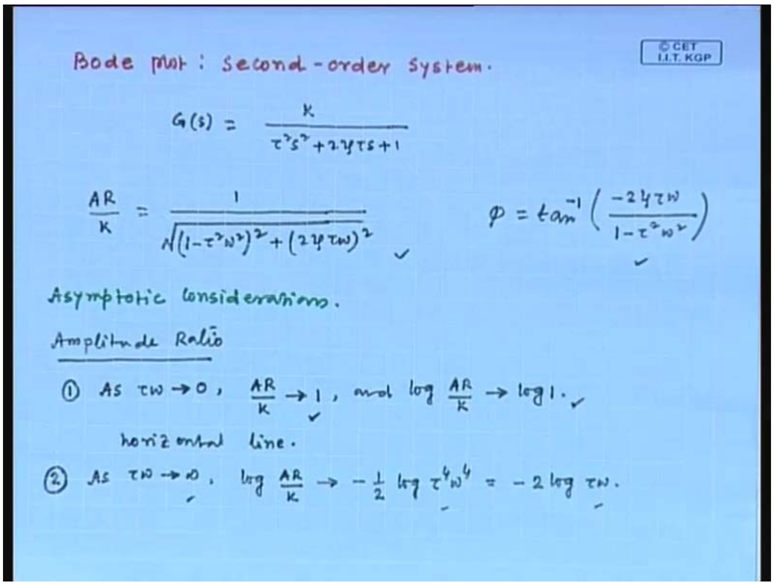

In the next we will discuss the prode plot of a second order system. So, the transfer function a second order system we considered earlier taking this form g s equals K divided by tau squire a squire plus 2 zeta tau s plus 1 and in the last class we derived the expirations for amplitude ratio and phi. So, they are amplitude ratio divided by k equals one divided by root over of one minus tau squire omega squire hole squire plus 2 zeta tau omega hole squire this expirations this expirations we derived earlier similarly the expiration of phi is return as tan inverse minus 2 zeta tau omega divided by 1 minus tau squire omega squire fine this 2 expirations we derived the earlier now we will discuss the asymptotic considerations in terms of first amplitude ratio forward by phase angle.

Asymptotic consideration first we will consider amplitude ratio as tau omega tends to zero amplitude ratio by K tends to one and log amplitude ratio by K tends to log 1 fine as tau omega tends to 0 we obtained amplitude ratio by K tends to 1 and accordingly log amplitude ratio by K tends to log 1.

Now, it is clear that the low frequency asymptote is a horizontal line it clearly says that it clearly says that this is a horizontal line next we will consider tau omega tends to infinity. So, as tau omega tends to infinity log amplitude ratio by K tends to minus 1 by2 log tau to the power 4 omega to the power 4 here we neglect all lower order terms, fine.

So, this is equal to basically minus two log tau omega as we considered tau omega tends to infinity we obtained log amplitude ratio by K tends to minus half log tau to the power

4 omega to the power four here all low order. Terms have been neglected. So, it is than becomes minus two log tau omega. So, this is a straight line with slope minus two. So, this is a straight line with slope minus 2, so the low frequency.

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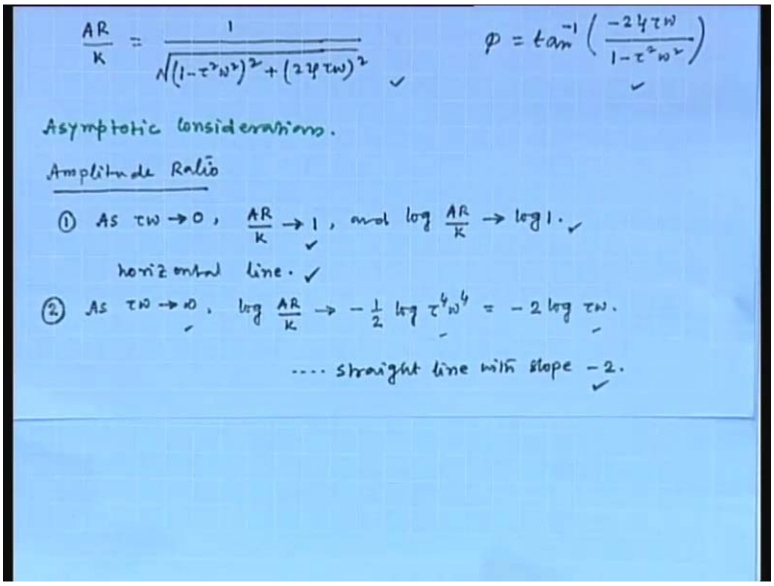

Asymptote is a horizontal line and the high frequency asymptote is a straight line with slope minus 2 for the first order system we obtained horizontal line as low frequency asymptote, but high frequency asymptote is a straight line with slope minus 1.

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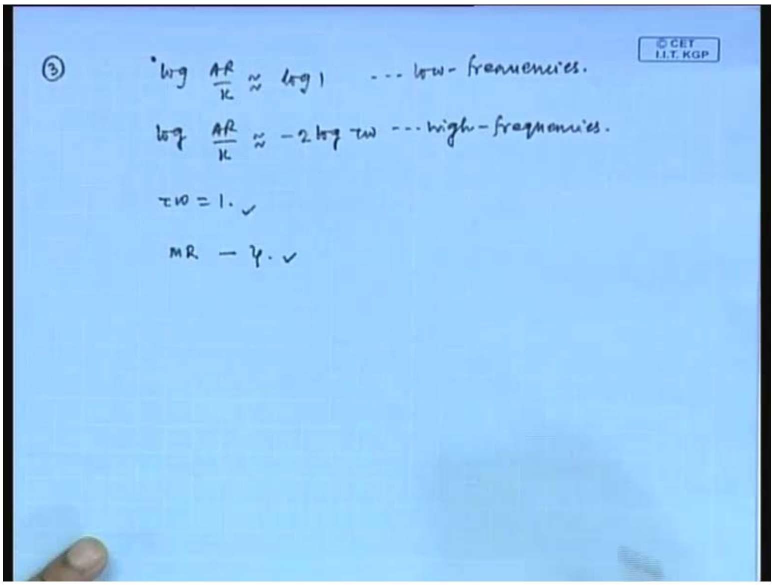

And for second order system the slope is minus two we now we can produce the plot of magnitude ratio verses tau omega before that we need to determined the point of intersection fine. So, this is say third step. So, we got basically log amplitude ratio by K which is approximately equal to log 1 this equation we obtain at low frequencies and for high frequencies. We obtained minus two log tau omega. So, equating these two equations we obtained again tau omega equals one again we obtain tau omega equals one, but here we cannot determine the exact values of magnitude ratio because that is also function of zeta here the exact value at corner frequency we can obtain ally knowing the value of zeta. So, here the magnitude ratio

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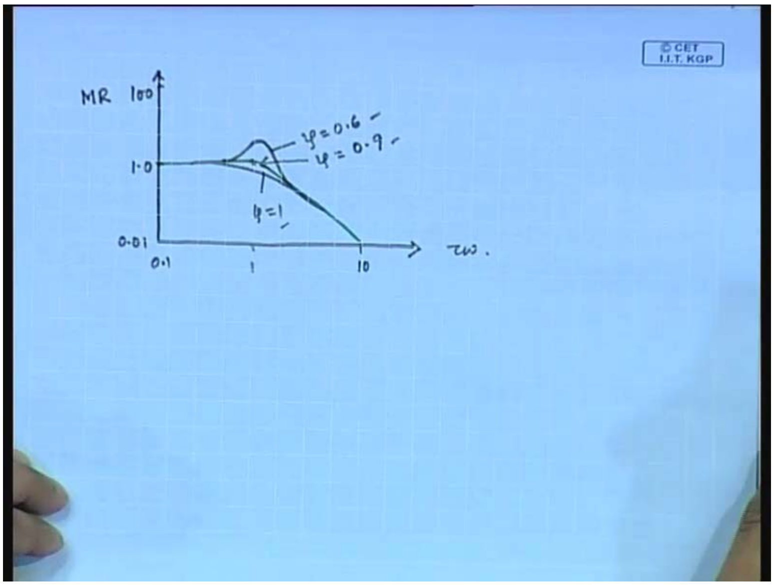

Can be obtain by knowing the value of zeta and this should be deflected in the prode plot fine next we will construct the prode plot and first we consider magnitude ratio verses tau omega say this is zero point one than one and this is ten zero point zero one the starting value of magnitude ratio next one is one and this is a 100. So, act low frequencies we obtained a horizontal line fine and at high frequencies we obtained a straight line with slope minus two it was mention that the magnitude ratio at the point of intersection depends on the value of zeta. So, accordingly we will considered the three different cases whirring the zeta values.

This type of response we obtained if zeta is say zero point six which is quit lace this one we obtained when zeta is say zero point nine and this plot we obtained zeta is one fine considering three different values of zeta we obtained three different plots as magnitude ratio verses tau omega.

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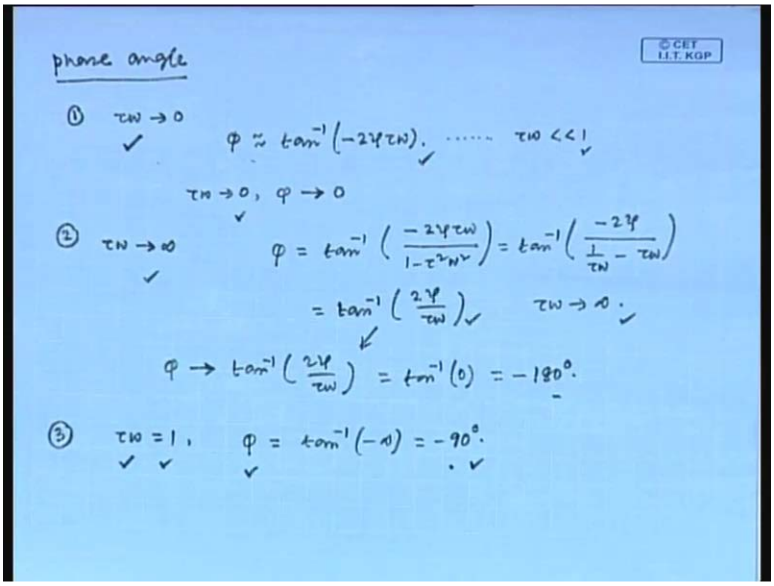

So, these are the three plots this tope one corresponds to zeta equals 0.6 next 1 corresponds to zeta equals 0.9 and this bottom was to 1 corresponds to that equals 1 next we will discuss the asymptotic considerations in term of phase angle next we will discuss the asymptotic consideration in term of phase angle when tau omega tends to 0 we can write phi equals tan inverse minus 2 zeta tau omega can we write this.

This we can write when tau omega becomes much less than unity when tau omega much less than unity fine now if we consider tau omega tends to 0 now if we consider tau omega tens to 0 this phi approaches 0 agree if tau omega is much less than 1 we can write the phi expression like this tan inverse minus 2 zeta tau omega now if we consider tau omega tends to 0 the phi approaches 0. Secondly, we consider tau omega tends to infinity. So, in this case we can write the phi expiration as 10 inverse minus 2 zeta tau omega divided by 1 minus tau squire omega squire this is the original expiration, we can write this as 10 inverse minus 2 zeta divided by 1 by tau omega minus tau omega agree. So, this is equal to tan inverse two zeta by tau omega when we considered tau omega tends to infinity agree. So, this one we can write in this form considering tau omega tends to infinity. So, than phi tends to 10 inverse 2 zeta by tau omega which is equal to tan inverse 0 agree this expirations we got now we are considering tau omega tends to infinity.

So, we can write tan inverse 0 and this is equal to minus 180 degree this is considered as minus 180 fine in the third case we will considered tau omega equals ,1 I mean the point of intersection tau omega equals 1 now in this case re gall is the value of zeta phi becomes tan inverse minus infinity which is minus 90, which is minus 90 tau omega equals 1 corresponds to the corner frequency and regal less of the value of the zeta we obtain phi equals tan inverse minus infinity; that means, this is minus 90 now you see initially we consider tau omega tense to 0 and this is tau omega equals 1. So, this is the lowest one this is the intermediate 1 and this is the highest 1 fine here the phi value is maximum this is intermediate and this is minimum. So, there is a train in prode plot fine.

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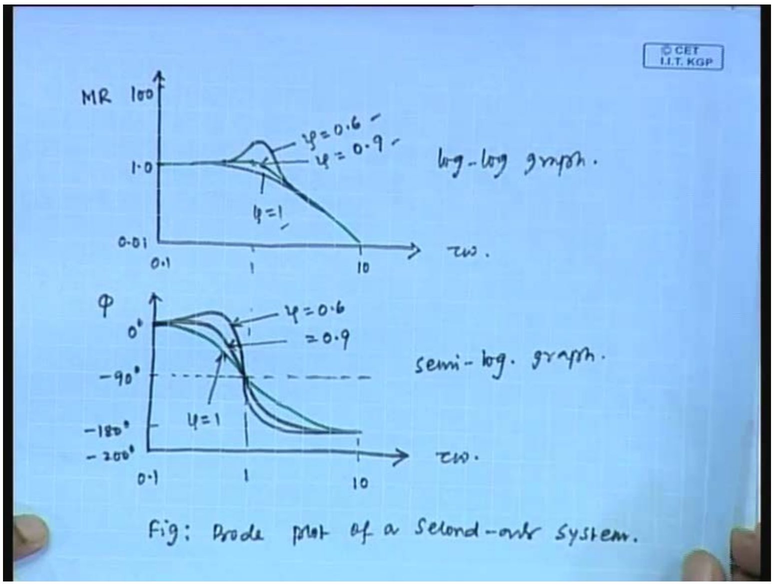

Now, we will try to produce the plot between phi verses tau omega and this is phi 0.1 this is indicating one and this is ten and this is suppose minus 200 degree minus 180 degree minus 90 degree 0 degree this plot also considered the different values of zeta This is one plot this is the second plot this is the third plot this top one we obtain for zeta zero point six second one we obtain I mean intermediate 1 we obtain considering zeta 0.9 and this one we obtained considering zeta equals 1 fine. So, this is the prode plot of a second order system the top one is plotted in using log log graph paper and the bottom one is obtain considering semi log graph paper, fine.

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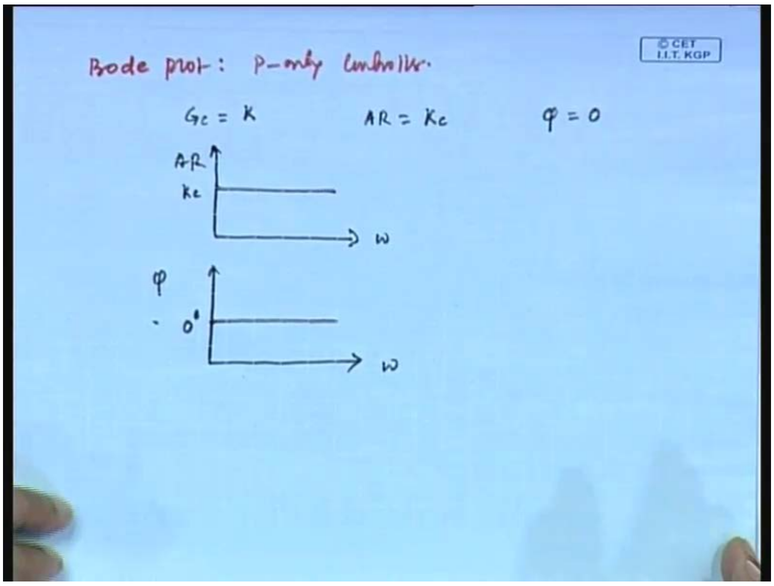

In the last example we will consider a feedback control system a feedback controller and we will construct the prode plot. So, next we vest to discuss the construction of prode plot for a p only controller the transfer function of a p only controller in K c I mean g c equal K c. So, we can write amplitude ratio equals K c and the phase angle we can write as phi equals 0. So, both are independent of omega. So, it is easy to develop the prode a plot this is indicating omega and this is indicating amplitude ratio since K c is independent of omega this should be a straight line agree in the bottom plot. We consider phi verses omega and phi remains 0 all the time I mean that is also a horizontal line having phi 0 degree. So, this is the prode plot of a p only controller.

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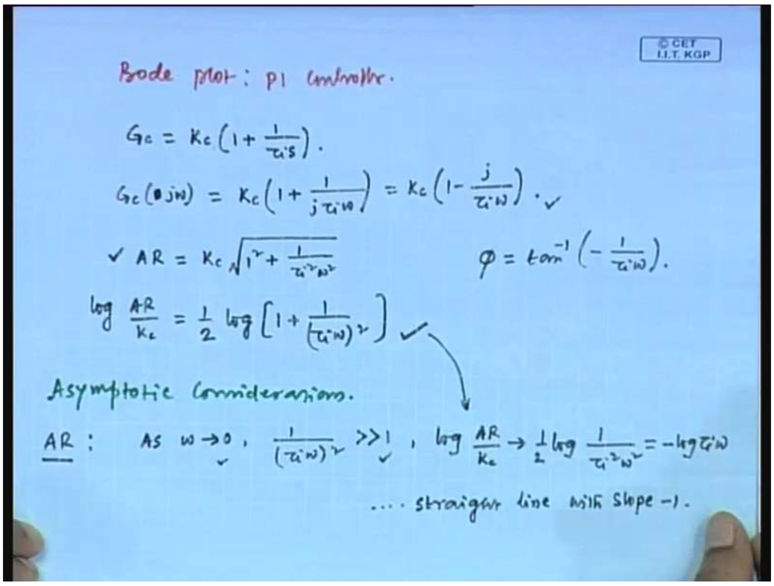

Next is the P i controller the P i controller has the transfer function and it can be written as G c equals K c equals one plus one divided by tau i s replacing a s by j omega replacing a s by j omega. We can write for the frequency response transfer function as K c multiplied by one plus one divided by j tau i omega we can write this as k c one minus j tau i omega agree. So, from this cartesian form it is straight forward to determine the expression for amplitude ratio and phi. So, amplitude ratio we obtain as K c root over of 1 square plus 1 divided by tau square omega square fine

Similarly, we can write the expression for phi phi equals tan inverse minus 1 divided by tau i omega this is the expression for phi now we can write this expression of amplitude ratio by taking algorithm as log amplitude ratio by K c equals half log 1 plus one divided by tau i omega whole square.

If we take the algorithm of this expression we can write log amplitude ratio by K c equals half log 1 plus one divided by tau i square omega square fine in the next we considered the asymptotic considerations, first we consider the amplitude ratio as omega tends to 0 1 by tau i omega whole square because much much higher than 1 agree.

So, we can write as log amplitude ratio by K c tends to half log 1 by tau i square omega square which is equal to minus log tau i omega agree if we considered omega tense to 0 we can say that one by tau i square omega is much much higher than one than this expression eldes this. So, the low frequency asymptote is a straight line.

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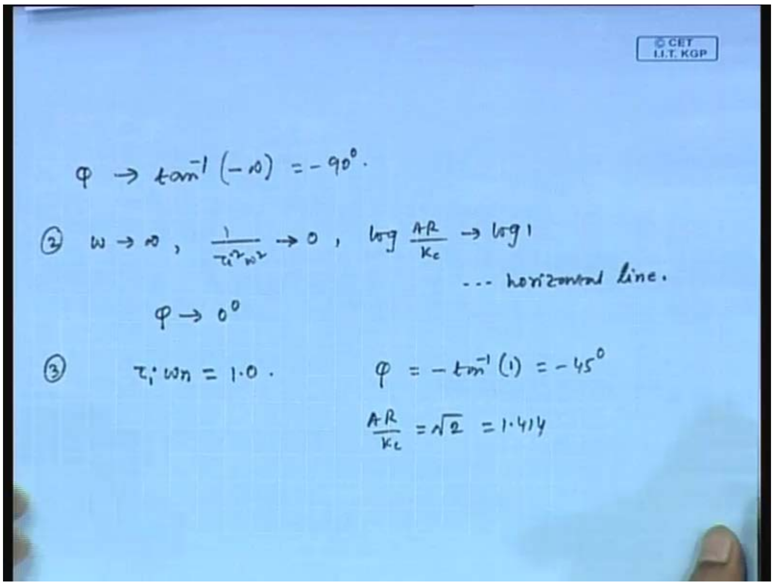

So, this is a straight line with slope minus 1 fine. So, what will be phi than if we consider omega tends to 0 phi becomes tan inverse minus infinity agree which is equal to minus 90 degree it is very straight forward, because we have this expression we are just considering omega tense to 0. In the next step we will consider omega tense to infinity than one by tau i square omega square approach a 0. Accordingly we can write log amplitude ratio by K c tense to log 1. So, at high the frequency into the horizontal line high frequency asymptote is a horizontal line and how much is phi if omega tense to infinity phi becomes 0 degree next one is the determination of corner frequency we obtained the corner frequency, when tau i omega n equals 1.0 fine accordingly phi becomes minus tan inverse 1 which is minus 45 degree and amplitude ratio by K c becomes root 2 which is 1.4 1 4, fine. So, these are the asymptote consideration an next we will make the plot amplitude ratio by K c and next one is phi verses tau i omega.

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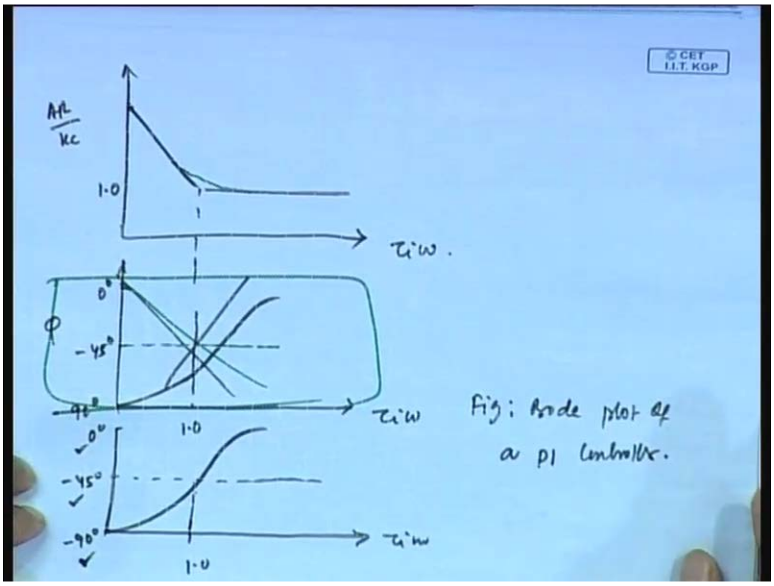

So, when omega tense to 0 we obtain straight line with minus 1 somewhat like this and this is indicating the corner frequency and when omega tends to infinity this is a straight line like this fine. So, we get this type of prode a plot similarly here, we obtained minus 90 we obtain like this plot for phi verses tau i omega here this this should be like this is 45 I am re join it tau i omega this is suppose 1 this is minus 90 degree

So, this is minus 45 and this is 0 degree. So, it will go like this fine. So, this is indicating the corner frequency at which phi is minus 45 and at low frequency this is minus 90 and at high frequency at high frequency this is 0. So, you just do not considered this 1 fine. So, this is the prode plot of a P i controller prode plot of a P i controller thank you any question minus 180. You know in the prode plot if there is no trained that cannot be used.